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Symbiotic Positioning, Navigation, and Timing

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***Abstract*—The explosive growth of Internet of Things (IoT) devices and location based services, along with the development of various 6G enabling technologies, are fueling the need for the design of alternative Positioning, Navigation, and Timing (PNT) solutions. In this paper, following this trend and inspired by the evolution of biological ecosystems, we introduce a novel symbiotic PNT solution, based on the principles of Game Theory and the exploitation of Reconfigurable Intelligent Surfaces (RISs). A set of actors, consisting of anchor nodes and RISs with known coordinates, collaborator nodes having a rough estimate of their positions, and targets of unknown positions, are cooperating to accurately determine the targets' positioning and timing. The key objective is to minimize the estimation error of each target and collaborator node, as well as of the overall examined system. The RISs' phase shifts optimization is performed to maximize the received signal strength of the signals reflected on the RISs and received by the collaborator nodes and the targets. Then, the optimization problem of the positioning and timing estimation error is formulated as a potential game among the targets and collaborator nodes, and the existence of at least one Nash Equilibrium is proven. Two algorithmic approaches, namely Asynchronous and Synchronous Best Response Dynamics, are introduced to determine the Nash Equilibrium, while the performance evaluation of the proposed approach is achieved via modeling and simulation.**

***Index Terms*—Positioning, Navigation, and Timing, Symbiotic Relationships, Potential Games, Game Theory.**

I. Introduction

Accurate Positioning, Navigation, and Timing (PNT) services are critical in several smart cities service domains, such as transportation, public safety, wireless communication, energy distribution, to name a few. Currently, the dominant PNT system is the Global Navigation Satellite System (GNSS), with the Global Positioning System (GPS) being the most representative service provider. However, GPS can suffer from unintentional or man-made interference to the satellite signals due to the long propagation distance, spoofing, jamming, etc., resulting in deteriorated GPS services or even GPS-denial. Thus, the design of alternative PNT solutions to complement or even substitute the GPS has been identified as a national planning objective in the USA [1]. In this paper, aligned with the latter vision, we introduce a novel symbiotic PNT solution, where the available nodes in the field collaborate among each other creating mutually benefiting relationships, in order to accurately determine their positioning and timing, following a game-theoretic approach. The proposed symbiotic PNT solution further exploits the key 6G technology of the Reconfigurable Intelligent Surfaces (RISs), as a means of further ameliorating the positioning and timing accuracy.

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A. Related Work

Several recent research works have focused on designing PNT solutions, characterized by high accuracy and robustness. A fingerprinting-based localization scheme is introduced in [2] by proposing an energy-efficient deep learning architecture for indoor localization. However, the main drawback of the fingerprinting-based localization schemes is that they heavily rely on the database of labeled data, constructed from an offline site survey, which makes them very expensive solutions. A multiple base stations PNT solution is proposed in [3], where a convolutional autoencoder model is used to determine the targets' position based on the received channel state information. Similarly, in [4], the authors focus on a small-cell environment by exploiting the Bluetooth Low Energy Received Signal Strength Indicator via a self-supervised machine learning model for indoor localization. Towards reducing the number of anchor nodes contributing to the PNT service, the authors in [5] provided a single-anchor localization scheme by exploiting the reflected signals in an indoor environment and utilizing their delay and angle of arrival in order to determine the target's position.

The key 6G technology of RISs has attracted great attention in the design of alternative PNT solutions. The RISs are characterized by some noticeable attributes, i.e., low-cost, easy deployment, control of the phase shifts of the reflected signals in a programmable manner, and passive operation. Also, RISs can provide a strong reflected signal and at the same time act as a reference point in a PNT system [6]. In [7], the RISs' phase shift optimization is performed to maximize the strength of the received signal. Capitalizing on this, the authors introduce a simultaneous localization and mapping scheme that minimizes the position error. Similarly, in [8], the authors optimize the reflected beamforming on the RISs by following a gradient descent method, aiming at minimizing the targets' positioning estimation error. A self-localization model is proposed in [9] by processing the reflected signals from multiple RISs, stemming from a pilot signal transmitted by the target.

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Limited research effort has been invested in the cooperative PNT solutions, where the targets collaborate among each other to improve the accuracy of their positioning and timing. The research in this field is still in its infancy. In [10], the authors introduce novel beamforming schemes aiming at optimizing the cooperative localization performance among multiple targets and minimizing the localization error.

B. Contributions & Outline

Aiming to make a step towards filling this research gap and inspired by the evolution of biological ecosystems, we introduce a novel *symbiotic positioning, navigation, and timing* (PNT) solution based on the principles of Game Theory and exploiting the key 6G technology of Reconfigurable Intelligent Surfaces (RISs). Specifically, we extend the concept of collaboration among the nodes in order to determine their positioning and timing to the level of creating a symbiotic mutualistic relationship among them. The nodes establish a mutualistic relationship founded on the service exchange basis, where they coordinate with each other to minimize their personal and the overall system's positioning and timing error. Indeed, this is an interesting mutualistic relationship given that each node cannot achieve its goal, i.e., accurate estimation of positioning and timing, by being isolated from its neighboring nodes. This paradigm leads to creating relationships between the involved entities that are beneficial to all parties (mutualistic), in a reciprocal and symbiotic way. The main contributions and key elements of this research work are summarized as follows.

