Reimagining and reinterpreting Cooper pairs, the Fermi sea, Pauli blocking and superfluidity: the Pauli principle in collective motion

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Identifying possible microscopic mechanisms underlying superfluidity has been the goal of various studies since the introduction of the original BCS theory. Recently a series of papers have proposed microscopic dynamics based on normal modes to describe superfluidity without the use of real-space Cooper pairs. Multiple properties were determined with excellent agreement with experimental data. The group theoretic basis of this general N-body approach has allowed the microscopic behavior underlying these results to be analyzed in detail. This reimagination is now used to reinterpret several interrelated phenomena including Cooper pairs, the Fermi sea, and Pauli blocking. This approach adheres closely to the early tenets of superconductivity/superfluidity which assumed pairing only in momentum space, not in real space. The Pauli principle is used, in its recently revealed role in collective motion, to select the allowed normal modes. The expected properties of superfluidity including the rigidity of the wave function, interactions between the fermions in different pairs, convergence of the momentum and the gap in the excitation spectrum are discussed.

Introduction. - Multiple studies in the field of condensed matter have sought to identify the microscopic mechanisms responsible for superconductivity/superfluidity in different systems. Some studies have proposed alternative mechanisms to the conventional understanding proposed in the original BCS theory[1– 10]. In particular, recent investigations[1–3] using an exact solution of the BCS Hamiltonian demonstrated that the fermions that form composite bosons, i.e. Cooper pairs, do not condense into a single state as originally assumed[11]. These studies and others suggest that nature may have more than one way to achieve superconductivity/superfluidity and that alternative microscopic descriptions could be of value.

In a series of recent papers, the superfluidity of ultracold superfluid Fermi gases[12–15] has been studied using a first-principles perturbation formalism called symmetry-invariant perturbation theory, SPT. This approach uses group theory and graphical techniques rather than a basis set or numerical methods to solve each perturbation order, in principle exactly. The first order equation is harmonic and has been solved exactly by determining the group theoretic N-body normal modes[16– 18]. The Pauli principle is applied without explicit antisymmetrization using an adiabatic transition from an independent particle regime to an interacting regime[19]. Despite the many-body approach with no two body pairing, the first-order results yielded close agreement with experiment without higher order corrections both at unitarity and across the BCS to unitarity transition[20–22]. This suggested the possibility that normal modes might provide an alternative microscopic basis for superfluid behavior that would differ from the conventional view that some of the fermions form loosely bound pairs that condense into a macroscopic occupation of the lowest state[23–36]. In addition to producing good agreement with multiple experimental results, normal mode dynamics offer an interesting microscopic explanation for universal behavior at unitarity[37, 38].

The possibility that normal modes can provide an alternative route to a macroscopic wave function with phase coherence over the entire ensemble without twobody pairs in real space, i.e. composite bosons, necessarily refocuses attention on the importance of inter-pair correlations which are due to the Pauli principle and have always been recognized as crucial for an accurate description of superconductivity/superfluidity. Based on these findings and the analysis of the microscopic basis underlying superfluidity using normal modes, the goal of this Letter is to offer an alternative interpretation of some of the seminal ideas behind conventional approaches to superfluidity, to reimagine the microscopic basis underlying superfluidity and to elucidate the role of the Pauli principle in the emergence and stability of collective behavior.

Background. - The Pauli principle dominates the inter-pair interactions in the BCS ansatz[1–3], and is critical to producing important properties of superfluidity/superconductivity including an energy gap in the excitation spectrum, the rigidity of the superfluid wave function that yields the Meissner effect, and the vanishing resistance to current flow. It is interesting that early work did not assume two-body pairing in real space. The highly successful BCS theory proposed in 1957[11] assumes that the fermions are paired in momentum space with $+k$ and $-k$ values, i.e. zero-momentum states. As stated in the 1957 paper, the BCS wave function describes the "coherence of large numbers of electrons," but does not propose that fermion pairs are localized into pseudomolecules that transition as in Bose-Einstein condensation[11]. As suggested by London in 1950, a superconductor is a “quantum structure on a macroscopic scale... a kind of solidification or condensation of the average momentum distribution” of the electrons. “It would not be due to distinct electrons at separate places having the same momentum”, but “it would arise from wave packets of wide extension in space assigning the same local momentum to the entire superconductor”[39]. These early concepts of superfluidity/superconductivity as well as the seminal properties: pairing in momentum space, the long-range order over macroscopic distances, a “rigidity” of the wave function, and the gap in the excitation spectrum[40, 41] are naturally manifested in a normal mode picture of superfluidity.

