Quantum Optimisation of Reconfigurable Surfaces in Complex Propagation Environments

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***Abstract*—The design and analysis of reconfigurable meta-surfaces operating within rich multi-path propagation fading is of crucial importance for the development of real-life programmable electromagnetic environments. We incorporate the effect of multi-path fading in an impedance-based model of wireless communication links assisted by reconfigurable surfaces. Previous work has shown that impedance-based channel models under rich multi-path propagation have an isomorphism with Sherrington-Kirkpatrick (SK) Hamiltonians. We focus on received power minimisation, which is equivalent to the hard-to-solve task task of finding the ground state of the SK Hamiltonian. It has been recently discovered that the Quantum Approximate Optimisation Algorithm (QAOA) predicts the ground state of SK Hamiltonians accurately. However, the landscape parameters of QAOA are dependent on the specific realisation of the random SK Hamiltonian, which hinders the full usage of quantum hardware to optimise reconfigurable surfaces dynamically. We show by Montecarlo simulations that a concentration property cures this impediment thus making QAOA an excellent candidate for surface optimisation under fast multi-path fading.**

***Index Terms*—RIS, Quantum Computing, Sherrington-Kirkpatrick Hamiltonian, QAOA, MIMO**

I. Introduction

The potentials of programmable meta-surfaces will be harnessed in future wireless communication systems if the research community is capable of providing advanced physics-based models and optimisation algorithms. Programmable meta-materials are artificial structures engineered to have desired properties to dynamically control and manipulate electromagnetic (EM) waves [1]. This new generation of meta-surfaces, also referred as reconfigurable meta-surfaces, or reconfigurable intelligent surfaces (RIS), are based realised loading meta-material cells, or groups of cells, with tuneable electrics components, e.g., p.i.n. diodes or varactors. Over the last few years, most of the studies in the literature have devised first electromagnetic principle models and studied the performance of reconfigurable meta-surfaces for beam-forming in quasi-free space [2], [3]. Advanced design and analysis of meta-surfaces for the next generation of mobile wireless networks beyond 5G pose two-challenges: i) self-consistent modelling of the interaction between meta-surface and dynamic propagation environments; ii) Optimised design of reconfigurable meta-surfaces in wireless channels with fading. The first challenge has been recently addressed in a series of works involving physics-based modelling that adopts scattering theory [4] and wave chaos theory [5]. The second challenge has been addressed from an algorithmic perspective by various authors in the electromagnetics [6] and wireless communication community [7]. A different angle has ben taken that concerns physics-based grounded on quantum graphs [8] and quantum computing [9], [10]. This recent effort deals with the optimisation of deterministic physics models through analog computation structures. Here, we focus on the fast optimisation of stochastic physical models through universal-gate quantum computing algorithms. In particular, we adopt the quantum approximate optimisation algorithm (QAOA) to find the ground state of an effective Sherrington-Kirckpatrick (SK) Hamiltonian on which the EM model of the meta-surface scattered energy is mapped onto. In other words, we tackle the quantum optimisation of large and dynamically reconfigurable meta-surface under rich multi-path fading through a physics-based end-to-end (E2E) MIMO communications model. We have recently showed how to exploit the mathematical analogy between received power in RIS-assisted wireless links and SK Hamiltonians to describe the spin state dynamics of arrays of reflective surface unit cells [11]. This analogy has been also discussed in the field of optical simulators of spin glasses (SG) through a spatial light modulator (SLM) [12], [13]. The quantum computation of our random SK Hamiltonian extends the workflow adopted for the radar cross section (RCS) of a binary RIS under oblique plane wave incidence [9]. Introducing random couplings in Ising Hamiltonians produce multi-stability that has been a concern already of the statistical physics community for several decades [14]. The ground state of the SK Hamiltonian was discovered in 1978 by *Giorgio Parisi*, who also elucidated the role of spin glasses in disordered systems. As a matter

