Dark dimension and the standard model landscape

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We study the landscape of lower-dimensional vacua of the standard model (SM) coupled to gravity in the presence of the so-called "dark dimension" of size $R_\perp$ in the micron range, focusing on the validity of the swampland conjecture forbidding the presence of nonsupersymmetric anti–de Sitter (AdS) vacua in a consistent quantum gravity theory. We first adopt the working assumption that right-handed neutrinos propagate in the bulk, so that neutrino Yukawa couplings become tiny due to a volume suppression, leading to naturally light Dirac neutrinos. We show that the neutrino Kaluza-Klein (KK) towers compensate for the graviton tower to maintain stable de Sitter (dS) vacua found in the past, but neutrino oscillation data set restrictive bounds on $R_\perp$ and therefore the first KK neutrino mode is too heavy to alter the shape of the radion potential or the required maximum mass for the lightest neutrino to carry dS rather than AdS vacua found in the absence of the dark dimension, $m_{1,\max} \lesssim 7.63$ meV. We also show that a very light gravitino (with mass in the meV range) could help relax the neutrino mass constraint $m_{1,\max} \lesssim 50$ meV. The differences for the predicted total neutrino mass $\sum m_\nu$ among these two scenarios are within reach of next-generation cosmological probes that may measure the total neutrino mass with an uncertainty $\sigma(\sum m_\nu) = 0.014$ eV. We also demonstrate that the KK tower of a very light gravitino can compensate for the graviton tower to sustain stable dS vacua and thus right-handed neutrinos can (in principle) be locked on the brane. For this scenario, Majorana neutrinos could develop dS vacua, which is not possible in the SM coupled to gravity. Finally, we investigate the effects of bulk neutrino masses in suppressing oscillations of the zero modes into the first KK modes to relax the oscillation bound on $R_\perp$.

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I. INTRODUCTION

Far off in the infrared, well below the electron mass threshold m_e, the structure of the standard model (SM) is really simple: it can be characterized by 4 bosonic degrees of freedom (2 from the photon and 2 from the graviton) plus 6 or 12 fermionic degrees of freedom depending on whether neutrinos are Majorana or Dirac, respectively. The other mass scale pertinent to the SM infrared world is the cosmological constant, $\Lambda \sim 10^{-120} M_p^4 \sim (0.25 \times 10^{-2}\ \mathrm{eV})^4$, where $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass.

Even though we do not know yet the transformation properties of the neutrinos under particle-antiparticle conjugation (i.e., whether neutrinos are Majorana or Dirac), other sectors of the worldwide neutrino program have reached precision stage. Data analyses from short- and long-baseline neutrino oscillation experiments, together with observations of neutrinos produced by cosmic ray collisions in the atmosphere and by nuclear fusion reactions in the Sun, provide the most sensitive insights to determine the extremely small mass-squared differences. Neutrino oscillation data can be well fitted in terms of two nonzero differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$ between the squares of the masses of the three ($i = 1, 2, 3$) mass eigenstates m_i; namely, $\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5}$ and $\Delta m_{32}^2 = (2.453 \pm 0.033) \times 10^{-3}\ \mathrm{eV}^2$ or $\Delta m_{32}^2 = (-2.536 \pm 0.034) \times 10^{-3}\ \mathrm{eV}^2$ [1]. In addition, the total neutrino mass $\sum m_\nu \equiv \sum_{i=1}^3 m_i$ can be determined (or more restrictively bounded) by analyzing the impact of cosmological relic neutrinos on the growth of structure formation. Assuming a Λ cold dark matter (CDM) cosmology, *Planck* temperature and polarization data lead to $\sum m_\nu < 0.26$ eV, but when the observations of the cosmic microwave background (CMB) are complemented with those of baryon acoustic oscillations (BAO) the bound becomes more restrictive

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$\sum m_\nu < 0.13$ eV [2]. Moreover, when CMB + BAO data are supplemented with supernovae type Ia luminosity distances and confronted with determinations of the growth rate parameter, the upper limit translates to $\sum m_\nu < 0.09$ eV [3]; see also [4,5]. Putting all of this together, we arrive at an intriguing experimental fact: the scale of neutrino masses, $m_i \lesssim 10^{-2}$ eV, is not far from that of the observed vacuum energy $\Lambda \sim m_i^4$. This happenstance could be the carrier of fundamental information on the possible connections between particle physics, cosmology, and quantum gravity.

The SM coupled to gravity has a unique four-dimensional vacuum (although possible metastable), but it has long been known that there may also exist lower-dimensional vacua stabilized by the Casimir energies of particles with masses $\ll m_e$ [6]. Such vacua can have both de Sitter (dS) as well as anti–de Sitter (AdS) geometries and their nitty gritty depends sensitively on the value of neutrino masses. In particular, if all neutrinos were Majorana and we compactify the low-energy effective theory down to three or two dimensions, then AdS SM vacua would appear for any values of neutrino masses consistent with experiment. It is noteworthy that these lower-dimensional vacua are virtually indistinguishable from the SM vacuum at distances $\gtrsim 30$ μm.

A seemingly different, but in fact closely related, subject has been the development of the Swampland program that lays out a set of constraints to distinguish effective theories which can be consistently coupled to quantum gravity in the ultraviolet (UV) from those which cannot [7]. These constraints have been formulated in the form of swampland conjectures [8–10]. A well-known swampland conjecture is the absence of nonsupersymmetric (SUSY) AdS vacua supported by fluxes in a consistent quantum gravity theory [11]. This conjecture, if correct, implies that if AdS SM lower dimensional vacua exist and are stable, then the four-dimensional SM itself could not be completed in the UV. Automatically, the conjecture then also implies that the minimal SM setting with Majorana neutrinos would be excluded. If neutrinos are Dirac, however, the conjecture constrains the mass of the lightest neutrino state, $m_i \lesssim \Lambda^{1/4}$ [12].[1] But of course, to avoid AdS vacua one can always extend the mass spectrum of the low-energy effective theory by adding fermionic degrees of freedom in the deep infrared region, e.g., from a very light gravitino [12]. In plain English, Majorana neutrinos, which in the SM are not consistent with the bounds from absence of AdS vacua, can be rescued by a very light gravitino, keeping the attractive seesaw mechanism for neutrino masses active. The requirements to avoid the instability of non-SUSY AdS vacua have been established in the so-called light fermion conjecture [14].

Another interesting aspect of the Swampland program is considerations regarding the behavior of effective theories with a cosmological constant. In particular, the distance conjecture [15] when generalized to dS space [16] suggests that the smallness of dark energy could signal a universe living at the boundary of the field space in quantum gravity with a proper distance given by $-\ln|\Lambda|$, in Planck units. A universal feature of these asymptotic corners in the string landscape of vacua is that they predict a light infinite tower of Kaluza-Klein (KK) states whose mass $m_{\rm KK}$ is correlated to Λ. Actually, by combining the generalized distance conjecture for dS with observational data, the smallness of the cosmological constant and astrophysical constraints led to a scenario with one mesoscopic dimension of micron scale [17]. This extra dimension, dubbed the dark dimension, opens up at the scale $m_{\rm KK} \sim \lambda^{-1}\Lambda^{1/4}$ of the tower, where the proportionality factor is estimated to be within the range $10^{-4} \lesssim \lambda \lesssim 10^{-1}$. Within this setup, the five-dimensional Planck scale (or species scale where gravity becomes strong [18,19]) is $\Lambda_{\rm QG} \sim m_{\rm KK}^{1/3} M_p^{2/3} \simeq 10^9$ GeV.

The dark dimension scenario enjoys a rich phenomenology:

(i) It provides a natural set up for right-handed neutrinos propagating in the bulk [17]. Within this framework we expect neutrino masses to occur in the range $10^{-4} < m_i/\text{eV} < 10^{-1}$, despite the lack of any fundamental scale higher than $\Lambda_{\rm QG}$. The suppressed neutrino masses are not the result of a seesaw mechanism, but rather because the bulk modes have couplings suppressed by the volume of the dark dimension (akin to the weakness of gravity at long distances) [20–24].

(ii) It encompasses a framework for primordial black holes [25,26] and KK gravitons [27] to emerge as interesting dark matter candidates.

(iii) It also encompasses an interesting framework for studying cosmology [28,29] and astroparticle physics [30,31].

(iv) It provides a profitable arena to accommodate a very light gravitino [32].

In light of this rich phenomenology that connects the various topics described above, in this paper we examine the landscape of three-dimensional vacua obtained from compactifying the SM to three dimensions in the presence of the dark dimension. The precise geometry (dS, AdS, or Minkowski) is driven by competing contributions to the effective lower-dimensional potential. The classical contributions include the four-dimensional cosmological constant and the curvature terms resulting from dimensional reduction, while the quantum contributions are determined by the Casimir energies of SM particles, as well as of KK excitations of fields propagating in the dark dimension. The fermionic degrees of freedom we consider in our study are those of left- and right-handed neutrinos with and without

[1]We note in passing that other swampland conjectures applied to the same class of lower-dimensional SM vacua lead to similar constraints on neutrino masses [13].

bulk masses, a very light gravitino, and the KK towers associated to the bulk fields.

The layout of the article is as follows. In Sec. II we review the 3D vacua obtained in the SM coupled to gravity from the interplay of Casimir forces and the cosmological constant. In Sec. III we describe the general structure of the dark dimension scenario, focusing on the 5D gravity sector along with the degrees of freedom associated to the zero modes and their corresponding KK towers that pop up in the 4D low-energy effective theory. After that, assuming bulk right-handed neutrinos, in Sec. IV we discuss upper limits on the lightest neutrino mass obtained by balancing bosonic and fermionic degrees of freedom of the effective radion potential, while imposing at the same time the absence of AdS vacua. In Sec. V we analyze how the presence of a very light gravitino could help modify the upper limit on the mass of the lightest neutrino state. In Sec. VI we analyze the effects of bulk neutrino masses in suppressing oscillations of the zero modes into the first KK modes, while equalizing the KK bosonic towers to those associated with the neutrino fields. We reserve Sec. VII for our conclusions.

II. COMPACTIFYING THE SM ON A CIRCLE

Consider the action of general relativity (GR) compactified on a circle of radius R,

$$\begin{aligned} S_{\rm GR} &= \int d^3x d\phi \sqrt{-g_{(4)}}\left(\frac{1}{2}M_p^2\mathcal{R}_{(4)} - \Lambda_4\right) \\ &\to \int d^3x \sqrt{-g_{(3)}}(2\pi r)\left[\frac{1}{2}M_p^2\mathcal{R}_{(3)} - \frac{1}{4}\left(\frac{R}{r}\right)^4 V_{\mu\nu}V^{\mu\nu} \right. \\ &\quad \left. - M_p^2\left(\frac{\partial R}{R}\right)^2 - \Lambda_4\left(\frac{r}{R}\right)^2\right], \end{aligned} \quad (1)$$

where $g_{(d)}$ is the determinant of the d-dimensional metric tensor, $\mathcal{R}_{(d)}$ the d-dimensional Ricci scalar, $V_{\mu\nu}$ the field strength of the graviphoton, $0 \leq \phi < 2\pi$, Λ_4 is the 4D cosmological constant, and where r is an arbitrary scale that we fix to the expectation value of the radion field R. For distances larger than R, there is an effective 3D theory with metric parametrized by

$$ds^2_{(4)} = \frac{r^2}{R^2}ds^2_{(3)} + R^2\left(d\phi^2 - \sqrt{2}M_p r V_\mu dx^\mu\right), \quad (2)$$

where V_μ is the graviphoton. From (1) it is straightforward to see that the classical potential of the radion coming from the 4D cosmological constant,

$$V_C(R) = 2\pi r\left(\frac{r}{R}\right)^2 M_p^2\Lambda_4 = 2\pi r\left(\frac{r}{R}\right)^2\Lambda, \quad (3)$$

is runaway, and makes the circle decompactify.