Symmetry-Invariant Perturbation Theory: a group theoretic and graphical approach to the general N-body problem. -

1) Overview. - Symmetry-invariant perturbation theory is a first-principles general many-body method with no adjustable parameters that employs group theory and graphical techniques to avoid the intensive numerical work typical in conventional many-body methods. If higher-order terms are small, the first-order normal mode solutions can offer physical insight into the underlying dynamics. The perturbation parameter is the inverse dimensionality of space. Using $1/D$ or $1/N$ expansions to study physical systems was originally developed by t'Hooft in quantum chromodynamics[42], and subsequently used by Wilson[43] in condensed matter to calculate critical exponents for $D = 3$ phase transitions starting from the $D = 4$ exact values. These techniques have now been used in multiple fields of physics from atomic and molecular physics [44–62] and condensed matter [43, 63–65], to quantum field theory[66–72].

The SPT formalism was developed to handle the large ultracold ensembles of interest in atomic physics/condensed matter and was initially applied to bosonic systems[16–18, 73–76]. Recently, this formalism was extended to ultracold Fermi gases[20–22] which are subject to Pauli constraints[19, 20, 22, 77]. Currently, this method is formulated through first order for $L = 0$ systems in three dimensions that are confined by spherically-symmetric potentials with general interaction potentials. The SPT approach uses symmetry to attack the N-scaling problem[16–18, 78, 79], rearranging the work required for an exact solution so the exponential scaling depends on the order of the series, not the value of N which is a simple parameter. To access maximal symmetry, a perturbation series is formulated about a large-dimension structure with a point group isomorphic to the symmetric group S_N, then evaluated for $D = 3$. This strategy allows the work at each order that scales exponentially to be extracted as a pure math problem (cf. the Wigner-Eckart theorem)[80, 81]. In principle, this problem can be solved exactly using group theoretic methods, and saved[82], with a significant reduction in numerical cost.

Since the perturbation does not involve the interaction strength, strongly interacting systems such as the unitary regime can be studied. This manybody approach does not provide a mechanism for the transition to diatomic molecules in the BEC regime. The BEC regime could, in principle, be described by including higher-order terms although many terms would probably be required undermining any physical insight.

Even the lowest order contains beyond-mean-field effects that produce excellent first-order results[20, 22, 73] as seen in earlier dimensional approaches[49, 83–86]. The formalism was tested on a fully-interacting model problem of harmonically-confined, harmonically-interacting particles[19, 75–77]. The SPT and the exact wave function agree to ten or more digits verifying this many-body formalism[75] and the forms for the group-theoretic, analytic N-body normal modes.

2) The SPT formalism. - Using D dimensional Cartesian coordinates, the N-body Schr¨odinger equation is:

$$H\Psi = \left[\sum_{i=1}^{N} h_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} g_{ij}\right]\Psi = E\Psi \quad , \qquad (1)$$

$$\begin{aligned} h_i &= -\frac{\hbar^2}{2m_i}\sum_{\nu=1}^{D}\frac{\partial^2}{\partial x_{i\nu}^2} + V_{\mathtt{conf}}\left(\sqrt{\sum_{\nu=1}^{D} x_{i\nu}^2}\right), \\ g_{ij} &= V_{\mathtt{int}}\left(\sqrt{\sum_{\nu=1}^{D}(x_{i\nu}-x_{j\nu})^2}\right), \end{aligned} \qquad (2)$$