1) We introduce a symbiotic environment consisting of anchor nodes, collaborator nodes, RISs, and targets. The anchor nodes and the RISs have known coordinates, while the collaborator nodes and the targets have a rough estimate and unknown position, respectively. All the involved entities collaborate with each other in a symbiotic manner to ultimately determine their accurate positioning and timing via minimizing the estimation error.
2) The RISs' phase shifts optimization is performed to maximize the received signal strength of the signals reflected on the RISs and received by the collaborator nodes and the targets. Then, the collaborator nodes and the targets measure their pseudoranges from the sources of the transmitted signals and perform an estimation of their positioning and timing. The minimization problem of the collaborator nodes' and the targets' estimation error is formulated as a potential game among them and the existence of a Nash Equilibrium is proven.
3) Two alternative algorithms are introduced to determine the Nash Equilibrium in a distributed manner following the principles of the Asynchronous and Synchronous Best Response Dynamics. Their drawbacks and benefits in terms of convergence time and PNT solution accuracy are quantified through a simulation-based analysis.

The rest of the paper is organized as follows. Section II presents the symbiotic environment and the RISs' phase shifts optimization. Section III formulates the symbiotic PNT problem as a potential game, proves the existence of a Nash Equilibrium, and subsequently, two algorithms are presented in order to determine such a point. A detailed comparative evaluation is presented in Section IV and finally, Section V concludes the paper.

II. System Model

A symbiotic environment is considered, consisting of a set of anchor nodes, RISs, collaborator nodes, and targets, denoted as $A = \{1, \dots, a, \dots, |A|\}$, $R = \{1, \dots, r, \dots, |R|\}$, $C = \{1, \dots, c, \dots, |C|\}$, and $U = \{1, \dots, u, \dots, |U|\}$, respectively [11]. The anchor nodes and the RISs have perfect knowledge of their coordinates $\mathbf{x}_a = (x_a, y_a, z_a)$ and $\mathbf{x}_r = (x_r, y_r, z_r)$, respectively [12]. On the other hand, the collaborator nodes have an estimate of their coordinates $\hat{\mathbf{x}}_c = (\hat{x}_c, \hat{y}_c, \hat{z}_c)$, and the targets' coordinates are unknown. All the four types of entities collaborate among each other by establishing a service-to-service mutualism, thus, creating a symbiotic PNT environment. The goal of this symbiotic relationship among them is to accurately determine their positioning and timing, while minimizing the estimation error.

A. Neighborhood Identification

Considering a target $u, \forall u \in U$, its goal is to accurately determine its positioning and timing. Initially, the target needs to identify its neighboring reference points, i.e., anchor nodes, RISs, and collaborator nodes, that contribute to its PNT service in a collaborative manner. Thus, the target broadcasts a ranging request beacon signal that is received by the neighboring anchor nodes $A_u \subseteq A$ and collaborator nodes $C_u \subseteq C$. Then, all the nodes $a \in A_u$ and $c \in C_u$ respond with a ranging reply beacon signal with fixed power $P = P_a = P_c$[W], including also digital information of their coordinates $\mathbf{x}_a, \hat{\mathbf{x}}_c$, respectively. Additionally, the anchor nodes include the information of the RISs coordinates $\mathbf{x}_r$ in their reply signal, considering the set of RISs R_u, which reside in the target's coverage area, as it is determined above. The target can measure the pseudoranges from a reference point $j = a, r, c, \forall a \in A_u, \forall c \in C_u, \forall r \in R_u$ based on the received power

$$P_{u,j} = P \frac{G_j^{trans} G_u^{rec}}{L_{u,j}} \quad (1)$$

where G_j^{trans} denotes the gain of the transmitting node's antenna, G_u^{rec} is the gain of the target's antenna, and $L_{u,j}$ follows the Okumura/Hata model for large cities scenarios [13],

$$\begin{aligned} L_{u,j} &= 69.55 + 26.16 \log f_c + (44.9 - 6.55 \log h_j{}^{trans}) \\ &\log d_{u,j} - 13.82 \log h_j^{trans} - 3.2[log(11.75 h_u)]^2 - 4.97[dB] \end{aligned} \quad (2)$$

where, f_c[Hz] is the carrier frequency, with $f_c \geq 400$MHz, h_j^{trans}[m] is the height of the reference points, h_u[m] is the target's u antenna's height, $d_{u,j}$[m] is the eventually measured pseudorange by target u from the reference points j. Thus, by following the above-described neighborhood identification process, each target u becomes aware of the coordinates $\mathbf{x}_a, \mathbf{x}_r, \hat{\mathbf{x}}_c$, and the corresponding pseudoranges $d_{u,a}, d_{u,c}, d_{u,r}, \forall a \in A_u, \forall c \in C_u, \forall r \in R_u$.

B. RISs Phase Shift Optimization

Towards improving the targets' accuracy of measuring their pseudoranges from the reference points and ultimately estimating their position and timing $\hat{\mathbf{P}}_u = (\hat{x}_u, \hat{y}_u, \hat{z}_u, \Delta\hat{t}_u)$, the strengths of the received signals should be improved. It is noted that $\Delta\hat{t}_u$ denotes the target's estimation regarding the clock offset among the anchor nodes' and the targets' clocks, while assuming that all the anchor nodes are synchronized among each other. Also, it is noted that among all the available reference points, only the anchor nodes have simultaneously perfect knowledge of their position and act as transmitters. Thus, each target can opportunistically select to optimize the RISs' phase shifts for the anchor node's $a^*, \forall a^* \in A_u$ strongest incoming signal. In this way, the RISs contribute to a constructive beam that will be received by the target with improved signal strength and further contribute to the accuracy of its PNT solution.

