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Letter

Anti-de Sitter → de Sitter transition driven by Casimir forces and mitigating tensions in cosmological parameters

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ABSTRACT

Over the last few years, low- and high-redshift observations set off tensions in the measurement of the present-day expansion rate H_0 and in the determination of the amplitude of the matter clustering in the late Universe (parameterized by S_8). It was recently noted that both these tensions can be resolved if the cosmological constant parametrizing the dark energy content switches its sign at a critical redshift $z_c \sim 2$. However, the anti-de Sitter (AdS) swampland conjecture suggests that the postulated switch in sign of the cosmological constant at zero temperature seems unlikely because the AdS vacua are an infinite distance apart from de Sitter (dS) vacua in moduli space. We provide an explanation for the required AdS → dS crossover transition in the vacuum energy using the Casimir forces of fields inhabiting the bulk. We then use entropy arguments to claim that any AdS → dS transition between metastable vacua must be accompanied by a reduction of the species scale where gravity becomes strong. We provide a few examples supporting this AdS → dS uplift conjecture.

It has been almost a century since the expansion of the Universe was established [1], but the Hubble constant H_0, which measures its rate, continues to encounter challenging shortcomings. Actually, the mismatch between the locally measured value of H_0 [2] and the one inferred from observations of the cosmic microwave background (CMB) on the basis of the Λ cold dark matter (CDM) model [3] has become the mainspring of modern cosmology, and many extensions of ΛCDM are rising to the challenge [4]. Concomitant with the H_0 tension, there is evidence of a growing tension between the CMB-preferred value [3] and weak gravitational lensing determination [5] of the weighted amplitude of matter fluctuations parametrized by S_8. It would be appealing and compelling if both the H_0 and S_8 tensions were resolved at once, but as yet none of the proposed extensions of ΛCDM have done so to a satisfactory degree [6].

Very recently, an empirical conjecture to simultaneously resolve the H_0 and S_8 tensions has been contemplated [7]. The conjecture, which is rooted on the graduated dark energy model [8–10], postulates that Λ may have switched sign (from negative to positive) at critical redshift $z_c \sim 2$. The so-called "Λ_sCDM model" then suggests that the Universe may have recently experienced a phase of rapid transition from anti-de Sitter (AdS) vacuum to de Sitter (dS) vacuum. However, the AdS swampland conjecture [11] indicates that AdS vacua are an infinite distance apart from dS vacua in moduli space. In fact, the AdS distance conjecture, taken at face value, implies that it is not possible to cross the barrier $\Lambda = 0$ and to have a transition from a negative to a positive cosmological constant. This follows because the distance in the space of metrics in terms of $\Lambda = 0$ is proportional to $-\log|\Lambda|$, and this expression diverges as $\Lambda \to 0$. However, we note that this no-go theorem is valid at zero temperature, where the number of light particles is (in general) constant. At finite temperature, particles can decay and hence the number of light particles can change. In this way the minima of the potential can be lifted. All in all, the postulated switch in sign of Λ at zero temperature seems, at first sight, unlikely. In this Letter we investigate whether quantum effects derived from the Casimir energy of fields

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inhabiting the bulk could produce the hypothesized AdS → dS crossover transition.[1]

Before proceeding, we pause to discuss the benefits of the Λ_sCDM model. The model simultaneously resolves the major cosmological tensions including: H_0, S_8, and SNe Ia absolute magnitude [7]. In addition, it can accommodate the BAO Lyman-α discrepancy [8].[2] Furthermore, for flat cosmology, $\Omega_m+\Omega_\Lambda \simeq 1$, from which it follows that the mean energy density in the universe is equal to the critical density, where Ω_m is the fractional nonrelativistic matter density and Ω_Λ is the cosmological-constant energy density. This implies that $\Omega_m > 1$ when $\Lambda < 0$, in agreement with the otherwise puzzling JWST observations [18].[3]

