

MULTICAST WITH MULTIPLE WARDENS IN IRS-AIDED COVERT DFRC SYSTEM

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ABSTRACT

Physical layer security is a common concern in dual-function radar communications (DFRC) because of sharing of information between different emitters. We study covert communications between a DFRC unit and multiple legitimate users, with assistance from an intelligent reflecting surface (IRS). The system has multiple targets that need to be detected, and each target is collocated with a warden trying to detect the ongoing communication. We seek to maximize the worst-case data rate across users under radar detection constraint and covertness constraint. To this end, we superpose artificial noise with our message signal so that the wardens' received signal statistics do not change significantly if communications suddenly starts. We formulate a highly non-convex optimization problem to determine the passive beamforming scheme for the IRS and active precoding scheme at the transmitter, and solve it using a combination of auxiliary matrices, alternating optimization, and a variant of stochastic gradient descent. Finally, we validate the proposed algorithm numerically.

Index Terms— Covert communications, dual-function radar-communications, intelligent reflecting surfaces, multicasting, wardens.

1. INTRODUCTION

Research on radar-communications coexistence seeks to alleviate spectrum congestion resulting from the growing demand for spectrum resources [1]. Dual-function radar-communications (DFRC) systems enable shared hardware and frequency bands for radar and communications [2], enhancing spectrum utilization, reducing costs, expanding device versatility, and improving overall performance [3]. However, this integration introduces security concerns due to the use of identical waveforms, making communications vulnerable to unauthorized detection [4]. Conventional encryption may prove insufficient when facing supercomputers and parallel computing by adversaries [5–7]. To address these challenges, covert communications [8–10] emerges as a solution, emphasizing securing communications from detection rather than solely safeguarding sensitive information. Common strategies for covert communications include techniques like jamming [5, 11, 12], leveraging node mobility [13], using relay or auxiliary nodes [14, 15], and employing intelligent reflecting surfaces (IRS) [16–18].

The threat of detection of communications is more pronounced in MIMO (multiple-input multiple-output) antenna-based DFRC systems [19]. It has been shown in [20] that when artificial noise

is permitted for use in MIMO communication, we can find an optimal jamming power to achieve the covert transmission rate with the given transmit signal power. Therefore, MIMO DFRC systems commonly transmit communications and artificial noise simultaneously to confuse potential wardens [19, 21].

It was shown in [22] that introducing an IRS can improve covert communications performance. This concept gained notable traction following the work [18] which demonstrated that flawless covertness can be achieved even with a single antenna transmitter, when accompanied by an IRS. Given its focus on reconfiguring wireless channels, IRS technology aligns well with transceiver design techniques [23]. Another recent paper [24] handled the multiple target/ED and multicast traffic problem for secrecy, but under perfect channel gain information at the transmitter and the IRS. We have seen [25] solve a problem similar to ours using symbol precoding and using different symbols for secure and public users, while [26] solves the problem with a single eavesdropper and uses game theory based optimization. Our work however relies on channel precoding and artificial noise to manipulate channel characteristics for covert communications in IRS-aided DFRC.

The main contributions in this paper are the following: (i) we formulate the problem of maximizing worst-case data rate to the legitimate users, under transmit power constraint, covertness constraints for multiple wardens, and SNR constraint for each target detection, and (ii) we solve the highly non convex problem using a combination of alternating optimisation, auxiliary matrices and simultaneous perturbation stochastic approximation (SPSA [27]).

Notation: We denote all sets in calligraphic font (e.g., $\mathcal{X}$) and $|\mathcal{X}|$ denotes its cardinality. Given any integers (m, n) and a $m \times n$ matrix $\mathbf{U}$, let $\mathbf{U}^T$ and $\mathbf{U}^H$ denote its transpose and conjugate transpose respectively. $|\mathbf{U}|$ denotes the determinant of a square matrix. Similarly given any integer m and a length- m vector $\mathbf{u}$, its transpose is denoted by $\mathbf{u}^T$. Hence a matrix can be expressed in the form $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n]$ where $\mathbf{u}_i$ denotes the i -th column vector. Let $\mathcal{CN}(0, \mathbf{\Sigma})$ denote a circularly symmetric complex Gaussian vector with zero mean and covariance $\mathbf{\Sigma}$.

2. SYSTEM MODEL

We consider a DFRC system comprising a $N_T \times N_T$ MIMO radar system, L authorized communications users each having N_R receive antennas, one $\sqrt{N} \times \sqrt{N}$ IRS, and K single antenna wardens collocated with K targets [28], as illustrated in Figure 1.