The channel gain of the direct communication link between the target u and the anchor node a^* is

$$h_{u,a^*} = L_{u,a^*}(d_{u,a^*}) \cdot \tilde{h} \tag{3}$$

where $L_{u,a^*}(d_{u,a^*})$ is given by Eq. 2 and $\tilde{h} \sim CN(0,1)$ captures the random scattering component represented by a zero-mean unit-variance complex Gaussian random variable. Focusing on the communication link between the anchor node a^*, and a RIS $r, r \in R_u$, the path loss component is derived as $PL_{a^*,r} = \rho(d_{a^*,r})^{\alpha}$, where ρ[dB] denotes the path loss at the reference distance 1 m, $d_{a^*,r}$ [m] is the distance between the anchor node a^* and the RIS r, and α is the path loss exponent [14]. We consider that the RISs are uniform linear arrays, and each RIS consists of $|M|$ reflecting elements, where $M = \{1, \dots, m, \dots, |M|\}$ denotes their set. Each RIS element m can control a phase shift $\omega_m \in [0, 2\pi), \forall m \in M$, and the corresponding diagonal reflection matrix of each RIS is $\Omega = \mathrm{diag}(e^{j\omega_1}, \dots, e^{j\omega_m}, \dots, e^{j\omega_{|M|}}) \in \mathbb{C}^{|M| \times |M|}$. Also, it is noted that the coordinates $\mathbf{x}_r = (x_r, y_r, z_r)$ refer to the first element $m = 1$ of each RIS. The channel gain coefficient of the communication link between the anchor node a^* and the RIS r is

$$\mathbf{h}_{a^*,r} = \sqrt{\frac{1}{PL_{a^*,r}}}[1, e^{-j\frac{2\pi}{\lambda}d_s\phi_{a^*,r}}, \dots, e^{-j\frac{2\pi}{\lambda}(|M|-1)d_s\phi_{a^*,r}}]^T \tag{4}$$

where λ [m] is the carrier wavelength, d_s[m] is the antenna separation, and $\phi_{a^*,r}$ is the cosine of the angle of arrival of the anchor nodes a^* signal to RIS r.

Furthermore, the channel gain of the communication link between the RIS r and the target u, is given as:

$$\mathbf{h}_{r,u} = L_{r,u}(d_{r,u})(\sqrt{\frac{k}{k+1}}\mathbf{h}_{r,u}^{LoS} + \sqrt{\frac{1}{1+k}}\mathbf{h}_{r,u}^{NLoS}) \tag{5}$$

where k denotes the Rician factor, and

$$\mathbf{h}_{r,u}^{LoS} = [1, e^{-j\frac{2\pi}{\lambda}d_s\phi_{r,u}}, \dots, e^{-j\frac{2\pi}{\lambda}(|M|-1)d_s\phi_{r,u}}]^T \tag{6}$$

and $\mathbf{h}_{r,u}^{NLoS} \sim CN(0,1)$ denote the Line of Sight (LoS) and non-LoS (NLoS) components, respectively, where $\phi_{r,u}$ is the cosine of the angle of departure of the signal from the RIS r to the target u. Therefore, the overall channel gain between the anchor node a^* and the target u is given as

$$G_u^{a^*,r} = |h_{u,a^*} + \mathbf{h}_{a^*,r}\mathbf{\Omega}\mathbf{h}_{r,u}|^2. \tag{7}$$

The targets' goal is to maximize the received signal strength given the transmission power P of the anchor node a^* and derive the optimal phase shifts $\omega^* = [\omega_1^*, \dots, \omega_m^*, \dots, \omega_{|M|}^*]$ of each RIS $r, \forall r \in R_u$. Thus, the optimization problem can be written as follows for each RIS $r \in R_u$.

$$\max_{\omega} |h_{u,a^*} + \mathbf{h}_{r,u}^H \mathbf{\Omega} \mathbf{h}_{a^*,r}|^2 \tag{8a}$$

$$s.t. \;\; 0 \leq \omega_m \leq 2\pi, \forall m \in M \tag{8b}$$

We set $v_m = e^{j\omega_m}, \forall m \in M$, thus, $\mathbf{v} = [v_1, \dots, v_m, \dots, v_{|M|}] \in \mathbb{C}^{|M| \times 1}$. Then, we substitute

$$\tilde{\mathbf{h}}_{a^*,r} = \mathbf{h}_{r,u}^H \,\mathrm{diag}(\mathbf{h}_{a^*,r}) \in \mathbb{C}^{1 \times |M|} \tag{9}$$

and rewrite the optimization problem (8a) – (8b), as follows.

$$\max_{\mathbf{v}} |h_{u,a^*} + \tilde{\mathbf{h}}_{a^*,r}\mathbf{v}|^2 \tag{10a}$$

$$s.t. \;\; |v_m| = 1, \forall m \in M \tag{10b}$$

Towards maximizing the quantity in Eq. 10a, the direct and reflected signal should be perfectly aligned and coherently combined. The optimal solution can be derived as follows,

$$\begin{aligned} \angle h_{u,a^*} &= -\angle\tilde{\mathbf{h}}_{a^*,r} + \angle\mathbf{v} \Rightarrow \\ \mathbf{w}^* = \angle\mathbf{v} &= \angle h_{u,a^*} + \angle\tilde{\mathbf{h}}_{a^*,r}. \end{aligned} \tag{11}$$

The optimization problem (10a) – (10b) is solved for each RIS $r, \forall r \in R_u$, and the target broadcasts the RISs optimal phase shifts, in order for the corresponding RISs controllers to tune the phase shifts of its elements appropriately. After the RISs' elements phase shift optimization is performed, the anchor nodes will send a second ranging reply signal at the same fixed transmission power level P. Thus, the target will receive a stronger signal from the anchor node with the strongest transmitted signal, contributing to more accurately estimating the corresponding pseudorange and ultimately improving the estimation of its positioning and timing $\hat{\mathbf{P}}_u$.