In order to demonstrate the main mechanism of the AdS → dS crossover transition we will first consider a simple toy model. This toy model already incorporates some of the main features that we will meet in the cosmological model, which will be discussed next, and it is also compatible with the dark dimension scenario [21]. We will start with an AdS potential, which is quadratic in the radius modulus R of a one-dimensional compactification, and which leads to AdS minimum[4]:

$$V_{\rm AdS}(R) = \frac{1}{R_0^6}(R-R_0)^2 - \frac{1}{R_0^4}\,. \tag{1}$$

This potential possesses a minimum at $R = R_0$ and the potential at its minimum satisfies

$$V_{\rm AdS}(R_0) = -\frac{1}{R_0^4}\,. \tag{2}$$

Hence this AdS minimum exhibits scale separation in four dimension, in agreement with the weak form of the AdS swampland conjecture. Next we consider a positive Casimir energy of the simple form

$$V_C(R,a) = \frac{a}{R^4}\,, \tag{3}$$

where a is a so far undetermined constant. Here $V_C(R)$ provides the potential uplift to dS, as the total potential is given by

$$\begin{aligned} V(R,a) &= V_{\rm AdS}(R) + V_C(R) \\ &= \frac{1}{R_0^6}(R-R_0)^2 - \frac{1}{R_0^4} + \frac{a}{R^4}\,. \end{aligned} \tag{4}$$

This potential possesses a minimum at $R = R_1$, for which one can show that it always satisfies $R_1 > R_0$. So the radius gets increased due to the uplift, and hence the associated species scale [22–26] decreases (see also the discussion below). The species entropy was introduced in [27], and it is defined in four space-time dimensions as $S_{\rm sp} = M_*^{-2}$. Following the law of thermodynamics, it is also argued in [27] that any dynamical motion in moduli space should always follow a path, for which $S_{\rm sp}$ does not decrease. With $M_* \simeq R^{-1/3}$, it follows that species thermodynamics allow only for transition processes where the radius of the extra dimension increases (or stays constant) due to the uplift.

The value and in particular the sign of the uplifted vacuum energy $V(R_1,a)$ now depends on the value of the constant a. Actually, one can show that there is critical value $a^c = 5^5/3^6 \simeq 4.28$ and an associated critical radius $R_1^c = 5R_0/3$, where the potential exactly vanishes:

$$V(R_1^c, a^c) = 0\,. \tag{5}$$

For $0 < a < a^c$ and $R_0 < R_1 < R_1^c$ the vacuum is negative: $V(R_1^c,a) < 0$; for $a > a^c$ and $R_1 > R_1^c$ the vacuum becomes positive, i.e. $V(R_1^c,a) > 0$. For example, for $a = 16$ it follows that the minimum is at $R_1 = 2R_0$, and the corresponding positive vacuum energy takes the value

$$V(R_1 = 2R_0, a = 16) = \frac{1}{R_0^4}\,. \tag{6}$$

Note that for $a - a^c \to 0^+$, the minimum R_1 is near R_0, consistently with the quadratic approximation given in (1). Thus, the parametric dependence between the vacuum energy and the radius at its minimum is the same as before the uplift, in accordance with the dark dimension scenario [21]. A point worth noting at this juncture is that the relation $|V| = R^{-4}$ is at the heart of vacuum energy calculations in string theory, see [28] for a recent discussion.

In summary we have demonstrated that by varying the parameter a of the Casimir energy, we could model a phase transition from negative vacuum energy to positive vacuum energy, i.e. from AdS to dS.[5] In the early universe a becomes a function of the temperature, $a = a(T)$, and at a certain critical temperature T_c, there could be a phase transition from AdS to dS, like we have described above. Alternatively, for a compact space without cosmic expansion, the dynamics of a could be driven by the particle decay widths of the spectrum in the deep infrared region.

Now, we turn to construct a cosmological model that can simultaneously resolve the H_0 and S_8 tensions; namely we compactify a 5-dimensional (5D) model of Einstein gravity equipped with a next-to-minimal hidden sector. The model is based on the idea that a compact dimension may have undergone a uniform rapid expansion, together with the 3D non-compact space, via regular exponential inflation described by a 5D de Sitter (or approximate) solution of Einstein equations [30]. The size of the compact space has grown from the fundamental length $\mathcal{O}(M_*^{-1})$ to the micron size, so that at the end of inflation the emergent 4D strength of gravity became much weaker, as it is measured today,

$$M_p^2 = 2\pi R_\perp M_*\,, \tag{7}$$

where M_p is the reduced Planck mass, $R_\perp$ the radius of the compact space at the end of inflation, and M_* the 5D Planck scale (or species scale where gravity becomes strong [22–26]). As a matter of fact, M_* is not a fundamental scale but only the energy/length region where gravity becomes strong and requires an UV completion, such as string theory. The string scale is the fundamental parameter which does not change, but as we discuss below, the species scale can vary depending the particle content and interaction at a given energy. In our case the variation is minimal and within the energy region where gravity becomes strong.