with h_i a single-particle Hamiltonian, $V_{\mathtt{int}}$ a general two-body interaction potential, $x_{i\nu}$ the ν^{th} Cartesian component of the i^{th} particle, and $V_{\mathtt{conf}}$ a sphericallysymmetric confining potential[16–18]. The Hamiltonian is transformed to internal coordinates, r_i and γ_{ij}, where

$$r_i = \sqrt{\sum_{\nu=1}^{D} x_{i\nu}^2}, \quad (1 \le i \le N),$$

$$\gamma_{ij} = cos(\theta_{ij}) = \left(\sum_{\nu=1}^{D} x_{i\nu}x_{j\nu}\right)/r_i r_j\,, \ (1 \le i < j \le N),$$

$$\bar{T} = \sum_{i=1}^{N}\Big(-\frac{1}{2}\frac{\partial^2}{\partial \bar{r}_i{}^2} - \frac{1}{2\bar{r}_i^2}\sum_{j\neq i}\sum_{k\neq i}\frac{\partial}{\partial\gamma_{ij}}(\gamma_{jk}-\gamma_{ij}\gamma_{ik})\frac{\partial}{\partial\gamma_{ik}}\Big)$$

$$\bar{U} = \sum_{i=1}^{N}\left(\frac{\delta^2 N(N-2) + (1-\delta(N+1))^2\left(\frac{\Gamma^{(i)}}{\Gamma}\right)}{8\bar{r}_i^2}\right)$$

are the N D-dimensional scalar radii r_i and the cosines γ_{ij} of the $N(N-1)/2$ angles between the radial vectors. A similarity transformation removes the first-order derivatives[87], and a scale factor is employed to regularize the large-dimension limit of the Schr¨odinger equation. Substituting the scaled variables and defining $\delta = 1/D$ as the perturbation parameter gives:

$$\bar{H}\Phi = \left(\delta^2\bar{T} + \bar{U} + \bar{V}_{\mathtt{conf}} + \bar{V}_{\mathtt{int}}\right)\Phi = \bar{E}\,\Phi\,. \qquad (3)$$

$$\bar{V}_{\mathtt{conf}} = \sum_{i=1}^{N} \frac{1}{2}\bar{r}_i^2, \quad \bar{V}_{\mathtt{int}} = \frac{\bar{V}_0}{1-3b'\delta} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} 1 - \tanh \Theta_{i,j}$$

The Gramian determinant Γ has elements γ_{ij} (see Appendix D in Ref [16]), and the $\Gamma^{(i)}$ determinant has the row and column of the i^{th} particle deleted.

Defining $\Theta_{ij} = \frac{\bar{c}_0}{1-3\delta}\left(\frac{\bar{r}_{ij}}{\sqrt{2}} - \bar{\alpha} - 3\delta\left(\bar{R} - \bar{\alpha}\right)\right)$, where

$\bar{r}_{ij} = q\bar{r}_i^2 + \bar{r}_j^2 - 2\bar{r}_i\bar{r}_j\gamma_{ij}$ is the interatomic separation,

$\bar{R}$ is the range of the square-well potential, and $\bar{\alpha}$ is a constant which softens the potential as $D \to \infty$, the form of $\bar{V}_{\mathtt{int}}$ reduces to a square well at $D = 3$ and is differentiable away from $D = 3$ to permit the dimensional analysis[16, 73]. The constant b' is chosen to yield an infinite scattering length at unitarity with $\bar{V}_0 = 1.0$. For weaker interactions in the BCS regime, $\bar{V}_0$ is scaled to smaller values. $R << \bar{a}_{ho}$ and is systematically reduced to extrapolate to zero-range interaction. When $D \to \infty$, the second derivatives drop out producing a static problem at zeroth order with an effective potential, $\bar{V}_{\mathtt{eff}}$:

$$\bar{V}_{\mathtt{eff}} = \sum_{i=1}^{N} \bar{U}(\bar{r}_i;\delta) + \bar{V}_{\mathtt{conf}}(\bar{r}_i;\delta) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \bar{V}_{\mathtt{int}}(\bar{r}_i,\gamma_{ij};\delta),$$

whose minimum is an infinite-dimensional maximallysymmetric structure with all $\bar{r}_i$ and γ_{ij} equal: for $D \to \infty$, $\bar{r}_i = \bar{r}_\infty (1 \le i \le N)$ and $\gamma_{ij} = \gamma_\infty (1 \le i < j \le N)$.

