An IRS-Assisted Secure Dual-Function Radar-Communication System

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***Abstract*—In dual-function radar-communication (DFRC) systems the probing signal contains information intended for the communication users, which makes that information vulnerable to eavesdropping by the targets. We propose a novel design for enhancing the physical layer security (PLS) of DFRC systems, via the help of intelligent reflecting surface (IRS) and artificial noise (AN), transmitted along with the probing waveform. The radar waveform, the AN jamming noise and the IRS parameters are designed to optimize the communication secrecy rate while meeting radar signal-to-noise ratio (SNR) constrains. Key challenges in the resulting optimization problem include the fractional form objective, the SNR being a quartic function of the IRS parameters, and the unit-modulus constraint of the IRS parameters. A fractional programming technique is used to transform the fractional form objective of the optimization problem into more tractable non-fractional polynomials. Numerical results are provided to demonstrate the convergence of the proposed system design algorithm, and also show the impact of the power assigned to the AN on the secrecy performance of the designed system.**

***Index Terms*—DFRC, physical layer security, IRS**

I. INTRODUCTION

Integrated sensing and communication (ISC) systems aim to perform both sensing and communication functions from a common platform [1]–[4]. Thus, ISC systems are prime candidates for next-generation wireless systems, such as unmanned aerial vehicles or autonomous vehicles, which are envisioned to achieve both high data rates and high sensing performance simultaneously. A special case of ISC systems is the Dual-Function Radar-Communication (DFRC) systems, which not only use a common platform for both communication and sensing but also use a shared waveform for these two functions [5]–[7]. In DFRC systems, user information is embedded in the probing waveform, resulting in higher spectral efficiency compared to general ISC systems. However, this also raises security concerns as the embedded communication information can potentially be intercepted by a target that is also an eavesdropper. This paper focuses on security issues associated with DFRC systems, approaching the design of the system from a physical layer security (PLS) perspective.

By exploiting the physical characteristics of the wireless channel, PLS design aims to enable the legitimate destination to obtain the source information successfully, while preventing an eavesdropper (ED) from decoding the information [8]. PLS design of communication systems has been well investigated [9]–[15]. One approach to ensure PLS is cooperative jamming, where trusted relays act as helpers and beamform artificial noise (AN), aiming to degrade the ED's channel [9]–[13]. Another approach is that the source beamforms AN along with the information for users, in a way that the users do not experience interference [14], [15]. PLS design for DFRC systems has been considered in [16], [17], where the DFRC system embeds AN in the transmit waveform, and a radar beamformer and the AN are jointly designed respectively to minimize the ED signal-to-noise-ratio (SNR) [16], and to maximize the weighted sum of normalized fisher information matrix determinant and normalized secrecy rate [17].

Here, we consider the problem of DFRC systems aided by intelligent reflecting surfaces (IRS). IRS can assist DFRC systems in overcoming performance limitations caused by path loss or blockage, which are likely to arise in the next-generation wireless systems. These systems often rely on high frequencies to achieve high communication rates and sensing performance. However, high frequencies experience high attenuation and blockage. IRS is a planar array that consists of passive elements, each of which can alter the phase of the incoming electromagnetic wave in a computer-controlled manner. These elements can cooperatively perform beamforming to increase the power level in intended directions or decrease it in unintended directions. IRS can also create additional links between the radar and the users, or between the radar and the targets, thus introducing more degrees of freedom (DoFs) for system design [18]–[22]. When there is no line-of-sight (LOS) between the radar and the target, the IRS can provide alternative paths for the radar signal to reach the target [18], [20].

In this paper, we investigate design for an IRS-aided secure DFRC system design from the PLS perspective. The radar transmits a precoded waveform along with additive precoded AN. The precoding matrices and the IRS parameter matrix are jointly designed in order to optimize the communication secrecy rate, reflecting secure communication performance, while maintaining a certain level of radar SNR, reflecting good target sensing performance. The secrecy rate is a non-convex function of ratios and the radar SNR is a non-convex fourth order function of the IRS parameter. For a fixed IRS parameter,

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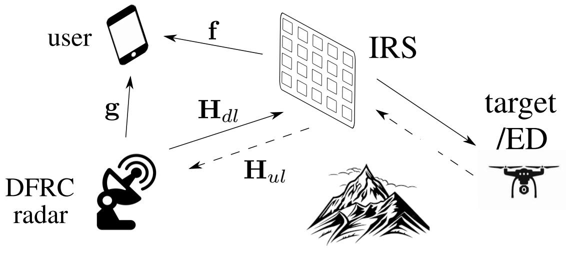

Fig. 1. IRS-assisted secure DFRC system.

the design of waveform and AN precoding matrices can be formulated as a quadratic programming problem. *The challenge mainly lies in optimizing with respect to the IRS parameter, i.e., in simultaneously considering the non-convex fractional objective of secrecy rate, and the non-convex high-order radar SNR.* To address the non-convex high-order radar SNR term we express it as a quadratic function of an auxiliary variable, which is quadratic in the IRS parameter. Subsequently, we replace the SNR with a lower bound that is linear in the auxiliary variable, and which is found via the minorization technique [23]. To address the non-convex fractional objective of secrecy rate and achieve a tractable design problem we invoke a fractional programming technique [24].