In the spirit of [31], we model the dark dimension as a space with two boundaries: an interval with end-of-the-world 9-branes attached at each end. After a suitable compactification of six dimensions the space boundaries can be approximated by 3-branes, and one of them can be identified with the Standard Model 3-brane which portrays our non-compact 4D world, at least up to energies $\mathcal{O}(10$ TeV) and perhaps much further. The line interval along the dark dimension can also be understood as a semicircular dimension endowed with $S^1/\mathbb{Z}_2$ symmetry.[6]

After the end of 5D inflation, the radion is stabilized in a local (metastable) vacuum by the Casimir energy of 5D fields. More concretely, the effective 4D potential of the radion R after the end of inflation takes the form

$$V(R) = \frac{2\pi\,\Lambda_5\,r^2}{R} + \left(\frac{r}{R}\right)^2 T_4 + V_C(R)\,, \tag{8}$$

[1] For related work, see [12].

[2] The recent investigation presented in [7] is based on: the *Planck* CMB data [13], the Pantheon+ SNe Ia sample [14], the data release of KiDS-1000 [5], and the (angular) transversal 2D BAO data on the shell [15,16], which are less model dependent than the 3D BAO data used in previous studies of Λ_sCDM. It is important to stress that the BAO 3D data sample assumes ΛCDM to determine the distance to the spherical shell, and hence could potentially introduce a bias when analyzing beyond ΛCDM models [17].

[3] See however [19] and [20].

[4] Such AdS potentials are expected to arise in the context of flux compactifications. More generally, the quadratic form is also justified around the minimum.

[5] Alternative uplift scenarios for dS were originally suggested by Kachru, Kallosh, Linde, and Trived [29].

[6] In the context of toroidal orientifold models compact dimensions are associated to $S^1/\mathbb{Z}_2$ symmetry which is needed for obtaining a 4D chiral model (S^1 does not lead to chirality).

where Λ_5 is the 5D cosmological constant, $r \equiv \langle R \rangle$, T_4 is the total brane tension, and

$$V_C(R) = \pi R \left(\frac{r}{R}\right)^2 \sum_i (-1)^{s_i} N_i \, \rho(R,m) \tag{9}$$

corresponds to the Casimir energy, and where the sum goes over all 5D states in the spectrum, N_i is the number of degrees of freedom of the i-th particle, $s_i = 0(1)$ for bosons (fermions), and the factor $(r/R)^2$ comes from the rescaling of the 4D metric in the transformation to the Einstein frame [32]. The Casimir energy density for a particle of mass m is given by

$$\rho(R,m) = -\sum_{n=1}^{\infty} \frac{2m^5}{(2\pi)^{5/2}} \frac{K_{5/2}(2\pi Rmn)}{(2\pi Rmn)^{5/2}}\,, \tag{10}$$

where $K_{5/2}(x)$ is the Bessel function [33].[7]

The Casimir contribution to the potential falls off exponentially at large R compared to the particle wavelength. Considering only the first two terms in (8) it is straightforward to see that the potential develops a maximum at

$$R_{\rm max} = -T_4/(\pi\Lambda_5)\,, \tag{11}$$

requiring a negative tension T_4.

Corrections to the vacuum energy due to Casimir forces are expected to become important in the deep infrared region. Indeed, as R decreases different particle thresholds open up,

$$V_C(R) = \sum_i \frac{\pi r^2}{32\pi^7 R^6} (N_F - N_B)\, \Theta(R_i - R)\,, \tag{12}$$

where $m_i = R_i^{-1}$, Θ is a step function, $N_F - N_B$ stands for the difference between the number of light fermionic and bosonic degrees of freedom [32]. Eventually, the fermionic degrees of freedom overwhelm the bosonic contribution, giving rise to possible minima, as long as $R_i < R_{\rm max}$.