Nevertheless, Λ_4 is so tiny that quantum corrections to the vacuum energy from the lightest SM modes could become important to stabilize the radion potential. The one-loop corrections to $V_C(R)$ are driven by the Casimir energy (inferred from loops wrapping the circle) of the lightest SM particles, which are UV insensitive and have been calculated in [6].

Altogether, if we compactify the SM + GR on a circle, the radion gets an effective potential of the form

$$V(R) = V_C(R) + \sum_i V_i(R), \quad (4)$$

where V_i denotes the contribution from the one-loop Casimir energy of the particle i. For a particle of mass m_i with N_i degrees of freedom, the contribution to the potential is given by

$$V_i(R) = (-1)^{s_i}\frac{N_i r^3 m_i^2}{4\pi^3 R^4}\sum_{n=1}^{\infty}\frac{K_2(2\pi R m_i n)}{n^2}\cos(n\theta_i), \quad (5)$$

where $s_i = 0(1)$ for fermions (bosons), θ_i is an angle defining the periodicity around the circle by a phase $e^{2\pi i\theta}$, and

$$K_\nu = \frac{1}{2}\int_0^\infty d\beta\, \beta^{\nu-1}\exp\left[-\frac{z}{2}\left(\beta + \frac{1}{\beta}\right)\right] \quad (6)$$

is the Bessel function.

For massless particles,

$$V_i(R) = 2\pi r\rho_i(R)\left(\frac{r}{R}\right)^2, \quad (7)$$

with

$$\rho_i(R) = (-1)^{s_i}\frac{1}{16\pi^6 R^4}\mathrm{Re}[\mathrm{Li}_4(e^{i\theta_i})], \quad (8)$$

where

$$\mathrm{Li}_n(z) = \sum_{k=1}^{\infty}\frac{z^k}{k^n} \quad (9)$$

is the polilogarithm. Throughout we consider particles with periodic boundary conditions; namely $\theta_i = 0$. A relevant relation is then $\mathrm{Li}_n(1) = \zeta(n)$, where $\zeta(z)$ is the Riemann zeta-function. Note that for a massive particle m_i, the Casimir energy density is exponentially suppressed by a factor of the form $\exp(-2\pi R m_i)$ at large m_i, and therefore only the light particles have to be considered in the sum of (4). All in all, in the case of the SM spectrum, we have:

(i) 2 massless bosonic degrees of freedom for the photon,

(ii) 2 massless bosonic degrees of freedom for the graviton,

(iii) 4 (2) fermionic degrees of freedom for each of the three Dirac (Majorana) neutrinos of masses m_1, m_2 and m_3.

The effective potential (4) can be recast as

$$V(R) \simeq V_C(R) - 4\left(\frac{r^3}{720\pi R^6}\right) + \sum_i \frac{N_i}{720\pi}\frac{r^3}{R^6}\Theta(R_i - R), \tag{10}$$

where $R_i = 1/m_i$ and $\Theta(x)$ is the step function, with $i = 1$, 2, 3. Note that in (10) we only take into account (nearly) massless 4D states and look for a 3D vacuum of toroidal compactification, where the 4D graviphoton is projected out. Note also that if we only consider the first two terms in the potential, $V(R)$ develops a maximum at

$$R_{\rm max} = \left(\frac{1}{120\pi^2\Lambda}\right)^{1/4} \simeq 11\ \mu\text{m}, \tag{11}$$

corresponding to a mass scale

$$m_{\rm max} = \frac{1}{2\pi R_{\rm max}} \simeq 2.11\ \text{meV}, \tag{12}$$

which is below about the neutrino mass scale. Then, as the value of R decreases the various neutrino thresholds open up and sooner or later overwhelm the bosonic contribution to $V(R)$. Thus, provided $R_i < R_{\rm max}$ the effective radion potential would develop minima. As can be seen from Eqs. (3) and (10), r is an overall normalization scale which does not influence the nature of AdS or dS vacua, and so following [6] in our calculations we set $2\pi r = 1\ \text{GeV}^{-1}$.

Before proceeding, we pause to note that matter effects provide the only means by which we can determine the sign of Δm_{ij}^2. Indeed, because of matter effects in the Sun, we know that $\Delta m_{21}^2 > 0$. However, the atmospheric mass splitting Δm_{32}^2 is essentially measured only via neutrino oscillations in vacuum and, as noted in the Introduction, its sign is unknown. This implies that as of today it is not possible to decide whether the ν_3 neutrino mass eigenstate is heavier or lighter than the ν_1 and ν_2 eigenstates. The scenario, in which the ν_3 is heavier, is referred to as the normal mass hierarchy or normal ordering (NO). The other scenario, in which the ν_3 is lighter, is referred to as the inverted mass hierarchy or inverted ordering (IO). It has been argued that the latest cosmological constraint, $\sum m_\nu < 0.09$ eV, provides Bayesian evidence for the NO [33]. However, whether Bayesian suspiciousness is enough to disfavor the IO is still a matter of debate, see e.g. [34,35]. Moreover, some cosmological parameters are correlated with the total neutrino mass, and so in beyond ΛCDM models that tend to ameliorate the Hubble constant tension the bound on $\sum m_\nu$ could be relaxed, see e.g. [36]. Herein, we will consider the two possibilities: for NO, we have $m_1 < m_2 < m_3$, whereas for IO, we have $m_3 < m_1 < m_2$.

Now, depending on the nature and on the masses of the neutrinos we can obtain different types of SM vacua. As an illustration, in Fig. 1 we show the landscape of vacua for Dirac neutrinos with normal ordering. The required maximum mass on the lightest neutrino state to avoid AdS vacua is $m_{1,\rm max} = 7.63$ meV. If neutrinos were Majorana particles, then AdS vacua would appear for any values of neutrino masses consistent with experiment. Hence, the AdS non-SUSY conjecture rejects the case of Majorana neutrinos if the low-energy effective theory is SM + GR.

It is well known that models in which the observed weakness of gravity at long distances is due to the existence of compact spatial dimensions [37,38] provide a compelling framework to explain Dirac neutrino masses [20–24]. Hence, it is of interest to investigate how new degrees of freedom that open up in 5D models of neutrino physics would modify the shape of $V(R)$.

III. EXTENDING THE LOW-ENERGY EFFECTIVE THEORY

We consider the compactification framework of [37,38], in which it is natural to assume that, in the case of an

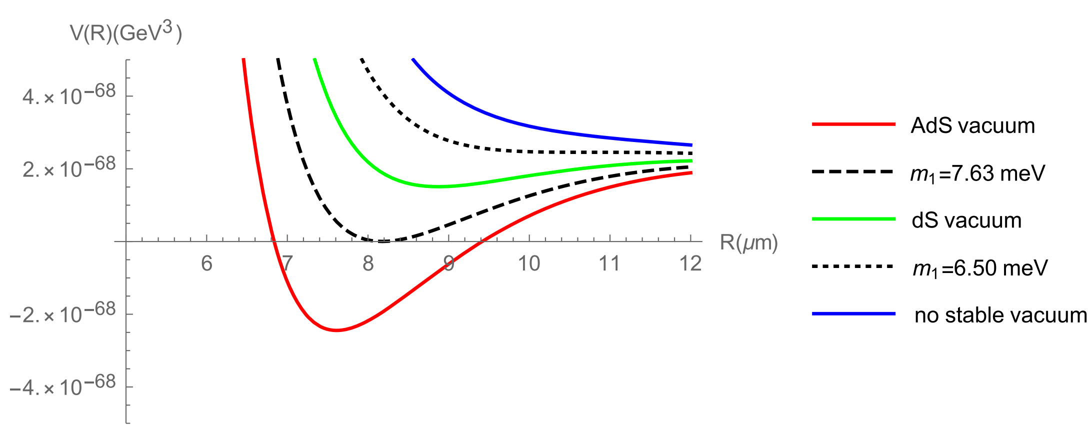

FIG. 1. Effective radion potential with Dirac neutrinos and NO.

orbifold compactification, the particles charged under the SM gauge group are locked on a 3-brane. Because SM gauge and matter fields live on the brane, the only long-range interaction which sees the extra dimensions is gravity. Herein, we restrict ourselves to the case of one mesoscopic extra dimension characterized by a length scale in the micron range, dubbed the dark dimension [17]. Actually, null results from searches of deviations from Newton's gravitational inverse-square law place an upper bound on the compactification radius, $R_\perp < 30\ \mu\text{m}$ [39].

A point worth noting at this juncture is that a connection has been made elsewhere [32] between the dark dimension and the scale of SUSY breaking. As shown in [32], the gravitino mass $m_{3/2}$ and the scale of SUSY breaking can directly be determined from the dark energy density. Furthermore, as also shown in [32], the dark dimension provides a cost-effective background to host a very light gravitino. Since this gravitino would modify the mass spectrum of the effective theory in the deep infrared region, it is of interest to study how the extra degrees of freedom modify the maximum and minima of the effective radion potential.

With this in mind, herein we consider 5D supergravity, which contains 8 bosonic degrees of freedom (5 for the graviton and 3 for a gauge field) and 8 fermionic degrees of freedom (2×4 for two gravitinos). From a 4D perspective, the degrees of freedom in the bosonic sector are: 2 for the graviton $+2$ for the graviphoton $+1$ for the radion $+2$ for the gauge field $+1$ for an extra scalar. This corresponds to $\mathcal{N} = 2$ SUSY: graviton + vector multiplet, each containing one Dirac spinor (2 gravitinos $+2$ Weyl fermions) + their KK excitations. The orbifold breaks SUSY to $\mathcal{N} = 1$. At the massive level the spectrum is divided by 2 (with cosine and sine wave functions). At the massless level there is a projection to $\mathcal{N} = 1$ leading to 4 bosonic and 4 fermionic degrees of freedom: spin 2 multiplet + a chiral multiplet counting the radion, its pseudoscalar partner, and the goldstino. SUSY breaking makes the gravitino massive by absorbing the goldstino and yields two scalars with different masses (of course all are set by $m_{3/2}$). The content of the gravity multiplet is summarized in Table I.