The literature on striking a balance between PLS and sensing performance for IRS-aided DFRC systems is sparse. In [25]–[27], the waveform design problems are formulated as quadratic programming problems, and respectively addressed by fractional programming technique [25], Riemannian conjugate gradient (RCG) algorithm [26], and first-order Taylor series approximation [27]. The IRS parameter design problems are respectively solved by minorization maximization (MM) [25], the RCG algorithm [26], and the numerical simultaneous perturbation stochastic approximation method [27]. As compared to the above literature, our work addresses the challenging fourth order SNR term, which is either not considered in [25], [26], or it is not convexified to facilitate a tractable problem in [27]. As compared to [25], [26], besides the radar waveform design, we consider the design of AN as well, which results in a different design problem.

II. SYSTEM MODEL

We consider an IRS-aided DFRC system as Fig. 1, where the DFRC is communicating with a user and is simultaneously tracking a non-line-of-sight (NLOS) target, who is also an eavesdropper. The DFRC has an N_T-antenna uniform linear array (ULA) transmitter, and a collocated N_R-antenna ULA receiver. The inter-antenna distance in both arrays is denoted by d. An N-element IRS is deployed to aid both the sensing and communication functionalities. The channels are assumed flat fading and perfectly known. The transmitted signal at the DFRC radar is

$$\mathbf{x} = \mathbf{w}s + \mathbf{W}_n\mathbf{n}, \tag{1}$$

where $\mathbf{w} \in \mathbb{C}^{N_T\times 1}$ denotes the precoder for information intended for the communication user, s represents the transmit waveform that contains the information for the user. s is assumed to be zero-mean white with unit variance; $\mathbf{W}_n \in \mathbb{C}^{N_T\times N_T}$ is the precoding matrix for the AN, and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_{N_T\times 1}, \sigma^2\mathbf{I}_{N_T})$ is the AN. The AN is used for jamming and detecting the target/ED.

Since there is no LOS between the radar and the target, the transmitted waveform arrives at the target via the DFRC-IRS-target path, and the target echo reaches the radar receiver via the target-IRS-DFRC path. The signal at the radar receiver is

$$\begin{aligned}\mathbf{y}_R &= \beta\mathbf{H}_{ul}\mathbf{\Phi}\mathbf{a}_I(\psi_a,\psi_e)\mathbf{a}_I^T(\psi_a,\psi_e)\mathbf{\Phi}\mathbf{H}_{dl}(\mathbf{w}s+\mathbf{W}_n\mathbf{n}) + \mathbf{n}_R\\ &= \mathbf{C}_T(\mathbf{w}s+\mathbf{W}_n\mathbf{n}) + \mathbf{n}_R,\end{aligned} \tag{2}$$

where β is the target reflection coefficient; $\mathbf{H}_{ul}$ and $\mathbf{H}_{dl}$ are the normalized IRS-DFRC and DFRC-IRS channels, respectively; $\mathbf{\Phi} = \mathrm{diag}([e^{j\phi_1}, e^{j\phi_2}, \cdots, e^{j\phi_N}])$ represents the IRS parameter matrix and is diagonal, where $\phi_n \in [0, 2\pi)$ is the phase shift of the n-th element of IRS; $\mathbf{a}_I(\psi_a,\psi_e)$ is the steering vector of IRS, and ψ_a and ψ_e are the azimuth and elevation angles of the target relative to the IRS, respectively; $\mathbf{n}_R \sim \mathcal{CN}(\mathbf{0}_{N_R\times 1}, \sigma_R^2\mathbf{I}_{N_R})$ indicates the additive white Gaussian noise (AWGN) at the DFRC receiving array, and σ_R^2 represents the power of noise per radar receive antenna.

The SNR at the radar receiver for detecting the target is

$$\gamma_R = \mathrm{tr}\big[\mathbf{C}_T\left[\mathbf{w}\mathbf{w}^H + \mathbf{W}_n\mathbf{W}_n^H\right]\mathbf{C}_T^H\big]/\sigma_R^2. \tag{3}$$

The received signal at the communication user is written as

$$\begin{aligned}y_u &= (\mathbf{g}^T + \beta_H^{\frac{1}{2}}\mathbf{f}^T\mathbf{\Phi}\mathbf{H}_{dl})(\mathbf{w}s+\mathbf{W}_n\mathbf{n}) + n_u\\ &= \mathbf{c}_u^T(\mathbf{w}s+\mathbf{W}_n\mathbf{n}) + n_u,\end{aligned} \tag{4}$$

where $\mathbf{g} \in \mathbb{C}^{N_T\times 1}$ is the channel between the user and DFRC transmitter; β_H is the path-loss of the DFRC-IRS channel; $\mathbf{f} \in \mathbb{C}^{N\times 1}$ denotes the user-IRS channel; $n_u \sim \mathcal{CN}(0, \sigma_u^2)$ is AWGN at the user.

The received signal at the target/ED is

$$\begin{aligned}y_{te} &= (\beta^{\frac{1}{2}}\mathbf{a}_I^T(\psi_a,\psi_e)\mathbf{\Phi}\mathbf{H}_{dl})(\mathbf{w}s+\mathbf{W}_n\mathbf{n}) + n_{te}\\ &= \mathbf{c}_{te}^T(\mathbf{w}s+\mathbf{W}_n\mathbf{n}) + n_{te},\end{aligned} \tag{5}$$

where $n_{te} \sim \mathcal{CN}(0, \sigma_{te}^2)$ is AWGN at the target/ED.
The achievable rate at the user and target/ED are respectively

$$R_u = \log(1+\mathrm{SINR}_u) = \log\left(1 + \frac{|\mathbf{c}_u^T\mathbf{w}|^2}{||\mathbf{c}_u^T\mathbf{W}_n||^2 + \sigma_u^2}\right), \tag{6}$$

$$R_{te} = \log(1+\mathrm{SINR}_{te}) = \log\left(1 + \frac{|\mathbf{c}_{te}^T\mathbf{w}|^2}{||\mathbf{c}_{te}^T\mathbf{W}_n||^2 + \sigma_{te}^2}\right). \tag{7}$$

Remark In the following we will assume that the target/eavesdropper angle is known. However, it does not need to be known exactly. As long as the target will fall within the beam that will be formulated by the proposed system, it will be detected and its angle will be fine-tuned. The new angle will be used in the subsequent communication/sensing phase. In this way, the proposed DFRC will be able to track moving targets.

