

A Penetrable Reactance Surface for Spherical Invisibility Cloaking

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Abstract—A penetrable tensorial metasurface on a grounded dielectric shell is presented for 3-D spherical invisibility cloaking. The spatially modulated impedance surface transforms the incident plane wave into a surface wave on the lit side and carries the power to the shadow side. Power is continuously released as a leaky wave with a wavefront consistent with the incident plane wave on the shadow side. A numerical design example for a four-wavelength-diameter conducting sphere is presented, and the cloaking effectiveness is validated by simulation.

I. INTRODUCTION

Electromagnetic invisibility cloaking requires meticulous manipulation of waves around an object, rendering it undetectable to external observers or sensors. A variety of techniques have been reported, such as transformation optics [1], scattering cancellation [2], and wave control using a metasurface [3]. These techniques are primarily focused on cloaking for 2-D cylindrical objects. Toward practical applications, 3-D cloaking of volumetric objects is needed. Achieving 3-D spherical cloaking is more challenging than 2-D cloaking because all three components of a field vector should be controlled. A multilayered isotropic media design for spherical cloaking has been presented [4]. Recently, an impenetrable impedance spherical surface has been numerically derived to conceal a large freestanding object [5].

In this work, a spatially modulated penetrable impedance surface is designed to effectively hide a large free-standing object. Introducing auxiliary surface waves (SWs), the total fields are built and optimized to satisfy the local and global lossless and gainless condition. Using the optimized fields, the penetrable tensorial surface impedance distribution is retrieved. A numerical metasurface concealing a 4-wavelength-radius perfect electric conductor (PEC) sphere is designed with its effectiveness confirmed through simulation.

II. COMPLETE FIELD SYNTHESIS

The proposed spherical cloaking concept is illustrated in Fig. 1. An x -polarized plane wave (PW) with $\mathbf{E}^i = \hat{x}E_0^i e^{-jkz}$ V/m (k = free-space wavenumber) illuminates a metasurface on a dielectric shell of a thickness t that surrounds a PEC sphere with a radius of $b = a - t$. The metasurface converts the incident PW into SWs on the lit side and transports the power to the shadow side before restoring the incident wave by continuous leaky-wave radiation on the shadow side. In spherical (r, θ, ϕ) coordinates, the incident

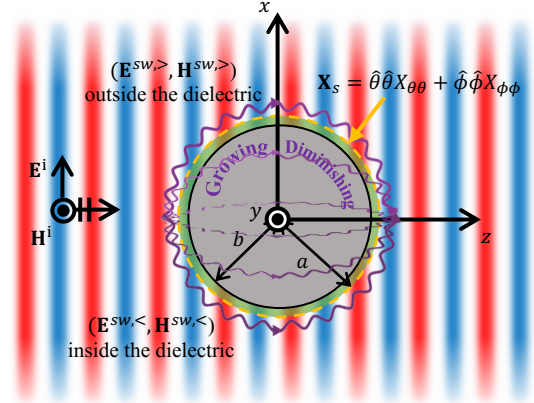


Fig. 1. Conceptual 3-D spherical cloaking with a penetrable metasurface characterized by $\mathbf{X}_s$.

PW fields are represented by the radial magnetic and electric vector potential components associated with TM and TE (to r) modes at the angular frequency ω

$$\begin{bmatrix} A_r^i \\ F_r^i \end{bmatrix} = E_0^i \begin{bmatrix} \cos \phi / \omega \\ \sin \phi / \omega \eta \end{bmatrix} \sum_{n=1}^{\infty} a_n \hat{J}_n(kr) P_n^1(\cos \theta), \quad (1)$$

where η is the free-space intrinsic impedance, $a_n = j^{-n}(2n+1)/n(n+1)$, $\hat{J}_n(\cdot)$ is the Riccati-Bessel function of the first kind, and $P_n^1(\cdot)$ is the associated Legendre function.