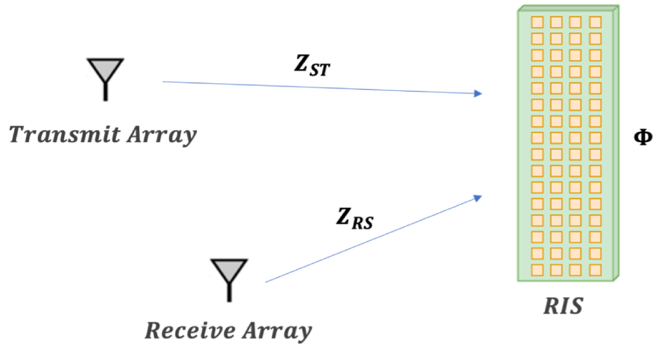

Fig. 1: RIS-assisted wireless communication in presence of rich uplink and downlink multi-path fading.

of fact, the fluctuation of the ground state is reduced as the temperature of magnetic spin chain systems is reduced below a critical temperature. When implemented in quantum computers, the energy fluctuation driven by the disordered spin chain underpins the fluctuation of the QAOA landscape parameters within the computing architectures. Therefore, the convergence of QAOA landscape parameters is dependent on the specific statistical realization of the SK Hamiltonian couplings, which creates a classical bottleneck to quantum simulations. Furthermore, a question arises on whether the ground state energy computed by QAOA specifically provides the Parisi ground state expected by the SK Hamiltonian. Recent work has shown that the convergence to the Parisi value is fast and accurate with a moderate depth of the quantum circuit [15]. We elaborate on this achievement and provide shot-based simulations combined with Montecarlo generation to study the existence of a concentration property that alleviates the classical landscape bottleneck.

II. Statistical Channel Model

A. MIMO Communication model overview

A cascaded communication model, based on impedance matrices that describe a multiple-input multiple-output (MIMO) system assisted by reconfigurable meta-surface, has been recently derived from first EM principles[11]. The model extends the quasi-free space channel transfer matrix in [17] to indoor propagation environments supporting rich multi-path fading. We treat the meta-surface unit cells as radiating elements with ports loaded with tuning circuitry. The radiated waves leaving the unit cells bounce around a large and complex reflective environments, and return back to the meta-surface. This mechanism establishes the intricate self- and mutual-coupling interactions among unit cells, which are captured by wave chaos theory and are described by the random coupling model (RCM). Figure 1 shows a RIS-assisted propagation channel with N_{T} antenna elements in the transmit array, N_{R} antenna elements in the receive array, and N_{S} unit cells in the RIS. The channel transfer matrix $\mathcal{H}_{\mathrm{E2E}}$ describing the transmission through the RIS reads in presence of LOS blockage reads

$$\mathcal{H}_{\mathrm{E2E}} = \mathcal{Y}_{\mathrm{RR}}\mathcal{Z}_{\mathrm{L}}\mathcal{Z}_{\mathrm{RS}}\Phi_{\mathrm{SS}}\mathcal{Z}_{\mathrm{ST}}\mathcal{Y}_{\mathrm{TT}} \tag{1}$$

where we have defined the following admittance matrices

$$\Phi_{\mathrm{SS}} = \left(\mathcal{Z}_{\mathrm{RIS}} + \mathcal{Z}_{\mathrm{SS}}\right)^{-1}, \tag{2}$$
$$\mathcal{Y}_{\mathrm{RR}} = \left(\mathcal{Z}_{\mathrm{L}} + \mathcal{Z}_{\mathrm{RR}}\right)^{-1}, \tag{3}$$
$$\mathcal{Y}_{\mathrm{TT}} = \left(\mathcal{Z}_{\mathrm{G}} + \mathcal{Z}_{\mathrm{TT}}\right)^{-1}, \tag{4}$$