The channel gain matrix $\mathbf{H}_{rtr,k} = \beta_k \mathbf{a}_R(\theta_k) \mathbf{a}_T(\theta_k)^T \in \mathbb{C}^{N_T \times N_T}$ is for the path from the radar to the k -th target and back to the radar. Here, β_k represents the complex reflectivity linked to atmospheric attenuation and the radar cross-section (RCS) of the target. The parameter θ_k corresponds to the azimuthal positioning of the target with respect to the radar. The vectors $\mathbf{a}_T(\theta_k) = [1, e^{j\frac{2\pi}{\lambda} d_r \sin(\theta_k)}, \dots, e^{j\frac{2\pi}{\lambda} (N_T-1) d_r \sin(\theta_k)}]^T$ and $\mathbf{a}_R(\theta_k) = [1, e^{j\frac{2\pi}{\lambda} d_r \sin(\theta_k)}, \dots, e^{j\frac{2\pi}{\lambda} (N_T-1) d_r \sin(\theta_k)}]^T$ sym-

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bolize the transmit and receive steering vectors associated with the k -th target. In these expressions, d_r and λ denote the spacing between radar antenna elements, and the wavelength of the transmitted signal, respectively.

$$\begin{aligned} \mathbf{r}(t) = & \sum_{k=1}^K \mathbf{H}_{rtr,k} \mathbf{x}(t - \tau_{rtr,k}) e^{j\omega_{rtr,k}t} \\ & + \sum_{k=1}^K \mathbf{H}_{tr,k} \mathbf{H}_{it,k} \Phi \mathbf{H}_{ri} \mathbf{x}(t - \tau_{ritr,k}) e^{j\omega_{ritr,k}t} \\ & + \sum_{k=1}^K \mathbf{H}_{ir} \Phi \mathbf{H}_{ti,k} \mathbf{H}_{rt,k} \mathbf{x}(t - \tau_{ritr,k}) e^{j\omega_{ritr,k}t} \\ & + \sum_{k=1}^K \mathbf{H}_{ir} \Phi \mathbf{H}_{ti,k} \mathbf{H}_{it,k} \Phi \mathbf{H}_{ri} \mathbf{x}(t - \tau_{ritr,k}) e^{j\omega_{ritr,k}t} + \mathbf{n}_T(t), \end{aligned} \quad (1)$$

The remaining channel matrices are denoted as follows: $\mathbf{H}_{ri} \in \mathbb{C}^{N \times N_T}$ signifies the radar-IRS channel, $\mathbf{H}_{ru,l} \in \mathbb{C}^{N_R \times N_T}$ denotes the radar-user channel for the l -th legitimate user, $\mathbf{H}_{rw,k} \in \mathbb{C}^{1 \times N_T}$ characterizes the channel between radar and the k -th warden, $\mathbf{H}_{iu,l} \in \mathbb{C}^{N_R \times N}$ represents the channel between the IRS and the l -th user, $\mathbf{H}_{it,k} \in \mathbb{C}^{1 \times N}$ signifies the channel between the IRS and the k -th target. $\mathbf{H}_{iw,k} \in \mathbb{C}^{1 \times N}$ depicts the IRS-warden channel, and since the target and warden are co-located, it can be used interchangeably with $\mathbf{H}_{it,k}$. Additionally, $\mathbf{H}_{ir} \in \mathbb{C}^{N_T \times N}$ indicates the IRS-radar channel, while $\mathbf{H}_{ti,k} \in \mathbb{C}^{N \times 1}$ captures the channel between the k -th target and IRS.

The radar system transmits both the information-bearing signal $\mathbf{m}(t) \in \mathbb{C}^{K \times 1}$ and introduces artificial noise (AN) $\mathbf{s}(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}) \in \mathbb{C}^{K \times 1}$ into its transmissions. This approach serves a dual purpose: it facilitates joint target detection and communications with users. In scenarios where direct communications with users is not needed, the radar exclusively emits AN to create ambiguity for wardens, making it challenging for them to distinguish whether communications is in progress. As a common message is transmitted to all users, the dimension of the message signal matches that of the artificial noise.

The transmitted signal $\mathbf{x}(t) \in \mathbb{C}^{N_T \times 1}$ emanating from the radar when engaging with authorized users is expressed as $\mathbf{x}(t) = \mathbf{W}\mathbf{m}(t) + \mathbf{B}_1\mathbf{s}(t)$. Here, $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_K] \in \mathbb{C}^{N_T \times K}$ denotes the pre-coding matrix responsible for information transmission, and $\mathbf{B}_1 = [\mathbf{b}_{11}, \mathbf{b}_{12}, \dots, \mathbf{b}_{1K}] \in \mathbb{C}^{N_T \times K}$ signifies the pre-coding matrix pertaining to the AN generated during active communications periods. Similarly the transmit signal when there is no information being transmitted is $\mathbf{x}(t) = \mathbf{B}_0\mathbf{s}(t)$ where $\mathbf{B}_0 = [\mathbf{b}_{01}, \mathbf{b}_{02}, \dots, \mathbf{b}_{0K}] \in \mathbb{C}^{N_T \times K}$ is the precoding matrix for the artificial noise produced.

We define the phase shift induced by the i -th reflection unit of the IRS as $\phi_i \in [0, \pi]$, and define $\Phi = [\phi_1, \phi_2, \dots, \phi_N]^T$. Also, $\Phi \triangleq \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \dots, e^{j\phi_N})$ is the phase shift matrix. Let $\tau_{(\cdot),k}$ and $\omega_{(\cdot),k}$ denote the range-time delay and Doppler shift specific to the k -th target for a given channel (indicated by the first subscript). The continuous-time received signal at the radar is expressed as in (1). The additive white Gaussian noise (AWGN) is given by $\mathbf{n}_T(t) \sim \mathcal{CN}(0, \sigma_T^2 \mathbf{I})$. Importantly, we ignore the received signal from the radar-IRS-target-IRS-radar pathway because it is weak.

