Dark dimension, the swampland, and the dark matter fraction composed of primordial near-extremal black holes

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(Received 8 February 2024; accepted 8 April 2024; published 10 May 2024)

In a recent publication we studied the decay rate of primordial black holes perceiving the dark dimension, an innovative five-dimensional (5D) scenario that has a compact space with characteristic length scale in the micron range. We demonstrated that the rate of Hawking radiation of 5D black holes slows down compared to 4D black holes of the same mass. Armed with our findings we showed that for a species scale of $\mathcal{O}(10^{10}\ \mathrm{GeV})$, an all-dark-matter interpretation in terms of primordial black holes should be feasible for black hole masses in the range $10^{14} \lesssim M/\mathrm{g} \lesssim 10^{21}$. As a natural outgrowth of our recent study, herein we calculate the Hawking evaporation of near-extremal 5D black holes. Using generic entropy arguments we demonstrate that Hawking evaporation of higher-dimensional near-extremal black holes proceeds at a slower rate than the corresponding Schwarzschild black holes of the same mass. Assisted by this result we show that if there were 5D primordial near-extremal black holes in nature, then a primordial black hole all-dark-matter interpretation would be possible in the mass range $10^5\sqrt{\beta} \lesssim M/\mathrm{g} \lesssim 10^{21}$, where β is a parameter that controls the difference between mass and charge of the associated near-extremal black hole.

DOI: 10.1103/PhysRevD.109.095008

I. INTRODUCTION

The swampland program aims at understanding which are the "good" low-energy effective field theories (EFTs) that can couple to gravity consistently (e.g., the landscape of superstring theory vacua) and distinguish them from the "bad" ones that cannot [1]. In theory space, the boundary setting apart the good theories from those downgraded to the swampland is characterized by a set of conjectures classifying the indispensable properties of an EFT to enable a consistent completion into quantum gravity. These conjectures provide a catwalk from quantum gravity to astrophysics, cosmology, and particle physics [2–4].

For instance, the distance conjecture (DC) predicts the appearance of infinite towers of states that become exponentially light and trigger the collapse of the EFT at infinite distance limits in moduli space [5]. Associated to the DC is the anti–de Sitter (AdS) distance conjecture, which correlates the dark energy density to the mass scale m characterizing the infinite tower of states, $m \sim |\Lambda|^\alpha$, as the negative AdS vacuum energy $\Lambda \to 0$, with α a positive constant of $\mathcal{O}(1)$ [6]. In addition, under the premise that this scaling behavior holds in de Sitter (dS)—or quasi dS—space, an unbounded number of massless modes also materialize in the limit $\Lambda \to 0$.

As demonstrated in [7], applying the AdS-DC to dS space could help elucidate the origin of the cosmological hierarchy $\Lambda/M_p^4 \sim 10^{-120}$, because it connects the size of the compact space $R_\perp$ to the dark energy scale $\Lambda^{-1/4}$ via $R_\perp \sim \lambda\Lambda^{-1/4}$, where the proportionality factor is estimated to be within the range $10^{-1} < \lambda < 10^{-4}$. Actually, the previous relation between $R_\perp$ and Λ derives from constraints by theory and experiment. On the one hand, since the associated Kaluza-Klein (KK) tower contains massive spin-2 bosons, the Higuchi bound [8] provides an absolute upper limit to α, whereas explicit string calculations of the vacuum energy (see e.g. [9–12]) yield a lower bound on α. All in all, the theoretical constraints lead to $1/4 \leq \alpha \leq 1/2$. On the other hand, experimental arguments (e.g., constraints on deviations from Newton's gravitational inverse-square law [13] and neutron star heating [14]) lead to the conclusion encapsulated in $R_\perp \sim \lambda \Lambda^{-1/4}$; namely, that there is one extra dimension of radius $R_\perp$ is in the micron range, and that the lower bound for $\alpha = 1/4$ is basically saturated [7]. A theoretical amendment on the connection between the cosmological and KK mass scales confirms $\alpha = 1/4$ [15]. Assembling all this together, we can conclude that the KK tower of the new (dark) dimension opens up at the mass scale $m_{\rm KK} \sim 1/R_\perp$. For the dark dimensions scenario, the five-dimensional (5D) Planck scale (or species scale where gravity becomes strong [16–19]) is given by

$$M_* \sim m_{\rm KK}^{1/3} M_p^{2/3}, \tag{1}$$

where M_p is the reduced Planck mass. Thus, since the size of dark dimension in the micron scale, $m_{\rm KK} \sim 1$ eV and so $10^9 \lesssim M_*/\text{GeV} \lesssim 10^{10}$.