Two minimum conditions: $\left(\frac{\partial \bar{V}_{\mathrm{eff}}}{\partial \bar{r}_i}\right)\Big|_\infty = 0, \left(\frac{\partial \bar{V}_{\mathrm{eff}}}{\partial \gamma_{ij}}\right)\Big|_\infty = 0$ yield two equations in $\bar{r}_\infty$ and γ_∞: $\bar{r}_\infty = \frac{1}{\sqrt{2}\sqrt{1+(N-1)\gamma_\infty}}$ and.

$$\frac{\gamma_\infty(2+(N-2)\gamma_\infty)}{(1-\gamma_\infty)^{3/2}\sqrt{1+(N-1)\gamma_\infty}} + \bar{V}_0 \operatorname{sech}^2\left(\Theta_\infty\right)\Theta'_\infty = 0$$

Expanding about the minimum $(r_\infty, \gamma_\infty)$: $\bar{r}_i = \bar{r}_\infty + \delta_{1/2}\bar{r}_i'$ and $\gamma_{ij} = \gamma_\infty + \delta_{1/2}\gamma_{ij}'$, sets up a power series in $\delta^{1/2}$. The first-order, $\delta = 1/D$, equation is harmonic and is solved exactly using group theory to obtain the N-body normal modes[16–18]. The first-order Hamiltonian, $\bar{H}_1$, is defined in terms of the constant matrices, G composed of kinetic energy terms, and **F** composed of potential terms, evaluated at the large dimension limit:

$$\bar{H}_1 = -\frac{1}{2}\partial_{\bar{y}'}{}^T \boldsymbol{G} \partial_{\bar{y}'} + \frac{1}{2}\bar{\boldsymbol{y}}'^T \boldsymbol{F} \bar{\boldsymbol{y}}' + v_o, \qquad (4)$$

with v_o a constant[16]. The FG matrix method[88], which has been used extensively in molecular physics, is used to obtain the normal-mode frequencies[16] and coordinates[17]. (See Appendix A in Ref. [16] for a brief summary.) Only five distinct frequencies, $\bar{\omega}$, are obtained. This large degeneracy is a manifestation of the very high degree of symmetry in the F and G matrices which are evaluated for the $D \to \infty$, maximallysymmetric structure with a single value for all $\bar{r}_\infty$ and γ_∞. These matrices are thus invariant under the N! particle interchanges of S_N and do not connect subspaces belonging to different irreducible representations (irreps) of S_N[89, 90], thus the normal coordinates transform under irreps of S_N.

Five irreps are involved: a 1-D radial and a 1-D angular irrep both labelled by the partition [N], an $(N-1)$-D radial and an $(N-1)$-D angular irrep both labelled by the partition $[N-1, 1]$, and one angular $N(N-3)/2D$ irrep labelled by $[N-2, 2]$. These irreps are given shorthand labels: $\mathbf{0}^-$, $\mathbf{0}^+$, $\mathbf{1}^-$, $\mathbf{1}^+$, and **2** respectively, (see Refs. [17, 18]), where the single normal mode of type $\mathbf{0}^+$ is a center of mass/symmetric bend motion; the single $\mathbf{0}^-$ mode is a breathing motion/symmetric stretch; the $N-1$ type $\mathbf{1}^+$ modes have particle-hole/single-particle angular excitation behavior; the $N-1$ type $\mathbf{1}^-$ modes exhibit particle-hole i.e. single-particle radial excitation behavior; and the $N(N-3)/2$ type **2** modes are phonon modes. Ref. [37] analyzes these motions in detail.

