Learning 3D matrix algebra using virtual and physical manipulatives: Qualitative analysis of the efficacy of the AR-Classroom

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Abstract. The AR-Classroom application aims to teach three-dimensional (3D) geometric rotations and their underlying mathematics using virtual and physical manipulatives. In an efficacy experiment, undergraduates completed six 3D matrix algebra rotation questions and were assigned to interact with virtual (N = 20) or physical (N = 20) manipulatives in the AR-Classroom. While completing these rotation questions, researchers documented the participants' reported thoughts, feelings, and perceptions (i.e., qualitative data). A thematic analysis of participants' reports revealed four prevalent themes regarding participants' learning experience: (1) Difficulty using traditional methods, (2) Reliance on resources, (3) Pattern recognition, and (4) Developing an understanding of 3D matrix algebra. Participants struggled to complete rotation matrices when only using information from the question and the model; when unsure how to solve the matrix, participants utilized any available resources. Moreover, participants could identify similarities among matrices, demonstrated after using AR-Classroom repeatedly. The findings indicate that the AR-Classroom may aid students in improving their mathematical skills. Suggestions for future research on the AR-Classroom and efficacy experiments are discussed.

Keywords: User Experience (UX), Augmented Reality (AR), Educational Technology, Embodied Learning, Math Learning.

1 Introduction

Traditional methods for teaching and learning mathematics are well-documented as complex for students and instructors. Students struggle to visualize mathematical shapes and objects, particularly in the three-dimensional space, and to grasp abstract concepts related to mathematical theory [1, 2]. Moreover, students struggle to understand geometry subjects, and their educators have problems finding suitable and practical teaching aids. These difficulties appear to persist across levels of school and often continue into higher education [3, 4, 5,], which can negatively impact students wanting to pursue majors within science, technology, engineering, or mathematics (STEM).

Most STEM instruction uses multiple representations to illustrate complex or abstract concepts, a practice built on evidence that multiple representations can enhance learning. While many interventions seek to integrate abstract concepts and embodied mechanisms to enhance learning [6], a recent review [7] showed that theories of conceptual learning and embodied learning often make conflicting predictions about the effectiveness of virtual and physical manipulatives alone. Thus, designing effective interventions that combine multiple forms of manipulatives and stimulation may allow for deeper learning of abstract mathematical concepts. From a pedagogical perspective, educational applications utilizing augmented reality (AR) technology can provide a situated and embodied approach to learning [8, 9, 10] as the learned knowledge occurs within a specific context and is marked by the embodiment, enhancing the learner's knowledge acquisition.

Educational applications using AR technology have the potential to provide an innovative solution to mathematical learning issues. AR allows teachers and students access to immersive and interactive learning experiences enhanced by the intentional integration of real and virtual stimuli [11, 12, 13]. A review by Ahmad and Junaini [14] found that AR usage in math teaching and learning provides students with an interactive learning process, increased understanding, and enhanced visualization. For the matrix algebra underlying geometric transformations, AR allowes learners to interact with simplifed complex and abstract mathematical theory information through virtual and physical stimuli. Combining AR technology and multiple forms of physical and virtual manipulatives, the AR-Classroom educational application may provide an effective intervention for learning geometric transformations and their underlying mathematical theory.

2 AR-Classroom

AR-Classroom aims to teach two-dimensional (2D) and three-dimensional (3D) geometric rotations and their mathematics. It provides a virtual and physical interactive environment to facilitate embodied learning, making it more engaging and straightforward to learn 3D matrix algebra. The AR-Classroom app comprises a virtual and physical workshop and a model registration tutorial. Users can perform rotations by manipulating the application's X-, Y-, and Z-axes sliders to rotate a virtual model (i.e., virtual workshop, Fig. 1). Alternatively, they can rotate the physical LEGO space shuttle (i.e., physical workshop, Fig. 2). The virtual and physical workshops of the AR-Classroom share similar features, such as a green wireframe model superimposed onto the LEGO space shuttle to visualize the rotation transformations, color-coded X-, Y-, and Z-axes lines, degree or radian representations of rotation angles, Z-axis direction manipulation, multiple types of model views, and 2D or 3D matrices.