We write the received signal at the k -th warden when there is ongoing communications as $\mathbf{z}_k(t) = \mathbf{H}_{rw,k}(\mathbf{W}\mathbf{m}(t - \tau_{rw,k}) + \mathbf{B}_1\mathbf{s}(t - \tau_{rw,k}))e^{j\omega_{rw,k}t} + \mathbf{H}_{iw,k}\Phi\mathbf{H}_{ri}(\mathbf{W}\mathbf{m}(t - \tau_{riw,k}) + \mathbf{B}_1\mathbf{s}(t - \tau_{riw,k}))e^{j\omega_{riw,k}t} + \mathbf{n}_{W,k}(t)$ where $\mathbf{n}_{W,k}(t) \sim \mathcal{CN}(0, \sigma_{W,k}^2 \mathbf{I})$ is the noise at the k -th warden. Similarly we write the received signal at the k -th warden when there is no ongoing communications as

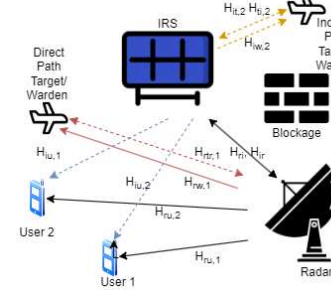


Fig. 1. MIMO DFRC system with 2 users and 2 targets.

$$\mathbf{z}_k(t) = \mathbf{H}_{rw,k}(\mathbf{B}_0\mathbf{s}(t - \tau_{rw,k}))e^{j\omega_{rw,k}t} + \mathbf{H}_{iw,k}\Phi\mathbf{H}_{ri}(\mathbf{B}_0\mathbf{s}(t - \tau_{riw,k}))e^{j\omega_{riw,k}t} + \mathbf{n}_{W,k}(t).$$

The received signal at the l -th user is $\mathbf{y}_l(t) = \mathbf{H}_{ru,l}(\mathbf{W}\mathbf{m}(t - \tau_{ru,l}) + \mathbf{B}_1\mathbf{s}(t - \tau_{ru,l}))e^{j\omega_{ru,l}t} + \mathbf{H}_{iu,l}\Phi\mathbf{H}_{ri}(\mathbf{W}\mathbf{m}(t - \tau_{riu,l}) + \mathbf{B}_1\mathbf{s}(t - \tau_{riu,l}))e^{j\omega_{riu,l}t} + \mathbf{n}_{U,l}(t)$ where $\mathbf{n}_{U,l}(t) \sim \mathcal{CN}(0, \sigma_{U,l}^2 \mathbf{I})$ is the noise at the l -th warden. The matrix characterizing the channel between radar and k -th warden is $\mathbf{G}_k = \alpha_k \mathbf{a}_T(\theta_k)^T$, where α_k is the path attenuation.

We seek to determine the optimal $\mathbf{W}$, $\mathbf{B}_1$, and $\mathbf{B}_0$ to meet covertness and target detection requirements. We assume perfect channel knowledge within the DFRC framework, where the detection of all targets (and hence the estimation of the 'sensing channel') is feasible while communications channel can be estimated a priori.

3. MULTICAST WITH MULTIPLE WARDENS

We consider static targets, and ignore Doppler shift and time delays [29]. We group the targets into two distinct sets $\mathcal{D}$ and $\mathcal{I}$ such that $|\mathcal{D}| + |\mathcal{I}| = K$, for which only direct and indirect paths are available, respectively. We assume that IRS-reflected signal can be ignored when a direct path is present. Let us represent $H_{dc,k} = H_{rtr,k}$ as the channel for the direct path concerning the k -th target, where $k \in \mathcal{D}$. With this, the Signal-to-Interference-plus-Noise Ratio (SINR) associated with the k -th target is

$$\text{SINR}_{dc,k} = \sigma_T^{-2} \{ \text{Tr}(\mathbf{H}_{dc,k} \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_{dc,k}^H) + \text{Tr}(\mathbf{H}_{dc,k} \mathbf{b}_{1k} \mathbf{b}_{1k}^H \mathbf{H}_{dc,k}^H) \}. \quad (2)$$

However, if an obstruction blocks the direct path ($k \in \mathcal{I}$):

$$\text{SINR}_{in,k} = \sigma_T^{-2} \{ \text{Tr}(\mathbf{H}_{in,k} \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_{in,k}^H) + \text{Tr}(\mathbf{H}_{in,k} \mathbf{b}_{1k} \mathbf{b}_{1k}^H \mathbf{H}_{in,k}^H) \}, \quad (3)$$

where, $\mathbf{H}_{in,k} = \mathbf{H}_{ir} \Phi \mathbf{H}_{ti,k} \mathbf{H}_{it,k} \Phi \mathbf{H}_{ri}$ is the channel gain for the indirect path for the k -th target.