To get further insights into this idea we envision a next-to-minimal hidden sector to characterize the deep infrared region. It consists of: the 5D graviton (contributing with $N_B = 5$), a massive gauge boson ($N_B = 4$), and three massive bulk fermions (each contributing with $N_F = 4$). These fermions could be identified with sterile neutrinos furnished by Dirac bulk masses, which weaken the bounds on the size of the fifth dimension from neutrino oscillation data [35]. Altogether, this gives a difference $N_F - N_B = 3$. Now, we would further assume that the gauge boson is unstable and decays into two neutrinos. The 5D gauge coupling has dimension of mass$^{-1/2}$, which we can write as $g/M_*^{1/2}$ where M_* is the 5D gravity scale (of order the string scale) and g a dimensionless parameter. The total decay width is then given by

$$\Gamma \sim \frac{g^2}{(2\pi)^4} \frac{m^2}{M_*}\,, \tag{13}$$

where m is the mass of the gauge boson. Taking $m \sim 100$ meV, $g \sim 10^{-4.5}$, and $M_* \sim 10^9$ GeV we obtain $\Gamma \sim 5 \times 10^{-42}$ GeV, which implies the gauge bosons decay at $z \sim z_c$. If this were the case, then for $z \lesssim z_c$ the difference in (12) would become $N_F - N_B = 7$. In Fig. 1 we show an example of the AdS → dS transition produced by the dark sector described above.

Now, the AdS → dS transition shown in Fig. 1 slightly deviates from the model analyzed in [7], because the fields characterizing the deep infrared region of the dark sector contribute to the effective number of relativistic neutrino-like species $N_{\rm eff}$ [36]. Using conservation of entropy, fully thermalized relics with g_* degrees of freedom contribute

[7] The Casimir energy per unit of volume is the same for Dirichlet and Neumann boundary conditions [34].

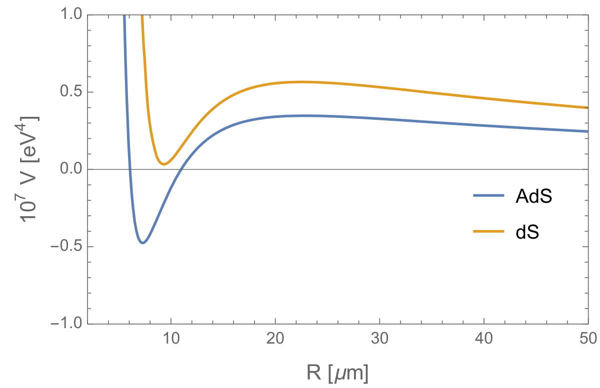

Fig. 1. The potential $V(R)$ for $\Lambda_5 = (22.6\ {\rm meV})^5$ and $|T_4| = (24.2\ {\rm meV})^4$ considering $N_F - N_B = 3$ (AdS) and $N_F - N_B = 7$ (dS).

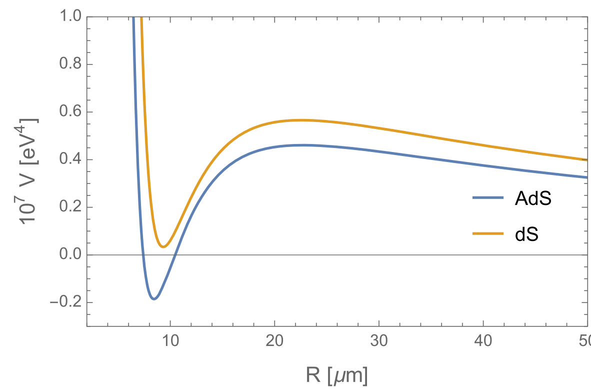

Fig. 2. The potential $V(R)$ for $\Lambda_5 = (22.6\ {\rm meV})^5$ and $|T_4| = (24.2\ {\rm meV})^4$ considering $N_F - N_B = 5$ (AdS) and $N_F - N_B = 7$ (dS).