III. SYSTEM DESIGN

We consider the design of the radar information precoder, $\mathbf{w}$, the artificial noise precoder, $\mathbf{W}_n$, and the IRS parameter matrix, $\mathbf{\Phi}$, as that of maximizing the secrecy rate, defined as $(R_u - R_{te})$, while satisfying certain constraints, i.e.,

$$(\mathbb{P}) \quad \max_{\mathbf{w},\mathbf{W}_n,\mathbf{\Phi}} \quad R_u - R_{te} \tag{8a}$$
$$\text{s.t.} \quad \text{tr}\left[\mathbf{w}\mathbf{w}^H + \mathbf{W}_n\mathbf{W}_n^H\right] \leq P_R \tag{8b}$$
$$|\mathbf{\Phi}_{n,n}| = 1, \ \ \forall n = 1, \cdots, N \tag{8c}$$
$$\gamma_R \geq \gamma_{R,th} \tag{8d}$$

where (8b) enforces that the total power of the waveform and AN stays below P_R; (8c) enforces unit modules elements in $\mathbf{\Phi}$. This is because the IRS is composed of passive elements, which can only change the phase of the impinging signal; (8d) ensures that the radar SNR will be above threshold $\gamma_{R,th}$.

The problem $(\mathbb{P})$ in (8) is challenging for the following reasons: (i) The design variables, $\mathbf{w}, \mathbf{W}_n, \mathbf{\Phi}$, are mutually coupled. (ii) The objective is the difference between two non-convex functions (R_u and R_{te}), and is non-convex. (iii) The radar SNR in (8d), γ_R, is quartic in $\mathbf{\Phi}$. Since γ_R is quadratic in $\mathbf{C}_T$ as (3), and $\mathbf{C}_T$ is second order in $\mathbf{\Phi}$ as (2). (iv) $\mathbf{\Phi}$ is subject to highly non-convex unit modulus constraints (UMC) as (8c).

To decouple the design variables, we divide the problem (8) into two sub-problems. The first one is optimization with respect to $\mathbf{w}$ and $\mathbf{W}_n$ for fixed $\mathbf{\Phi}$, and the second one is optimizations with respect to $\mathbf{\Phi}$ for fixed $\mathbf{w}$ and $\mathbf{W}_n$. These two sub-problems are solved in an alternating way until a convergence condition is met [28].

A. First sub-problem: Solve for $\mathbf{w}$ and $\mathbf{W}_n$ by fixing $\mathbf{\Phi}$

The first sub-problem is formulated as follows:

$$(\mathbb{P}_1) \quad \max_{\mathbf{w},\mathbf{W}_n} \quad R_u - R_{te} \tag{9a}$$
$$\text{s.t.} \quad \text{tr}\left[\mathbf{w}\mathbf{w}^H + \mathbf{W}_n\mathbf{W}_n^H\right] \leq P_R. \tag{9b}$$
$$\gamma_R \geq \gamma_{R,th}. \tag{9c}$$

Note that R_u, R_{te}, and γ_R are all second order functions of $\mathbf{w}$ and $\mathbf{W}_n$, as (6), (7) and (3), respectively. However, both R_u and R_{te} are fractional forms of the design variables. Here, we invoke the quadratic transform technique [24] to recast R_u and R_{te} into non-fractional functions of $\mathbf{w}$ and $\mathbf{W}_n$. Thereby, the problem $(\mathbb{P}_1)$ is modeled as a non-fractional quadratic programming problem. The Lagrangian dual expressions of R_u and R_{te} in (6) and (7) are respectively

$$\begin{aligned} R_u &= (1+\gamma_u)\,|\mathbf{c}_u^T\mathbf{w}|^2/\left(|\mathbf{c}_u^T\mathbf{w}|^2 + ||\mathbf{c}_u^T\mathbf{W}_n||^2 + \sigma_u^2\right) - \gamma_u \\ &+ \log(1+\gamma_u) = -\alpha_u^2(|\mathbf{c}_u^T\mathbf{w}|^2 + ||\mathbf{c}_u^T\mathbf{W}_n||^2 + \sigma_u^2) \\ &+ 2\alpha_u\sqrt{1+\gamma_u}|\mathbf{c}_u^T\mathbf{w}| + \log(1+\gamma_u) - \gamma_u, \end{aligned} \tag{10}$$

$$\begin{aligned} R_{te} &= (1+\gamma_{te})\,|\mathbf{c}_{te}^T\mathbf{w}|^2/\left(|\mathbf{c}_{te}^T\mathbf{w}|^2 + ||\mathbf{c}_{te}^T\mathbf{W}_n||^2 + \sigma_{te}^2\right) - \gamma_{te} \\ &+ \log(1+\gamma_{te}) = -\alpha_{te}^2(|\mathbf{c}_{te}^T\mathbf{w}|^2 + ||\mathbf{c}_{te}^T\mathbf{W}_n||^2 + \sigma_{te}^2) \\ &+ 2\alpha_{te}\sqrt{1+\gamma_{te}}|\mathbf{c}_{te}^T\mathbf{w}| + \log(1+\gamma_{te}) - \gamma_{te}, \end{aligned} \tag{11}$$