III. Symbiotic Positioning, Navigation, and Timing

A. Problem Formulation

In this section, we introduce a symbiotic PNT solution based on the principles of Game Theory that: (i) determines the targets' $u, \forall u \in U$, accurate positioning and timing $\hat{\mathbf{P}}_u$, (ii) minimizes the estimation error of the targets' $\hat{\mathbf{P}}_u, \forall u \in U$, and the collaborators' $\hat{\mathbf{P}}_c$ estimated positioning and timing in a distributed manner, and (iii) minimizes the overall estimation error in the system.

We define the Euclidean distance of the positioning and timing between the targets and the reference points as follows,

$$\hat{d}(\hat{\mathbf{P}}_u, \hat{\mathbf{P}}_j) = \begin{cases} ||\hat{\mathbf{P}}_u - \mathbf{P}_j||, \text{ if } \; j = a, r, \forall a \in A_u, \forall r \in R_u \\ ||\hat{\mathbf{P}}_u - \hat{\mathbf{P}}_j||, \text{ if } \; j = c, \forall c \in C_u \end{cases} \tag{12}$$

where, the initial estimation of $\hat{\mathbf{P}}_u, \forall u \in U$, and $\hat{\mathbf{P}}_j, j = c, \forall c \in C_u$ can be determined based on the analysis presented in Section II, complemented with the multilateration technique [15]. The estimation error of the positioning and timing can be determined as follows,

$$\epsilon(\hat{\mathbf{P}}_u, \hat{\mathbf{P}}_j) = [d_{u,j} - \hat{d}(\hat{\mathbf{P}}_u, \hat{\mathbf{P}}_j)]^2 \quad (13)$$

$\forall u \in U, \forall j = a, r, c, \forall a \in A_u, r \in R_u$, where $d_{u,j}$[m] are the pseudoranges between the target u and the reference points $j = a, r, c, \forall a \in A_u, \forall r \in R_u, \forall c \in C_u$ as they have been measured following the analysis presented in Section II-A. Obviously, if the targets' $\hat{\mathbf{P}}_u, \forall u \in U$, and the reference points' $\hat{\mathbf{P}}_j, \forall j = a, r, c, \forall a \in A_u, \forall r \in R_u, \forall c \in C_u$ positioning and timing are accurately determined, then $\epsilon(\hat{\mathbf{P}}_u, \hat{\mathbf{P}}_j) \rightarrow 0$. It is noted that we denote the reference points' positioning and timing as $\hat{\mathbf{P}}_j, \forall j = a, r, c, \forall a \in A_u, \forall r \in R_u, \forall c \in C_u$ for notation convenience, while the positioning and timing for the anchor nodes and RIS is perfectly known, i.e., $\hat{\mathbf{P}}_j = \mathbf{P}_j, \forall j = a, r, \forall a \in A_u, \forall r \in R_u$. Thus, the goal of each target (similarly, and of each collaborator node) is to minimize its personal experienced estimated error, that is:

$$\min_{\hat{\mathbf{x}}_u} \sum_{j \in A_u \cup R_u \cup C_u} \epsilon(\hat{\mathbf{P}}_u, \hat{\mathbf{P}}_j), \forall u \in U \quad (14)$$

The overall goal of the system is to minimize the overall positioning and timing estimation error within the examined symbiotic PNT system.

$$\min_{\hat{\mathbf{x}}_u} E(\hat{\mathbf{x}}_u, \hat{\mathbf{x}}_j) = \sum_{\forall u \in U \cup C_u} \sum_{\forall j \in A_u \cup R_u \cup C_u} \epsilon(\hat{\mathbf{x}}_u, \hat{\mathbf{x}}_j) \quad (15)$$

B. Problem Solution

Towards solving the optimization problems (14) and (15), we formulate a non-cooperative game among the nodes that have an estimation regarding their positioning and timing, i.e., $N = U \cup C$ considering the targets and the collaborator nodes. We denote the non-cooperative Symbiotic PNT (SPNT) game as $G = [N, \{S_n\}_{\forall n \in N}, \{U_n\}_{\forall n \in N}]$, where $N = U \cup C$ is the set of players, S_n is their strategy set with strategy $s_n = (\hat{x}_n, \hat{y}_n, \hat{z}_n, \Delta\hat{t}_n)$, and

$$U_n(s_n) = \sum_{j \in A_u \cup R_u \cup C_u} \epsilon(\hat{\mathbf{P}}_u, \hat{\mathbf{P}}_j). \quad (16)$$

Definition 1: **(Nash Equilibrium – NE)** A strategy vector $\mathbf{s}^* = (s_1^*, \dots, s_n^*, \dots, s_{|N|}^*)$ is a Nash Equilibrium for the game G, iff

$$U_n(s_n^*, \mathbf{s}_{-n}^*) \leq U_n(s_n^{'}, \mathbf{s}_{-n}^*) \quad (17)$$

$\forall s_n^{'} \in S_n, \forall n \in N$, where $\mathbf{s}_{-n}^* = [s_1^*, \dots, s_{n-1}^*, s_{n+1}^*, \dots, s_{|N|}^*]$.