Early universe phenomena and the nature of dark matter are among the most strategic science cases of theoretical high energy physics. As potentially the first density perturbations to collapse during the early universe, primordial black holes (PBHs) provide our earliest landmarks to probe the very early universe, at energies between the QCD phase transition and the Planck scale. The corresponding energy scales are out of the reach of existing cosmological probes. Much of the parameter space characterizing the PBH abundance has been constrained by existing probes, but a large window remains open, where PBHs around asteroid mass (10^{-15} to $10^{-10} M_\odot$) could make up the entirety of dark matter [20–23]. The detection of PBHs could provide a cornerstone for our perception of the physics processes in the very early universe. This significant reward motivates new investigations on this subject.

In previous work [24,25], we first calculated the decay rate of PBHs perceiving the dark dimension and demonstrated that the rate of Hawking radiation slows down compared to 4D black holes of the same mass. Then, we used this result to show that the mass range supporting a 5D PBH all-dark-matter interpretation is extended compared to that in the 4D theory by 3 orders of magnitude in the low mass region. As a natural outgrowth of this work, herein we study the Hawking evaporation of near-extremal 5D black holes. More concretely, we generalize the 4D results obtained in [26] to d-dimensions. We then discuss the impact of our findings in assessing the dark matter fraction that could be composed of PBHs. Since Hawking evaporation of near-extremal 5D black holes proceeds at a slower rate than the corresponding Schwarzschild black holes of the same mass, we show herein that near extremality could further relax the lower mass bound range of a PBH all-dark-matter interpretation.

The outline of the paper is as follows. In Sec. II we summarized the results of our previous work. In Sec. III we provide an overview of near-extremal black holes and discuss the various charges that can potentially bring together the inner and outer horizons. In Sec. IV we lay out a proof of principle for primordial near-extremal black holes investigating a model in which the charge leading to extremality is carried by dark electrons living in the bulk. In Sec. V, we first adopt generic entropy arguments to derive the scaling behavior of the decay rate of higher-dimensional near-extremal black holes. After that, armed with our findings we investigate how near-extremal black holes perceiving the dark dimension could modify the constraints on a PBH all-dark-matter interpretation. The paper wraps up in Sec. VI with some conclusions.

II. PRIMORDIAL BLACK HOLE DARK MATTER INTERPRETATION

It has long been speculated that black holes could be produced from the collapse of large amplitude fluctuations in the early universe [27–30]. For an order of magnitude estimate of the black hole mass M, we first note that the cosmological energy density scales with time t as $\rho \sim 1/(Gt^2)$ and the density needed for a region of mass M to collapse within its Schwarzschild radius is $\rho \sim c^6/(G^3 M^2)$, so that PBHs would initially have around the cosmological horizon mass [20]

$$M \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23}\ \text{s}}\right)\ \text{g}, \tag{2}$$

with $M_p = 1/\sqrt{8\pi G}$. This means that a black hole would have the Planck mass ($M_p \sim 10^{-5}$ g) if they formed at the Planck time (10^{-43} s), $1 M_\odot$ if they formed at the QCD epoch (10^{-5} s), and $10^5 M_\odot$ if they formed at $t \sim 1$ s, comparable to the mass of the holes thought to reside in galactic nuclei. This back-of-the-envelope calculation suggests that PBHs could span an enormous mass range. Despite the fact that the mass spectrum of these PBHs is yet to be shaped, on cosmological scales they would behave like a typical cold dark matter particle.

Nevertheless, an all-dark-matter interpretation in terms of PBHs is severely constrained by observations [20–23]. Of relevance to our investigation, the extragalactic γ-ray background [31] and the spectrum of the cosmic microwave background (CMB) [32] constrain PBH evaporation of black holes with masses $\lesssim 10^{17}$ g, whereas the non-observation of microlensing events from the MACHO [33], EROS [34], Kepler [35], Icarus [36], OGLE [37] and Subaru-HSC [38] collaborations constrain black holes with masses $\gtrsim 10^{21}$ g.