where γ_u, α_u, γ_{te}, and α_{te} are auxiliary variables, whose values are updated in each iteration. Then, the objective can be re-written as

$$R_u - R_{te} = c + \Re(\mathbf{v}^T\mathbf{w}) + \text{tr}\left[\mathbf{M}(\mathbf{W}_n\mathbf{W}_n^H + \mathbf{w}\mathbf{w}^H)\right], \tag{12}$$

where

$$c = \log(1+\gamma_u) - \gamma_u - \log(1+\gamma_{te}) + \gamma_{te} + \alpha_{te}^2\sigma_{te}^2 - \alpha_u^2\sigma_u^2, \tag{13}$$
$$\mathbf{v} = 2\alpha_u\sqrt{1+\gamma_u}\mathbf{c}_u - 2\alpha_{te}\sqrt{1+\gamma_{te}}\mathbf{c}_{te}, \tag{14}$$
$$\mathbf{M} = \alpha_{te}^2\mathbf{c}_{te}^*\mathbf{c}_{te}^T - \alpha_u^2\mathbf{c}_u^*\mathbf{c}_u^T. \tag{15}$$

Thereby, a quadratic programming problem with non-fractional objective and constraints is formulated. We invoke semidefinite relaxation (SDR) to address the re-formulated problem. By letting $\mathbf{R}_w = \mathbf{w}^H\mathbf{w}$ and $\mathbf{R}_{W_n} = \mathbf{W}_n^H\mathbf{W}_n$, the Problem $(\mathbb{P}_1)$ in (9) becomes

$$(\overline{\mathbb{P}}_1) \quad \max_{\mathbf{w},\mathbf{R}_w\mathbf{R}_{W_n}} \quad \Re(\mathbf{v}^T\mathbf{w}) + \text{tr}[\mathbf{M}(\mathbf{R}_{W_n} + \mathbf{R}_w)] + c \tag{16a}$$
$$\text{s.t.} \quad \text{tr}[\mathbf{R}_w + \mathbf{R}_{W_n}] \leq P_R \tag{16b}$$
$$\text{tr}\left[\mathbf{C}_T^H\mathbf{C}_T\left[\mathbf{R}_w + \mathbf{R}_{W_n}\right]\right]/\sigma_R^2 \geq \gamma_{R,th} \tag{16c}$$
$$\mathbf{R}_w \succcurlyeq \mathbf{w}\mathbf{w}^H \tag{16d}$$

where c in the objective (16a) is irrelevant to the design variables, and thus can be dropped; $\mathbf{R}_w$ and $\mathbf{R}_{W_n}$ are set as positive semidefinite matrices in the optimization process. The Problem $(\overline{\mathbb{P}}_1)$ in (16) is a linear programming problem, of which the optimal solution, say $\mathbf{w}^*$ and $\mathbf{R}_{W_n}^*$, can be obtained by numerical solvers, for example CVX toolbox [29]. The optimal solution for $\mathbf{W}_n$ is obtained by calculating the square root matrix of $\mathbf{R}_{W_n}^*$.

B. Second sub-problem: Solve for $\mathbf{\Phi}$ by fixing $\mathbf{w}$ and $\mathbf{W}_n$

The design problem of IRS parameter, $\mathbf{\Phi}$, is as follows

$$(\mathbb{P}_2) \quad \max_{\mathbf{\Phi}} \quad R_u - R_{te} \tag{17a}$$
$$\text{s.t.} \quad |\mathbf{\Phi}_{n,n}| = 1, \ \ \forall n = 1, \cdots, N \tag{17b}$$
$$\gamma_R \geq \gamma_{R,th} \tag{17c}$$

Besides the fractional form objective, the problem $(\mathbb{P}_2)$ in (17) is subject to challenging non-convex UMC on $\mathbf{\Phi}$ as (17b). Moreover, the γ_R in (17c) is a non-convex fourth order function of $\mathbf{\Phi}$, and this term needs to be convexified to make the problem solvable.

We re-write the effective channel for the user as

$$\begin{aligned} \mathbf{c}_u^T &= \mathbf{g}^T + \beta_H^{\frac{1}{2}}\mathbf{f}^T\mathbf{\Phi}\mathbf{H}_{dl} = \mathbf{g}^T + \boldsymbol{\phi}^T\beta_H^{\frac{1}{2}}\text{diag}(\mathbf{f})\mathbf{H}_{dl} \\ &= \mathbf{g}^T + \boldsymbol{\phi}^T\mathbf{D}, \end{aligned} \tag{18}$$

where $\boldsymbol{\phi} = \text{diag}(\mathbf{\Phi})$ is the column vector that contains all diagonal elements of $\mathbf{\Phi}$, and $\mathbf{D} = \beta_H^{\frac{1}{2}}\text{diag}(\mathbf{f})\mathbf{H}_{dl}$. Similarly, the effective channel for the ED/target is re-written as

$$\begin{aligned} \mathbf{c}_{te}^T &= \beta^{\frac{1}{2}}\mathbf{a}_I^T(\psi_a, \psi_e)\mathbf{\Phi}\mathbf{H}_{dl} = \boldsymbol{\phi}^T\beta^{\frac{1}{2}}\text{diag}(\mathbf{a}_I(\psi_a, \psi_e))\mathbf{H}_{dl} \\ &= \boldsymbol{\phi}^T\mathbf{E}, \end{aligned} \tag{19}$$