2.1 Previous Research on AR-Classroom

Previous user studies on similar educational technology applications for learning geometric transformations and their mathematics have informed the development of AR-Classroom and its research. The iPad application BRICKxAR/T, developed by Shaghaghian and colleagues [15], used AR to display the mathematical concepts behind geometric transformations by visualizing the entries within transformation matrices. Using an iterative approach, Aguilar and colleagues [16] conducted two usability tests to investigate the usability of the BRICKxAR/T app's AR and non-AR workshops, the first to evaluate usability in its starting condition and the second to investigate the impact of changes made based on the original usability study's findings. Guided by the BRICKxAR/T development and usability studies, the AR-Classroom app was developed as further described in Yeh et al. [17].

Several usability tests on the AR-Classroom assessed user-app interactions, the functionality of app features, overall ease of use, and the effectiveness of iterative changes made to the app. First, Aguilar et al. [18] conducted usability tests of the AR-Classroom in its starting version; based on the findings from this test, recommendations were formulated to address issues and enhance users' experience. Next, a second usability test was conducted to investigate how changes made to the app based on the first usability test impacted its discoverability and usability. The changes made to the virtual and physical workshops of AR-Classroom improved usability and enhanced user-app interactions. However, based on the results of the updated usability test, there were still salient issues in user-app interactions. Finally, Aguilar et al. [19] conducted a third usability test to investigate the cumulative impact of changes made to the AR-Classroom. Through this iterative approach to usability testing, the current version of the AR-Classroom is deemed satisfactory, as it demonstrates an improved user experience, increased ease of use, and an overall increase in users' understanding of the app's functionality.

The data and procedures derived from previous studies on the BRICKxAR/T and AR-Classroom apps informed the development and execution of the present learning experiment and the importance of qualitative data for answering questions about the impact of AR educational technology on students’ learning.

2.2 Phenomenological research

Using a phenomenological approach, the current study explored participants' qualitative experience while interacting with the AR-Classroom's virtual or physical workshop to learn 3D matrix algebra. Phenomenological research is a qualitative approach that assumes understanding concepts, opinions, or experiences depends on individuals' descriptive reports of the occurrence [20, 21]. Qualitative research guided by phenomenology investigates the profound knowledge and experience of the participants through their description of how the experience made them think, feel, and act during specific situations rather than using the exploration of numeric patterns that traditionally provide a broad overview of findings. Phenomenological research design is beneficial for topics in which the researcher needs to go deep into the audience's thoughts, feelings, and experiences, such as Human-Computer Interactions [22], and can broaden our understanding of the complex phenomena involved in learning, behavior, and communication prevalent within the STEM disciplines [23, 24, 25].

2.3 Learning Experiment using AR-Classroom

The present study investigated users' experiences learning 3D matrix algebra using AR-Classroom (i.e., the phenomenon) via two experimental groups (virtual or physical). The study focused on the qualitative data (i.e., user-reported experience and observations) collected from the experiment. In contrast, another paper will focus on quantitative differences between the experimental and active control groups [26]. Using the qualitative data, the paper answers two research questions on the AR-Classroom's efficacy: 1) How effective is the AR-Classroom in teaching introductory 3D matrix algebra? 2) What mathematical concepts related to matrix algebra do students learn from using the AR-Classroom application?