Towards showing the existence of at least one NE for the game G, we use the theory of potential games [16].

Definition 2: **(Exact Potential Game)** The non-cooperative game G is an exact potential game, if

$$\Phi(s_n, \mathbf{s}_{-n}) - \Phi(s_n^{'}, \mathbf{s}_{-n}) = U_n(s_n, \mathbf{s}_{-n}) - U_n(s_n^{'}, \mathbf{s}_{-n}) \quad (18)$$

$\forall s_n^{'} \in S_n, \forall n \in N$ where $\Phi(s_n, \mathbf{s}_{-n})$ denotes the potential function.

Theorem 1: The SPNT game G is an exact potential game, with potential function

$$\Phi(s_n, \mathbf{s}_{-n}) = \frac{E(s_n, \mathbf{s}_{-n})}{2}. \quad (19)$$

Proof: Consider that node n updates unilaterally its position to $s_n^{'}$, while the rest of the nodes $N - \{n\}$, keep their positions estimates $\mathbf{s}_{-n}$ unaltered. Then, we have:

$$U_n(s_n, \mathbf{s}_{-n}) - U_n(s_n^{'}, \mathbf{s}_{-n}) = \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) - \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n^{'}, \hat{\mathbf{x}}_j)$$

where for notation convenience, we set $N_n = A_n \cup R_n \cup C_n$. We analyze the potential function, as follows,

$$\begin{aligned}
\Phi(s_n, \mathbf{s}_{-n}) &= \frac{1}{2} \sum_{\forall n \in N} \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) \\
&= \frac{1}{2} [\sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) + \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\forall j \in N_k} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j)] \\
&= \frac{1}{2} [\sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) + \sum_{\substack{\forall k \in N \\ k \neq n}} [(\sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j)) + \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n)]] \\
&= \frac{1}{2} [\sum_{j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) + \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j) \\
&\quad + \sum_{\substack{\forall k \in N \\ k \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n)]
\end{aligned} \quad (20)$$

If two nodes k, n are not neighbors, then, they cannot measure their pseudoranges, thus, $\epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n) = 0, k, n \notin N_n$. Thus, the last term of the potential function can be analyzed as follows:

$$\begin{aligned}
\sum_{\substack{\forall k \in N \\ k \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n) &= \sum_{\forall k \in N_n} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n) + \underbrace{\sum_{\substack{\forall k \notin N_n \\ k \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n)}_{= 0} \\
&= \sum_{\forall k \in N_n} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n)
\end{aligned}$$

Thus, we rewrite the potential function as follows [17]:

$$\begin{aligned}
\Phi(s_n, \mathbf{s}_{-n}) &= \frac{1}{2} [\sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) + \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j) \\
&\quad + \sum_{\forall k \in N_n} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_n)] = \frac{1}{2} [2 \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) \\
&\quad + \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j)] \\
&= \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) + \frac{1}{2} \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j).
\end{aligned}$$

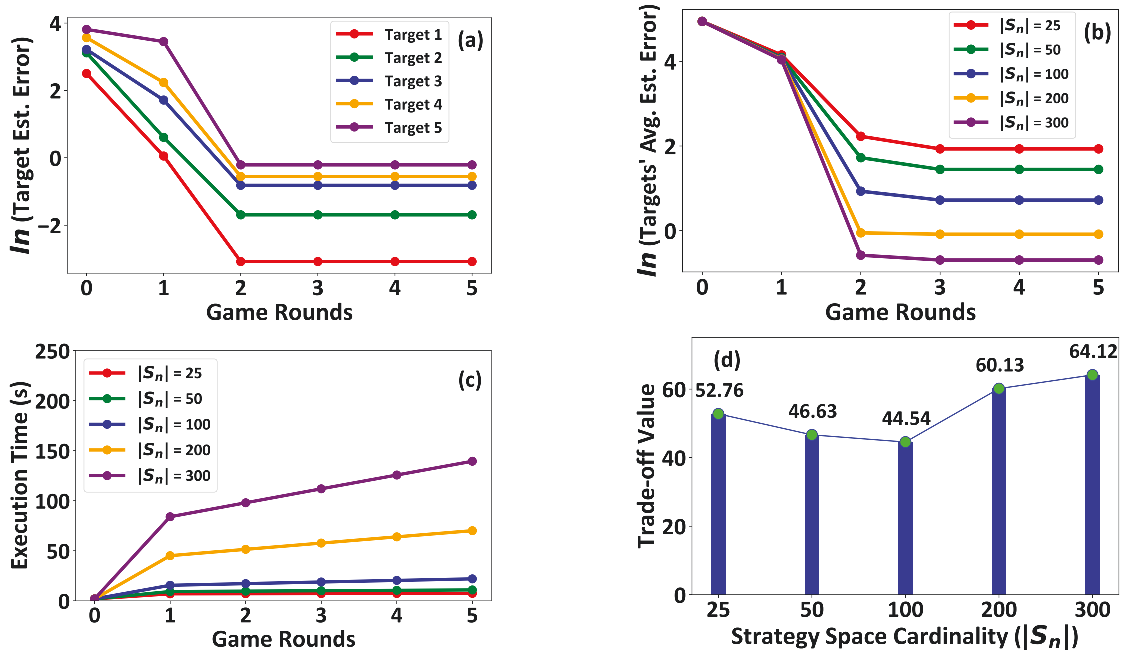

Fig. 1: Asynchronous Best Response Dynamics (ABRD)-based symbiotic PNT solution.