Microscopic black holes of Schwarzschild radii smaller than the size of the dark dimension are: bigger, colder, and longer-lived than a usual 4D black hole of the same mass [39]. Indeed, Schwarzschild black holes radiate all particle species lighter than or comparable to their temperature, which in four dimensions is related to the mass of the black hole by

$$T_s = \frac{M_p^2}{8\pi M} \sim \left(\frac{M}{10^{16}\ \mathrm{g}}\right)^{-1} \mathrm{MeV}, \tag{3}$$

whereas for five dimensional black holes the temperature mass relation is found to be [24]

$$T_s \sim \frac{1}{r_s} \sim \left(\frac{M}{10^{12}\ \mathrm{g}}\right)^{-1/2} \mathrm{MeV}, \tag{4}$$

where

$$r_s(M) \sim \frac{1}{M_*}\left[\frac{2}{3\pi}\frac{M}{M_*}\right]^{1/2} \tag{5}$$

is the 5D Schwarzschild radius [40]. The numerical estimate of (4) applies to the dark dimension scenario with $M_* \sim 10^{10}$ GeV, which is consistent with astrophysical observations [41,42].[1] It is evident that 5D black holes are colder than 4D black holes of the same mass. The Hawking radiation causes a 4D black hole to lose mass at the following rate [43]

$$\begin{aligned}\left.\frac{dM}{dt}\right|_{\mathrm{evap}} &= -\frac{M_p^2}{30720\pi M^2}\sum_i c_i(T_s)\tilde{f}\Gamma_s \\ &\sim -7.5\times 10^{-8}\left(\frac{M}{10^{16}\ \mathrm{g}}\right)^{-2}\sum_i c_i(T_s)\tilde{f}\Gamma_s\ \mathrm{g/s},\end{aligned} \tag{6}$$

whereas a 5D black hole has an evaporation rate of [24]

$$\begin{aligned}\left.\frac{dM}{dt}\right|_{\mathrm{evap}} &\sim -9\pi^{5/4}\zeta(4)T_s^2\sum_i c_i(T_s)\tilde{f}\Gamma_s \\ &\sim -\frac{27\Lambda^{1/4}M_p^2}{64\pi^{3/4}\lambda M}\sum_i c_i(T_s)\tilde{f}\Gamma_s \\ &\sim -1.3\times 10^{-12}\left(\frac{M}{10^{16}\ \mathrm{g}}\right)^{-1}\sum_i c_i(T_s)\tilde{f}\Gamma_s\ \mathrm{g/s},\end{aligned} \tag{7}$$

where $c_i(T_s)$ counts the number of internal degrees of freedom of particle species i of mass m_i satisfying $m_i \ll T_s$, $\tilde{f} = 1$ $(\tilde{f} = 7/8)$ for bosons (fermions), and where $\Gamma_{s=1/2} \approx 2/3$ and $\Gamma_{s=1} \approx 1/4$ are the (spin-weighted) dimensionless greybody factors normalized to the black hole surface area [44]. In the spirit of [45], we neglect KK graviton emission because the KK modes are excitations in the full transverse space, and so their overlap with the small (higher-dimensional) black holes is suppressed by the geometric factor $(r_s/R_\perp)$ relative to the brane fields. Thus, the geometric suppression precisely compensates for the enormous number of modes, and the total contribution of all KK modes is only the same order as that from a single brane field. On top of that, the 5D graviton has 5 helicities, but the spin-1 helicities do not have zero modes, because we assume the compactification has $S^1/\mathbb{Z}_2$ symmetry and so the ± 1 helicities are projected out. The greybody factors of spin-2 particles strongly suppress massless graviton emission on the brane $\Gamma_{s=2}/\Gamma_{s=1/2} \lesssim 10^{-3}$, and the emission of ± 1 helicities in the bulk is also suppressed; see, e.g., Fig. 2 of Ref. [46]. Contribution from the spin-0 depends on the radion mass. Since the addition of one scalar does not modify the order of magnitude calculations of this work, throughout we neglect the graviton emission. At the end of the concluding section we comment on the feasibility of detecting graviton emission on the brane. Now, comparing (6) and (7) it is easily seen that 5D black holes live longer than 4D black holes of the same mass.

Integrating (7) we can parametrize the 5D black hole lifetime as a function of its mass and temperature,

$$\tau_s \sim 13.8\left(\frac{M}{10^{12}\ \mathrm{g}}\right)^2\left(\frac{6}{\sum_i c_i(T_s)\tilde{f}\Gamma_s}\right)\ \mathrm{Gyr}, \tag{8}$$