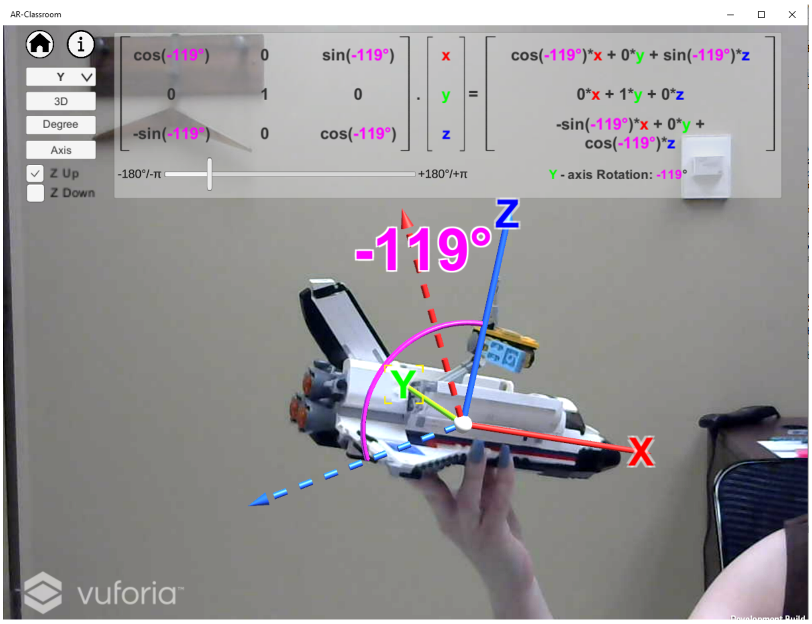

Fig. 1. AR-Classroom: Virtual workshop Y-axis rotation with degrees, and axis visualization (i.e., dotted axes for the LEGO space shuttle’s body frame).

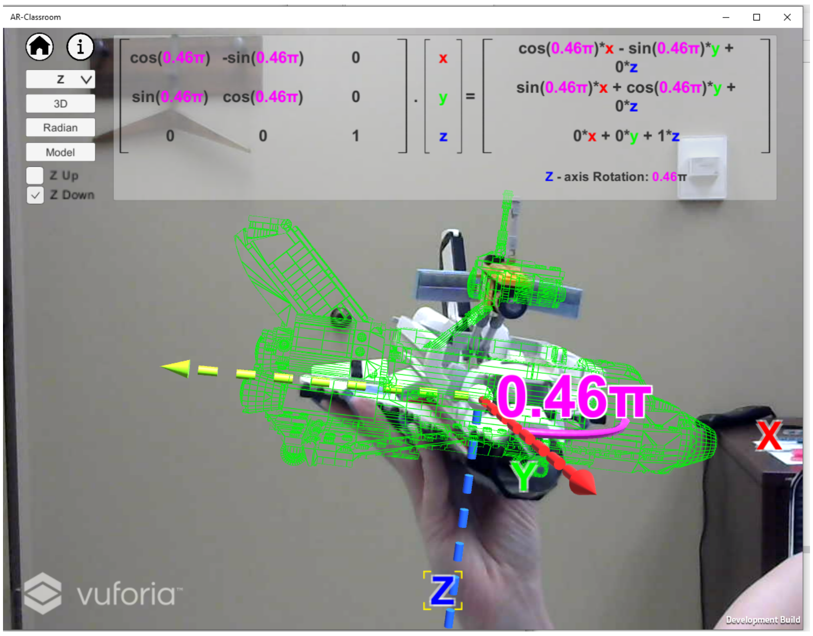

Fig. 2. AR-Classroom: Physical workshop Z-axis rotation with radians, and model visualization (i.e., green wireframe).

3 Methods

Participants were recruited via a research sign-up system in the Department of Psychological and Brain Sciences at Texas A&M University. The experiment took 2 hours, and participants received research credit for participation. A total of 60 participants were included in the experiment, with each participant being randomly assigned to the virtual condition (N = 20), the physical condition (N = 20), or the control condition (N = 20). For this study, this paper focuses on the forty participants in the two experimental groups to investigate the qualitative findings of learning using the AR-Classroom app. Participants in the virtual condition (N = 20) were mostly in their freshmen year, with the mean age being approximately 19 years old, and with twelve identifying as male and eight as female. The majority of the virtual condition participants reported experience with 2D matrices, and an even split of participants with and without 3D matrices experience. The physical condition participants shared similar characteristics as the majority were in their freshmen year, with the mean age being approximately 19 years old; however, eight identified as male and twelve as female, and all had experience with both 2D and 3D matrices.