We take the difference of the potential function for two strategies $s_n, s_n^{'}$, as follows:

$$\begin{aligned}
&\Phi(s_n, \mathbf{s}_{-n}) - \Phi(s_n^{'}, \mathbf{s}_{-n}) \\
&= \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) + \frac{1}{2} \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j) \\
&- [\sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n^{'}, \hat{\mathbf{x}}_j) + \frac{1}{2} \sum_{\substack{\forall k \in N \\ k \neq n}} \sum_{\substack{\forall j \in N_k \\ j \neq n}} \epsilon(\hat{\mathbf{x}}_k, \hat{\mathbf{x}}_j)] \\
&= \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n, \hat{\mathbf{x}}_j) - \sum_{\forall j \in N_n} \epsilon(\hat{\mathbf{x}}_n^{'}, \hat{\mathbf{x}}_j) \\
&= U_n(s_n, \mathbf{s}_{-n}) - U_n(s_n^{'}, \mathbf{s}_{-n})
\end{aligned}$$

Thus, G is an exact potential game and has at least one Pure Nash Equilibrium [16]. ■

Towards determining the Pure Nash Equilibrium, we introduce two algorithmic approaches based on the principles of Best Response Dynamics (BRD), i.e., the Asynchronous BRD (ABRD) and the Synchronous BRD (SBRD).

Given that the non-cooperative game G is an exact potential game, the convergence of the ABRD and SBRD to a Nash Equilibrium is guaranteed [16].

IV. Numerical Results

In this section, a detailed evaluation of the proposed symbiotic PNT solution is realized via modeling and simulation, in order to demonstrate its benefits and tradeoffs. Initially, the pure performance of the proposed model is presented in Section IV-A, demonstrating the operational characteristics of both the ABRD and SBRD algorithms. Subsequently, a scalabilty analysis is demonstrated in Section IV-B to capture the efficiency and robustness of the proposed alternative PNT solution, complemented by a detailed comparative evaluation between the ABRD and SBRD algorithms towards capturing their operational tradeoffs. Unless otherwise explicitly stated, the values of the key parameters used throughout our evaluation are as follows: $|A| = 9$, $|R| = 5$, $|C| = 4$, $P = 16$ [W], $G_j^{trans} = 0$ [dB], $G_u^{rec} = 0$ [dB], $f_c = 400$ [MHz], $h_j^{trans} = 1.5$ [m], $\rho = 100$ [dB], $\alpha = 2.8$, $|M| = 300$, $d_s = \lambda/2$, and $\kappa = 2.8$ [18]. The evaluation was conducted in a Dell Tower Desktop with Intel i7 11700K 3.6GHz processor, 32 GB available RAM.

Algorithm 1 Asynchronous BRD (ABRD) Algorithm

Input: $\mathbf{P}_a$, $\forall a \in A$, $\mathbf{P}_r$, $\forall r \in R$, $\hat{\mathbf{P}}_c$, $\forall c \in C$
Output: s^*
Initialization: $ite = 0$, $Convergence = 0$, $\mathbf{s}^{ite=0}$ randomly selected strategy.
while $Convergence == 0$ **do**
    $ite = ite + 1$;
    Select randomly a node $n \in N = U \cup C$
    The selected target determines s_n^{*ite} *(Eq. 14) and determines* $U_n(s_n^{*ite}, \mathbf{s}_{-n}^{*ite})$*, given* $\mathbf{s}_{-\mathbf{n}}^{\mathbf{ite-1}}$
    if $|U_n(s_n^{*ite}, \mathbf{s}_{-n}^{ite-1}) - U_n(s_n^{*ite+1}, \mathbf{s}_{-n}^{ite})| \leq \delta$, δ small positive number, $\forall n \in N$ **then**
        $Convergence = 1$
    end if
end while