$$\Delta N_{\rm eff} = g_* \left(\frac{43}{4g_s}\right)^{4/3} \begin{cases} 4/7 & \text{for bosons} \\ 1/2 & \text{for fermions} \end{cases}, \tag{14}$$

where g_s denotes the effective degrees of freedom for the entropy of the other thermalized relativistic species that are present when they decouple [37]. The 5D graviton has 5 helicities, but the spin-1 helicities do not have zero modes, because we assume the compactification has $S^1/\mathbb{Z}_2$ symmetry and so the ± 1 helicities are projected out. The spin-0 is the radion and the spin-2 helicities form the massless (zero mode) graviton. This means that for the 5D graviton, $g_* = 3$. The ± 1 helicities of the gauge field are odd and do not have zero modes, only the two scalars are even, so $g_* = 2$. The (bulk) left-handed neutrinos are odd, but the right-handed neutrinos are even and so each counts as a Weyl neutrino, for a total $g_* = 2 \times 3$. Assuming that the dark sector decouples from the Standard Model sector before the electroweak phase transition we have $g_s = 106.75$. This gives $\Delta N_{\rm eff} = 0.27$, consistent with the 95%CL bound reported by the Planck Collaboration [3].

We can also envision a more complex dark sector in which the gauge field is replaced by a complex scalar with Yukawa coupling to the neutrinos (that could provide them with mass). The spectrum has addition of an extra complex scalar, such that before its decay $N_F - N_B = 5$ and after decay $N_F - N_B = 7$. In Fig. 2 we show the corresponding AdS → dS transition for this scenario. For this scenario, $\Delta N_{\rm eff} = 0.27$.

The next-to-minimal dark sectors that can accommodate the AdS → dS crossover transition can be nicely combined with the dark dimension proposal to elucidate the radiative stability of the cosmological hierarchy $\Lambda/M_p^4 \sim 10^{-120}$ [21]. As can be seen in Figs. 1 and 2, the connection is possible for a compactification radius $R_\perp^{(\rm AdS)} \sim 8\ \mu$m, which implies the Kaluza-Klein (KK) tower of the dark dimension opens up at $m_{\rm KK}^{(\rm AdS)} \sim 155$ meV. After the vacuum undergoes the AdS → dS

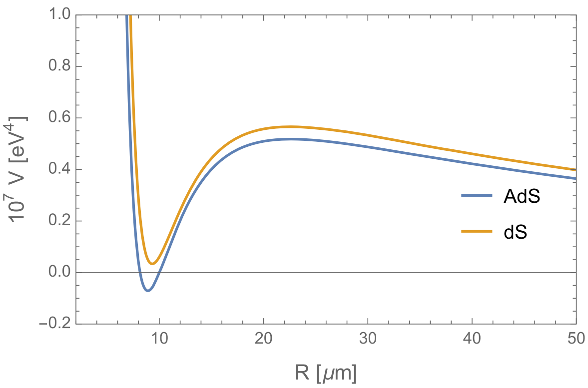

Fig. 3. The potential $V(R)$ for $\Lambda_5 = (22.6\ \text{meV})^5$ and $|T_4| = (24.2\ \text{meV})^4$ considering $N_F - N_B = 6$ (AdS) and $N_F - N_B = 7$ (dS).

transition the compact space slightly enlarges to $R_\perp^{(\text{dS})} \sim 9\ \mu$m and so $m_{\text{KK}}^{(\text{dS})} \sim 138$ meV. This in turn makes a shift of the species scale,

$$M_* \sim (m_{\text{KK}}^{(i)})^{1/3} M_p^{2/3}, \tag{15}$$

where $i =$ AdS, dS. Putting all this together, we have this interesting observation that as the species scale goes down correspondingly the species entropy [27] goes up, and we can even speculate that this has a deeper origin: since the entropy should increase we argue that *any AdS to dS transition in which the minimum is shifted towards smaller values of R are rule out.* Note that the reduction in the species scale can be also understood as a result of fixing the 4D M_p. Fixing instead the 5D scale M_* amounts to increase M_p and thus reduce the 4D strength of gravity since the radius becomes bigger. This alternative interpretation is constrained by observations, see e.g. [38].