3.1 Procedures

The two experiment conditions followed similar procedures, except for completing different workshops. Participants completed a pre-test with questions regarding demographic information, previous experience with matrix algebra, measures of spatial visualization abilities, and math abilities and confidence. After completing the pre-test, participants watched an introductory video on matrix algebra that provided a brief overview of key concepts and terminology as a primer for students and a second video on setting up the LEGO space shuttle while interacting with the AR-Classroom. After watching the videos, the AR-Classroom application was run on the desktop computer with a webcam, and participants were given the LEGO space shuttle Depending on the participant's assigned experimental condition, rotations about the X-, Y-, and Z-axes are performed either by manipulating the rotation slider for the virtual condition or by physically rotating the LEGO space shuttle for the physical condition. Interacting with one of the workshops on the AR-Classroom, participants completed six 3D matrix algebra questions in a rotation booklet while being recorded.

The rotation booklet consisted of three problems on 90-degree and three problems on 30-degree rotations about the X-, Y-, and Z-axes (Fig. 3). The rotation booklet questions were guided by a scaffolding teaching approach in which learners are guided towards a greater understanding, skill acquisition, and learning independence through less direct instruction of a concept [27]. Each of the three sets progressively provided less supporting information as participants worked through the rotation booklet. The first 90-degree rotation question was thoroughly explained so the participant understood how each of the nine values in the matrix was created. The second rotation question was presented with a shortened explanation, requiring students to use what they learned from the first rotation. The third rotation question contained the most basic explanation. The 30-degree rotation questions followed this same progression. Participants were instructed to first complete Matrix R (Fig. 3, Bottom) only using the instructions in the rotation booklet and the LEGO space shuttle (Fig. 3, Top), after doing so, they were then prompted to use the AR-Classroom to check their work and complete Matrix R' (Fig. 3, Bottom) and compare their answers for both.

While working through the problems, participants were instructed to think aloud and explain what they were trying to do, if the task was easy or challenging, why they found it easy or challenging, and any general thoughts related to their experience with the app. During the experiment, a research assistant documented the participant’s feedback. After interacting with the app, participants completed a post-test with the same spatial visualization abilities, math abilities, and confidence measures from the pre-test.

Rotation #2 - Rotation by 90 degrees counterclockwise about the y-axis

2. We would like to understand the matrix which describes a rotation by 90° counterclockwise about the y-axis.

Step A: Fill in the matrix in the answer booklet using the LEGO model and instructions below

Again, let this matrix be $R = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. Hold the shuttle so you are looking at the tip of its left wing (the y-axis).

(a) Notice that the z-axis (the antenna) is still up but the x-axis (the nose) is now on your left. Rotate the shuttle by 90° counterclockwise about the y-axis. The wings stay along the y-axis, i.e. $(0, W, 0)$ moves to $(0, W, 0)$.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix}$$

(b) The antenna moves to where the nose was, i.e. $(0, 0, A)$ moves to $(A, 0, 0)$.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ A \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ 0 \end{pmatrix}$$

(c) The nose moves to down, opposite to where the antenna was, i.e. $(N, 0, 0)$ moves to $(0, 0, -N)$.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} N \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -N \end{pmatrix}$$

(d) Use this information to fill in the entries in Matrix R on Answer Sheet 2.

Answer Sheet #2: Rotation by 90° counterclockwise about the y-axis.

Step A: Fill in Matrix R using the instructions in the Rotation Booklet and the Lego Shuttle:

Matrix R

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Step B: Fill in Matrix R' using the AR Classroom – Workshop 1 Virtual and the Lego Shuttle:

Hold the shuttle in front of the AR Classroom App and again rotate it **90° counterclockwise about the y-axis**. Remember to set the AR Classroom: Dimension = 3D, Angle = Degrees

Matrix R'

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

STEP C: Does the Matrix R' displayed by the AR Classroom agree with the Matrix R? If not, ask the experimenter for help.