The SW fields outside and inside the dielectric are associated with the vector potential components

$$\begin{bmatrix} A_r^{\text{sw},>} \\ F_r^{\text{sw},>} \end{bmatrix} = E_0^i \begin{bmatrix} \cos \phi / \omega \sum_{n=1}^{\infty} b_n \hat{H}_n^{(2)}(kr) P_n^1(\cos \theta) \\ \sin \phi / \omega \eta \sum_{n=1}^{\infty} c_n \hat{H}_n^{(2)}(kr) P_n^1(\cos \theta) \end{bmatrix}, \quad (2)$$

$$\begin{bmatrix} A_r^{\text{sw},<} \\ F_r^{\text{sw},<} \end{bmatrix} = E_0^i \begin{bmatrix} \cos \phi / \omega \sum_{n=1}^{\infty} B_n(k_d r) P_n^1(\cos \theta) \\ \sin \phi / \omega \eta \sum_{n=1}^{\infty} C_n(k_d r) P_n^1(\cos \theta) \end{bmatrix}, \quad (3)$$

where $k_d = k/\sqrt{\epsilon_r}$, $B_n(k_d r) = d_n \hat{H}_n^{(1)}(k_d r) + e_n \hat{H}_n^{(2)}(k_d r)$, $C_n(k_d r) = f_n \hat{H}_n^{(1)}(k_d r) + g_n \hat{H}_n^{(2)}(k_d r)$, and $\hat{H}_n^{(1)}(\cdot)$ and $\hat{H}_n^{(2)}(\cdot)$ are the Riccati-Hankel functions of the first and second kind, respectively. Using Maxwell's equations and the boundary conditions, the coefficients of $B_n(k_d a)$ and $C_n(k_d a)$ are found in terms of b_n and c_n , respectively.

We build two auxiliary SW sets for the radial field components that propagate along the spherical surface at $r = a$ in

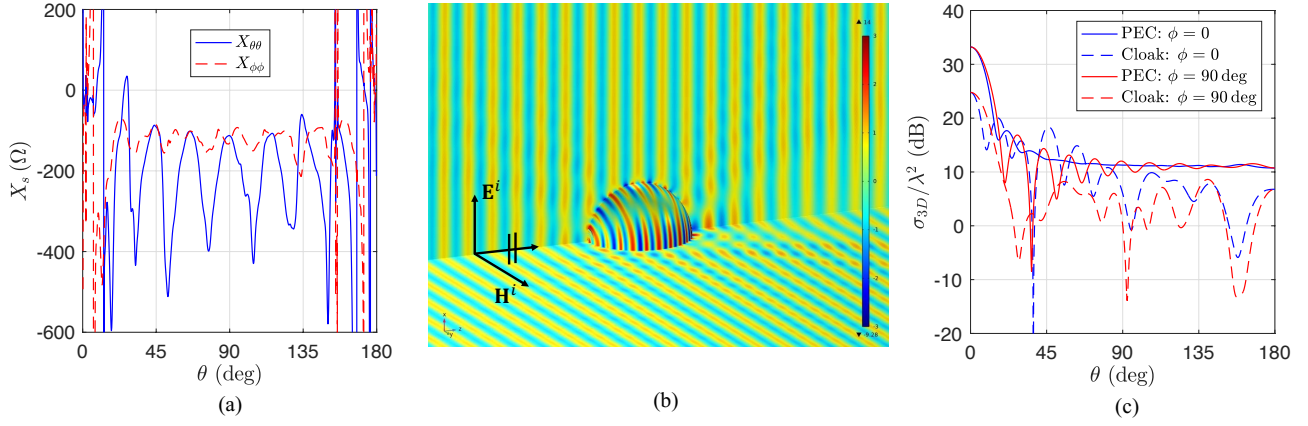


Fig. 2. Metasurface cloaking design for a 4λ -diameter PEC sphere. (a) Optimized surface reactance $X_{\theta\theta}$ and $X_{\phi\phi}$. (b) Snapshot of the total E_x field component. (c) Comparison of the RCS between cloaked and uncloaked PEC spheres in the xz - and yz -planes.