In our analysis we first drop the extra scalar assuming it becomes heavier and keep only the radion. We analyze two scenarios, one in which the radion is very light and another in which the radion is heavy and does not partake in carving $V(R)$. Then we consider that the pseudoscalar partner of the radion (the axion) is also light. The graviphotons Z_μ and $A_\mu^{(0)}$ are taken into account at the massive level, because they have a sine wave function that vanishes at the brane position and therefore there is no contribution of the zero modes to $V(R)$. In Sec. IV we will consider that the gravitino is heavy and plays no role in the shape of $V(R)$. After that in Sec. V we study the impact of a very light gravitino on the determination of the effective radion potential.

TABLE I. Gravity multiplet content.

	Bosons					Fermions			
5D									
Field	g_{MN}			A_M		$\psi_{1,M}$		$\psi_{2,M}$	
Spin	2			1		3/2		3/2	
dof	5			3		4		4	
m	0			0		0		0	
$\downarrow$ compactification on $\mathcal{S}^1$ $\downarrow$									
4D zero modes ($\mathcal{N} = 2$)									
Field	$g_{\mu\nu}^{(0)}$	Z_μ	$R_\perp$	$A_\mu^{(0)}$	$A_\perp$	$\psi_{1,\mu}^{(0)}$	$\psi_{1,\perp}$	$\psi_{2,\mu}^{(0)}$	$\psi_{2,\perp}$
Spin	2	1	0	1	0	3/2	1/2	3/2	1/2
dof	2	2	1	2	1	2	2	2	2
m	0	0	0	0	0	0	0	0	0
4D KK modes $(n \in \mathbb{N}^*)$									
Field	$g_{\mu\nu}^{(n)}$			$A_\mu^{(n)}$		$\psi_{1,\mu}^{(n)}$		$\psi_{2,\mu}^{(n)}$	
Spin	2			1		3/2		3/2	
dof	5			3		4		4	
m	$n/R_\perp$			$n/R_\perp$		$n/R_\perp$		$n/R_\perp$	
$\downarrow$ with the action of $\mathbb{Z}_2$ and SUSY breaking $\downarrow$									
4D zero modes ($\mathcal{N} = 1$)									
Field	$g_{\mu\nu}^{(0)}$		$R_\perp$		$A_\perp$	$\psi_{1,\mu}^{(0)}$			
Spin	2		0		0	3/2			
dof	2		1		1	4			
m	0		$m_{R_\perp}$		$m_{A_\perp}$	$m_{3/2}$			
4D KK modes $(n \in \mathbb{N}^*)$									
Field	$g_{\mu\nu}^{(n)}$			$A_\mu^{(n)}$		$\psi_{1,\mu}^{(n)}$		$\psi_{2,\mu}^{(n)}$	
Spin	2			1		3/2		3/2	
dof	5			3		4		4	
m	n/R			n/R		$n/R + m_{3/2}$		$n/R + m_{3/2}$	

IV. BULK RIGHT-HANDED NEUTRINOS

Throughout this section we proceed on the working assumptions that gravitinos (and the SUSY mass spectrum) are heavy and that neutrino masses derive from three 5D fermion fields $\Psi_\alpha \equiv (\psi_{\alpha L}, \psi_{\alpha R})$, which are SM singlets and interact on our brane with the three active left-handed neutrinos $\nu_{\alpha L}$ and the Higgs doublet preserving lepton number, where the indices $\alpha = e, \mu, \tau$ indicate the generation 24–20]]. From the viewpoint of 4D observers on the brane, each of the singlet fermion fields can be decomposed into an infinite tower of KK states, $\psi^\kappa_{L(R)}$, with $\kappa = 0, \pm 1, \ldots, \pm\infty$. The right-handed fields ψ_R^κ combine with the left-handed bulk states ψ_L^κ to assemble Dirac mass terms, which come from the quantized internal momenta in the dark dimension. In addition, there is a mixing between the bulk states and the active left-handed neutrinos through Dirac-like mass terms. Note that the bulk fields can be redefined as $\nu_{\alpha R}^{(0)} \equiv \psi_{\alpha R}^{(0)}$ and $\nu_{\alpha L(R)}^{(n)} \equiv (\psi_{\alpha L(R)}^{(n)} + \psi_{\alpha L(R)}^{(-n)})/\sqrt{2}$, and so after electroweak symmetry breaking the mass terms of the Lagrangian read

$$\mathcal{L}_{\rm mass} = \sum_{\alpha,\beta} m^D_{\alpha\beta}\left[\bar{\nu}^{(0)}_{\alpha L}\nu^{(0)}_{\beta R} + \sqrt{2}\sum_{n=1}^{\infty}\bar{\nu}^{(0)}_{\alpha L}\nu^{(n)}_{\beta R}\right] + \sum_{\alpha}\sum_{n=1}^{\infty} m_n \bar{\nu}^{(n)}_{\alpha L}\nu^{(n)}_{\alpha R} + \text{H.c.} = \sum_{i=1}^{3}\bar{\mathbb{N}}_{iR}\mathbb{M}_i\mathbb{N}_{iL} + \text{H.c.}, \quad (13)$$

where $m^D_{\alpha\beta}$ is a Dirac mass matrix, $m_n = n/R_\perp = n m_{\rm KK}$,

$$\mathbb{N}_{iL(R)} = \left(\nu_i^{(0)}, \nu_i^{(1)}, \nu_i^{(2)}, \cdots\right)^T_{L(R)}, \quad \text{and}$$

$$\mathbb{M}_i = \begin{pmatrix} m_i^D & 0 & 0 & 0 & \cdots \\ \sqrt{2}m_i^D & 1/R_\perp & 0 & 0 & \cdots \\ \sqrt{2}m_i^D & 0 & 2/R_\perp & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (14)$$

and where m_i^D are the elements of the diagonalized Dirac mass matrix $= \text{diag}(m_1^D, m_2^D, m_3^D)$. Greek indices from the beginning of the alphabet run over the three active flavors ($\alpha, \beta = e, \mu, \tau$), Roman lower case indices over the three SM families ($i = 1, 2, 3$), and n over the KK modes ($n = 1, 2, 3, \ldots, +\infty$). Note that $\psi^{(0)}_{\alpha L}$ is projected out from the orbifold. For the configuration at hand,

$$m_i^D = \frac{y_i v}{\sqrt{\pi R_\perp \Lambda_{\rm QG}}}, \quad (15)$$

where y_is are the five-dimensional Yukawa couplings localized on the SM brane and where $v = 246/\sqrt{2}$ GeV $=$ 174 GeV is the Higgs vacuum expectation value. For the neutrinos, we therefore have 4 fermionic degrees of freedom for each $n \in \mathbb{N}$ and for each $i = 1, 2, 3$ leading to three towers of neutrinos of masses $m_i^{(n)} = \lambda_i^{(n)}/R_\perp$ where $\lambda_i^{(n)}$ are solutions of the transcendental equation [20–24],

$$\lambda_i^{(n)} - \pi\left(m_i^{\rm D} R_\perp\right)^2 \cot\left(\pi\lambda_i^{(n)}\right) = 0. \quad (16)$$

Next, we make contact with experiment to develop some sense for the orders of magnitude involved. Bearing that in mind, we impose the *cosmological constraint* on the sum of neutrinos masses. More precisely, on the sum of the three zero modes of the three neutrino towers,

$$\frac{\lambda_1^{(0)}}{R_\perp} + \frac{\lambda_2^{(0)}}{R_\perp} + \frac{\lambda_3^{(0)}}{R_\perp} < \sum m_\nu. \quad (17)$$

In addition, we consider the Δm^2_{ij} measured by neutrino oscillation experiments. We remind the reader that Δm^2_{ij} are the mass squared differences between the three zero modes of the three neutrinos towers. For NO, we can write

$$\left(\lambda_2^{(0)}\right)^2 = R_\perp^2 \Delta m^2_{21} + \left(\lambda_1^{(0)}\right)^2 \quad (18)$$

and

$$\left(\lambda_3^{(0)}\right)^2 = R_\perp^2 \Delta m^2_{32} + R_\perp^2 \Delta m^2_{21} + \left(\lambda_1^{(0)}\right)^2. \quad (19)$$

Note that the Δm^2_{ij}s constrain the parameter space due to the fact that we need the $\lambda_i^{(0)}$s to be smaller than $1/2$ in order to have solutions of (16). By imposing the Δm^2_{ij} constraint we arrive at

$$\left(\lambda_1^{(0)}\right)^2 < \frac{1}{4}, \quad (20)$$

$$R_\perp^2 \Delta m^2_{21} + \left(\lambda_1^{(0)}\right)^2 < \frac{1}{4}, \quad (21)$$

and

$$R_\perp^2 \Delta m^2_{32} + R_\perp^2 \Delta m^2_{21} + \left(\lambda_1^{(0)}\right)^2 < \frac{1}{4}. \quad (22)$$

Combining (17) with (20)–(22) we obtain

$$\lambda_1^{(0)} + \sqrt{R_\perp^2 \Delta m^2_{21} + \left(\lambda_1^{(0)}\right)^2} + \sqrt{R_\perp^2 \Delta m^2_{32} + R_\perp^2 \Delta m^2_{21} + \left(\lambda_1^{(0)}\right)^2} < R_\perp \sum m_\nu. \quad (23)$$

For the IO case, it is more convenient to keep $\lambda_3^{(0)}$ instead of $\lambda_1^{(0)}$ (because ν_3 is then the lightest neutrino) and the oscillation constraint is now

$$\left(\lambda_3^{(0)}\right)^2 < \frac{1}{4}, \quad (24)$$

$$-R_\perp^2 \Delta m^2_{21} - R_\perp^2 \Delta m^2_{32} + \left(\lambda_3^{(0)}\right)^2 < \frac{1}{4}, \quad (25)$$

and

$$-R_\perp^2 \Delta m^2_{21} + \left(\lambda_3^{(0)}\right)^2 < \frac{1}{4}. \quad (26)$$

Combining (17) with (24)–(26) we obtain

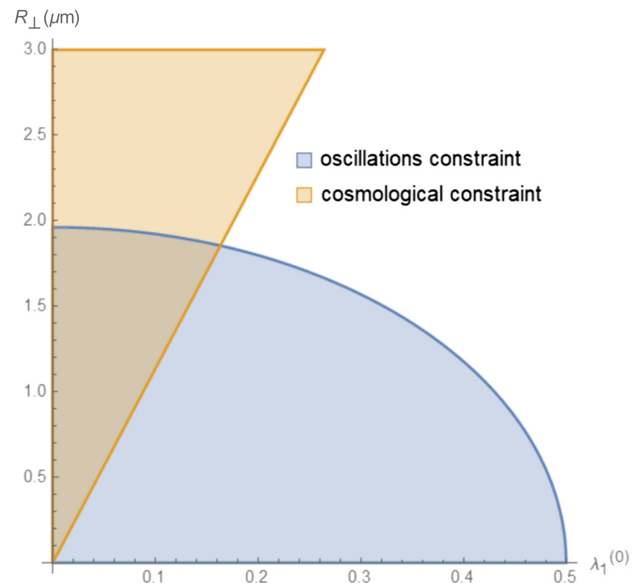

FIG. 2. Constraints imposed by measurements of Δm_{ij}^2 in oscillation experiments and the inferred $\sum m_\nu$ for NO via cosmological observations.

$$\lambda_3^{(0)} + \sqrt{-R_\perp^2 \Delta m_{32}^2 + \left(\lambda_3^{(0)}\right)^2} + \sqrt{-R_\perp^2 \Delta m_{21}^2 - R_\perp^2 \Delta m_{32}^2 + \left(\lambda_3^{(0)}\right)^2} < R_\perp \sum m_\nu. \quad (27)$$

Constraints on the $\lambda_1^{(0)} - R_\perp$ plane for the NO are encapsulated in Fig. 2. To estimate the allowed region of the parameter space we have adopted the ΛCDM cosmological constraint, $\sum m_\nu < 0.09$ eV. As previously noted, the ΛCDM cosmological constraint is in tension with the IO.