where $\mathbf{E} = \beta^{\frac{1}{2}}\text{diag}(\mathbf{a}_I(\psi_a, \psi_e))\mathbf{H}_{dl}$. By invoking (14), (18), and (19), the first term in the transformed non-fractional objective in (16), $\Re(\mathbf{v}^T\mathbf{w})$, can be re-written as

$$\Re(\mathbf{v}^T\mathbf{w}) = \Re(\boldsymbol{\phi}^T\boldsymbol{\eta}) + c_1, \tag{20}$$

where

$$\boldsymbol{\eta} = 2(\alpha_u\sqrt{1+\gamma_u}\mathbf{D} - \alpha_{te}\sqrt{1+\gamma_{te}}\mathbf{E})\mathbf{w}, \tag{21}$$
$$c_1 = 2\Re(\alpha_u\sqrt{1+\gamma_u}\mathbf{g}^T\mathbf{w}). \tag{22}$$

Furthermore, c_1 is taken as a constant in the second sub-problem. Likewise, by referring to (15), (18), and (19), the second item of the converted objective in (16), $\text{tr}[\mathbf{MR}_{W_n}]$, is re-expressed as

$$\begin{aligned}\text{tr}[\mathbf{MR}_{W_n}] &= \alpha_{te}^2\mathbf{c}_{te}^T\mathbf{W}_n\mathbf{W}_n^H\mathbf{c}_{te}^* - \alpha_u^2\mathbf{c}_u^T\mathbf{W}_n\mathbf{W}_n^H\mathbf{c}_u^* \\ &= \boldsymbol{\phi}^T\mathbf{L}_1\boldsymbol{\phi}^* - \Re(\boldsymbol{\phi}^T\boldsymbol{\mu}) - c_2,\end{aligned} \tag{23}$$

where

$$\mathbf{L}_1 = \alpha_{te}^2\mathbf{E}\mathbf{W}_n\mathbf{W}_n^H\mathbf{E}^H - \alpha_u^2\mathbf{D}\mathbf{W}_n\mathbf{W}_n^H\mathbf{D}^H, \tag{24}$$
$$\boldsymbol{\mu} = 2\alpha_u^2\mathbf{D}\mathbf{W}_n\mathbf{W}_n^H\mathbf{g}^*, \tag{25}$$
$$c_2 = \alpha_u^2\mathbf{g}^T\mathbf{W}_n\mathbf{W}_n^H\mathbf{g}^*. \tag{26}$$

Likewise, c_2 is taken as a constant in the design of $\boldsymbol{\Phi}$ or $\boldsymbol{\phi}$. Similarly, $\text{tr}[\mathbf{MR}_w]$ can be re-written as

$$\text{tr}[\mathbf{MR}_w] = \boldsymbol{\phi}^T\overline{\mathbf{L}}_1\boldsymbol{\phi}^* - \Re(\boldsymbol{\phi}^T\overline{\boldsymbol{\mu}}) - \bar{c}_2, \tag{27}$$

where

$$\overline{\mathbf{L}}_1 = \alpha_{te}^2\mathbf{E}\mathbf{w}\mathbf{w}^H\mathbf{E}^H - \alpha_u^2\mathbf{D}\mathbf{w}\mathbf{w}^H\mathbf{D}^H, \tag{28}$$
$$\overline{\boldsymbol{\mu}} = 2\alpha_u^2\mathbf{D}\mathbf{w}\mathbf{w}^H\mathbf{g}^*, \tag{29}$$
$$\bar{c}_2 = \alpha_u^2\mathbf{g}^T\mathbf{w}\mathbf{w}^H\mathbf{g}^*. \tag{30}$$

Thereby, the objective, $R_u - R_{te}$ can be re-written as

$$\begin{aligned}R_u - R_{te} &= \Re(\mathbf{v}^T\mathbf{w}) + \text{tr}[\mathbf{M}(\mathbf{R}_{W_n} + \mathbf{R}_w)] + c \\ &= \boldsymbol{\phi}^T(\mathbf{L}_1 + \overline{\mathbf{L}}_1)\boldsymbol{\phi}^* + \Re[\boldsymbol{\phi}^T(\boldsymbol{\eta} - \boldsymbol{\mu} - \overline{\boldsymbol{\mu}})] + \text{const},\end{aligned} \tag{31}$$

which is a quadratic function of $\boldsymbol{\phi}$.

Next, we need to transform the radar SNR, γ_R, in (17c) to make $(\mathbb{P}_2)$ solvable. Recall that the radar channel $\mathbf{C}_T = \beta\mathbf{H}_{ul}\boldsymbol{\Phi}\mathbf{a}_I(\psi_a, \psi_e)\mathbf{a}_I^T(\psi_a, \psi_e)\boldsymbol{\Phi}\mathbf{H}_{dl}$. By denoting $\mathbf{U} = \boldsymbol{\Phi}\mathbf{a}_I(\psi_a, \psi_e)\mathbf{a}_I^T(\psi_a, \psi_e)\boldsymbol{\Phi}$, γ_R can be re-arranged as