A symmetry coordinate vector, S, is defined:

$$S = \begin{pmatrix} \boldsymbol{S}_{\bar{\boldsymbol{r}}'}^{[N]} \\ \boldsymbol{S}_{\overline{\boldsymbol{\gamma}}'}^{[N]} \\ \boldsymbol{S}_{\bar{\boldsymbol{r}}'}^{[N-1,\ 1]} \\ \boldsymbol{S}_{\overline{\boldsymbol{\gamma}}'}^{[N-1,\ 1]} \\ \boldsymbol{S}_{\overline{\boldsymbol{\gamma}}'}^{[N-2,\ 2]} \end{pmatrix} = \begin{pmatrix} W_{\bar{\boldsymbol{r}}'}^{[N]}\bar{\boldsymbol{r}}' \\ W_{\overline{\boldsymbol{\gamma}}'}^{[N]}\overline{\boldsymbol{\gamma}}' \\ W_{\bar{\boldsymbol{r}}'}^{[N-1,\ 1]}\bar{\boldsymbol{r}}' \\ W_{\overline{\boldsymbol{\gamma}}'}^{[N-1,\ 1]}\overline{\boldsymbol{\gamma}}' \\ W_{\overline{\boldsymbol{\gamma}}'}^{[N-2,\ 2]}\overline{\boldsymbol{\gamma}}' \end{pmatrix}, \qquad (5)$$

where the transformation matrices $W_{\bar{\boldsymbol{r}}'}^{[\alpha]}$ and $W_{\overline{\boldsymbol{\gamma}}'}^{[\alpha]}$ are determined using the theory of group characters to decompose $\bar{r}'$ and γ' into basis functions that transform under the five irreps of S_N[17]. The FG method is applied to determine the normal modes, The normal coordinates in the [N] and [N−1,1] sectors have mixed radial and angular behavior. The [N − 2,2] normal modes are purely angular since this sector has no $\bar{r}'$ symmetry coordinates.

The extent of radial/angular mixing depends on the first-order Hamiltonian terms.

The energy through first-order in $\delta = 1/D$ [16, 49]:

$$\overline{E} = \overline{E}_\infty + \delta\left[\sum_{\mu=\{\mathbf{0}^\pm,\,\mathbf{1}^\pm,\,\mathbf{2}\}} (n_\mu + \frac{1}{2}d_\mu)\bar{\omega}_\mu \, + \, v_o\right], \quad (6)$$

has the form of a harmonic energy in terms of the normal

mode frequencies, where$\bar{}_{\mathrm{eff}}|$, μ labels the five types of

normal modes$\bar{E}_\infty$ is the energy at the mini-

mum of V_∞

(irrespective of the value of N, see Ref. [16] and Ref.[15] in [17]), n_μ is the total normal mode quanta with frequency $\bar{\omega}_\mu$; and v_o is a constant (defined in Ref. [16], Eq.(125)). The five roots have multiplicities: $d_{\mathbf{0}^+} = 1, d_{\mathbf{0}^-} = 1, d_{\mathbf{1}^+} = N-1, d_{\mathbf{1}^-} = N-1, d_{\mathbf{2}} = N(N-3)/2$. Eq. (6) defines the ground state energy as well as the spectrum of excited states by assigning normal mode quantum numbers consistent with the Pauli principle. The allowed assignments are determined by finding a correspondence between the normal mode states $|n_{\mathbf{0}^+}, n_{\mathbf{0}^-}, n_{\mathbf{1}^+}, n_{\mathbf{1}^-}, n_{\mathbf{2}}>$ and the non-interacting states of the three dimensional harmonic oscillator ($V_{\mathrm{conf}}(r_i) = \frac{1}{2}m\omega_{ho}^2 {r_i}^2$) which have known restrictions due to antisymmetry. These two spectrums are related in the double limit $D \to \infty$, $\omega_{ho} \to \infty$ where both representations are valid. At this double limit, the radial and angular characters separate resulting in two conditions[19, 20]:

$$2n_{\mathbf{0}^-} + 2n_{\mathbf{1}^-} = \sum_{i=1}^{N} 2\nu_i\,, \;\; 2n_{\mathbf{0}^+} + 2n_{\mathbf{1}^+} + 2n_{\mathbf{2}} = \sum_{i=1}^{N} l_i\,, \; (7)$$