with $\mathcal{Z}_{\mathrm{G}}$ internal generator impedance matrix at transmitter side, $\mathcal{Z}_{\mathrm{L}}$ load impedance matrix at receiver side, $\mathcal{Z}_{\mathrm{TT}}$ active impedance matrix of the transmit array, $\mathcal{Z}_{\mathrm{RR}}$ active impedance matrix of the receive array, $\mathcal{Z}_{\mathrm{RS}}$ mutual impedance matrix between receive array and RIS, $\mathcal{Z}_{\mathrm{ST}}$ mutual impedance matrix between RIS and transmit array, $\mathcal{Z}_{\mathrm{SS}}$ active impedance array of the RIS, $\mathcal{Z}_{\mathrm{RIS}}$ equivalent impedance matrix of the tuneable circuit loading the RIS unit cells. Note that $\mathcal{Z}_{\mathrm{RIS}}$ can be non-diagonal if the circuitry is designed so that to interconnect unit cells. It is worth noticing that the reflection phase in canonical communication models is replaced by the admittance matrix Φ_{SS}, which includes the active impedance matrix $\mathcal{Z}_{\mathrm{SS}}$ of the array of unloaded RIS unit cells, as well as the equivalent impedance matrix $\mathcal{Z}_{\mathrm{RIS}}$ of the tuning circuitry. In presence of multi-path fading, the field radiated by the antennas is reflected back from the environment, and the interaction between transmit/receive array and RIS is perturbed by the interference between a multitude of scattered waves. Wave scattering in large and complex cavities is described by wave chaos theory. Inherently, the mutual coupling impedance matrices $\mathcal{Z}_{RS}$, $\mathcal{Z}_{ST}$, and $\mathcal{Z}_{\mathrm{SS}}$ become (complex-valued) random matrices. The RCM incorporates complex wave scattering within the impedance matrix of antennas and apertures. radiating inside cavities with irregular geometry and distributed losses (captured by an average scalar parameter α). Specifically, the entries of the uplink and downlink impedance matrices, $\mathcal{Z}_{\mathrm{RS}}$ and $\mathcal{Z}_{\mathrm{ST}}$ respectively, read

$$\begin{aligned}\mathcal{Z}\left(n,m\right) \approx \mathcal{Z}^{LOS}\left(n,m\right) + \\ \sqrt{\mathcal{R}^{fs}\left(n,n\right)}\,\xi\left(n,m\right)\left(\alpha\right)\sqrt{\mathcal{R}^{fs}\left(m,m\right)},\end{aligned} \tag{5}$$

For moderate values of absorption losses in the environment, i.e., $\alpha \geq 1$ we have that ξ follows a zero mean Gaussian distribution and standard deviation that scales with $1/\sqrt{\alpha}$. Also, according to the first-principle model in [18], the term $\mathcal{Z}^{LOS}$ in (5) captures the single (short-orbit) LOS channels between RIS unit cells and transmit/receive array elements in the far-field approximation. It is worth remarking that [19]

$$\left\langle \mathcal{Z}\left(n,n\right)\right\rangle = j\mathcal{X}^{fs}\left(n,n\right) + \mathcal{R}^{fs}\left(n,n\right) \triangleq \mathcal{Z}^{fs}\left(n,n\right), \tag{6}$$

with $\mathcal{Z}^{fs}\left(n,n\right)$ free-space (fs) active impedance of the n-th antenna element.

B. Derivation of Sherrington-Kirkpatrick Hamiltonian

The MIMO communication model (1) with RCM prescriptions encoded in (5) can be written an an effective SK

Hamiltonian. By assuming an inter-distance between RIS unit cells of $\lambda/2$, it is known that the mutual coupling is reduced and the RIS active impedance is approximately diagonal [11]

$$\Phi_{\mathrm{SS}}\left(n,n\right)=R\left(n\right)\ s\left(n\right), \tag{7}$$

with RIS unit cell (port) phase factor

$$s\left(n\right)=e^{-j\phi(n)}. \tag{8}$$

The calculation of the received power

$$\mathrm{H}_{\mathrm{RIS}}=\|\mathcal{H}_{\mathrm{E2E}}\|_{\mathrm{F}}=\mathrm{Tr}\left[\mathcal{H}_{\mathrm{E2E}}^{\dagger}\,\mathcal{H}_{\mathrm{E2E}}\right], \tag{9}$$

where $\|\ldots\|_{\mathrm{F}}$ indicates the Frobenius norm, yields the SK Hamiltonian

$$\mathrm{H}_{\mathrm{RIS}}=\sum\nolimits_{n_1=1}^{N_S}\sum\nolimits_{n_2=1}^{N_S} s\left(n_1\right)s\left(n_2\right)\tilde{\mathcal{J}}\left(n_1,n_2\right), \tag{10}$$