where we have used (4) to estimate that $T_s \sim 1$ MeV and therefore $c_i(T_s)$ receives a contribution of 6 from neutrinos, 4 for electrons, and 2 from photons, yielding $\sum_i c_i(T_s)\tilde{f}\Gamma_s = 6$. Armed with (8) we can estimate the bound on the 5D PBH abundance by a simple rescaling procedure of the $d = 4$ bounds on the fraction of dark matter composed of primordial black holes f_{PBH}. The key point for such a rescaling is that for a given photon energy, or equivalently a given Hawking temperature, we expect a comparable limit on f_{PBH} for both $d = 4$ and $d = 5$. For example, from (3) and (4) we see that the constraint of $f_{\mathrm{PBH}} \lesssim 10^{-3}$ for 4D black holes with $M_{\mathrm{BH}} \sim 10^{16}$ g, should be roughly the same for the abundance of 5D black holes with $M_{\mathrm{BH}} \sim 10^{12}$ g. Now, since in $d = 4$ for $M_{\mathrm{BH}} \sim 10^{17}$ g we have $f_{\mathrm{PBH}} \sim 1$, this implies the same abundance for 5D black holes of $M_{\mathrm{BH}} \sim 10^{14}$ g. By duplicating this procedure for heavier black holes we conclude that for a species scale of $\mathcal{O}(10^{10}$ GeV), an all-dark-matter interpretation in terms of 5D black holes must be feasible for masses in the range

$$10^{14} \lesssim M/\mathrm{g} \lesssim 10^{21}. \tag{9}$$

[1]We have taken the highest possible value of M_* to remain conservative in the estimated bound on f_{PBH}.

This range is extended compared to that in the 4D theory by 3 orders of magnitude in the low mass region.

At this stage, it is worthwhile to point out that a stunning coincidence is that the size of the dark dimension $R_\perp \sim$ wavelength of visible light. This means that the Schwarzschild radius of 5D black holes is well below the wavelength of light. For pointlike lenses, this is the critical length where geometric optics breaks down and the effects of wave optics suppress the magnification, obstructing the sensitivity to 5D PBH microlensing signals [38].

III. NEAR-EXTREMAL BLACK HOLES

Asymptotically flat, static, and spherically symmetric charged (or rotating) black holes can be categorized as generalizations of the popular Schwarzschild metric. Such charged black holes carry additional quantum numbers, which make their properties change drastically and unique new phenomena arise. A far reaching hallmark of rotating black holes or those which are electrically (and/or magnetically) charged is their thermodynamical property dubbed extremality (i.e., zero temperature). Extremal black holes are in essence stable gravitational objects with finite entropy but vanishing temperature, and so the contribution to the gravitational energy completely originates in the electromagnetic charges and/or rotational angular momentum/spin.[2] Extremality also implies that the inner (Cauchy) and outer (event) horizons do coincide, leading to a vanishing surface gravity. The Reissner-Nordström (RN) metric describes the simplest extremal black hole, which has its mass equal to its charge in appropriate units.

It has long been suspected that any electromagnetic charge or spin would be lost very quickly by any 4D black hole population of primordial origin. On the one hand, the electromagnetic charge of a Reissner-Nordstrom (RN) black hole is spoiled by the Schwinger effect [49], which allows pair-production of electron-positron pairs in the strong electric field outside the black hole, leading to the discharge of the black hole and subsequent evaporation [50,51]. On the other hand, a rapidly rotating Kerr black hole [52] spins down to a nearly nonrotating state before most of its mass has been given up, and therefore it does not approach to extremal when it evaporates [53]. All in all, near-extremal primordial RN black holes or Kerr black holes are not expected to prevail in the universe we live in.

Adding to the story, it was pointed out in [54] that primordial black holes could grow by absorbing unconfined quarks and gluons. Given Debye screening, the quark-gluon plasma must be color neutral on long length scales $l \gg \lambda_D$, but could have a nontrivial distribution of color charge across shorter length scales $l \sim \lambda_D$. In particular, there could exist regions with net color charge, whose spatial extent is set by $\lambda_D(T)$ [55,56]. If this were the case, then black holes would acquire a net color charge [54]. However, after the QCD confinement transition, the medium would cease to screen the primordial black hole enclosed charge ($\lambda_D \to \infty$), and therefore it would become energetically (very) costly for any primordial black hole to maintain its color charge.

An alternative interesting possibility is to envision a scenario where the black hole is charged under a generic unbroken $U(1)$ symmetry (dark photon), whose carriers (dark electrons with a mass m_e' and a gauge coupling e') are always much heavier than the temperature of the black hole [57]. This implies that the charge Q does not get evaporated away from the black hole and remains therefore constant. Strictly speaking, the pair production rate per unit volume from the Schwinger effect can be slowed down by arbitrarily decreasing e', whereas the weak gravity conjecture (WGC) imposes a constraint on the charge per unit mass; namely, for each conserved gauge charge there must be a sufficiently light charge carrier such that

$$e' q/m_{e'} \geq \sqrt{4\pi}\sqrt{(d-3)/(d-2)} M_p^{-(d-2)/2}, \quad (10)$$

where q is the integer-quantized electric charge of the particle and M_p is the reduced Planck mass [58]. Setting $e = e' = \sqrt{4\pi\alpha}$ the (4D) Schwinger effect together with the WGC lead to a bound on the minimum black hole mass of near extremal black holes with evaporation time longer than the age of Universe, $M_{ne} \gtrsim 5 \times 10^{15}\ \mathrm{g}(m_{e'}/10^9\ \mathrm{GeV})^{-2}$ [57].