Within a multicast framework, the radar emits a shared message which is intended for all users. We represent the channel matrix linking the radar and the l -th user as $\mathbf{F}_l$. The received signal at the l -th user, considering the message vector $\mathbf{m}(t) = [m_1(t), m_2(t), \dots, m_K(t)]^T$, is described by $\mathbf{y}_l(t) = \mathbf{F}_l(\mathbf{W}\mathbf{m}(t) + \mathbf{B}_1\mathbf{s}(t)) + \mathbf{n}_l(t)$ where $\mathbf{n}_l(t) \sim \mathcal{CN}(0, \sigma_{R,l}^2 \mathbf{I})$ is the receiver noise. Hence, the rate attained by the l -th user is

$$R_{u,l} = \log \det(\mathbf{I} + (\sigma_{R,l}^2 \mathbf{I} + \mathbf{F}_l \mathbf{B}_1 \mathbf{B}_1^H \mathbf{F}_l^H)^{-1} (\mathbf{F}_l \mathbf{W} \mathbf{W}^H \mathbf{F}_l^H)). \quad (4)$$

Let H_0 be the hypothesis that message signal has not been transmitted and H_1 be the hypothesis that message has been transmitted. The probability distributions of the received signal value at the k -th warden under each hypothesis is given by:

$$p_{0,k} \equiv f(\mathbf{z}_k | H_0) = \mathcal{CN}(0, \Sigma_{0,k}) \text{ \& } p_{1,k} \equiv f(\mathbf{z}_k | H_1) = \mathcal{CN}(0, \Sigma_{1,k}), \quad (5)$$

where

$$\begin{aligned}\Sigma_{0,k} &= \mathbf{H}_{rw,k}(\mathbf{B}_0\mathbf{B}_0^H)\mathbf{H}_{rw,k}^H \\ &+ \mathbf{H}_{iw,k}\Phi\mathbf{H}_{ri}(\mathbf{B}_0\mathbf{B}_0^H)\mathbf{H}_{ri}^H\Phi^H\mathbf{H}_{iw,k}^H + \sigma_{W,k}^2\mathbf{I},\end{aligned}\quad (6)$$

$$\begin{aligned}\Sigma_{1,k} &= \mathbf{H}_{rw,k}(\mathbf{W}\mathbf{W}^H + \mathbf{B}_1\mathbf{B}_1^H)\mathbf{H}_{rw,k}^H \\ &+ \mathbf{H}_{iw,k}\Phi\mathbf{H}_{ri}(\mathbf{W}\mathbf{W}^H + \mathbf{B}_1\mathbf{B}_1^H)\mathbf{H}_{ri}^H\Phi^H\mathbf{H}_{iw,k}^H + \sigma_{W,k}^2\mathbf{I}.\end{aligned}\quad (7)$$

We seek to maximize the overall probability of error for the wardens, which is calculated as the sum of the probability of false alarm P_{FA} and the probability of missed detection P_{MD} . For a given covertness parameter ϵ we seek to ensure $P_{FA} + P_{MD} > 1 - \epsilon$.

Therefore the likelihood ratio can be computed as,

$$\Lambda(\mathbf{z}) = \frac{|\Sigma_{0,k}|}{|\Sigma_{1,k}|} \exp(-\mathbf{z}^H(\Sigma_{1,k}^{-1} - \Sigma_{0,k}^{-1})\mathbf{z}). \quad (8)$$

3.1. Hypothesis testing and covertness

Each warden performs a likelihood ratio test to determine the true hypothesis. The likelihood ratio at the k -th warden is computed as

$$\Lambda(\mathbf{z}_k) = \frac{|\Sigma_{0,k}|}{|\Sigma_{1,k}|} \exp(-\mathbf{z}_k^H(\Sigma_{1,k}^{-1} - \Sigma_{0,k}^{-1})\mathbf{z}_k). \quad (9)$$

Assuming that the wardens have a single receiver antenna, we substitute $\Sigma_{1,k} = \sigma_{1,k}^2$ and $\Sigma_{0,k} = \sigma_{0,k}^2$ where $\sigma_{0,k}$ and $\sigma_{1,k}$ are scalars.

Case I: If $\sigma_{1,k}^2 > \sigma_{0,k}^2$, after some simplification [30], the likelihood ratio test reduces to

$$|z_k| \underset{H_0}{\overset{H_1}{\gtrless}} \sqrt{k \ln \frac{|\sigma_{0,k}|}{|\sigma_{1,k}|} \left(\frac{1}{\sigma_{1,k}^2} - \frac{1}{\sigma_{0,k}^2} \right)^{-1}} \doteq \eta. \quad (10)$$

Since $|z_k|$ in (10) is Rayleigh distributed [31], we can compute the corresponding $P_{FA} = \mathbb{P}(|z_k| > \eta | H_0) = \exp(-\frac{\eta^2}{2\sigma_{0,k}^2})$ and $P_{MD} = \mathbb{P}(|z| < \eta | H_1) = 1 - \exp(-\frac{\eta^2}{2\sigma_{1,k}^2})$. The constraint on $P_{FA} + P_{MD}$ becomes $\exp(-\frac{\eta^2}{2\sigma_{1,k}^2}) - \exp(-\frac{\eta^2}{2\sigma_{0,k}^2}) < \epsilon$. Through some simple analysis we can prove that $\exp(-\frac{\eta^2}{2\sigma^2})$ is uniformly continuous in σ^2 . Hence, covertness is ensured if we can substitute the corresponding constraint with $\sigma_{1,k}^2 - \sigma_{0,k}^2 < \delta$ for some suitable $\delta > 0$.