For RIS-assisted MIMO systems, the couplings $\tilde{\mathcal{J}}\left(n_1,n_2\right)$ in (10) have the form of a generalised Hopfield model. Furthermore, it has been already shown that generalised Hopfield converges to a non-zero mean Gaussian distribution on account of the central limit theorem [11], [12]. Optimising the performance of a RIS-assisted communication channel requires maximisation/minimisation of (10) under appropriate constraints. In real-life environments where the RIS operates in a multi-path fading environment, the unit cells would configure as spatially disordered (optimised) reflective states that change with fade duration. This dynamic optimisation is no doubt computationally intensive and time consuming, hence a question arises on whether quantum algorithms help addressing this challenge. We already demonstrated that for fast multi-path fading channel hardening alleviates the requirement of dynamic ground state search of the SK Hamiltonian, hence removing the link between the (classical) channel state information and (quantum) hardware landscape. We prove the existence of a concentration property that makes QAOA independent on the classical fluctuation of the SK Hamiltonian. Both channel hardening and concentration help predicting a robust state for the RIS under channel variability.

III. Quantum Approximate Optimisation Algorithm: A Micro Primer

Without lack of generality, we tackle the general problem of minimisation of SK Hamiltonians of the form (10). This is equivalent to maximising (10) with negative couplings $-\tilde{\mathcal{J}}\left(n_1,n_2\right)$. The SK Hamiltonian can be implemented in a universal gate quantum computer through QAOA, which runs through P hardware layers to achieve the associated ground state eigenvalue. The QAOA iterates a specific Ansatz composed of two unitary operators with p^{th}-layer paramaters (γ_p,β_p). The iterated application of the Ansatz to P-layers produces the landscape parameters (β,γ) that are dictated by the numerical convergence of the Hamiltonian to the ground state. The optimisation of these parameters is classical and pertains the rotation angles of quantum state, in this case generated by the SK Hamiltonian dynamics, at each hardware layer. The entire procedure is depicted in Fig. 2. From a

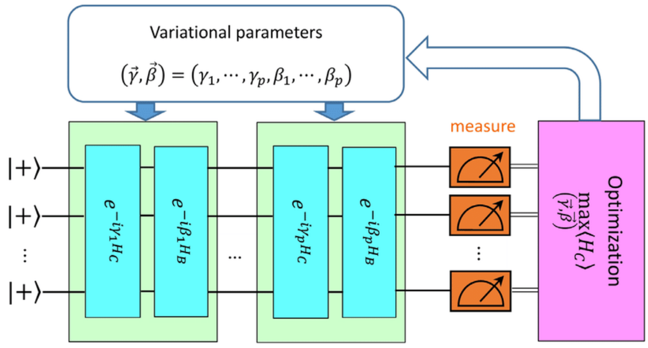

Fig. 2: QAOA ansatz and optimization

mathematical standpoint, we prepare the quantum computer in the state with uniform superposition of bit strings by applying a Hadamard gate to each qubit in the zero state [20]. Then, for each layer p of QAOA, we evolve the system with the cost Hamiltonian for some angle γ,

$$U_p=\exp\left(-i\gamma_p H_C\right), \tag{11}$$

and then evolve the system with a driver Hamiltonian

$$H_D=\frac{1}{2}\sum_i\sigma_i^x, \tag{12}$$

for an angle β

$$V_p=\exp\left(-i\beta_p H_D\right), \tag{13}$$

where σ^x is the Pauli X matrix. Repeated applications of the cost and driver dynamics evolves the system to the quantum state $\beta,\psi\rangle$

$$|\beta,\gamma\rangle=V_pU_p\cdots V_2U_2V_1U_1|\psi\rangle \tag{14}$$

Functional evaluation is performed by Monte-Carlo estimation of the cost Hamiltonian with samples drawn from a Gaussian distribution. In the original QAOA algorithm the control (landscape) parameters (β,γ) are optimised such that the functional, the expectation of the cost Hamiltonian for a given instance of the problem

$$\min_{\beta,\gamma}\langle\beta,\gamma|\mathcal{H}_{\mathcal{SK}}|\beta,\gamma\rangle \tag{15}$$

is minimized, with $\mathcal{H}_{\mathcal{SK}}=\mathrm{H}_{\mathrm{RIS}}$ in this work. This leads to the ground state eigenvalue λ_m for the single realisation (instance) of the SK Hamiltonian. However, due to coupling fluctuation, this introduces a potentially expensive training step for every query associated with the instance, which poses a computational bottleneck. This is alleviated by the use of a single set of landscape parameters for the optimisation of whole statistical ensemble of SK Hamiltonians. Results become accurate upon onset of a concentration property with respect to large statistical ensembles of SK Hamiltonians [15].