The latest chapter in the story is courtesy of dS backgrounds. If a black hole is embedded in a dS background, there is an additional bound on $m_{e'}$ from the festina lente (FL) conjecture [59]. This is because the RN-dS line element comprises two horizons accessible to an observer outside the black hole: (i) the familiar event horizon of the charged black hole and (ii) the cosmological horizon. Usually, the black hole and the cosmological horizons would have different temperatures, and so they cannot be in thermal equilibrium. Considering large black holes whose size is comparable to the dS radius and demanding their evaporation avoids superextremality leads to the festina lente bound: for every charged state in the theory,

$$m_{e'}^4 \gtrsim (e' q)^2 \frac{(d-1)(d-2)}{2 M_p^{2-d} \ell_d^2}, \quad (11)$$

where ℓ_d is the dS radius. This bound is satisfactorily satisfied in our universe for the electron.

IV. HIGHER-DIMENSIONAL SCHWINGER PAIR PRODUCTION

The metric of a d-dimensional RN-like dS black hole has the form

$$ds^2 = U(r) dt^2 - U^{-1}(r) dr^2 - d\Omega_{d-2}^2, \quad (12)$$

[2] Actually, when the temperature reduces to zero, the entropy reduces to the logarithm of the number of degenerate ground states, which is zero if the ground state is not degenerate [47,48].

where $d\Omega_{d-2}$ is the line element of a flat space of $d-2$ dimensions in spherical coordinates and

$$U(r) = 1 - \frac{2M}{M_*^{d-2} r^{d-3}} + \frac{(e'Q)^2}{4\pi M_*^{d-2} r^{2d-6}} - \frac{r^2}{\ell_d^2}, \quad (13)$$

where M_* is the d-dimensional Planck scale and the coupling e' is taken as a parameter. A well motivated scenario emerges if the SM has charges under the $U(1)$ field, such that e' becomes of the order of SM gauge couplings divided by the square root of the volume of the internal space.

Before proceeding, we pause to note that the FL inequality (11) remains the same in any number of dimensions, since the gauge coupling has units of Energy$^{2-d/2}$ [60]. However, it is important to stress that the FL bound only applies to black holes of size comparable to the cosmological horizon and therefore it is not of direct interest for scales smaller than $R_\perp$. The dS-WGC is relevant to black holes with a horizon radius smaller than $R_\perp$ [61]. However, for large ℓ_4 values, dS-WGC constraints on the particle spectrum can also be safely neglected. Hereafter, we proceed under the assumption of a (nearly) flat 5D Minkowski background and neglect the last term in (13). We further assume that (10) is satisfied. For details, see the Appendix.

For d dimensions, the Schwinger probability (per unit volume and unit time) of pair creation in a constant electric field is found to be

$$\Gamma_d = \frac{(2J+1)}{(2\pi)^{d-1}} \sum_{n=1}^{\infty} (-1)^{(2J+1)(n+1)} \left(\frac{e'E'}{n}\right)^{d/2} \exp\left(-\frac{\pi n m_{e'}^2}{e'E'}\right), \quad (14)$$

where E' is the dark electric field, $e'Q \to \sqrt{4\pi} M/M_*^{(d-2)/2}$, and J is the spin of the produced particles [62]. Throughout, the arrow indicates we are considering near-extremal rather than extremal black holes. A point worth noting at this juncture is that for $d > 6$ the RN-dS solution is gravitationally unstable [63,64] and so we focus our calculation on the interesting case of $d = 5$ that characterizes the dark dimension scenario [7]. For $d = 5$, the spin J is half-integer and so (14) can be rewritten as

$$\Gamma_5 = \frac{1}{8\pi^4} \sum_{n=1}^{\infty} \left(\frac{e'E'}{n}\right)^{5/2} \exp\left(-\frac{\pi n m_{e'}^2}{e'E'}\right). \quad (15)$$

It is of interest to make a comparison between the outer horizon radius of the 5D black hole,

$$r_{+,5d} = \left[\frac{M + \sqrt{M^2 - (e'Q)^2 M_*^3/(4\pi)}}{M_*^3}\right]^{1/2} \to \sqrt{\frac{M}{M_*^3}}, \quad (16)$$

and that of a 4D black hole $r_{+,4d} \to M/M_p^2$ with the same M and $e'Q$. It follows that $r_{+,5d} > r_{+,4d} \Leftrightarrow M < M_p^4/M_*^3$, which if we take $M_* \sim 10^9$ GeV implies that $M < 10^{45}$ GeV and $r_{+,5d} < 1$ μm. This in turn entails that for the length scale of interest, the outer horizon of a 5D RN black hole is larger than the corresponding 4D black hole. If this were the case, then the electric field strength in the outer horizon would be smaller and it would be easier to suppress Schwinger production pairs in five than in four dimensions.