where ν_i and l_i are the radial and orbital angular momentum quantum numbers of the three dimensional oscillator, and $n_i = 2\nu_i + l_i$ is the ith particle energy level quanta defined by: $E = \sum_{i=1}^{N}\left[n_i + \frac{3}{2}\right]\hbar\omega_{ho} = \sum_{i=1}^{N}\left[(2\nu_i + l_i) + \frac{3}{2}\right]\hbar\omega_{ho}$. This strategy is analogous to Landau's use of the non-interacting system to set up the correct Fermi statistics as interactions adiabatically evolve in Fermi liquid theory[91].

For ultracold systems, the lowest angular and radial modes are occupied i.e. phonon, $n_{\mathbf{2}}$, and single-particle radial excitation modes, $n_{\mathbf{1}^-}$, yielding:

$$2n_{\mathbf{1}^-} = \sum_{i=1}^{N} 2\nu_i, \quad 2n_{\mathbf{2}} = \sum_{i=1}^{N} l_i \quad . \qquad (8)$$

3) Application of SPT to ultracold fermions. - During the last nine years, the SPT approach has been used to investigate superfluidity for ultracold Fermi gases. Properties at unitarity as well as from BCS to unitarity were obtained in close agreement with experiment including ground state energies[20], critical temperatures[21], excitation frequencies[21, 38], thermodynamic entropies and energies[21, 22] as well as the lambda transition in the specific heat[22], a well known signature of the onset of superfluidity. (For a brief discussion of these calculations including graphs showing the agreement with experiment and theory and an explanation of the microscopic basis underlying these results, see Supplemental Information Section I. A-D.)

The Fermi sea and Pauli blocking: a collective viewpoint. - From an independent particle view, degenerate Fermi systems have all the lowest energy states filled, with a "Fermi surface" dividing the filled from the unfilled levels. This "sea" of fermions exists in energy space, with the scale of energies defined by the Fermi energy which is the largest occupied energy in the system. The role of the Fermi sea is to Pauli-block states below the Fermi energy, thus the behavior of such systems is dominated by the Pauli principle which determines their general structure through the filling of the states.

Using the current approach and assuming superfluid temperatures near $T = 0$, the concept of a Fermi sea is now defined from a collective viewpoint, assuming that the particles are in a collective mode allowed by the Pauli principle. The Pauli restrictions originate in the independent particle picture, but are transferred to the collective picture through an adiabatic evolution of the system to the collective mode as interparticle interactions turn on. The occupations of the lowest states in the independent particle picture are responsible for the restrictions on the number of quanta permitted in the collective motion of the ensemble. For ultracold systems at $T = 0$, only phonon modes are occupied so the Fermi sea of occupied independent states becomes an energy minimum of the phonon collective mode, with lower energy phonon modes unoccupied, i.e. Pauli-blocked from occupation.

The Fermi energy in the independent view is the energy of the highest occupied independent state, while in the collective view, the Fermi energy is the energy of the lowest occupied phonon mode. The quantum numbers in these two regimes are discussed in the Supplemental Information, Section I.E.

Cooper pairs: a critical concept. - The concept of Cooper pairs has been called one of the pillars of the microscopic theory of superconductivity, a concept that opened up the route to a successful theory that could explain the physical effects of zero resistivity, the existence of a gap in the excitation spectrum, and the Meissner effect among others.

Since the original BCS theory was introduced, the concept of Cooper pairs has evolved to include a more nuanced understanding of the role played by this pairing. Although it was recognized from the beginning that Cooper pairs were not simple bosons, it was widely assumed that these composite bosons condensed into a single lowest state. Multiple studies have since argued that the condensation involves multiple states[1–10]. By restricting configurations in the original BCS calculations to pairs of states with $+k$ and $-k$, i.e. zero momentum states, a "coherent lowering of the energy" was obtained[41]. This was "consonant" with "London's concept of a condensation in momentum"[41]. Theoretically, the pairs are created by two fermion creation operators which do not satisfy Bose statistics. This is essential to the success of the theory which must include many-body effects to yield an energy gap and long-range order over macroscopic distances. As reviewed by Bardeen in his Nobel address, "A theory involving a true many-body interaction between the electrons seemed to be required to account for superconductivity"[92].