Case II: Similarly, if $\sigma_{0,k}^2 > \sigma_{1,k}^2$, the inequality is reversed as $|z_k| \underset{H_0}{\overset{H_1}{\gtrless}} \eta$. The expressions of P_{FA} and P_{MD} are altered as $P_{FA} = \mathbb{P}(|z_k| < \eta | H_0) = 1 - \exp(-\frac{\eta^2}{2\sigma_{0,k}^2})$ and $P_{MD} = \mathbb{P}(|z_k| > \eta | H_1) = \exp(-\frac{\eta^2}{2\sigma_{1,k}^2})$. Hence in this case, covertness is ensured if $\sigma_{0,k}^2 - \sigma_{1,k}^2 < \delta$ for a suitable $\delta > 0$.

Let $\mathbf{Q} \doteq \mathbf{W}\mathbf{W}^H + \mathbf{B}_1\mathbf{B}_1^H - \mathbf{B}_0\mathbf{B}_0^H$. Combining the above two cases, we write the covertness constraint at the k -th warden as: $|\mathbf{H}_{rw,k}\mathbf{Q}\mathbf{H}_{rw,k}^H + \mathbf{H}_{iw,k}\Phi\mathbf{H}_{ri}\mathbf{Q}\mathbf{H}_{ri}^H\Phi^H\mathbf{H}_{iw,k}^H| < \delta$.

4. THE OPTIMIZATION ALGORITHM

We seek to jointly optimize the worst-case data rates to the users, while achieving SNR constraints for radar detection under both hypotheses as well as covertness requirement, under the total power constraint at the transmitter under both hypotheses. Optimization is done iteratively over the matrices $\mathbf{W}$, Φ , $\mathbf{B}_0$ and $\mathbf{B}_1$. This non-convex domain of feasibility is transformed into a convex one by

means of linearizing the Signal-to-Noise Ratio (SNR) constraints. Additionally, optimizing over the matrix Φ presents a high level of nonconvexity which is handled by a variant of stochastic gradient ascent.

The optimization problem as follows:

$$\begin{aligned}\text{maximize}_{\mathbf{W}, \mathbf{B}_0, \mathbf{B}_1, \Phi} \quad & \min_{1 \leq l \leq L} R_{u,l}\end{aligned}\quad (11a)$$

$$\text{s.t.} \quad \text{Tr}(\mathbf{W}\mathbf{W}^H) + \text{Tr}(\mathbf{B}_1\mathbf{B}_1^H) \leq P$$

$$\text{Tr}(\mathbf{B}_0\mathbf{B}_0^H) \leq P$$

$$\text{Tr}(\mathbf{H}_{dc,k}\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_{dc,k}^H) + \text{Tr}(\mathbf{H}_{dc,k}\mathbf{b}_{1k}\mathbf{b}_{1k}^H\mathbf{H}_{dc,k}^H) \geq \gamma_k, \quad k \in D$$

$$\text{Tr}(\mathbf{H}_{in,k}\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_{in,k}^H) + \text{Tr}(\mathbf{H}_{in,k}\mathbf{b}_{1k}\mathbf{b}_{1k}^H\mathbf{H}_{in,k}^H) \geq \gamma_k, \quad k \in I$$

$$\text{Tr}(\mathbf{H}_{dc,k}\mathbf{b}_{0k}\mathbf{b}_{0k}^H\mathbf{H}_{dc,k}^H) \geq \gamma_k, \quad k \in D \quad (11b)$$

$$\text{Tr}(\mathbf{H}_{in,k}\mathbf{b}_{0k}\mathbf{b}_{0k}^H\mathbf{H}_{in,k}^H) \geq \gamma_k, \quad k \in I \quad (11c)$$

$$|\mathbf{H}_{rw,k}\mathbf{Q}\mathbf{H}_{rw,k}^H + \mathbf{H}_{iw,k}\Phi\mathbf{H}_{ri}\mathbf{Q}\mathbf{H}_{ri}^H\Phi^H\mathbf{H}_{iw,k}^H| < \delta \quad \forall k. \quad (11d)$$

Optimizing over auxiliary matrices: It follows from [32, Lemma 4.1] that

$$R_{u,l} = \max_{\mathbf{W}_b \succ 0, \mathbf{U}_b} (\log \det \mathbf{W}_b - \text{Tr}(\mathbf{W}_b \mathbf{E}_b(\mathbf{U}_b, \mathbf{W}, \mathbf{B}_1))) + \text{constant}, \quad (12)$$

where $\mathbf{W}_b, \mathbf{U}_b$ are auxiliary matrices of appropriate dimensions, and

$$\begin{aligned}\mathbf{E}_b(\mathbf{U}_b, \mathbf{W}, \mathbf{B}_1) &= (\mathbf{I} - \mathbf{U}_b^H \mathbf{F}_l \mathbf{W})(\mathbf{I} - \mathbf{U}_b^H \mathbf{F}_l \mathbf{W})^H \\ &+ \mathbf{U}_b^H (\sigma_{R,l}^2 \mathbf{I} + \mathbf{F}_l \mathbf{B}_1 \mathbf{B}_1^H \mathbf{F}_l^H) \mathbf{U}_b.\end{aligned}\quad (13)$$