V. HIGHER-DIMENSIONAL NEAR-EXTREMAL BLACK HOLE DECAY RATE

The suppression of the near-extremal black hole decay rate with respect to that of Schwarzschild black holes of the same mass advertised in the Introduction is evident in the order of magnitude calculation that follows.

For a d-dimensional spacetime, the relation between the black hole entropy S and its mass M is [65]

$$S = 4\pi M r_s/(d-2) \sim (M/M_*)^{(d-2)/(d-3)}. \quad (17)$$

For a Schwarzschild black hole, the temperature scales with entropy as

$$T_s \sim M_* S^{-1/(d-2)} \quad (18)$$

and the black hole decay rate scales as

$$\Gamma_s \sim T_s. \quad (19)$$

For near-extremal black holes, however, the temperature scales as

$$T_{ne} \sim \frac{c}{S} \quad (20)$$

where $c = \sqrt{M^2 - (e'Q)^2 M_*^{d-2}/(4\pi)}$ [66]. For $(e'Q)M_*^{(d-2)/2}/(2M\sqrt{\pi}) \ll 1$, it follows that $c \sim M$, which leads to the nonextremal relation between S and c, i.e., $c \sim M_* S^{(d-3)/(d-2)}$. However, for the near-extremal case with $M \sim (e'Q)M_*^{(d-2)/2}/\sqrt{4\pi}$, the scaling of the c and of the temperature in terms of S is considered in [67], and one has to expand the square root to see that the leading term cancels and the subleading term provides

$$c \sim M_* \sqrt{\beta} S^{(d/2-2)/(d-2)}, \quad (21)$$

which in turn leads to

$$T_{ne} \sim M_* \sqrt{\beta} S^{-d/(2d-4)} = M_* S^{-1/(d-2)} \sqrt{\beta/S}, \quad (22)$$

with $M = M_* S^{(d-3)/(d-2)}(1 + 2S^{-1})$ and $e'QM_*^{(d-2)/2}/\sqrt{4\pi} = M_* S^{(d-3)/(d-2)}[1 + (2+\beta)S^{-1}]$, and where β is an order-one parameter that controls the differences between the masses and charges of particle species and hence also the difference between mass and charge of the associated near-extremal black hole. Therefore,

$$T_{ne} = T_s \sqrt{\beta/S} \tag{23}$$

and so it follows that

$$\Gamma_{ne} \sim T_{ne} = \sqrt{\beta/S}\Gamma_s. \tag{24}$$

Altogether, the evaporation rate of near-extremal black holes would be suppressed by a factor of $\sqrt{\beta/S}$ with respect to that of Schwarzschild black holes of the same mass.

Next, in line with our stated plan, we investigate how near-extremal black holes could modify the PBH range given in Eq. (9). To do so, we consider a black hole with $M \sim 10^5$ g. From (4) we see that such a black hole has a temperature $T_s \sim 4$ GeV. This means that $c_i(T_s)$ receives a contribution of 2 from photons, 6 from neutrinos, 12 from charged leptons (electrons, muons, and taus), 48 from quarks (up, down, strange, and charm), and 24 from gluons, yielding $\sum_i c_i(T_s)\tilde{f}\Gamma_s = 45$. Substituting these figures into (6) we find that the lifetime of a 10^5 g Schwarzschild black hole is $\tau_s \sim 10^{-5}$ yr. For a near extremal black hole of the same mass, the temperature would be $T_{ne} \sim 10^{-5}\sqrt{\beta}$ eV, where we have used (23). Bearing this in mind we find that the lifetime of the near-extremal black hole would be $\tau_{ne} \sim 15/\sqrt{\beta}$ Gyr. Now, the temperature of the near-extremal black hole is below the CMB temperature and hence there are no constraints from electromagnetic signals. The bound simply comes from the black hole survival probability. Then, a rough order of magnitude estimate suggests that if there were 5D primordial near-extremal black holes in nature, then a PBH all-dark-matter interpretation would be possible in the mass range

$$10^5\sqrt{\beta} \lesssim M/\mathrm{g} \lesssim 10^{21}. \tag{25}$$

Note that by tuning the β parameter we can have a PBH all-dark-matter interpretation with very light 5D black holes. Note also that

$$\hat{c} = c/M \sim \sqrt{\beta/S}, \tag{26}$$

which quantifies the near-extremality, is very small because of the large entropy.