$$\begin{aligned}\gamma_R &= \text{tr}\left[\mathbf{C}_T^H\mathbf{C}_T\left[\mathbf{w}\mathbf{w}^H + \mathbf{W}_n\mathbf{W}_n^H\right]\right]/\sigma_R^2 \\ &= \beta^2/\sigma_R^2\text{tr}\left[\mathbf{U}^H\mathbf{H}_{ul}^H\mathbf{H}_{ul}\mathbf{U}\mathbf{H}_{dl}(\mathbf{w}\mathbf{w}^H + \mathbf{W}_n\mathbf{W}_n^H)\mathbf{H}_{dl}^H\right] \\ &\overset{(a)}{=} \beta^2/\sigma_R^2\mathbf{u}^H\mathbf{Z}\mathbf{u},\end{aligned} \tag{32}$$

where in step (a), $\text{tr}\left[\mathbf{U}^H\mathbf{A}\mathbf{U}\mathbf{B}^T\right] = \mathbf{u}^H(\mathbf{B}\otimes\mathbf{A})\mathbf{u}$ is referred to, $\mathbf{Z} = [\mathbf{H}_{dl}(\mathbf{w}\mathbf{w}^H + \mathbf{W}_n\mathbf{W}_n^H)\mathbf{H}_{dl}^H]^T\otimes(\mathbf{H}_{ul}^H\mathbf{H}_{ul})$, and $\mathbf{u} = \text{vec}(\mathbf{U})$. Note that both $\mathbf{U}$ and $\mathbf{u}$ are quadratic in $\boldsymbol{\Phi}$ or $\boldsymbol{\phi}$. To create a lower bound for γ_R that is quadratic in $\boldsymbol{\phi}$, we need to havea bound which is linear in $\mathbf{u}$. For this we invoke MM [23] as

$$\begin{aligned}\gamma_R &= \beta^2/\sigma_R^2\mathbf{u}^H\mathbf{Z}\mathbf{u} \\ &\geq \tilde{\gamma}_R = \beta^2/\sigma_R^2(\mathbf{u}^H\mathbf{Z}\mathbf{u}_t + \mathbf{u}_t^H\mathbf{Z}\mathbf{u} - \mathbf{u}_t^H\mathbf{Z}\mathbf{u}_t),\end{aligned} \tag{33}$$

where $\mathbf{u}_t = \text{vec}[\boldsymbol{\Phi}_t\mathbf{a}_I(\psi_a, \psi_e)\mathbf{a}_I^T(\psi_a, \psi_e)\boldsymbol{\Phi}_t]$, where $\boldsymbol{\Phi}_t$ is the solution of $\boldsymbol{\Phi}$ in t-th/previous iteration, the current iteration index is $(t+1)$, and $\tilde{\gamma}_R$ is the surrogate function for γ_R. In addition, the first term of $\tilde{\gamma}_R$ is re-expressed as

$$\begin{aligned}\mathbf{u}^H\mathbf{Z}\mathbf{u}_t &\overset{(b)}{=} \text{vec}(\mathbf{U})^H\text{vec}(\mathbf{V}) = \text{tr}\left[\mathbf{U}^H\mathbf{V}\right] \\ &\overset{(c)}{=} \text{tr}\left[\boldsymbol{\Phi}^H\mathbf{a}_I^*(\psi_a, \psi_e)\mathbf{a}_I^H(\psi_a, \psi_e)\boldsymbol{\Phi}^H\mathbf{V}\right] \\ &= \boldsymbol{\phi}^H\{[\mathbf{a}_I^*(\psi_a, \psi_e)\mathbf{a}_I^H(\psi_a, \psi_e)]\circ\mathbf{V}^T\}\boldsymbol{\phi}^*,\end{aligned} \tag{34}$$

where $\mathbf{u} = \text{vec}(\mathbf{U})$; $\mathbf{V}$ is set such that $\text{vec}(\mathbf{V}) = \mathbf{Z}\mathbf{u}_t$; $\mathbf{U} = \boldsymbol{\Phi}\mathbf{a}_I(\psi_a, \psi_e)\mathbf{a}_I^T(\psi_a, \psi_e)\boldsymbol{\Phi}$. Similarly, the second term in $\tilde{\gamma}_R$, $\mathbf{u}_t^H\mathbf{Z}\mathbf{u}$, can be re-written as

$$\mathbf{u}_t^H\mathbf{Z}\mathbf{u} = \boldsymbol{\phi}^T\{[\mathbf{a}_I(\psi_a, \psi_e)\mathbf{a}_I^T(\psi_a, \psi_e)]\circ\mathbf{Y}^T\}\boldsymbol{\phi}, \tag{35}$$

where the matrix $\mathbf{Y}$ satisfies $\text{vec}(\mathbf{Y}) = \mathbf{Z}^T\mathbf{u}_t^*$. Thereby, $\tilde{\gamma}_R$ in (33) can be written as

$$\tilde{\gamma}_R = \boldsymbol{\phi}^H\mathbf{L}_2\boldsymbol{\phi}^* + \boldsymbol{\phi}^T\mathbf{L}_3\boldsymbol{\phi} - \beta^2/\sigma_R^2\mathbf{u}_t^H\mathbf{Z}\mathbf{u}_t, \tag{36}$$

where the third term is not relevant to the variable $\boldsymbol{\phi}$, and

$$\mathbf{L}_2 = \beta^2/\sigma_R^2\{[\mathbf{a}_I^*(\psi_a, \psi_e)\mathbf{a}_I^H(\psi_a, \psi_e)]\circ\mathbf{V}^T\}, \tag{37}$$
$$\mathbf{L}_3 = \beta^2/\sigma_R^2\{[\mathbf{a}_I(\psi_a, \psi_e)\mathbf{a}_I^T(\psi_a, \psi_e)]\circ\mathbf{Y}^T\}. \tag{38}$$