These early concepts are consistent with a collective picture, a rigid macroscopic wave function with long range order extending over the entire ensemble. The condensation of the frequency to a single value as the particles adopt the collective motion of a normal mode results in the expected convergence of the momentum to two values, $+k$ and $-k$, as the particles slosh back and forth in lockstep. This normal mode picture retains Cooper pairing, a concept critical to the development of BCS theory, but redefines it, not as a two-body phenomenon in real space, but rather as a many-body phenomenon that consolidates the momentum of an ensemble to two equal and opposite values. This is consistent with both the early BCS concepts and the recognized need for a fully interacting many-body wave function.

The seminal properties of superfluidity as supported by the microscopic dynamics of the normal mode picture. -

1) "Rigidity" of the wave function. - This property of superconductivity has been called "a striking manifestation of a subtle form of quantum rigidity on the macroscopic scale"[93]. It prevents a moderate external magnetic field from modifying the wave function and is also responsible for the gap in the excitation spectrum. Collective motions in the form of normal modes naturally provide rigid harmonic motion with the particles moving in lockstep with the same frequency and phase. In the SPT formalism these synchronized, collisionless motions are eigenfunctions of an approximate Hamiltonian and thus possess some degree of stability. They provide simple, quantum macroscopic wave functions with phase coherence over the entire ensemble. The microscopic behavior of the particles in a normal mode as they execute rigid, harmonic motions is explored briefly in Supplemental Information Section II and in detail in Ref. [37].

2) Interactions between the fermions in different pairs due to the Pauli principle. - The interactions between the fermion constituents of different "bosonic" Cooper pairs in BCS theory are due to the Pauli principle. Thus the fermions in BCS theory play a dual role: creating composite bosons that are assumed to condense to the lowest state; and simultaneously retaining their fermionic nature giving rise to inter-pair interactions from the Pauli principle. These inter-pair interactions in conventional BCS approaches are known to be crucial to producing superfluidity/superconductivity. This dual role calls into question the importance of two-body pairing as the underlying microscopic dynamic compared to the many-body correlations which are critical to the emergence of superconductivity/superfluidity.

The collective picture reimagines this as simply the motion of N interacting fermions in a macroscopic normal mode wave function. This normal mode function at ultracold temperatures is a very low energy phonon mode. Thus it is not necessary to produce composite bosonic entities that condense to the lowest state to produce a macroscopic quantum wave function, as fermions can occupy one or more of the closely spaced phonon modes to form a quantum, macroscopic function with collective behavior. The many-body synchronous motion is subject to the Pauli principle which controls the dynamics at all strengths from BCS to unitarity.

3) "Solidification or condensation of the average momentum distribution"; arising "from wave packets of wide extension in space assigning the same local momentum to the entire superconductor": the Uncertainty Principle. - These quotes from London[39, 40] are manifested in the BCS ansatz by assuming that the particles pair into $+k$ and $-k$ pairs with a resulting lowering of the energy. The collective picture assumes many-body motion of the fermions in a phonon/compressional normal mode. As the particles begin to move in sync with a single frequency and phase, the spatial extent of the normal mode expands, while the single frequency of motion means that the average momentum of the fermions is converging toward a single absolute value as predicted by London and others. This convergence in momentum space and the corresponding expansion of the wave packet in position space is expected from the uncertainty principle. The SPT normal mode microscopic dynamics underlying this condensation are analyzed briefly in Supplemental Information Section II.C. and in detail in Ref. [38].