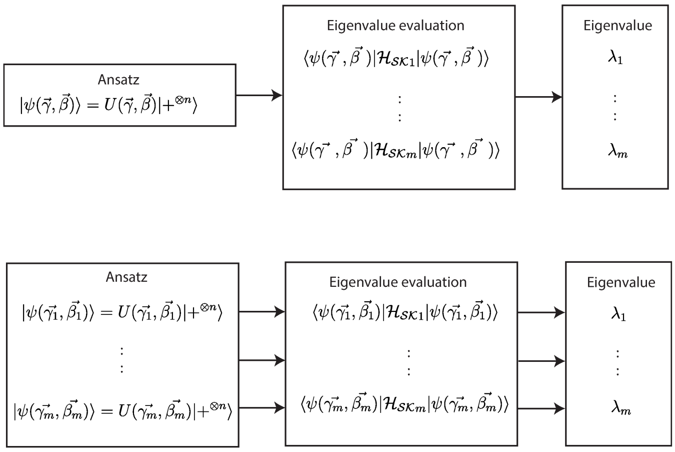

Fig. 3: Decoupling of classical Montecarlo generation of the SK Hamiltonians from QAOA simulations that retrieve their ground state.

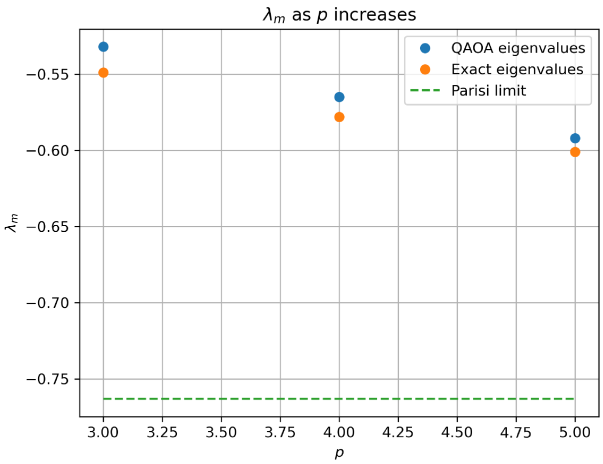

Fig. 4: Comparison between the average ground-state minimum eigenvalue (blue dots) computed by QAOA, the theoretical eigenvalue (orange dots), and the Parisi limit (dashed line) for an increasing circuit depth p.

IV. Numerical Results

A. Quantum Montecarlo simulations

The maximisation/minimisation of (10) implies a stochastic optimisation problem that is more involved than the Ising Hamiltonian optimisation [9]. In particular, we perform a numerical analysis of the random couplings via Montecarlo simulations of an arbitrary MIMO system. The caveat in this simulations is that the total number of radiating elements in both the transmit and receive array should be smaller than the number of the (mutually uncoupled) unit cells in the RIS. This aspect is important to avoid memory effects in the SK Hamiltonian that would create a deviation from Gaussian of the random couplings resulting from multi-stability underpinned by a spin glass state [14]. The distinctive different between QAOA of SK Hamiltonians and classical Metropolis-Hastings (MH) or simulated annealing (SA) optimisation is that the choice of the landscape parameters of the quantum computer is related to the Montecarlo generation of the (random) Hamiltonian couplings. This hybrid classical-quantum optimisation creates a bottleneck in searching the optimal RIS state that causes the response of the RIS to the EME variability to increase. Our idea is centred around the decoupling of Monte Carlo generation from QAOA execution. This is visually depicted in the scheme of Fig. 3. Inherently, it has been recently proposed that a unique set of landscape parameters can be used to run QAOA optimisation on a (large) ensemble of SK Hamiltonians [15].