VI. CONCLUSIONS

We have studied the decay rate of near-extremal black holes within the context of the dark dimension. Using generic entropy arguments we have demonstrated that Hawking evaporation of higher-dimensional near-extremal black holes proceeds at a slower rate than the corresponding Schwarzschild black holes of the same mass. Armed with our findings we have shown that if there were 5D primordial near-extremal black holes in nature, then a PBH all-dark-matter interpretation would be possible in the mass range $10^5\sqrt{\beta} \lesssim M/\mathrm{g} \lesssim 10^{21}$, where β is a parameter that controls the difference between mass and charge of the associated near-extremal black hole.

The possible existence of near-extremal PBHs evaporating today remains an open question. We have discussed herein an interesting possibility in which the black hole is charged under a generic unbroken $U(1)$ symmetry of the dark dimension, whose carriers are always much heavier than the temperature of the black hole, and so the charge does not get evaporated away from the black hole and remains therefore constant. Alternatively, it has been speculated in [68] that PBHs may have been formed with a spin above the Thorne's limit $a_* < 0.998$ of astrophysical objects [69], and actually near the Kerr extremal value $a_* < 1$ set by the third law of thermodynamics [70], where $a_* = a/M$, with $a \equiv J_k/M$ the spin parameter and J_k the black hole angular momentum. If this were the case, then PBHs may be still spinning today. Further investigation along these lines is obviously important to be done.

We end with an observation. The spectrum of graviton emission from black hole evaporation peaks at a frequency which is an order one factor times the temperature of a Schwarzschild black hole, $\omega_{\mathrm{peak}} \sim T_s$ [71]. For ultralight black holes $M \sim 10M_*$, the spectrum peaks at $\omega_{\mathrm{peak}} \sim M_*(M_*/M)^{1/2} \sim 10^8$ GeV. It was recently speculated that in scenarios with large extra-dimensions graviton emission from ultralight PBHs may be observed by future gravitational wave detectors [46]. Here we generalized the estimate of [46] to the dark dimension scenario. First, we note that after accounting for the redshift in energy density and frequency due to the cosmological expansion between evaporation and today the gravitational wave spectrum of a $10M_*$ PBH would have a peak at a frequency of $10^{12} \lesssim f/\mathrm{Hz} \lesssim 10^{14}$; see Fig. 4 of Ref. [46]. This frequency is in the range of JURA [72] and OSQAR II [73] experiments. Second, the gravitational wave energy density can be estimated from Fig. 5 of Ref. [46] and is given by $10^{-8} < \Omega_{\mathrm{GW}} h^2 < 10^{-6}$. Finally, we note that such a gravitational wave energy density is orders of magnitude below the current sensitivity of JURA and OSQAR II [71].

ACKNOWLEDGMENTS

The work of L. A. A. is supported by the U.S. National Science Foundation (NSF Grant No. PHY-2112527). The work of D. L. is supported by the Origins Excellence Cluster and by the German-Israel-Project (DIP) on Holography and the Swampland. I. A. is supported by the Second Century Fund (C2F), Chulalongkorn University.

APPENDIX

For completeness, in this Appendix we provide a concise summary of the salient characteristics of 4D RN-dS black holes. For $d = 4$, the blackening function (13) is given by

$$U(r) = 1 - \frac{2GM}{r} + \frac{(e'Q)^2 G}{4\pi r^2} - \frac{r^2}{\ell_4^2}. \tag{A1}$$

With the change of variables $\mathcal{M} = GM$ and $\mathcal{Q} = \sqrt{G}Qe'/\sqrt{4\pi}$ we can rewrite (A1) in the compact form

$$U(r) = 1 - \frac{2\mathcal{M}}{r} + \frac{\mathcal{Q}^2}{r^2} - \frac{r^2}{\ell_4^2}. \tag{A2}$$

4D RN-dS configurations generally admit three horizons, which are located at $r = r_h$ where (A2) vanishes, i.e., $U(r)|_{r=r_h} = 0$, yielding a quartic polynomial. The number of real roots is dictated by the sign of the discriminant locus D of the quartic polynomial

$$\frac{D}{16} = \frac{\mathcal{M}^2}{\ell_4^2} - \frac{\mathcal{Q}^2}{\ell_4^2} - \frac{27\mathcal{M}^4}{\ell_4^4} + \frac{36\mathcal{M}^2\mathcal{Q}^2}{\ell_4^4} - \frac{8\mathcal{Q}^4}{\ell_4^4} - \frac{16\mathcal{Q}^6}{\ell_4^6}. \tag{A3}$$