the $-\theta$ -direction as

$$E_r^{\text{sw}}(\theta) = E_1(\theta)e^{jk_{ce}\theta} + \sum_{n=2}^{\infty} E_n(\theta)e^{j\psi_n(\theta)}, \quad (4)$$

$$H_r^{\text{sw}}(\theta) = H_1(\theta)e^{jk_{ch}\theta} + \sum_{n=2}^{\infty} H_n(\theta)e^{j\psi_n(\theta)}, \quad (5)$$

where $E_1(\theta)$ and $H_1(\theta)$ are real-valued envelopes of the dominant SW terms having phase constants in the invisible region (i.e., $k_{ce/h} > k$). In (4)–(5), higher-order SW terms are represented by the complex-valued envelopes, E_n and H_n , and $\psi_n(\theta) = nk_{ce/h}a\theta - (n-1)(\angle E_r^i$ or $\angle H_r^i)$. Once b_n and c_n are determined from (4)–(5), we can retrieve all fields components associated with the SWs on the surface, $\mathbf{E}^{\text{sw}}(r = a)$, $\mathbf{H}^{\text{sw}}(r = a^+)$, and $\mathbf{H}^{\text{sw}}(r = a^-)$. The H -field discontinuity arises from the induced current on the metasurface.

The envelopes are optimized for the total fields to satisfy the pointwise lossless and gainless condition on the sphere surface at $r = a$, written as

$$\Delta S_r = \hat{r} \cdot \frac{1}{2} \text{Re} \{ (\mathbf{E}^i + \mathbf{E}^{\text{sw}}) \times (\Delta \mathbf{H}^i + \Delta \mathbf{H}^{\text{sw}})^* \} \rightarrow 0, \quad (6)$$

where $\Delta \mathbf{H}^i = \mathbf{H}^i(r = a^+) - \mathbf{H}^i(r = a^-)$ and $\Delta \mathbf{H}^{\text{sw}} = \mathbf{H}^{\text{sw},>}(r = a^+) - \mathbf{H}^{\text{sw},<}(r = a^-)$. The optimization minimizes a metric that measures satisfaction of (6) in an average sense [3]. Once the optimization completes, the spherical surface is characterized by the surface reactance tensor $\mathbf{X}_s(\theta, \phi) = \hat{\theta}\hat{\theta}X_{\theta\theta}(\theta) + \hat{\phi}\hat{\phi}X_{\phi\phi}(\theta) = \hat{\theta}\hat{\theta} \text{Re}\{E_{t\theta}\Delta H_{t\theta}^*\}/\text{Im}\{\Delta H_{t\theta}^*\Delta H_{t\phi}\} + \hat{\phi}\hat{\phi} \text{Re}\{E_{t\phi}\Delta H_{t\phi}^*\}/\text{Im}\{\Delta H_{t\theta}^*\Delta H_{t\phi}\}$.

III. NUMERICAL EXAMPLE

A reactance surface for cloaking a 4λ -diameter ($\lambda =$ free-space wavelength) PEC sphere is designed as a numerical example. The target frequency is 10 GHz and a dielectric shell with a relative permittivity $\epsilon_r = 10$ and a thickness of 2 mm is selected. For both polarizations, the carrier propagation constants are chosen to be $k_{ce} = k_{ch} = 2k$.

The optimized surface reactance profiles are plotted in Fig. 2(a). The two reactance profiles oscillate predominantly in the capacitive regime. Scattering simulation is performed using COMSOL MULTIPHYSICS with $\mathbf{X}_s$ numerically enforced at $r = a$. Figure 2(b) shows a snapshot of the total E_x field component, when a unit x -polarized, $+z$ -propagating PW illuminates the cloaked sphere. Low backscattering is observed, and the incident wave is approximately restored in the forward direction ($\theta = 0^\circ$). In the $E(\phi = 0^\circ)$ and $H(\phi = 90^\circ)$ planes, the bistatic radar cross section (RCS) normalized by λ^2 at 10 GHz is compared for cloaked and uncloaked in Fig. 2(c). The RCS is reduced in most directions. In particular, the RCS is reduced by 8.44 dB and 3.95 dB in the forward and backward directions, respectively.

IV. CONCLUSION

A penetrable tensor reactance surface on a grounded spherical-shell dielectric substrate has been presented for 3-D spherical cloaking. Introducing and optimizing SWs, a lossless and passive surface is achieved and characterized by a spatially modulated tensor reactance. Full-wave simulation using a numerically defined surface confirms RCS reduction. The tensor reactance surface may be realized by an array of modulated conductor patterns on the dielectric shell.

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