A more restrictive constraint arises from oscillations of the active zero modes into the first KK states of the towers. Such a disappearance effect has not been observed by neutrino oscillation experiments and so experimental data place a bound on the compactification radius, $R_\perp < 0.1$ μm [40,41]. This in turn implies that the first KK modes in the towers have a mass of at least $\mathcal{O}(10 \text{ eV})$. Consequently, the KK excitations are too massive to counterbalance the effect of the massless bosons, only affecting the radion potential for very small R. As a result, within this setup we would expect that the constraints on the maximum mass of the lightest neutrino coincide with those predicted by the SM + GR. Actually, the constraint on the neutrino mass should become stronger in the presence of a light radion field.

We have scanned the parameter space varying the radion mass $m_{R_\perp}$ and m_1 assuming the NO of neutrino masses and assuming the axion is heavy. If we further assume that the radion is heavy and does not partake in carving $V(R)$, then for $m_1 > 7.63$ meV an AdS vacuum is formed, whereas for $6.50 < m_1/\text{meV} < 7.63$ a dS vacuum is obtained, and if $m_1 < 6.50$ meV there is no vacuum. If we instead consider the opposite limit in which the radion is almost massless, we find that if $m_1 > 5.28$ meV an AdS vacuum is formed, while for $4.36 < m_1/\text{meV} < 5.28$ a dS vacuum is obtained, and if $m_1 < 4.36$ meV there is no vacuum. At this point a reality check is in order. Substituting the maximum mass of the lightest neutrino that can avoid an AdS vacuum ($m_{1,\max} = 7.63$ meV) and the upper limit of the compactification radius ($R_\perp = 10$ eV) into (15) and (16) we obtain $y_1 \sim 10^{-4}$.

We have also scanned the parameter space varying $m_{R_\perp}$ and m_3, but considering the IO of neutrino masses. In this case, if the radion is heavy the values of m_3 demarcating the transitions between geometries with no vacuum, with dS vacua, and with AdS vacua are as follows: (i) for $m_3 > 2.51$ meV an AdS vacuum is formed; (ii) for $2.04 < m_3/\text{meV} < 2.51$ a dS vacuum is obtained; (iii) for $m_1 < 2.04$ meV there is no vacuum. On the other hand, if the radion is light and becomes relevant in the determination of the critical points of $V(R)$, then the lightest neutrino must be massless, and the minimum radion mass to avoid an AdS vacuum is $m_{R_\perp} = 25.09$ meV, whereas to support a dS vacuum $m_{R_\perp} < 27.88$ meV.

Next, in line with our stated plan, we consider the case of NO Dirac neutrinos, assuming a massless radion and massless axion. For $m_1 > 2.82$ meV an AdS vacuum is formed, while for $1.90 < m_1/\text{meV} < 2.82$ a dS vacuum is obtained, and for $m_1 < 1.90$ meV there is no stable vacuum.

In summary, if we assume that right-handed neutrinos propagate in the bulk (so that the Yukawa couplings become tiny because of a volume suppression) then their KK towers can compensate for the graviton tower to avoid AdS vacua. However, neutrino oscillation data set restrictive bounds on $R_\perp$ and therefore the first KK neutrino mode is too heavy to alter the shape of the radion potential or $m_{1,\max}$ from those predicted by the SM + GR when compactified down to 3D.

In closing, we note that in Table I and in our general presentation of the mass spectrum in Sec. III, we made the assumption that the modulino is the goldstino; i.e., it is part of the massive zero mode of the gravitino. At this stage, it is worthwhile to point out that the above consideration is actually model dependent as, e.g., the goldstino could be the fermion of a chiral multiplet if we have F-term SUSY breaking. In models with high-scale SUSY breaking the gravitino and the modulino are heavy [42]. However, in models with low-scale SUSY breaking, the modulino could be very light. An interesting scenario emerges if the modulino is almost massless. On the one hand, if the radion and the axion are also almost massless, the modulino fermionic degrees of freedom get canceled by the bosonic degrees of freedom of the radion and axion, and the shape of the potential is the same as that considering a heavy goldstino with the radion and axion also as heavy particles.

TABLE II. Maximum gravitino mass necessary to avoid an AdS vacuum for Dirac neutrinos.[a]

NO			IO		
m_1	$m_{R_\perp}$	$m_{3/2}$	m_3	$m_{R_\perp}$	$m_{3/2}$
50 meV	No radion	2.51 meV	30 meV	No radion	3.42 meV
40 meV	No radion	2.98 meV	25 meV	No radion	3.94 meV
30 meV	No radion	3.77 meV	20 meV	No radion	4.72 meV
20 meV	No radion	5.39 meV	15 meV	No radion	6.02 meV
15 meV	No radion	7.19 meV	10 meV	No radion	8.77 meV
10 meV	No radion	12.16 meV	5 meV	No radion	18.65 meV
5 meV	No radion	Whatever	1 meV	No radion	Whatever
0 meV	No radion	Whatever	0 meV	No radion	Whatever
50 meV	Massless	Impossible	30 meV	Massless	Impossible
40 meV	Massless	Impossible	25 meV	Massless	Impossible
30 meV	Massless	Impossible	20 meV	Massless	Impossible
20 meV	Massless	1.86 meV	15 meV	Massless	2.67 meV
15 meV	Massless	4.12 meV	10 meV	Massless	5.38 meV
10 meV	Massless	7.56 meV	5 meV	Massless	10.59 meV
5 meV	Massless	Whatever	1 meV	Massless	17.94 meV
0 meV	Massless	Whatever	0 meV	Massless	18.60 meV

[a]Impossible means that there is always a stable AdS vacuum and whatever that there is no constraint.

On the other hand, it could be that the radion and the axion are heavy. If this were the case, for NO of Dirac neutrinos the existence of AdS vacua would be avoided if $m_1 <$ 14.49 meV and for IO if $m_3 < 11.14$ meV. This scenario (with a massless modulino and heavy radion and axion) also allows to avoid AdS vacua if neutrinos are Majorana particles: for NO the presence of AdS vacua can be avoided if $m_1 < 9.61$ meV and for IO if $m_3 < 3.43$ meV.

V. A VERY LIGHT GRAVITINO

In line with our stated plan, we now turn to consider the addition of a very light gravitino in the mass spectrum. Before proceeding, we pause to note that various mechanisms have been suggested for SUSY breaking, which span a wide range of gravitino masses: very light, light, and heavy; a review can be found, e.g., in [43]. For gauge-mediated SUSY breaking [44], with scale $M_{\rm SUSY} = 10$ TeV, the minimum gravitino mass is $m_{3/2} \sim 0.1$ eV [32]. However, scenarios with tiny masses have also been considered in the literature, see e.g. [45]. Herein we adopt the lower bound on the gravitino mass coming from the LHC experiment, $m_{3/2} \gtrsim$ 1 meV [46]. Whichever point of view one may find more convincing, it seems most conservative at this point to depend on experiment (if possible) to resolve the issue.

A. Cosmological inference of the long-distance effective field theory

If the gravitino is very light then it would contribute with fermionic degrees of freedom to the sum in (10) and can help relaxing the bound on $m_{1,\rm max}$. The different mass scales of m_1 (or m_3), $m_{3/2}$, and $m_{R_\perp}$ are summarized in Table II. It is of interest to see whether the modifications induced on $V(R)$ by the gravitino contribution can be discerned by future cosmological probes measuring $\sum m_\nu$.

Future observations from the Simons Observatory [47], when complemented with BAO from DESI [48] and Rubin LSST weak lensing data [49], will allow a determination of the total neutrino mass with an uncertainty $\sigma(\sum m_i) = 40$ meV, and with expected improvements in the determination of the optical depth the sensitivity will refine to $\sigma(\sum m_i) = 20$ meV [50]. Future measurements of the lensing power spectrum (or cluster abundances) by CMB-S4 [51], when supplemented with BAO from DESI and the *Planck* measurement of the optical depth, will provide a constraint on the sum of neutrino masses at the level $\sigma(\sum m_\nu) = 24$ meV, and with expected improvements in the determination of the optical depth the sensitivity will refine to $\sigma(\sum m_\nu) = 14$ meV [52]. The proposed probe of inflation and cosmic origins (PICO), in combination with BAO from DESI (or Euclid) will reach a sensitivity of $\sigma(\sum m_\nu) = 14$ meV [53]. This implies that CMB-S4 and the proposed CMB satellite PICO will be sensitive to a 4σ detection of the minimum sum predicted by the NO. Far into the future, measurements of the gravitational lensing of the CMB and the thermal and kinetic Sunyaev-Zel'dovich effect on small scales by the millimeter-wave survey CMB-HD may reach a sensitivity of $\sigma(\sum m_\nu) = 13$ meV, corresponding to a 5σ detection on the sum of the neutrino masses [54].

Altogether, this suggests that future cosmological observations will be able to pin down whether the mass of the lightest neutrino is $m_1 > 7.63$ meV and at the same time will inform us about the possible existence of a very light gravitino or other fermionic degrees of freedom in the deep infrared region.

TABLE III. Maximum gravitino mass necessary to avoid an AdS vacuum for Majorana neutrinos.[a]

NO			IO		
m_1	$m_{R_\perp}$	$m_{3/2}$	m_3	$m_{R_\perp}$	$m_{3/2}$
50 meV	No radion	2.31 meV	30 meV	No radion	3.05 meV
40 meV	No radion	2.72 meV	25 meV	No radion	3.45 meV
30 meV	No radion	3.36 meV	20 meV	No radion	4.02 meV
20 meV	No radion	4.56 meV	15 meV	No radion	4.87 meV
15 meV	No radion	5.67 meV	10 meV	No radion	6.28 meV
10 meV	No radion	7.71 meV	5 meV	No radion	8.76 meV
5 meV	No radion	12.21 meV	1 meV	No radion	10.97 meV
0 meV	No radion	18.86 meV	0 meV	No radion	11.14 meV
50 meV	Massless	Impossible	30 meV	Massless	Impossible
40 meV	Massless	Impossible	25 meV	Massless	Impossible
30 meV	Massless	Impossible	20 meV	Massless	Impossible
20 meV	Massless	Impossible	15 meV	Massless	Impossible
15 meV	Massless	2.13 meV	10 meV	Massless	2.69 meV
10 meV	Massless	4.49 meV	5 meV	Massless	5.17 meV
5 meV	Massless	7.67 meV	1 meV	Massless	6.81 meV
0 meV	Massless	10.84 meV	0 meV	Massless	6.93 meV

[a]Impossible means that there is always a stable AdS vacuum.