So far, the objective (17a) and constraint (17c) in $(\mathbb{P}_2)$ are transformed into non-fractional quadratic form. By invoking (31) and (36), $(\mathbb{P}_2)$ can be re-written as

$$\begin{aligned}(\overline{\mathbb{P}}_2)\quad &\max_{\boldsymbol{\phi}}\quad \boldsymbol{\phi}^T(\mathbf{L}_1 + \overline{\mathbf{L}}_1)\boldsymbol{\phi}^* + \Re[\boldsymbol{\phi}^T(\boldsymbol{\eta} - \boldsymbol{\mu} - \overline{\boldsymbol{\mu}})] &\text{(39a)}\\ &\text{s.t.}\quad |\boldsymbol{\phi}_{n,1}| = 1,\ \ \forall n = 1,\cdots,N &\text{(39b)}\\ &\qquad\ \boldsymbol{\phi}^H\mathbf{L}_2\boldsymbol{\phi}^* + \boldsymbol{\phi}^T\mathbf{L}_3\boldsymbol{\phi} \geq \gamma'_{R,th} &\text{(39c)}\end{aligned}$$

where $\gamma'_{R,th} = \gamma_{R,th} + \beta^2/\sigma_R^2\mathbf{u}_t^H\mathbf{Z}\mathbf{u}_t$. Now, $(\overline{\mathbb{P}}_2)$ is a quadratic programming problem with UMC on the elements of the variable $\boldsymbol{\phi}$ as (39b). We solve $(\overline{\mathbb{P}}_2)$ with SDR by relaxing the UMC as

$$\begin{aligned}(\overline{\overline{\mathbb{P}}}_2)\ &\max_{\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\phi}}\quad \text{tr}\left[(\mathbf{L}_1 + \overline{\mathbf{L}}_1)\mathbf{R}_1^*\right] + \Re[\boldsymbol{\phi}^T(\boldsymbol{\eta} - \boldsymbol{\mu} - \overline{\boldsymbol{\mu}})] &\text{(40a)}\\ &\text{s.t.}\quad [\mathbf{R}_1]_{n,n} = 1, \forall n = 1,\cdots,N &\text{(40b)}\\ &\qquad |[\mathbf{R}_2]_{n,n}| \leq 1, \forall n = 1,\cdots,N &\text{(40c)}\\ &\qquad \text{tr}[\mathbf{L}_2\mathbf{R}_2^*] + \text{tr}[\mathbf{L}_3\mathbf{R}_2] \geq \gamma'_{R,th} &\text{(40d)}\\ &\qquad \mathbf{R}_1 \succcurlyeq \boldsymbol{\phi}\boldsymbol{\phi}^H &\text{(40e)}\\ &\qquad \mathbf{R}_2 \succcurlyeq \boldsymbol{\phi}\boldsymbol{\phi}^T &\text{(40f)}\end{aligned}$$

The solved $\boldsymbol{\phi}$ is substituted into the sub-problem 1 of next iteration. In addition, the overall optimization algorithm for jointly designing $\mathbf{w}$, $\mathbf{W}_n$ and $\boldsymbol{\Phi}$, is summarized as Algorithm 1. Furthermore, $\text{obj}^{(t+1)}$ is the objective, or secrecy rate ($R_u - R_{te}$), obtained in the $(t+1)$-th iteration. ε is the indicator of error tolerance.

Algorithm 1: Secrecy rate maximization

Result: Return $\mathbf{w}$, $\mathbf{W}_n$ and $\mathbf{\Phi}$.

Initialization: $\mathbf{\Phi} = \mathbf{\Phi}_0$, $\mathbf{w} = \mathbf{w}_0$, $\mathbf{W}_n = \mathbf{W}_{n,0}$, $\gamma_u = \gamma_{u,0}$, $\alpha_u = \alpha_{u,0}$, $\gamma_{te} = \gamma_{te,0}$, $\alpha_{te} = \alpha_{te,0}$, $t = 0$;

while (1) **do**

// Auxiliary variables update

$\alpha_u = \sqrt{1+\gamma_u}|\mathbf{c}_u^T\mathbf{w}|/\left(|\mathbf{c}_u^T\mathbf{w}|^2+||\mathbf{c}_u^T\mathbf{W}_n||^2+\sigma_u^2\right)$;

$\gamma_u = |\mathbf{c}_u^T\mathbf{w}|^2/\left(||\mathbf{c}_u^T\mathbf{W}_n||^2+\sigma_u^2\right)$;

$\alpha_{te} = \sqrt{1+\gamma_{te}}|\mathbf{c}_{te}^T\mathbf{w}|/\left(|\mathbf{c}_{te}^T\mathbf{w}|^2+||\mathbf{c}_{te}^T\mathbf{W}_n||^2+\sigma_{te}^2\right)$;

$\gamma_{te} = |\mathbf{c}_{te}^T\mathbf{w}|^2/\left(||\mathbf{c}_{te}^T\mathbf{W}_n||^2+\sigma_{te}^2\right)$;

// First sub-problem

Solve $(\overline{\mathbb{P}}_1)$ of (16) to obtain $\mathbf{w}$ and $\mathbf{R}_{W_n}$;

Obtain $\mathbf{W}_n$ as the square root matrix of $\mathbf{R}_{W_n}$;

// Second sub-problem

Substitute $\mathbf{w}$, $\mathbf{W}_n$, found in the first sub-problem into the second sub-problem;

Obtain ϕ by solving $(\overline{\overline{\mathbb{P}}}_2)$ in (40);