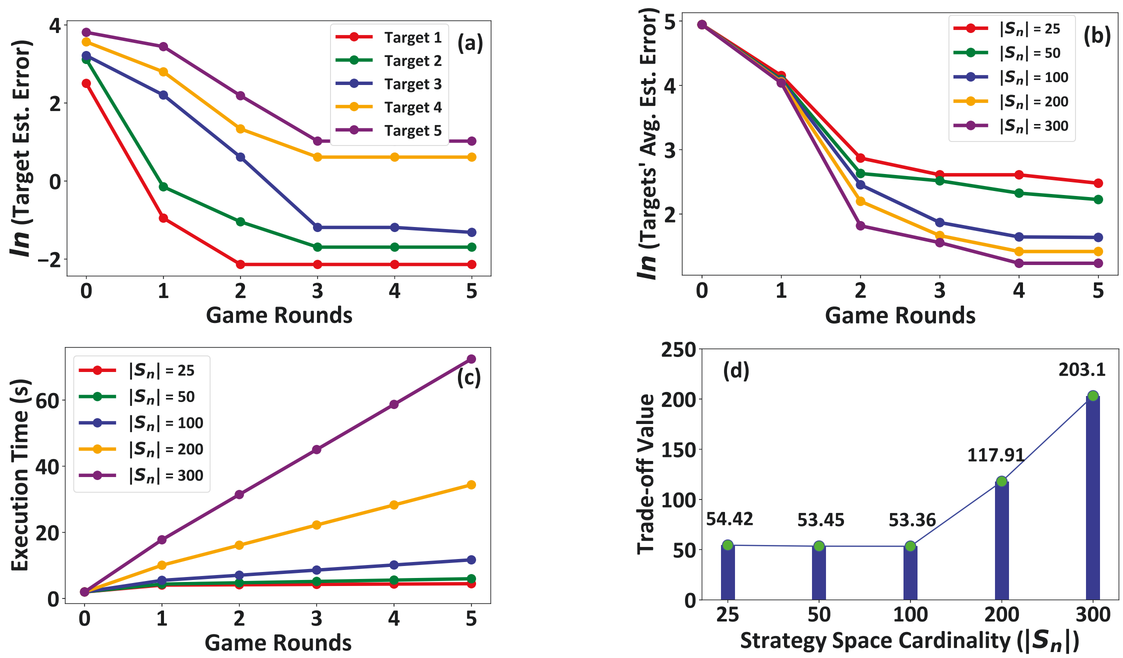

Fig. 2: Synchronous Best Response Dynamics (SBRD)-based symbiotic PNT solution.

Algorithm 2 Synchronous BRD (SBRD) Algorithm

Input: $\mathbf{P}_a, \forall a \in A$, $\mathbf{P}_r, \forall r \in R$, $\hat{\mathbf{P}}_c, \forall c \in C$
Output: s^*
Initialization: $ite = 0$, $Convergence = 0$, $\mathbf{s}^{ite=0}$ randomly selected strategy.
while $Convergence == 0$ **do**
  $ite = ite + 1$;
  for all $n \in N = U \cup C$ **do**
    Determine s_n^{*ite} *(Eq. 14) and* $U_n(s_n^{*ite}, \mathbf{s}_{-n}^{ite-1})$*, given* $\mathbf{s}_{-\mathbf{n}}^{\mathbf{ite-1}}$
  end for
  if $|U_n(s_n^{*ite}, \mathbf{s}_{-n}^{ite-1}) - U_n(s_n^{*ite+1}, \mathbf{s}_{-n}^{ite})| \leq \delta$, δ small positive number, $\forall n \in N$ **then**
    $Convergence = 1$
  end if
end while

A. Pure Performance and Operation

Fig. 1a and Fig. 2a show the targets' positioning and timing estimation error in a logarithmic scale as a function of the ABRD and SBRD game rounds, respectively. Similarly, Fig. 1b-1c and Fig. 2b-2c present the targets' average estimation error in a logarithmic scale and the algorithm's execution time, considering different cardinality of all the targets' strategy space. Fig. 1d and Fig. 2d illustrate the corresponding trade-off value, which is defined as the multiplication between the overall estimation error and the algorithm's execution time, as a function of the targets' strategy space cardinality. It is noted that in the considered topology, the higher the target's ID, the fewer reference points, i.e., anchor nodes, collaborator nodes, and RISs, reside in its neighborhood.

The following main observations are derived and hold true for both symbiotic PNT solutions, i.e., ABRD and SBRD. The potential game converges to a Nash Equilibrium (Fig. 1a and Fig. 2a), where the targets that reside in a more favorable position in terms of available reference points in their neighborhood achieve lower estimation error. Also, as the targets' strategy space becomes more discretized, i.e., higher cardinality of the targets' strategy space, the accuracy of the symbiotic PNT solution improves (Fig. 1b and Fig. 2b), at the cost of higher execution time in order to converge to the Nash Equilibrium (Fig. 1c and Fig. 2c). The results reveal that there is an optimal cardinality of the target's strategy space that balances the trade-off between the accuracy of the symbiotic PNT solution and the corresponding execution time of the ABRD and SBRD algorithms in order to determine the targets' positioning and timing (Fig. 1d and Fig. 2d). Also, by comparing the ABRD and SBRD algorithms we observe that the SBRD algorithm achieves lower execution time by sacrificing the accuracy of the symbiotic PNT solution.

B. Scalability and Comparative Analysis

In this section, we perform a detailed scalability analysis of the proposed symbiotic PNT solution under an increasing number of targets and collaborator nodes in order to demonstrate its efficiency and robustness. The proposed symbiotic PNT solution is compared to the multilateration technique following the Iterative Least Square algorithm [15], where the targets and collaborator nodes determine their position based on the signals received by the anchor nodes within the area

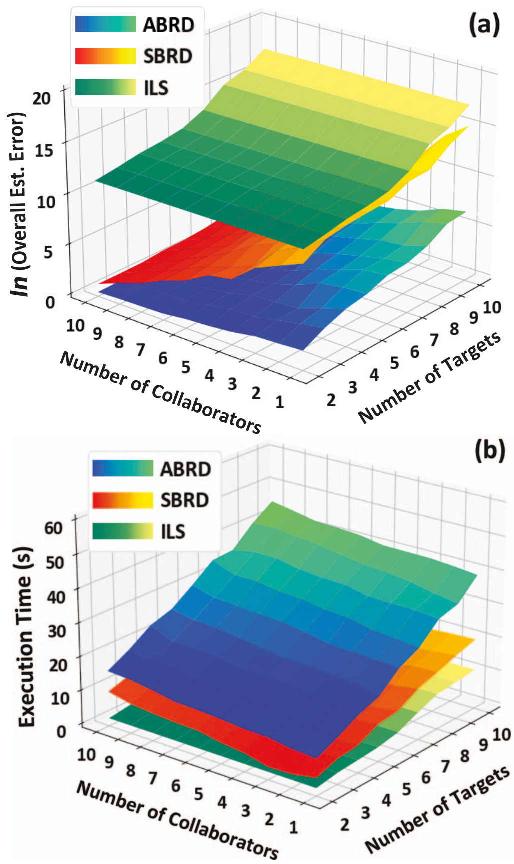

Fig. 3: Scalability analysis of the ABRD, SBRD-based symbiotic PNT solutions and the ILS algorithm.