B. Neutrinos locked on the brane

If the gravitino is very light, neutrinos could, in principle, be locked on the brane. If this were the case, it is of interest to investigate whether the gravitino could help Majorana neutrinos to avoid the existence of AdS vacua. As a first step of this investigation we assume neutrinos are Majorana and duplicate the scanning procedure carried out for Dirac neutrinos to establish the mass scales of m_1 (or m_3), $m_{3/2}$, and $m_{R_\perp}$ that can avoid AdS vacua. The results are summarized in Table III.

Now, within this scenario neutrinos do not have KK towers, but to avoid AdS vacua we still have to compensate for the bosonic towers from the gravity multiplet (whose components propagate into the bulk). As noted in Sec. III we have adopted $\mathcal{N} = 2$ (broken) SUSY in the bulk, namely 8 degrees of freedom for each layer of the gravity towers; see Table I. Note that the gravitino tower is shifted from the bosonic towers by $m_{3/2}$. As a consequence, if this shift is too big, the bosonic modes could create stable AdS vacua. Now, because of the orbifold compactification, some of the zero modes can be projected out. The most natural choice is to consider only the graviton and the radion zero modes among the bosons, and of course the gravitino. We also assume that the scalar superpartners of neutrinos are heavy to contribute to the potential.

For convenience, we define $X = R_\perp m_{3/2}$. We begin by assuming neutrinos are Dirac. On the one hand, if $X = 1$, then the gravitino tower (including the zero mode) exactly cancels the bosonic towers except for the first layer. Therefore, we are left with 9 bosonic degrees of freedom against 12 fermionic ones. This implies that if $X = 1$, the fermions will always “win” at the end. The region of the parameter space which can develop an AdS vacuum is determined by the mass of the neutrinos (and of the radion). On the other hand, if $X = 2$, then the gravitino tower (including the zero mode) exactly cancels the bosonic towers except for the first two layers. We therefore have 17 bosonic degrees of freedom against 12 fermionic ones. This implies that if $X = 2$ the bosons will always win at the end and so the potential is unbounded from below. Of course $X < 1$ would also work and even relax the constraint on $m_{1,\max}$ like in the analysis of Sec. V A. Consequently, it seems that the interesting range of the gravitino mass to compensate for the bosonic towers is $0 \leq X < 2$.

In the case of three Majorana neutrinos however, we already know that we need to add new light fermionic degrees of freedom to avoid the AdS vacua. Therefore, for Majorana neutrinos the interesting range is $0 \leq X < 1$. Using the results of Tables II and III we can obtain the maximal values of X needed to avoid a stable AdS vacuum. These values are given in Table IV. By comparing Tables III and IV we conclude that in the presence of a very light gravitino Majorana neutrinos can support stable dS vacua.

In summary, if the gravitino is very light, then its KK tower can counterbalance the bosonic towers to avoid AdS vacua in 4D → 3D compactifications. This implies that the right-handed neutrinos and their left-handed counterparts can be both localized on the brane. Besides, the gravitino zero mode could help modify the 3D Casimir vacua for the case of Majorana neutrinos to become viable. The maximum gravitino mass needed to avoid the AdS vacuum for Majorana and Dirac neutrinos, assuming NO and IO, is summarized in Fig. 3.

VI. BULK NEUTRINO MASSES

It has been pointed out that bulk neutrino masses allow relaxing the bounds on $R_\perp$ [55–57]. Considering that fact,

TABLE IV. Maximum $m_{3/2}$ and X necessary to avoid an AdS vacuum.[a]

	NO			IO		
	$R_\perp$	$m_{3/2}$	X	$R_\perp$	$m_{3/2}$	X
Dirac	5 μm	54.26 meV	1.375	5 μm	18.14 meV	0.460
	10 μm	22.35 meV	1.133	10 μm	10.15 meV	0.514
	15 μm	12.72 meV	0.967	15 μm	6.13 meV	0.466
	20 μm	8.71 meV	0.883	20 μm	4.36 meV	0.442
	25 μm	6.56 meV	0.831	25 μm	3.38 meV	0.428
	30 μm	5.25 meV	0.798	30 μm	2.77 meV	0.421
Majorana	5 μm	10.79 meV	0.273	5 μm	6.83 meV	0.173
	10 μm	8.08 meV	0.409	10 μm	3.63 meV	0.184
	15 μm	4.89 meV	0.372	15 μm	2.17 meV	0.165
	20 μm	3.46 meV	0.351	20 μm	1.54 meV	0.156
	25 μm	2.67 meV	0.338	25 μm	1.19 meV	0.151
	30 μm	2.17 meV	0.330	30 μm	0.98 meV	0.149

[a]We have assumed that the radion and lightest neutrino are massless.

we now add Dirac masses for the three 5D neutrino fields. For simplicity, we swap to an intermediate mass basis Ψ_i in which the flavor mixing has been already diagonalized. The kinetic and mass terms in the Lagrangian take the form

$$\mathcal{L} \supset \sum_{i=1}^{3} [i\bar{\Psi}_i \Gamma^A \overleftrightarrow{\partial}_A \Psi_i - c_i \bar{\Psi}_i \Psi_i], \tag{28}$$

where c_is are the bulk mass parameters and $\Gamma^A = (\gamma^\mu, i\gamma^5)$. For convenience, we consider real c_i and define the mass matrix $\mathbb{M}_i$ by

$$\mathbb{M}_i = \begin{pmatrix} vY_0^i & 0 & \cdots & 0 \\ vY_1^i & m_1^i & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ vY_N^i & 0 & \cdots & m_N^i \end{pmatrix}, \tag{29}$$

where

$$Y_0^i = y_i \sqrt{\frac{2}{\Lambda_{\rm QG}}} \sqrt{\frac{c_i}{e^{2c_i R_\perp \pi} - 1}} \tag{30}$$

and

$$Y_n^i = y_i \sqrt{\frac{2}{\Lambda_{\rm QG} \pi R_\perp}} \sqrt{\frac{n^2}{n^2 + c_i^2 R_\perp^2}} \tag{31}$$

are the 4D Yukawa couplings (localized on the SM brane), with $(m_n^i)^2 = (n/R_\perp)^2 + c_i^2$ [57]. We can then compute

$$\begin{aligned} \lim_{N\to\infty} \det\left(\mathbb{M}_i^\dagger \mathbb{M}_i - \frac{\alpha_{i,n}^2}{R_\perp^2} \mathbb{1}\right) &= \left[\frac{c_i \xi_i^2}{e^{2c_i R_\perp \pi} - 1} - \frac{\alpha_{i,n}^2}{R_\perp^2} + \frac{\xi_i^2}{2\pi R_\perp} (\pi\sqrt{\alpha_{i,n}^2 - c_i^2 R_\perp^2} \cot(\pi\sqrt{\alpha_{i,n}^2 - c_i^2 R_\perp^2}) \right. \\ &\quad \left. - c_i R_\perp \pi \coth(c_i R_\perp \pi))\right] \prod_{j=1}^{\infty} \left((m_j^i)^2 - \frac{\alpha_{i,n}^2}{R_\perp^2}\right) \\ &= \left[-\frac{c_i \xi_i^2}{2} - \frac{\alpha_{i,n}^2}{R_\perp^2} + \frac{\xi_i^2}{2R_\perp} \sqrt{\alpha_{i,n}^2 - c_i^2 R^2} \cot\left(\pi\sqrt{\alpha_{i,n}^2 - c_i^2 R_\perp^2}\right)\right] \prod_{j=1}^{\infty} \left((m_j^i)^2 - \frac{\alpha_{i,n}^2}{R_\perp^2}\right), \end{aligned} \tag{32}$$

where $\xi_i = v y_i \sqrt{2/\Lambda_{\rm QG}}$. Note that the $\alpha_{i,n}^2/R_\perp^2$ solutions of this equation are also the eigenvalues of $\mathbb{M}_i \mathbb{M}_i^\dagger$. In the limit $c_i \to 0$, we recover from the bracket the usual transcendental equation:

$$\alpha_{i,n} - \pi (m_i^{\rm D} R_\perp)^2 \cot(\pi \alpha_{i,n}) = 0. \tag{33}$$

Note that the equation

$$-\frac{c_i\xi_i^2}{2}-\frac{\alpha_{i,n}^2}{R_\perp^2}+\frac{\xi_i^2}{2R_\perp}\sqrt{\alpha_{i,n}^2-c_i^2R_\perp^2}\cot\left(\pi\sqrt{\alpha_{i,n}^2-c_i^2R_\perp^2}\right)=0 \tag{34}$$

has two different behaviors. If $(\alpha_{i,n}/R_\perp)^2 \geq c_i^2$ it has an infinite number of solutions (the KK tower) but if we look for solutions lighter than the mass in the bulk, namely $(\alpha_{i,n}/R_\perp)^2 < c_i^2$, it becomes

$$-\frac{c_i\xi_i^2}{2}-\frac{\alpha_{i,n}^2}{R_\perp^2}+\frac{\xi_i^2}{2R_\perp}\sqrt{c_i^2R_\perp^2-\alpha_{i,n}^2}\coth\left(\pi\sqrt{c_i^2R_\perp^2-\alpha_{i,n}^2}\right)=0, \tag{35}$$

which can have at most one solution. Actually, the function defined by

$$f(\alpha_{i,n})=-\frac{c_i\xi_i^2}{2}+\frac{\xi_i^2}{2R_\perp}\sqrt{c_i^2R_\perp^2-\alpha_{i,n}^2}\coth\left(\pi\sqrt{c_i^2R_\perp^2-\alpha_{i,n}^2}\right) \tag{36}$$

is monotonically decreasing on $[0,|c_i|R_\perp)$, whereas $\alpha_{i,n}^2/R_\perp^2$ is monotonically increasing. Moreover, we have

$$f(0)=\frac{\xi_i^2}{2}(|c_i|\coth(\pi|c_i|R_\perp)-c_i)>\frac{\xi_i^2}{2}(|c_i|-c_i)\geq 0 \tag{37}$$

and

$$\lim_{\alpha_{i,n}\to|c_i|R} f(\alpha_{i,n}) = (m_i^{\mathrm{D}})^2(1-\pi c_i R_\perp), \tag{38}$$

so that Eq. (35) has a unique solution if and only if

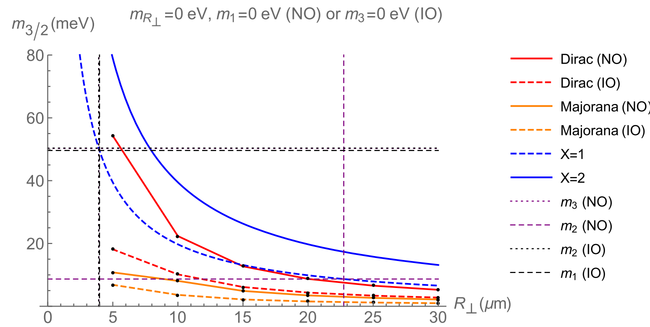

FIG. 3. Maximum gravitino mass needed to avoid the AdS vacuum.