The optimal solution can be calculated as:

$$\mathbf{U}_{b,l,k}^* = (\sigma_{R,l}^2 \mathbf{I} + \mathbf{F}_l \mathbf{B}_1 \mathbf{B}_1^H \mathbf{F}_l^H + \mathbf{F}_l \mathbf{W} \mathbf{W}^H \mathbf{F}_l^H)^{-1} \mathbf{F}_l \mathbf{W}, \quad (14)$$

$$\mathbf{W}_{b,l,k}^* = (\mathbf{E}_b(\mathbf{U}_{b,l,k}^*, \mathbf{W}, \mathbf{B}))^{-1}. \quad (15)$$

Optimizing over $(\mathbf{W}, \mathbf{B}_1, \mathbf{B}_0)$ given Φ : Here, the SNR and secrecy rate constraints lead to a non-convex feasible region. We linearize the SNR constraints by first-order Taylor series approximation around some initial approximations $\tilde{\mathbf{w}}_k$ and $\tilde{\mathbf{b}}_{1k}$. We define $f_{dc}(\mathbf{w}_k, \mathbf{b}_{1k}) = \text{Tr}(\mathbf{H}_{dc,k}\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_{dc,k}^H) + \text{Tr}(\mathbf{H}_{dc,k}\mathbf{b}_{1k}\mathbf{b}_{1k}^H\mathbf{H}_{dc,k}^H)$. Then it can be approximated as, $f_{dc}(\mathbf{w}_k, \mathbf{b}_{1k}) \approx f_{dc}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k}) + \Re(\nabla_{\mathbf{w}_k, \mathbf{b}_{1k}}^T f_{dc}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k})(\mathbf{d}_k - \tilde{\mathbf{d}}_k))$ where, $\mathbf{d}_k = [\mathbf{w}_k^T \ \mathbf{b}_{1k}^T]^T$ and $\nabla_{\mathbf{w}_k, \mathbf{b}_{1k}} f_{dc}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k}) = [\frac{\partial f_{dc}}{\partial \mathbf{w}_k^H} \ \frac{\partial f_{dc}}{\partial \mathbf{b}_{1k}^H}]^T$. Here, $\frac{\partial f_{dc}}{\partial \mathbf{w}_k^H} = 2\mathbf{w}_k^H \mathbf{H}_{dc,k} \mathbf{H}_{dc,k}^H$ and $\frac{\partial f_{dc}}{\partial \mathbf{b}_{1k}^H} = 2\mathbf{b}_{1k}^H \mathbf{H}_{dc,k} \mathbf{H}_{dc,k}^H$.

Similarly, $f_{dc}(\mathbf{b}_{0k}) = \text{Tr}(\mathbf{H}_{dc,k}\mathbf{b}_{0k}\mathbf{b}_{0k}^H\mathbf{H}_{dc,k}^H)$. Then, $f_{dc}(\mathbf{b}_{0k}) \approx f_{dc}(\tilde{\mathbf{b}}_{0k}) + \Re(\nabla_{\mathbf{b}_{0k}}^T f_{dc}(\tilde{\mathbf{b}}_{0k})(\mathbf{b}_{0k} - \tilde{\mathbf{b}}_{0k}))$ where, $\nabla_{\mathbf{b}_{0k}} f_{dc}(\tilde{\mathbf{b}}_{0k}) = \frac{\partial f_{dc}}{\partial \mathbf{b}_{0k}^H}$ and $\frac{\partial f_{dc}}{\partial \mathbf{b}_{0k}^H} = 2\mathbf{b}_{0k}^H \mathbf{H}_{dc,k} \mathbf{H}_{dc,k}^H$.

In order to linearize the constraint (11d) we can express the matrix products as:

$$\mathbf{W}\mathbf{W}^H = \mathbf{w}_1\mathbf{w}_1^H + \mathbf{w}_2\mathbf{w}_2^H + \dots + \mathbf{w}_K\mathbf{w}_K^H.$$

$$\mathbf{B}_1\mathbf{B}_1^H = \mathbf{b}_{11}\mathbf{b}_{11}^H + \mathbf{b}_{12}\mathbf{b}_{12}^H + \dots + \mathbf{b}_{1K}\mathbf{b}_{1K}^H. \quad (16)$$

$$\mathbf{B}_0\mathbf{B}_0^H = \mathbf{b}_{01}\mathbf{b}_{01}^H + \mathbf{b}_{02}\mathbf{b}_{02}^H + \dots + \mathbf{b}_{0K}\mathbf{b}_{0K}^H.$$

With this expansion, we express the LHS of constraint (11d) as a function of the collection of vectors $\mathbf{V} = \{\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{b}_{11}, \mathbf{b}_{12}, \dots, \mathbf{b}_{01}, \mathbf{b}_{02}, \dots, \mathbf{b}_{0K}\}$. Since the receivers have a single antenna,

the final value on the LHS is a scalar, which enables us to use first-order Taylor series approximation to linearize this function. Let $\hat{f}_k(\mathbf{V}) = |\mathbf{H}_{rw,k} \mathbf{Q} \mathbf{H}_{rw,k}^H + \mathbf{H}_{iw,k} \Phi \mathbf{H}_{ri} \mathbf{Q} \mathbf{H}_{ri}^H \Phi^H \mathbf{H}_{iw,k}^H|$.