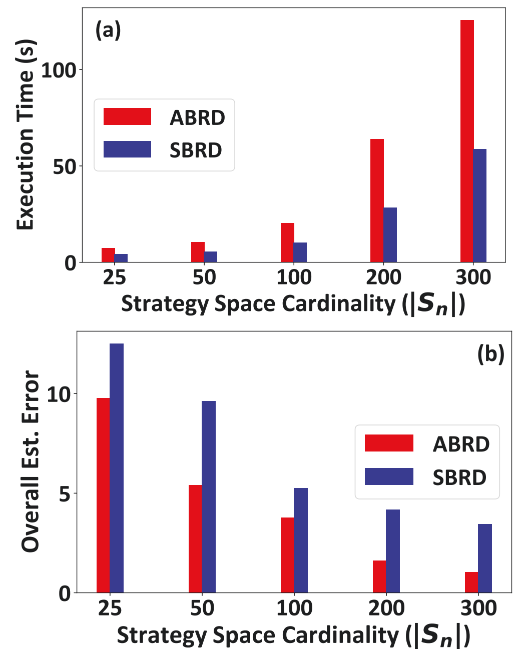

Fig. 4: Comparative analysis of the ABRD and SBRD-based symbiotic PNT solutions.

where they reside. We provide a detailed comparative analysis of the proposed symbiotic PNT solution under the ABRD and SBRD in order to demonstrate their tradeoffs in terms of execution time and estimation error.

Specifically, Fig. 3a-3b present the ABRD, SBRD, and ILS algorithms' overall estimation error in logarithmic scale and execution time, for an increasing number of targets and collaborator nodes, respectively. The results reveal that as the number of targets increases, the overall estimation error and execution time also increase for both the ABRD and SBRD algorithms. Also, it is observed that the overall estimation error and the execution time have very similar increase rate for both the ABRD and the SBRD algorithms with respect to the increasing number of targets. Moreover, the ABRD algorithm outperforms the SBRD algorithm in terms of achieved overall estimation error, at the cost of higher execution time, while the ILS algorithm presents substantially the worst results in terms of overall estimation error.

Focusing on the scalability scenario of an increasing number of collaborators for fixed number of targets (Fig. 3b), the results show that the overall estimation error decreases and the execution time increases for both the ABRD and SBRD algorithms. This observation is expected as the targets can more accurately determine their positioning and timing for an increasing number of reference points, i.e., collaborator nodes, at an expected cost of higher execution time. Also, we observe that the decrease rate of the total estimation error and the increase rate of the execution time are similar for both algorithms. Aligned with the observation made for increasing number of targets, the ABRD algorithm achieves better overall positioning and timing accuracy compared to the SBRD algorithm, at the cost of higher execution time.

Moreover, we study how the proposed symbiotic PNT solution behaves with respect to the targets' increasing strategy space under both algorithmic implementations, i.e., ABRD and SBRD algorithms. Fig. 4a-4b present the ABRD and SBRD algorithms' execution time and overall estimation error, respectively, as a function of the size of the targets' strategy space. The results show that as the size of the strategy space increases, the execution time of both algorithms increases (Fig. 4a), while the accuracy of the symbiotic PNT solution improves (Fig. 4b). Focusing on the comparative analysis between the two algorithms, we observe that a twelve-fold increase of the size of the targets' strategy space, results in approximately twelve-fold and ten-fold increase of the ABRD and SBRD execution times (Fig. 4a), respectively, and a ten-fold and four-fold decrease of the overall estimation error of the ABRD and SBRD algorithm, respectively (Fig. 4b). Thus, we can conclude that in scenarios, where the execution time of the PNT solution is critical, i.e., scenarios of high mobility of the targets, the SBRD algorithm is a more feasible and suitable option. In contrast, in scenarios where the accuracy of the PNT solution is more valuable than the execution time of the PNT mechanism (e.g., lower mobility of the targets), the ABRD algorithm appears as more appropriate choice.

V. Conclusion

In this paper, a novel symbiotic positioning, navigation, and timing (PNT) solution is introduced based on the principles of Game Theory and exploiting the key 6G technology of RIS. Specifically, the targets, anchor nodes, RISs, and collaborator nodes cooperate with each other in order to accurately determine the targets' positioning and timing, while minimizing the estimation error of each target and collaborator node, as well as of the overall examined system. The optimization problem of the positioning and timing estimation error is formulated as a potential game among the targets and collaborator nodes, and the existence of at least one Nash Equilibrium is proven. Two algorithmic approaches, based on the principles of Best Response Dynamics, are introduced in order to determine the Nash Equilibrium. A detailed simulation-based evaluation is provided to demonstrate the operational characteristics, as well as the tradeoffs of the proposed symbiotic PNT solution.

Part of our current and future work refers to the design of a self-PNT solution that eliminates the need for anchor nodes, where the targets and the collaborator nodes determine their positioning and timing by exploiting their own transmitted signal being reflected on the available RISs in the area.

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