$$(m_i^{\mathrm{D}})^2(1-\pi c_i R_\perp) < c_i^2; \tag{39}$$

namely, if

$$c_i \notin \left[\frac{1}{2}(m_i^{\mathrm{D}})^2\pi R_\perp\left(-1-\sqrt{1+\frac{4}{(m_i^{\mathrm{D}})^2\pi^2R_\perp^2}}\right),\right.$$
$$\left.\frac{1}{2}(m_i^{\mathrm{D}})^2\pi R_\perp\left(-1+\sqrt{1+\frac{4}{(m_i^{\mathrm{D}})^2\pi^2R_\perp^2}}\right)\right]$$

and has no solution otherwise. This means that even if the spectrum is shifted by adding a mass in the bulk, we can still have a light zero mode. The eigenvectors of $\mathbb{M}_i^\dagger\mathbb{M}_i$ are given by

$$\tilde{\mathbb{V}}_{i,\beta_n}=\tilde{\gamma}_{i,\beta_n}\begin{pmatrix}1 & vY_1^i\frac{m_1^i}{\beta_n-(m_1^i)^2} & vY_2^i\frac{m_2^i}{\beta_n-(m_2^i)^2} & \cdots\end{pmatrix}^\intercal, \tag{40}$$

and the eigenvectors of $\mathbb{M}_i\mathbb{M}_i^\dagger$ are given by

$$\mathbb{V}_{i,\beta_n}=\mathbb{M}_i\tilde{\mathbb{V}}_{i,\beta_n}=\gamma_{i,\beta_n}\begin{pmatrix}vY_0^i & vY_1^i\left(1+\frac{(m_1^i)^2}{\beta_n-(m_1^i)^2}\right) & vY_2^i\left(1+\frac{(m_2^i)^2}{\beta_n-(m_2^i)^2}\right) & \cdots\end{pmatrix}^\intercal, \tag{41}$$

where $\beta_n=\alpha_{i,n}^2/R_\perp^2$ is the associated eigenvalue and where $\tilde{\gamma}_{i,\beta_n}$ and γ_{i,β_n} are normalization factors. We are interested in eigenvectors satisfying $\tilde{\mathbb{V}}_{i,\beta_n}^\intercal\tilde{\mathbb{V}}_{i,\beta_n}=\mathbb{1}$ and $\mathbb{V}_{i,\beta_n}^\intercal\mathbb{V}_{i,\beta_n}=\mathbb{1}$, which lead to

$$\tilde{\gamma}_{i,\beta_n}^2=\frac{2\Lambda_{\mathrm{QG}}}{2\Lambda_{\mathrm{QG}}+v^2y_i^2\left[-\frac{\cot(\pi R_\perp\sqrt{\beta_n-c_i^2})}{\sqrt{\beta_n-c_i^2}}+\pi R_\perp\csc^2\left(\pi R_\perp\sqrt{\beta_n-c_i^2}\right)\right]}, \tag{42}$$

and

$$\gamma_{i,\beta_n}^2=\frac{4\Lambda_{\mathrm{QG}}\sqrt{\beta_n-c_i^2}}{v^2y_i^2\left[\csc(\pi R_\perp\sqrt{\beta_n-c_i^2})\right]^2}\left[2\sqrt{\beta_n-c_i^2}(\pi R_\perp\beta_n-c_i)+2c_i\sqrt{\beta_n-c_i^2}\right.$$
$$\left.\times\cos\left(\pi R_\perp\sqrt{\beta_n-c_i^2}\right)+(\beta_n-2c_i^2)\sin\left(\pi R_\perp\sqrt{\beta_n-c_i^2}\right)\right]^{-1}. \tag{43}$$

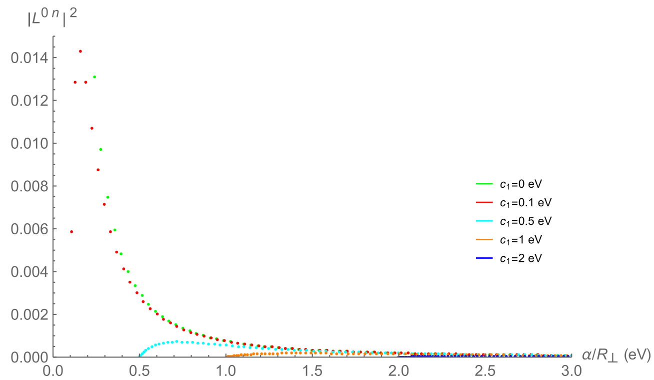

FIG. 4. Pattern of KK masses and mixings for the lightest zero mode assuming $y_1 = 0.001$ and fiducial values of c_i and $R_\perp = 5$ μm.

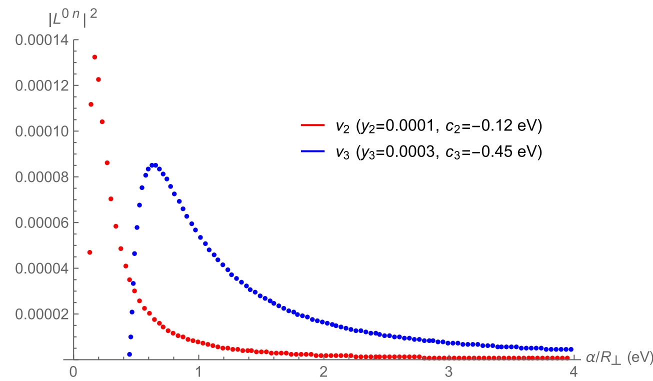

FIG. 5. Pattern of KK masses and mixings for the heaviest zero modes assuming NO and a compactification radius $R_\perp = 5$ μm. We have taken $c_2 = -0.12$ eV, $c_3 = -0.45$ eV, $y_2 = 0.0001$, and $y_3 = 0.0003$.

With this in mind, the masses and mixing between the zero mode of neutrino species i and the nth KK mode are characterized by[2]

$$|L^{0n}|^2 = \tilde{\gamma}^2_{i,\beta_n}, \quad n \in \mathbb{N}. \tag{44}$$

Using (34) we can express $\tilde{\gamma}^2_{i,\beta_n}$ as

$$\tilde{\gamma}^2_{i,\beta_n} = \frac{8(m_i^D)^2 \pi R_\perp (c_i^2 - \beta_n)}{4c_i (m_i^D)^4 \pi^2 R_\perp^2 + 8(m_i^D)^2 \pi R_\perp (c_i^2 - \beta_n) - \mathcal{A} - 4\pi R_\perp \beta_n^2}, \tag{45}$$

with $\mathcal{A} = 2(m_i^D)^2 \pi R_\perp \{-2 + \pi R_\perp [4c_i + 2(m_i^D)^2 \pi R_\perp]\}\beta_n$.

In Fig. 4 we show possible examples of the oscillation pattern of KK masses and mixings for fiducial values c_i, y_i, and $R_\perp$. We can see that for $c_1 = 0.1$ eV there is a strong suppression of the mixing with the first two KK modes, and $|L^{0n}|$ peaks is at $\alpha_{1,3}/R_\perp = 156$ meV, with $|L^{03}|^2 = 0.014$. This is in sharp contrast with the case for $c_1 = 0$, in which $|L^{0n}|$ peaks in the first KK at $\alpha_{1,1}/R_\perp = 46$ meV, with $|L^{01}|^2 = 0.22$. For higher values of c_1, there is suppression of a larger number of KK modes and at the peak $|L^{0n}|^2$ becomes even smaller. For the neutrino towers to be able to compensate the bosonic towers, the mass of the lightest neutrino zero mode should be very small, e.g., for $c_1 = 0.1$ eV and $R_\perp = 5$ μm we have $\alpha_{1,0}/R_\perp \sim 2.5 \times 10^{-2}$ meV. As shown in Fig. 5, the second and third zero modes also have a strong suppression on the mixing with the first two and ten KK modes, respectively. For ν_2 and ν_3, $|L^{0n}|$ peaks in the third and eleventh KK mode at $\alpha_{2,3}/R_\perp = 169$ and $\alpha_{3,11}/R_\perp = 625$ meV, with $|L^{03}|^2 = 0.00013$ and $|L^{011}|^2 = 0.0000849$. This corresponds to $\Delta m^2_{21} \sim 7.2 \times 10^{-5}$ eV^2, $\Delta m^2_{32} \sim 2.4 \times 10^{-3}$ eV^2, and $\sum m_\nu \sim 58$ meV in good agreement with observations. See the Appendix for further details.

Now, for $R_\perp \gtrsim 10$ μm, the masses of the bosonic KK become of the order of the neutrino zero modes, and therefore close to $|c|$ if we need to enforce a suppression of zero-mode oscillations into the first few KK states. If this were the case, the neutrino towers (whose masses can be approximated by $\sqrt{(n/R_\perp)^2 + c_i^2}$, $n > 0$) would be significantly shifted from the bosonic towers (whose masses can be approximated by $n/R_\perp$). This implies that the neutrino towers would not be sufficient to cancel the influence of the bosonic towers in carving $V(R)$. But again, to balance the bosonic towers a very light gravitino may come to the rescue.

VII. CONCLUSIONS

The Swampland program has made the striking proposal that if the low-energy effective theory is the minimal SM extension accommodating neutrino masses, then neutrinos cannot be Majorana particles. This is because the *sharpened* version of the weak gravity conjecture forbids the existence of non-SUSY AdS vacua supported by fluxes in a consistent quantum gravity theory, and if neutrinos are Majorana, when the SM + GR are compactified down to 3D, then AdS vacua appear for any values of neutrino masses consistent with experiment. However, this is not the case if neutrinos are Dirac particles, for which the SM + GR compactification down to 3D sets a limit on the required maximum mass of the lightest neutrino to carry dS rather than AdS vacua. Motivated by these astonishing results we have studied the landscape of lower-dimensional vacua that arise in the SM coupled to gravity enriched with the dark dimension. The results of our investigation can be summarized as follows:

(i) If right-handed neutrinos propagate in the bulk (so that their Yukawa couplings become tiny due to a volume suppression) then their KK towers can

[2]Note that contrary to what is stated in [57], the left rotation that diagonalizes the mass matrix $\mathbb{M}_i$ in the intermediate basis relates to $\mathbb{M}_i^\dagger \mathbb{M}_i$ rather than $\mathbb{M}_i \mathbb{M}_i^\dagger$.

compensate for the graviton tower to avoid AdS vacua. However, data from neutrino oscillation experiments set restrictive bounds on the compactification radius and so the first KK neutrino modes are too heavy to alter the shape of the radion potential or the maximum mass of the lightest neutrino state from those predicted by the SM + GR when compactified down to 3D.