Applying Taylor series approximation around $\hat{\mathbf{V}}$ we get $\hat{f}_k(\mathbf{V}) = \hat{f}_k(\hat{\mathbf{V}}) + \Re(\nabla_{\hat{\mathbf{V}}}^T \hat{f}_k(\hat{\mathbf{V}})(\mathbf{d}_\sigma - \hat{\mathbf{d}}_\sigma))$, where $\mathbf{d}_\sigma = [\mathbf{w}_1^T, \dots, \mathbf{b}_{11}^T, \dots, \mathbf{b}_{01}^T, \dots, \mathbf{b}_{0K}^T]^T$ and $\nabla_{\hat{\mathbf{V}}} \hat{f}_k = [\frac{\partial \hat{f}_k}{\partial \mathbf{w}_1^H} \dots \frac{\partial \hat{f}_k}{\partial \mathbf{w}_K^H} \frac{\partial \hat{f}_k}{\partial \mathbf{b}_{11}^H} \dots \frac{\partial \hat{f}_k}{\partial \mathbf{b}_{1K}^H} \frac{\partial \hat{f}_k}{\partial \mathbf{b}_{01}^H} \dots \frac{\partial \hat{f}_k}{\partial \mathbf{b}_{0K}^H}]^T$. This yields the revised optimization problem,

$$\begin{aligned} & \underset{\mathbf{W}, \mathbf{B}_0, \mathbf{B}_1, \Phi}{\text{maximize}} \quad \lambda \\ & \text{subject to} \quad \text{Tr}(\mathbf{W}\mathbf{W}^H) + \text{Tr}(\mathbf{B}_1\mathbf{B}_1^H) \leq P \\ & \quad \text{Tr}(\mathbf{B}_0\mathbf{B}_0^H) \leq P \\ & \quad f_{dc}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k}) + \Re(\nabla_{\mathbf{w}_k, \mathbf{b}_{1k}}^T f_{dc}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k})(\mathbf{d}_k - \tilde{\mathbf{d}}_k)) \geq \gamma_k, \quad k \in D \\ & \quad f_{in}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k}) + \Re(\nabla_{\mathbf{w}_k, \mathbf{b}_{1k}}^T f_{in}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{b}}_{1k})(\mathbf{d}_k - \tilde{\mathbf{d}}_k)) \geq \gamma_k, \quad k \in I \\ & \quad f_{dc}(\tilde{\mathbf{b}}_{0k}) + \Re(\nabla_{\mathbf{b}_{0k}}^T f_{dc}(\tilde{\mathbf{b}}_{0k})(\mathbf{b}_{0k} - \tilde{\mathbf{b}}_{0k})) \geq \gamma_k, \quad k \in D \\ & \quad f_{in}(\tilde{\mathbf{b}}_{0k}) + \Re(\nabla_{\mathbf{b}_{0k}}^T f_{in}(\tilde{\mathbf{b}}_{0k})(\mathbf{b}_{0k} - \tilde{\mathbf{b}}_{0k})) \geq \gamma_k, \quad k \in I \\ & \quad |\hat{f}_k(\tilde{\mathbf{V}}) + \Re(\nabla_{\hat{\mathbf{V}}}^T \hat{f}_k(\tilde{\mathbf{V}})(\mathbf{d}_\sigma - \tilde{\mathbf{d}}_\sigma))| < \delta \quad \forall k \\ & \quad R_{u,l} \geq \lambda \geq 0 \quad \forall l. \end{aligned} \quad (17)$$

This convex problem is solved by any standard solver.

Optimizing Φ for given $\mathbf{W}, \mathbf{B}_1, \mathbf{B}_0$: The SNR constraint for the direct radar-target-radar links and the secrecy rates do not depend on the phase shift matrix Φ . We denote $h_{1k}(\Phi) = \mathbf{H}_{in,k} \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_{in,k}^H + \mathbf{H}_{in,k} \mathbf{b}_{1k} \mathbf{b}_{1k}^H \mathbf{H}_{in,k}^H$ and $h_{2k}(\Phi) = \mathbf{H}_{in,k} \mathbf{b}_{0k} \mathbf{b}_{0k}^H \mathbf{H}_{in,k}^H$. Also, let $h_{3k}(\Phi) = |\mathbf{H}_{rw,k} \mathbf{Q} \mathbf{H}_{rw,k}^H + \mathbf{H}_{iw,k} \Phi \mathbf{H}_{ri} \mathbf{Q} \mathbf{H}_{ri}^H \Phi^H \mathbf{H}_{iw,k}^H|$. This results in the following optimization problem,

$$\underset{\Phi}{\text{maximize}} \quad \min_{k \in \mathcal{I}} (\text{Tr}(h_{1k}(\Phi)) - \gamma_k, \text{Tr}(h_{2k}(\Phi)) - \gamma_k, \delta - h_{3k}(\Phi)). \quad (18)$$