For $D \geq 0$, the quartic polynimial has four real-valued roots. However, one of them is always negative and therefore unphysical. Then, the spacetime can have a maximum of three causal horizons, which are dubbed: the Cauchy (a.k.a. inner) horizon r_-, the event (a.k.a. outer) horizon r_+, and the cosmological horizon r_c. Note that r_+ and r_c are the horizons which are accessible to an observer outside of the black hole.

Following [74], we define the phase space of 4D RN-dS black holes as the 3D parameter space spanned by the mass $\mathcal{M}$, charge $\mathcal{Q}$ and de Sitter radius ℓ_4. To respect the cosmic censorship conjecture [75], we require $\mathcal{M} \geq 0$ and $D \geq 0$, which ensures that all three horizons are real and satisfy $r_- \leq r_+ \leq r_c$. We also require $\ell_4 \geq 0$ to exclude AdS. We refer to the region that respects these conditions as physical phase space D.

The confluence of two or the three horizons defines an extremal limit at the boundary of the physical phase space, that we denote by ∂D and is characterized by $D = 0$. There are three extremal limits dubbed cold ($r_- = r_+$), Narai ($r_+ = r_c$) and ultracold ($r_- = r_+ = r_c$). The near horizon geometry for each of the extremal limits are $\mathrm{AdS}_2 \times S^2$, $\mathrm{dS}_2 \times S^2$, and $\mathrm{Mink}_2 \times S^2$, see e.g. [76]. In Fig. 1 we show the space of 4D RN-dS solutions. The shaded area is usually referred to as "shark fin" due to its shape.

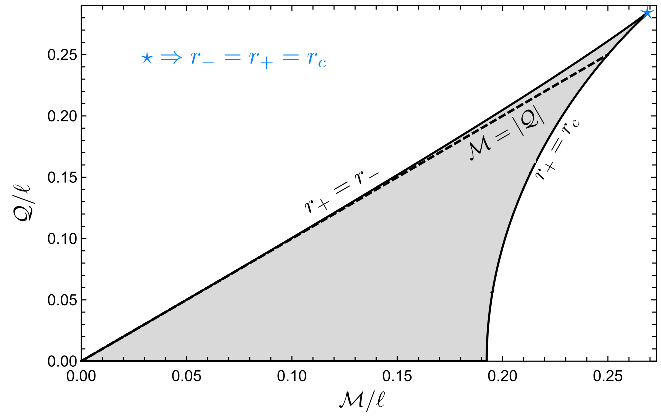

FIG. 1. The family of 4D RN-dS black holes. The gray shaded region represents the physical phase space of subextremal solutions. The boundary of this allowed region has two branches: the left (or cold) branch corresponds to RN-dS extremal black holes and the right branch corresponds to charged Nariai black holes, for which the event and cosmological horizons coincide. The blue star where the two branches intersect, stands for the ultracold solution. The dashed line indicates the lukewarm solutions with $\mathcal{M} = |\mathcal{Q}|$, where the Cauchy and event horizons have the same temperature. The $\mathcal{Q} = 0$ axis of neutral black holes indicates a big crunch singularity. Adapted from [59].

The ultracold near-extremal limit of the shark fin diagram is a moduli space point that represents Minkowski spacetime and lies at an infinite distance of any other spacetime independently of the geodesic path used to reach it [77]. The distance to the ultracold geometry is then consistent with the AdS-DC [6]. On the other hand, the geometric distance of any spacetime in the 4D RN-dS family to the origin is finite [77]. This implies that black holes will evaporate back to empty de Sitter space if the FL bound is satisfied.

On the other hand, in the small curvature limit the dS-WGC implies that there is at least one state with mass m and charge q satisfying

$$m^2 < \frac{(e'q)^2}{4\pi G} - \frac{(e'q)^4}{12\pi^2\ell_4^2} - \frac{G(e'q)^6}{32\pi^3\ell_4^4} + \mathcal{O}\left(\frac{1}{\ell_4^6}\right), \tag{A4}$$

which reproduces the known WGC in flat space

$$e'q > \sqrt{4\pi G}m\left(1 + \frac{G^2m^2}{2\ell_4^2} + \cdots\right), \tag{A5}$$

for $\ell_4 \to \infty$ [61].

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