Fig. 3. Example of 3D Matrix Algebra Rotation Questions (90 degrees about the y-axis; Top) and Rotation Booklet (virtual condition; Bottom).

4 Results

Qualitative investigation provides a detailed description of participants' experiences during an experiment. Thematic analysis, a method for analyzing qualitative data, provides a systematic way for researchers to identify meaningful patterns and themes within the data collected. The analysis aims to understand the complexity of meanings in the data by searching for meanings and determining how these patterns can be organized into themes to explain experiences [28, 29]. Thematic analysis findings can uncover the deeper meanings of the experiences being studied and help identify common experiences shared by the group, which can then be used to conclude the overall phenomenon being studied.

For the present study, the thematic analysis included transcribing materials, coding data into themes, and summarizing the participants' learning experience using AR-Classroom (Table 1). Derived themes highlighted four broad findings related to matrix algebra learning:

1. Difficulty using traditional methods
2. Reliance on resources
3. Pattern recognition
4. Developing understanding of 3D matrix algebra.

Table 1. Participants' matrix algebra learning experience using AR-Classroom (N=40).

Theme	Description	Significant Example(s)
Difficulty using traditional methos	The connection between geometric transformations in a 3D coordinate space and matrix operations were difficult when solving without the AR-Classroom's aid.	"I'm not sure how I was supposed to be getting the numbers..." "So, like as far as filling this in and saying x, y and y and then there's the whole matrix I guess that's where I'm confused."
Reliance on resources	Participants needed assistance completing the matrix using only the worksheet and model and relied heavily on previous notes and rotation questions to find solutions.	"[I] don't really understand, just using past notes." "The instructions are telling me the same things as I have here [notes], the same formulas."
Pattern recognition	Participants demonstrated a rudimentary understanding of the correlation between rotations and changes in matrices and patterns in three-dimensional rotations after using the AR-Classroom repeatedly.	"It follows the same formula. Pretty much if you can pick up the formula, you can do it even if you don't understand." "….this question had a 001 here there should be a 001 on this too."
Developing understanding of 3D matrix algebra	After repeated use of the AR-Classroom application, participants were able to provide a more detailed reasoning for how they solved the matrix and displayed a greater understanding of the rotations.	"I didn't try to find [a] pattern I just kind of remembered from before."

4.1 Difficulty Using Traditional Methods

For participants in both the virtual and physical conditions (N = 40), the findings of the thematic analysis revealed that participants found it difficult to relate how matrix operations correspond to geometric transformations and they lacked an understanding of the trigonometric functions involved in spatial rotations. For rotation question 1, Matrix R, participants either failed to fill out the entire matrix or created a rotation matrix incorrectly (N = 18), often putting in numbers and letters unrelated to the 90-degree rotation counterclockwise about the X-axis. Though participants appeared to approach question 2, Matrix R, with less apprehension than the first rotation question, they were still unable to solve the transformation matrix correctly. Participants reported being confused about where the numbers go in the columns based on the rotation (N = 6); "I'm not sure how I was supposed to be getting the numbers...". The main concerns documented related to difficulty completing the transformation matrix using traditional methods, as participants expressed great difficulty understanding how the 3D coordinates of the space shuttle pre- and post-rotation are represented in a 3D matrix. One participant in the physical condition expressed their disconnected understanding between the axes and the matrix by stating, "So, like as far as filling this in and saying x, y, and y and then there's the whole matrix, I guess that's where I'm confused."