$\mathbf{\Phi} = \text{diag}(\phi)$;

// Condition of termination

if $\left((t \geq t_{\max}) \,||\, (|\left[\text{obj}^{(t+1)} - \text{obj}^{(t)}\right]/\text{obj}^{(t)}| \leq \varepsilon)\right)$ **then**

`Break;`

end

$t = t + 1$;

end

TABLE I
SYSTEM CONFIGURATIONS

Parameter	Value
Error tolerance indicator of Algorithm 1, ε [dB]	-20
Maximum number of iterations allowed, $t_{\max}$	20
Rician channels $\mathbf{g}$, $\mathbf{f}$, $\mathbf{H}_{dl}$, and $\mathbf{H}_{ul}$ [dB]	0
Noise power at the radar receiver, σ_R^2 [dBm]	0
Noise power at the communication user, σ_u^2 [dBm]	0
Noise power at the target/ED, σ_{te}^2 [dBm]	0
Inter-antenna distance at radar transmitter/receiver, d	$\lambda/2$
Number of antennas of radar transmitter, N_T	16
Number of antennas of radar receiver, N_R	16
Number of IRS elements, N	25
Target coefficient $|\beta|$ [dB]	-40
SNR threshold at radar receiver $\gamma_{R,th}$ [dB]	-11
Radar power budget P_R [dBm]	30

IV. NUMERICAL RESULTS

In this section, we present numerical results to demonstrate the convergence of the proposed algorithm. While AN helps PLS, it results in loss of communication signal power. Let ω be ratio of user information power over the total power of user information and AN. We show via simulations of ω, on the system performance. The channels are simulated as Rician. The parameters of the simulation are shown as in Table I.

Fig. 2 demonstrates the convergence of our proposed algorithm. The solid blue line represents the mean of the convergence curves of the objective (secrecy rate), which are obtained with 30 different channel realizations. The light blue shaded area around the solid line shows the variance among the 30 distinct realizations. It is observed that, the objective, $R_u - R_{te}$, reaches convergence in few iterations. In addition, the convergence holds for different values of ω.

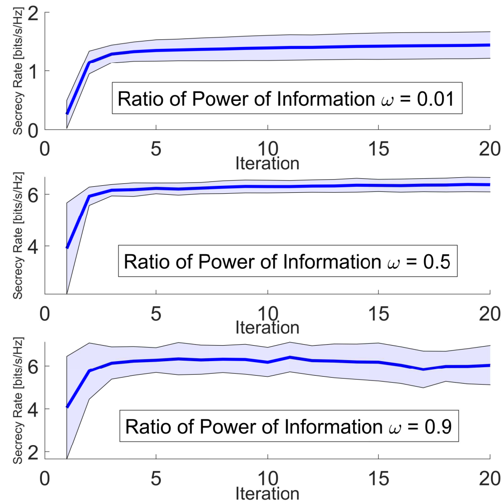

Fig. 2. Convergence of the proposed algorithm.

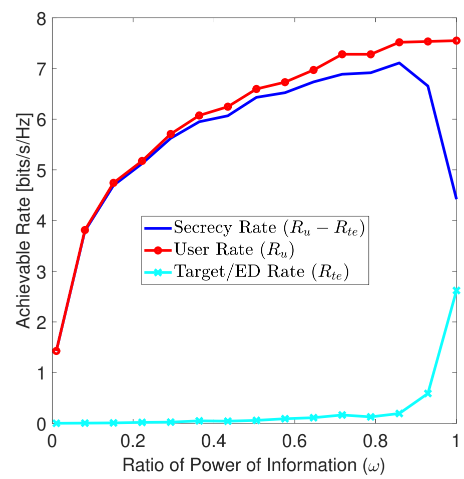

Fig. 3. The impact of ω on the achievable rates.

In Fig. 3, the influence of the power ratio of user information, ω, on the achievable rates, is investigated. When ω is small, low power is allocated to the user information, and high power to the AN. As a result, the user rate (R_u) and rate at the target/ED (R_{te}) are both low, and the secrecy rate ($R_u - R_{te}$) is also low in this case. When ω increases, both R_u and R_{te} are enhanced. R_u increases fast when ω is small, and slow when ω is large. Meanwhile, R_{te} rises slowly when ω is low, and increases fast when ω is large. Therefore, with the increase of ω, the secrecy rate, $R_u - R_{te}$, increases at first, since R_u increases faster than R_{te} in the beginning (See Fig. 3 when ω is less than 0.86). Afterwards, R_{te} rises faster, so the secrecy rate drops. When ω is 1, only user information embedded waveform is transmitted towards and illuminate the target/ED for the sensing task, which leads to high ED rate, and decreased secrecy rate.

V. CONCLUSIONS

In this paper, we have considered an IRS-aided DFRC system, where the target to be sensed acts as an eavesdropper. We have proposed an alternating optimization algorithm to jointly design the transmitted waveform for user information, the AN, and the IRS parameter matrix, to maximize the secrecy rate subject to certain non-convex constraints. We have invoked an efficient fractional programming technique, quadratic transform, to convert the fractional objective of secrecy rate, to a more mathematically tractable non-fractional form. Thereby, the design of waveform of user information and AN was transformed to a non-fractional quadratic programming problem. The IRS design problem, which contains a challenging fourth order function of the IRS parameter, was degraded to a second order one via the effective bounding technique, MM, to deliver a tractable IRS optimization problem. The numerical results have demonstrated the convergence of the proposed algorithm, and the influence of the user information power ratio on the achievable rates.

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