4) Gap in the excitation spectrum. - During the early 1950's increasing evidence appeared for an energy gap at the Fermi surface. This motivated the BCS ansatz of assuming that only zero momentum pairs contributed leading to a lowering of the energy of the lowest state[41]. In the normal mode picture, there is a natural gap between the phonon mode and the next higher mode which is a single particle excitation mode that increases from extremely small in the weakly interacting BCS regime to a maximum in the unitary regime. (See Fig. 2 in Ref. [38].) Thus, the gap is reimagined, not as originating from the excitation of a fermion out of a two-body bosonic entity, but as the excitation of a single particle out of the synced motion of the phonon mode as the ensemble adopts a new collective motion. This gap provides stability for superfluid behavior particularly as it widens as unitarity is approached. It also leads to a value for the first excited state of the ensemble as determined by the Pauli principle and an estimate of the critical temperature that agrees with both the BCS estimate in weaker interaction regimes as well as more intensive T matrix calculations of T_C near the strongly interacting unitary regime[21]. Gaps also exist between the other types of normal modes (See Fig. 2 in Ref. [38]) that could provide stability for collective behavior if techniques to prevent the transfer to other modes exist or could be engineered. Temperature is likely to play a role in controlling the dynamics of such structures, but other mechanisms may supercede the effect of temperature to perhaps allow a high temperature system to sustain stable collective behavior which could be harnessed to bring the desired benefits of quantum engineering on a macroscopic scale.

The role of the Pauli principle: the transition from Fermi to Bose statistics at low temperature. - The Pauli principle has always been recognized as critical to an accurate description of superconductivity/superfluidity. In the SPT approach, the Pauli principle controls the dynamics of the collective behavior of the ensemble. Multiple closely spaced phonon modes may be occupied consistent with the results from the exact solution of the BCS Hamiltonian. As N increases, these low-energy phonon states become infinitesimally closer and closer in energy merging toward the ground state. This allows a natural transition from Fermi statistics to Bose statistics as seen in the BCS-BEC crossover as true bosonic entities, i.e. diatomic molecules, form and occupy this ground state.

Conclusions. - In this paper, the Pauli principle's role in collective motion as documented in previous investigations of ultracold Fermi gases has led to a reimagination and reinterpretation of the seminal concepts of superfluidity. The perturbation method used to obtain these results, SPT, is solved exactly with no adjustable parameters to yield normal modes at first order. This method takes advantage of group theory to obtain a detailed microscopic view of the underlying dynamics.

Normal mode behavior is ubiquitous in our universe at all energy and length scales manifesting the widespread existence of vibrational forces that occur in different media and across many orders of magnitude. The successful use of normal modes to determine multiple properties of ultracold Fermi gases in excellent agreement with both experimental data and theoretical results has demonstrated that two-body pairing assumptions in real space are not necessary to describe superfluidity in theory. This suggests that reimagining the underlying microscopic basis for superfluid behavior in different regimes could be worthwhile. Correlations between the pairs have always been known to be critical to describing superfluid behavior. These correlations refocus the physics away from the importance of two-body pairing to a many-body picture.

The collective perspective also offers an explanation for universal behavior in the unitary regime. The absence of interparticle interactions typically defines a regime of independent particles, but can also be the defining characteristic of a strongly correlated ensemble that has particles moving in lockstep with fixed interparticle distances, i.e. angular normal modes. Both types of ensembles are independent of microscopic details despite the very different dynamics underlying their properties.

This Letter is an attempt to investigate and perhaps enlarge the role of the Pauli principle in achieving superfluidity. It is perhaps not surprising that there could be multiple routes to achieve superfluidity. Whether the model of superfluidity described in this Letter could be realized in nature or in the lab is an open and certainly interesting question.

Particle statistics are known to be powerful organizational, driving forces in the emergence of collective states of matter for both bosons and fermions with simple behavior emerging from the complexity of the microscopic world. The Pauli principle plays a fundamental role in organizing fermion matter in our universe, providing stability as it intervenes in a wide range of phenomena from the structure of atoms to the physics of neutron stars[94]. Restricting the permutation symmetry of indistinguishable particles and controlling the occupation of identical fermions, it is responsible for the prevalence of degenerate Fermi systems at all scales, forms the foundation of the periodic table and exists at the core of quantum field theory. Understanding the role of the Pauli principle in collective behavior could offer insight into dynamics that could support the emergence and stability of organized behavior in our universe.

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