Regarding participants' understanding of the trigonometry underlying the rotations, participants needed help utilizing and relating the trigonometric functions (sine, cosine) to the matrices and their corresponding geometric transformations (N = 12). Trigonometric functions presented an additional difficulty for participants with weaker math backgrounds throughout all presented tasks; some clarification was needed with the function of sign usage (+/-) in the matrices. This difficulty was further exposed as participants had trouble completing a 30-degree X-axis rotation because there was confusion about whether a value was positive or negative (N = 4), often resulting in failure to have the right solution (N = 15). Participants often attempted clockwise rather than counterclockwise rotation (N = 6) and rotated on the incorrect axis (N = 11). Participants often asked the experimenter, "Do I rotate it like this or?" or "Which way do I like to rotate it? This way or the other way?" Participants in both conditions appeared to demonstrate less confidence in their approach to completing a 30-degree rotation matrix about the X-, Y-, or Z-axis. Even though they had completed three 90-degree rotations about these axes in the previous questions, when the problems shifted from 90-degree to 30-degree rotation, participants in the virtual condition struggled the most with six participants needing clarification about what numbers to use in the matrix.

4.2 Reliance on Resources

Another prevalent theme in both conditions was a reliance on previous notes taken, instructions provided, and participants' answers to earlier rotation questions. Participants needed assistance completing the matrix using only the worksheet and model and, therefore, relied heavily on previous notes and rotation questions to find solutions. This was initially observed in their understanding and ability to solve the first transformation matrix as four participants in the virtual and four in the physical condition read through the instructions and then flipped back to the front page of the rotation booklet to reread notes, unit circles, or matrix information. Moreover, participants needed help starting the rotation questions. They would prompt the experimenters to provide more detailed instructions on matrix algebra, such as which types of values should go where on the matrix and how those numbers are obtained (N = 11).

As participants progressed through each question, their utilization of the resources provided persisted and adapted as they reached the 30-degree rotation questions. Participants began to connect that the introductory video provided the answers for solving problems. If participants recognized this (and took notes), they could fill out the corresponding parts with the angles in the question. Thus, participants utilized previous notes and answers as a template for completing the problem rather than solving it for themselves (N = 15). Participants used previous information and notes; one participant even stated, "The instructions are telling me the same things as I have here [notes], the same formulas." By the last rotation question, half of the participants still utilized notes from the videos and previous matrices as a template for completing the problem rather than demonstrating an understanding of the math involved (N = 10).

4.3 Pattern Recognition

In conjunction with utilizing their notes and past problems to complete the matrix rotation, participants also began to identify patterns in the types of matrices. "It follows the same formula. Pretty much if you can pick up the formula, you can do it even if you don't understand." When solving for rotation question 3, Matrix R, participants in both conditions began to use pattern recognition to complete the matrix (N = 11). For this question, rather than solving the matrix by learning how the rotations work, participants would use earlier problems and the patterns of the numbers to guess how to complete the matrix (N = 7). However, as participants progressed to question 4, participants understood what a 30-degree rotation looked like and recognized that it was similar to the previous problems (N = 12). It was clear that participants recognized the similarities between the 90- and 30-degree sets as one noted, "It's pretty similar to the first problem except instead of 90, you're just doing 30." By the final rotation question, several participants in the virtual condition (N = 5) demonstrated a clear understanding that regardless of the degree of rotation, rotations about the same axis will have the same values in some elements in the matrix "….this question had a 0 0 1 here there should be a 0 0 1 on this too."

4.4 Developing Understanding of 3D Matrix Algebra

After interacting with the AR-Classroom to complete four of the six rotation matrix questions, participants in both conditions were able to provide more detailed reasoning for how they solved the matrix and displayed a greater understanding of the rotations. In the physical condition, participants were able to understand that rotations along one of the three principal axes could be represented as a rotation of the shuttle's cross-section on a plane formed by the other two axes (e.g., a rotation along the Z-axis could be viewed from the perspective of looking directly at the XY plane). In comparison, participants in the virtual condition demonstrated a greater understanding of counterclockwise rotation as participants seemed to be less confused than on previous questions about the direction the rotation needs to be (N = 5). Finally, rather than relying on previously identified patterns, virtual condition participants finished filling out the final Matrix R (i.e., 30-degree counterclockwise rotation about the Z-axis) very quickly, with little help from the model or previous notes (N = 12), a participant stated: "I didn't try to find [a] pattern I just kind of remembered from before."