Given that the objective function $\tilde{f}(\Phi)$ exhibits nonconvex behavior with respect to Φ , we employ simultaneous perturbation stochastic approximation (SPSA [27]). In SPSA, we iteratively update the i -th component of $\Phi(t)$ as

$$\phi_i(t+1) = \phi_i(t) + a(t) \times \frac{\tilde{f}(\Phi^+(t)) - \tilde{f}(\Phi^-(t))}{2c(t)\Delta_i(t)}. \quad (19)$$

where the iterate $\tilde{\Phi}(t)$ undergoes perturbation in two opposing directions: $\tilde{\Phi}^+(t) = \tilde{\Phi}(t) + c(t)\Delta(t)$ and $\tilde{\Phi}^-(t) = \tilde{\Phi}(t) - c(t)\Delta(t)$ and the perturbation vector $\Delta(t) \in \mathbb{R}^{N \times 1}$ with a zero mean is generated independently. The SPSA iteration is executed until a suitable stopping criterion is fulfilled, which ensures a certain level of accuracy or convergence.

Algorithm 1 Optimize data rate in Covert DFRC

Require: All channel gains and noise co-variances, P , γ_k where, $k \in D, k \in I$

Ensure: $\mathbf{W}, \mathbf{B}_1, \mathbf{B}_0, \tilde{\Phi}$

Initialization: $\mathbf{W}(0), \mathbf{B}_1(0), \mathbf{B}_0(0), \tau = 0$.

For $\tau = 1, 2, 3, \dots$ do

1. Compute $\mathbf{W}_b^*(\tau-1), \mathbf{U}_b^*(\tau-1)$ for all l for given $\mathbf{W}(\tau-1), \mathbf{B}_1(\tau-1), \mathbf{B}_0(\tau-1), \tilde{\Phi}$ using 14 and 15.

2. Compute $\mathbf{W}(\tau), \mathbf{B}_1(\tau), \mathbf{B}_0(\tau)$ for given $\mathbf{U}_b^*(\tau)$ and $\mathbf{W}_b^*(\tau)$ using 17.

3. Compute $\tilde{\Phi}$ using multiple iterations of SPSA.

Stop when percentage increase in $R_{u,l}$ is below desired threshold.

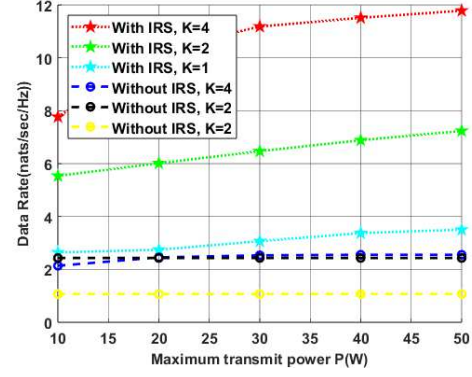


Fig. 2. Achieved data rate against different maximum power limits.

5. NUMERICAL EXPERIMENTS

We consider a system with $N_T = N_R = 8$, $N = 16$, and $N_R = 4$. We simulate 3 scenarios, first a single warden at $\theta = 72^\circ$, second another warden at $\theta = 78^\circ$ and finally two additional wardens ($K = 4$) at $\theta = 74^\circ$ and $\theta = 76^\circ$. We consider the path loss and RCS coefficients as $\alpha_i = 0.1$ and $\beta_i = 0.1 \forall i \in \{1, 2, 3, 4\}$. The covertness parameter δ is considered to be 0.1. We consider the SINR threshold for target detection $\gamma_k = 0.2$. The Gaussian noise variables have all been considered to have unit variance. We independently selected coefficients for each channel from circularly symmetric complex Gaussian random variables with a mean of zero and a variance of one. The results in Figure 2 reveal three interesting facts: (i) IRS helps in increasing the worst-case data rate, (ii) data rate increases with an increase in power budget P , and (iii) more targets might lead to more data rate to the users. The third point might appear counter-intuitive at the beginning. This is due to increased power consumption for jamming more targets and maintaining high detection error both during communications and when there's no communication. Increased data-rate is due to more power diverted to communication. The interior point optimizer SDPT3 solves the convex optimization problem in polynomial time taking roughly 10 iterations while the number of SPSA iterations grows linearly with increasing number of IRS elements.

6. SUMMARY

In this paper, we have optimized the worst case data rate to the users under radar detection constraint and covertness constraint to establish a theoretical bound. The problem was highly non-convex, and we have solved it by using a combination of techniques such as alternating optimization, SPSA and auxiliary matrices. We can observe that employing IRS in the system greatly increases the data rate achieved by each user, under radar detection constraint and covertness constraint. However compared to other techniques that do not rely on artificial noise our method naturally tends to consume more power. In future, we plan to study this system in presence of propagation delay and Doppler shift. The current solution depends on wardens having a single receive antenna, a limitation we aim to address in future updates.

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