B. Concentration property

The choice of the optimal set of QAOA angles has been devised using deep learning methods in the MaxCut optimisation problem [20]. More recently, it has been shown that the QAOA converges to the Parisi eigenvalue with a low circuit depths [15] thanks to the concentration property of the associated ansatz. The study in [15] considers an SK Hamiltonians with Bernoulli couplings fluctuating between $+1$ and -1 with equiprobability. In this work, we perform Qiskit [16] based simulations on an SK Hamiltonians with couplings following a non-central Gaussian distribution with mean value equal to 10 and unit variance. The convergence of the average ground-state (minimum) eigenvalue to the exact (Parisi) eigenvalue is shown in Fig. 4 for an increasing quantum circuit depth p. In Fig. 5 we show that the landscape parameters have reduced fluctuation with increasingly large ensemble distribution, which achieves the statistical concentration property. Furthermore, the eigenvalue associated with the ground state of the SK Hamiltonian has small error with respect to the multivariate set of optimised landscape parameters for a large statistical ensemble of instances (here we used 128 instances). In all the simulations we have adopted a low number of qubits ($n = 3$), which is important in lieu of implementation on practical quantum computing architectures where the hardware noise increases with an increasing number of qubits.

V. Conclusion

Using quantum algorithms to design optimised large antenna arrays and surfaces constitute a viable way forward in computational electromagnetics. However, the real-life operation of surfaces needs to account for the effect of the environment on the end-to-end RIS-assisted link at design stage. The electromagnetic propagation variability can be included in the channel gains of programmable environments through wave chaos theory, and recasted into a random Hamiltonian whose optimisation is a hard task. We find that searching for the optimal average RIS states state translates into searching for the average ground state of an SK Hamiltonian. Environment-aware design of the RIS becomes possible by using Monte Carlo generation in quantum optimisation: The coupling of the two operations create a bottleneck that is detrimental to achieve a quantum advantage over classical algorithms. We

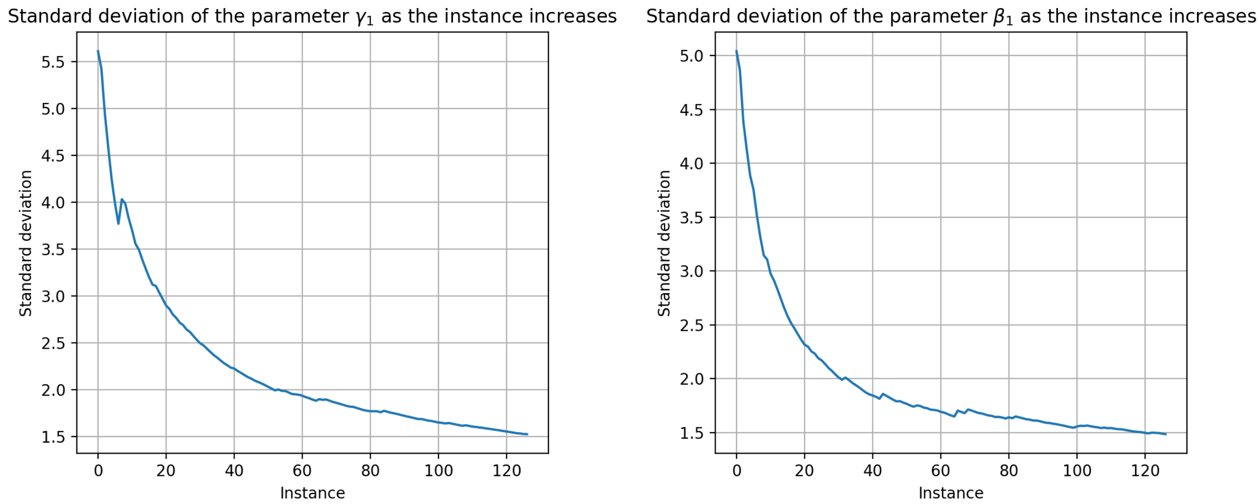

Fig. 5: Variance of two selected quantum circuit parameters: Decay with an increasing number of instances indicates statistical concentrations that decouples the Montecarlo generation from QAOA execution.

have found evidences of a statistical concentration property that decouples the Monte Carlo simulations from the QAOA execution. These findings support the prospects that QAOA will be an effective method for solving electromagnetics engineering problems on near-term quantum computers.

Acknowledgment

Funded by the EPSRC under Grant EP/V048937/1, and the Royal Society under Grant INF\R2\192066.

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