(ii) A very light gravitino (with mass in the meV range) could help relax the neutrino mass constraint. The difference between the predicted total neutrino mass $\sum m_\nu$ by SM + GR and SM + GR in the presence of a very light gravitino propagating through the dark dimension is within reach of next-generation cosmological probes that will measure the total neutrino mass with an uncertainty $\sigma(\sum m_\nu) = 0.014$ eV.

(iii) If the gravitino is very light, then its KK tower can compensate for the graviton tower to avoid AdS vacua and thus right-handed neutrinos can (in principle) be locked on the brane. For this scenario, Majorana neutrinos could develop dS vacua.

(iv) Bulk neutrino masses can suppress the mixing with the first KK mode in the neutrino towers and relax the oscillation bound on the compactification radius, but at the expense of shifting the KK neutrino towers to higher masses. However, there is a neutrino zero mode that can stay light in each tower to accommodate neutrino oscillation data and the cosmological bound.

As a by-product of our investigation focusing on the validity of the non-SUSY AdS instability conjecture we end up with a prediction of Swampland phenomenology within the framework of the dark dimension: either the gravitino is very light or else neutrinos have to be Dirac with the right-handed states propagating into the bulk, so that the neutrino towers can compensate the contribution of the graviton KK modes to the potential.

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APPENDIX

For completeness, in this Appendix we calculate the survival probability in the presence of bulk masses. In order to be able to compare the cases with and without bulk masses and clearly see the strong suppression of the mixing with the first few KK modes, we have to look at a value of $R_\perp$ in which the scenario without bulk masses is excluded. Indeed, because of the constraint from oscillation data, it is not possible to build a coherent spectrum without bulk masses for $R_\perp \lesssim 0.4$ μm for NO [28]. We will therefore

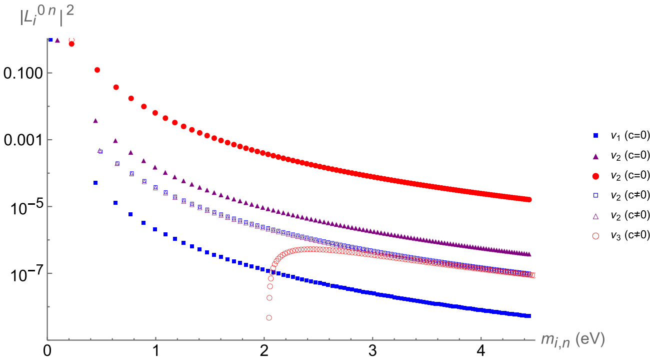

FIG. 6. Neutrino spectrum.

build an explicit example for NO with $R_\perp = 1$ μm, and a choice of realistic and simple parameters. Namely, we choose $m_1^{(0)} = 1$ meV which together with the value of $R_\perp$ fully determine the Yukawa couplings, and then the whole spectrum, when there are no masses in the bulk. Namely, it gives $y_1 \simeq 2 \times 10^{-5}$, $y_2 \simeq 2 \times 10^{-4}$ and $y_3 \simeq 1.3 \times 10^{-3}$, or equivalently $m_1^D \simeq m_1^{(0)} \simeq 1$, $m_2^D \simeq 9$ and $m_3^D \simeq 57$ meV. When we turn on the masses in the bulk, the choice of $R_\perp$ and of the values of the zero mode masses do not determine the spectrum as we still have two correlated parameters c_i and y_i for each i. Here we make the simple choice of $y_1 = y_2 = y_3 = 10^{-4}$. The bulk masses are thus determined and are found to be $c_1 \simeq 140$ meV, $c_2 \simeq -124$ meV and $c_3 \simeq -4.177$ eV. The spectrum is therefore fully determined and is presented in Fig. 6.

We can now compare the survival probability in the three cases: no neutrino in the bulk (SM case), and neutrinos in the bulk with and without bulk masses, all as functions of L/E, where L is the experiment baseline, E is the neutrino energy. To this end, we first define the relation between the flavor and intermediate bases:

$$\nu_{\alpha,0}^L = U_{\alpha i} \nu_{i,0}^L, \qquad \Psi_\alpha = R_{\alpha i} \Psi_i, \tag{A1}$$

where $U_{\alpha i}$ is the usual Pontecorvo-Maki-Nagakawa-Sakata matrix [58–60] for the standard three flavor neutrino model and R is a matrix that diagonalizes the bulk masses and Yukawa couplings. The oscillation amplitude (in vacuum) among active neutrinos is given by

$$\mathcal{A}(\nu_{\alpha,0} \to \nu_{\beta,0}; L) = \sum_{i,n} \mathcal{U}_{\alpha i}^{0n} (\mathcal{U}_{\beta i}^{0n})^* \exp\left(i \frac{m_{i,n}^2 L}{2E} \right), \tag{A2}$$

where $\mathcal{U}_{\alpha i}^{0n} = U_{\alpha i} L_i^{0n}$, and where and the superscripts indicating left handedness have been dropped. It is noteworthy that the other entries of $\mathcal{U}_{\alpha i}^{nm}$ [57] are not observable, because the sterile neutrinos do not couple to the electroweak gauge bosons. The survival probability of flavor α a distance L is given by

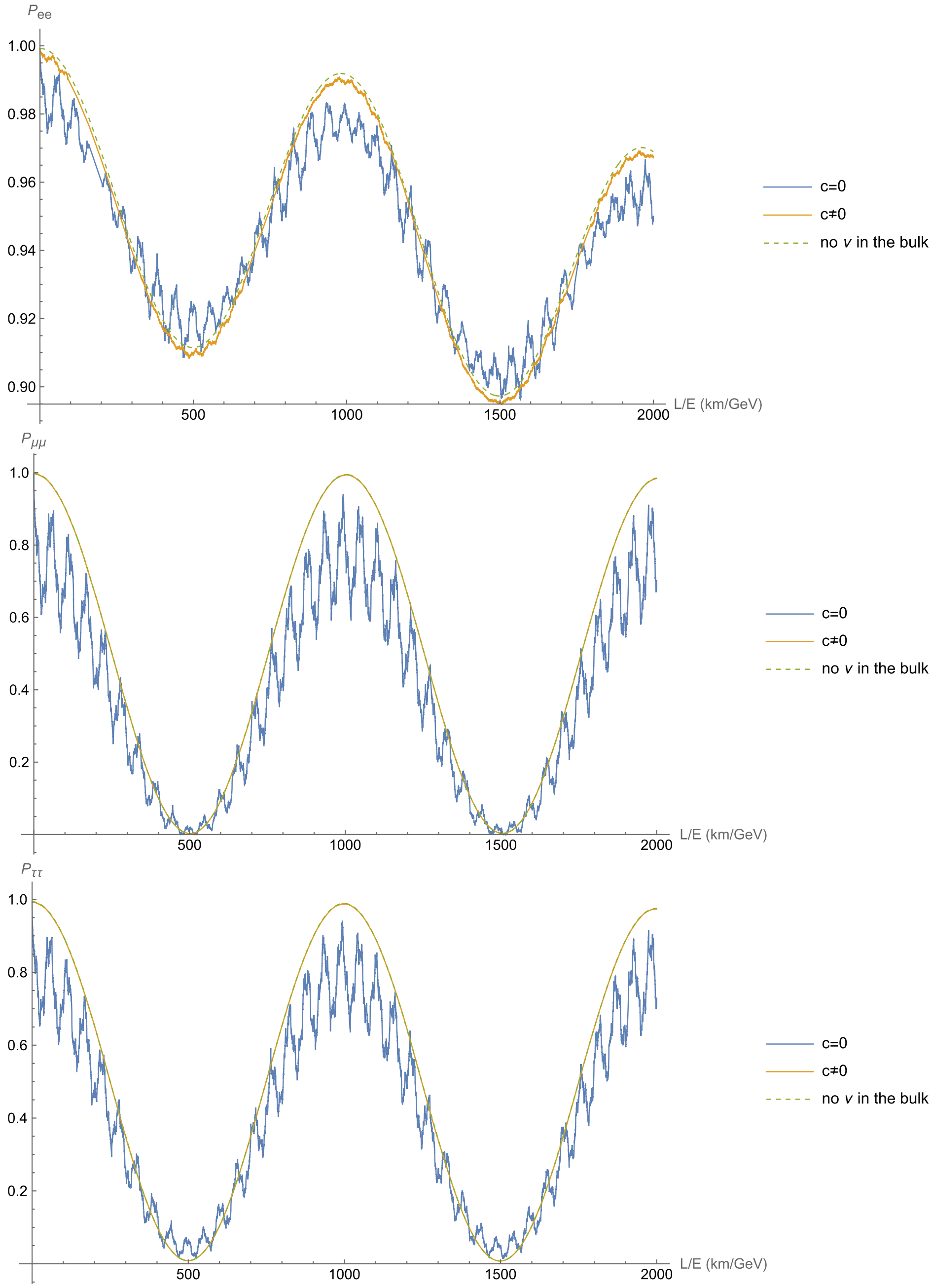

FIG. 7. Probability amplitudes P_{ee}, $P_{\mu\mu}$, and $P_{\tau\tau}$ for $L/E < 2000$ km/GeV.

$$P_{\alpha\alpha} \equiv P(\nu_{\alpha,0} \to \nu_{\alpha,0}) = |\mathcal{A}(\nu_{\alpha,0} \to \nu_{\alpha,0}; L)|^2. \quad \text{(A3)}$$

In Figs. 7 and 8 we show the survival probability of the different flavors as a function of L/E showing different scales. We can see that the case with bulk masses is way more similar to the SM case than the case without bulk masses. In order to refine this statement, in Figs. 9 and 10 we show the differences of probabilities with respect to the SM (without bulk neutrinos) scenario. We can see that even though the case without bulk masses has huge differences with respect to the SM case, this is no longer true once we include bulk masses. Moreover, we remind the reader that when we include bulk masses, choosing $R_\perp$ and the masses of the zero modes does not fully determine the spectrum. We decided to display simple and generic values of the parameters here to see the natural behavior of the system, but it is definitely possible to adjust the values of y_i and c_i to have the same zero modes spectrum, but to reduce even more the effect of the KK towers.