5 Discussion

The present study investigated AR-Classroom's efficacy in teaching introductory 3D matrix algebra and what mathematical concepts related to matrix algebra students learn from using the app. Four prevalent themes emerged from a thematic analysis of the participants' learning experience: (1) Difficulty using traditional methods, (2) Reliance on resources, (3) Pattern recognition, and (4) Developing understanding of 3D matrix algebra.

Participants displayed less confidence and more apprehension when solving the matrix rotation without the aid of the AR app. They often relied on their resources, such as notes or previous problems or began searching for patterns to ease the cognitive load it may take to solve the rotation. Such findings are not surprising, as previous literature asserts that students often use memorized formulas to solve problems and have difficulty solving problems that require visualization skills [30]. However, the AR-Classroom app eliminates the need for solid spatial skills, allowing students to visualize and physically manipulate complex spatial relationships and abstract concepts. Additionally, recognizing the patterns underlying spatial rotations is a foundational component of geometry learning, as previous research demonstrates that students often struggle with connecting spatial rotations and their underlying mathematical theories [31].

Finally, the connection between geometric transformations in a 3D coordinate space and matrix operations was easier when solving the problem with the AR-Classroom's aid. After using the AR-Classroom across multiple rotation questions, participants demonstrated a rudimentary understanding of the correlation between the rotations and changes in matrices and patterns in three-dimensional rotations. Therefore, 3D matrix algebra and geometry learning using the AR-Classroom app helps with learning by helping students understand the appearance of 3D objects in different directions and involve them in the learning process through interaction with virtual and physical manipulatives within a real-world environment.

5.1 Limitations and Future Research

The study examined the qualitative data extracted from the AR-Classroom's efficacy experiment using a rigorous qualitative methodology protocol to investigate participants' learning experiences using AR-Classroom. The data was collected via two researchers' observations of the participants during the intervention with corresponding recordings and analyzed by examining the researchers' notes, transcribing video recordings, coding results to identify patterns, and generating themes to describe participants' experiences. However, additional qualitative methods such as post-intervention interviews regarding the students' acquired knowledge of 3D matrix algebra and open-ended survey questions about their strategies for learning during the intervention will strengthen the present findings and provide additional prevalent themes. Future qualitative studies on the AR-Classroom should implement additional qualitative data collection methods to provide a robust understanding of learning using the application. The AR-Classroom's efficacy in teaching 3D matrix algebra must be further studied using different methodological approaches and content-specific research questions. For example, efficacy could be gauged using a longitudinal study to examine the educational gains on targeted rotation concepts through repeated use of the AR-Classroom application, or by conducting a case study of implementing the intervention in a STEM or math-based classroom. Expanding how our research team conceptualizes and measures learning outcomes using the AR-Classroom provides further validation of the app's potential success as a learning tool.

6 Conclusion

The paper reviewed the qualitative findings of the AR-Classroom efficacy experiment and provided recommendations for future research on AR-Classroom and similar technologies. The initial findings from the present study and its quantitative counterpart (Burte et al., Submitted to HCII 2024) suggest that matrix algebra learning interventions delivered by AR-Classroom may be helpful and lead to improvements in mathematical skills. After repeatedly using the AR-Classroom, students recognized the patterns and similarities between types of spatial rotations and their representations. They demonstrated a fundamental understanding of the mathematical theory underlying 3D spatial rotations. Guided by the data-informed and iterative approach previously used to improve the educational technology AR-Classroom and its predecessor, BRICKxAR/T, our research team, and tech development team will use the present findings to develop new versions of the app targeted at the current studies identified strategies students use to learn matrix algebra and develop a new learning experiment with revised procedures, measures, and data collection approaches to validate the app's efficacy further.

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