Another possible course of action to accommodate an AdS → dS transition (with $N_F - N_B = 6$ in the AdS phase and $N_F - N_B = 7$ in the dS phase) is to assume that a real scalar φ has a potential with two local minima with very small difference in vacuum energy and bigger curvature (mass) of the lower one. More concretely, the potential is given by

$$V(\varphi) = \xi\varphi - \frac{1}{2}m^2\varphi^2 + \frac{\zeta}{3!}\varphi^3 + \frac{\lambda}{4!}\varphi^4, \tag{16}$$

where the parameters ξ, m^2, ζ, and λ are real and positive. The potential has two minima (φ_{fv} and φ_{tv}, the false and true vacuum, respectively) whose free-energy difference is non-zero. We fix the model parameters by requiring

$$m_{\varphi_{\text{fv}}} \sim \sqrt{V_{\varphi\varphi}(\varphi_{\text{fv}})} \sim 100\ \text{meV}, \tag{17}$$

$$m_{\varphi_{\text{tv}}} \sim \sqrt{V_{\varphi\varphi}(\varphi_{\text{tv}})} \sim 500\ \text{meV}, \tag{18}$$

and

$$\Delta V \equiv V(\varphi_{\text{fv}}) - V(\varphi_{\text{tv}}) \ll (\Lambda_5)^{4/5}, \tag{19}$$

where $V_\varphi = \partial V/\partial\varphi$. The fourth degree of freedom is adjusted by requiring the height of the barrier to accommodate a decay rate of the false vacuum to be

$$\Gamma = 2\,\hbar\ \text{Im}\left\{\lim_{T\to\infty}\frac{1}{T}\ \ln\left[Z(T)\right]\right\} \sim 5\times 10^{-42}\ \text{GeV}, \tag{20}$$

where $Z(T) = \int [dx]\ e^{-S_E/\hbar}$ is the path integral, $[dx]$ is a functional measure, and S_E the Euclidean action [39,40]. Finally, we take $\Lambda_5 \sim (22.6\ \text{meV})^5$ and $|T_4| = (24.2\ \text{meV})^4$ to obtain the vacuum energy densities of the AdS and dS phases shown in Fig. 3. For this scenario, $\Delta N_{\text{eff}} = 0.25$.

At this stage, it is constructive to connect with contrasting and complementary perspectives to comment on two caveats of our analysis: *(i)* although the cosmological models are motivated by swampland constraints and string constructions, they have no concrete UV completion; *(ii)* another aspect of this analysis which may seem discrepant at first blush is the fact that the (one loop) Casimir energy is a zero-T effect. However, in the example of the real scalar (which not unexpectedly is at zero temperature) the argument to understand the transition is essentially the same as the one in finite temperature models because the number of light degrees of freedom changes due to a different transition of the 5D scalar field. In other words, the model avoids finite temperature requirements and relies on an ordinary vacuum decay in five dimensions. This obviously implies that the AdS vacuum is not a true vacuum. The vacuum in the radius modulus is determined by the contribution to the Casimir potential of the number of light degrees of freedom. This number changes discontinuously due to an ordinary vacuum decay of a 5D scalar field which satisfies the swampland AdS conjecture (for instance it can be dS to dS). This change drives the AdS to dS transition in the radius modulus, which is therefore discontinuous as in first order transitions.

It is of interest to further investigate the impact of adding extra-relativistic degrees of freedom into the Λ_sCDM model. In particular, a value of $\Delta N_{\text{eff}} \neq 0$ would accelerate the universe before the CMB epoch, and hence could modify the critical redshift z_c for the AdS → dS crossover transition. A study along this line is presented in an accompanying paper where we use the Boltzmann solver `CLASS` [41,42] in combination with `MontePython` [43,44] (and the latest results from the *Planck* mission [13], Pantheon+ SNe Ia [14], BAO [15,16], and KiDS-1000 [5]) to perform a Monte Carlo Markov Chain study of the Λ_sCDM + N_{eff} model and determine regions of the parameter space that can simultaneously resolve the H_0 and S_8 tensions [45].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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