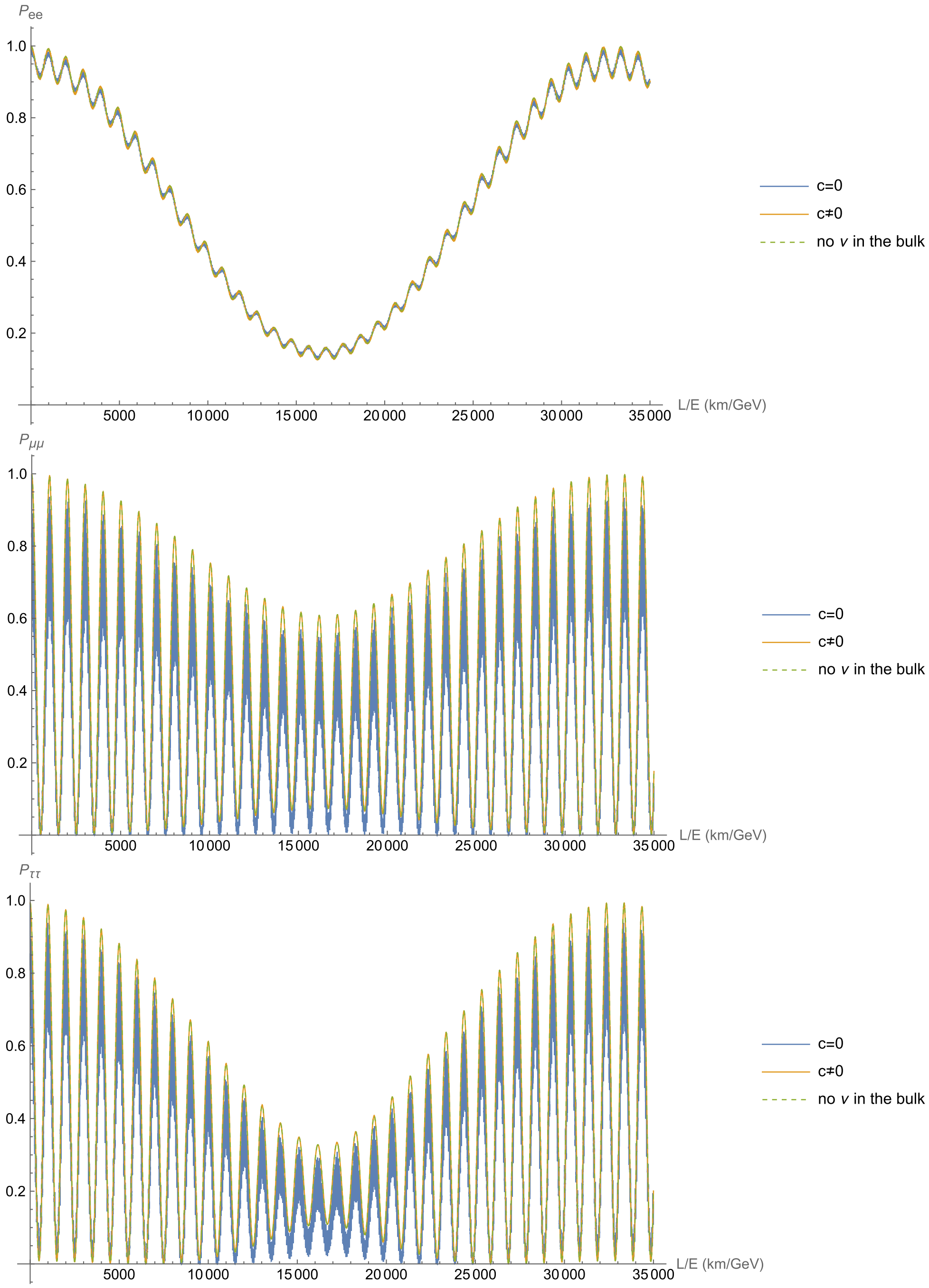

FIG. 8. Probability amplitudes P_{ee}, $P_{\mu\mu}$, and $P_{\tau\tau}$ for $L/E < 35000$ km/GeV.

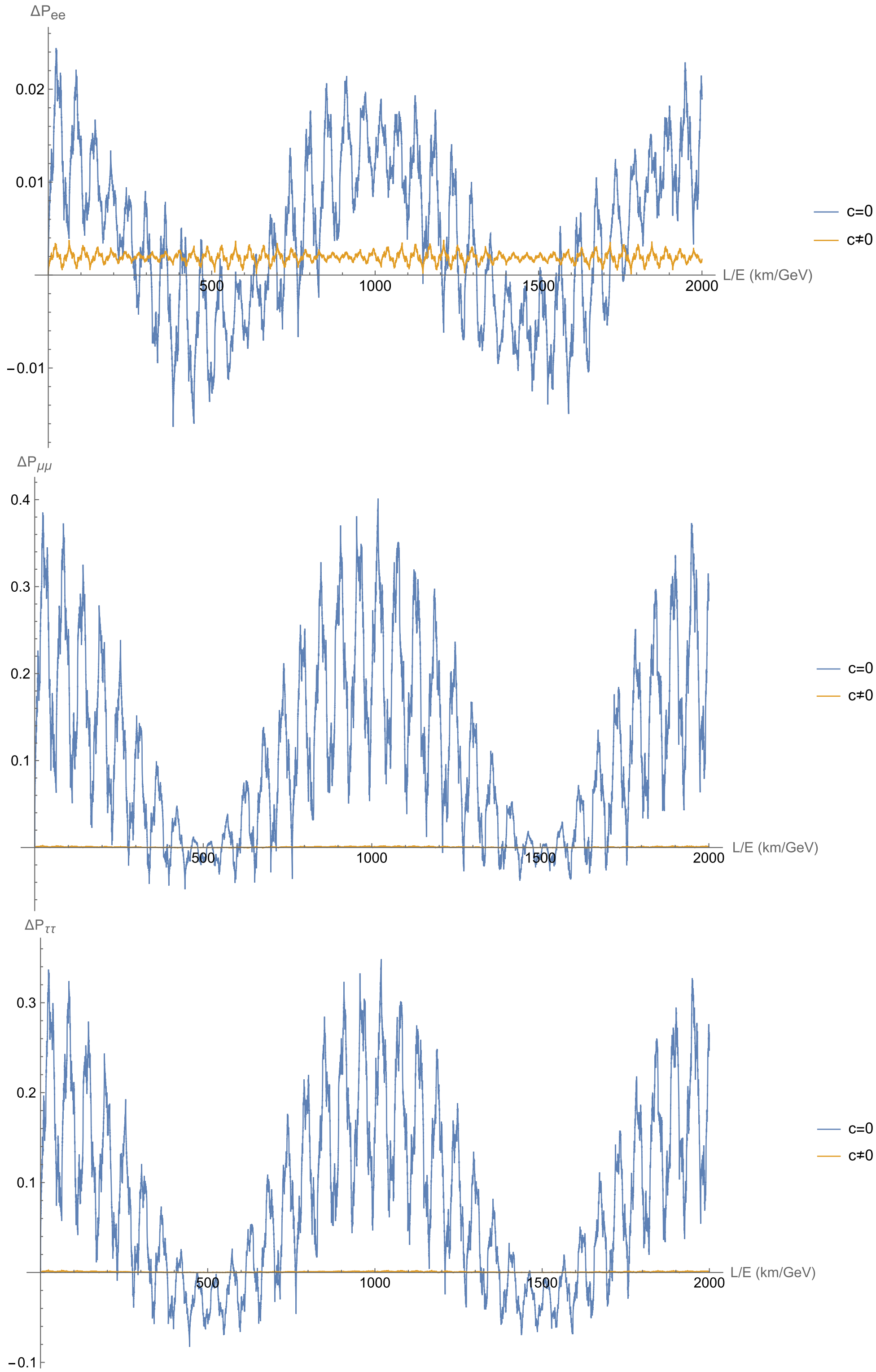

FIG. 9. Deviations of the probability amplitudes from the scenario without bulk neutrinos for $L/E < 2000$ km/GeV.

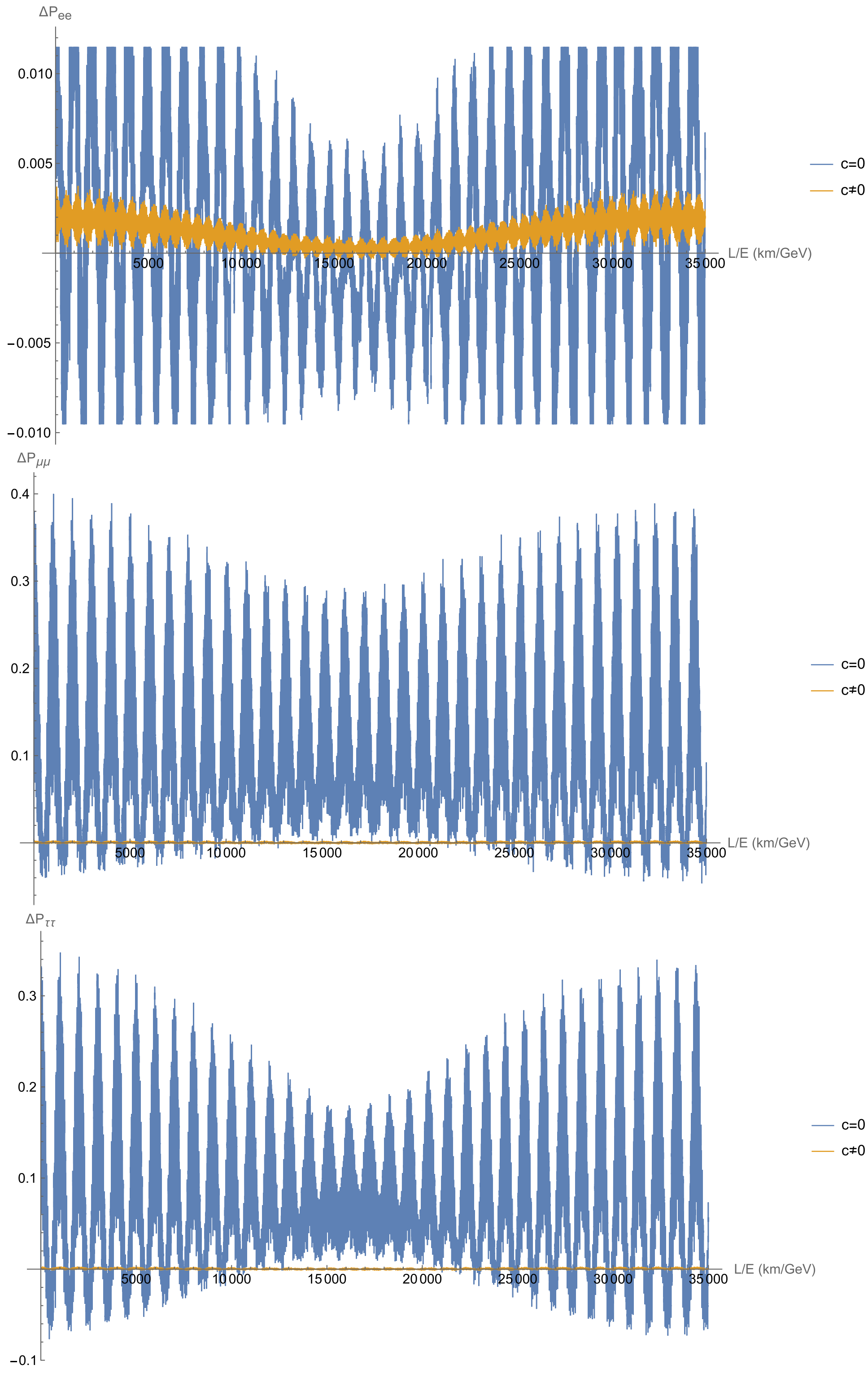

FIG. 10. Deviations of the probability amplitudes from the scenario without bulk neutrinos for $L/E < 35000$ km/GeV.

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