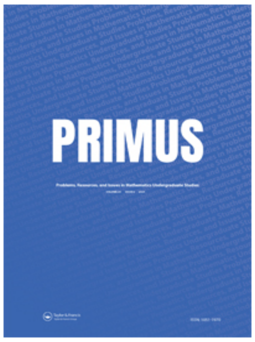

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A Research Introduction for Early College Students: Based on the OSU–HBCU Pilot Project

Anne Fernando, Katharine Gurski, and Timmy Ma

ABSTRACT
We joined an initiative to engage faculty and undergraduate students in research by scheduling regular early college-level mathematics research presentations with pre- and post-lecture exercises. While our program featured six talks that were predominantly in the mathematical biosciences, the concept of creating themed topics for presentations can be extended to other areas of mathematical research, and the length of the program can also be scaled. This article discusses three of the presentations, including QR codes to access their recordings. The purpose of this article is to present the materials of the talks from this initiative and how faculty members can use the materials for potential explorations with students. The hope is the audience may use these, as presented, for study with students, with minimal start-up investment and to use this as a model for future undergraduate research efforts.

KEYWORDS
Mathematical biology; HBCU; initial exposure to research; professional development; collaboration across institutions

1. INTRODUCTION

As faculty mentors of a pilot program to encourage undergraduate students to pursue graduate school, we wanted to present this paper to aid other faculty members in inspiring their students to start mathematical research. This pilot program was intended for those students lacking research experience and having insufficient mathematical background to be selected for a competitive REU program. Just as an introduction to proofs class bridges the gap between calculus and more abstract math, this pilot program is a bridge from regular "answer in the back of the textbook" problems to more open-ended problem explorations.

The President's Council of Advisors on Science and Technology and the National Academy of Science have agreed that providing research experiences to undergraduate students and directing them toward careers in STEM is a priority of science education in the 21st century [9,12]. However, Mendoza et al. [8] have noted that undergraduate research, despite being recognized as a high priority is often not an accessible opportunity for minority students. Smith et al. [15] recently noted that programs such as networking, seminars, summer bridge programs, field trips, and workshops – in addition to undergraduate research opportunities – are crucial for

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facilitating student success and perseverance in STEM fields. This implies that a "bridge to research" seminar series with directed problems could be effective both as a means to increase retention in mathematics as well as help ready students for an REU program.

We recognize that a key obstacle to student research at many schools is the high teaching load of faculty. The Subcommittee on Undergraduate Research, in a report to the MAA Committee on the Undergraduate Program in Mathematics [7] estimated that for a single student's academic year research project "that the cost in terms of faculty time averages about 3 hours per week for the year, including actual meeting time, time spent thinking about what direction to take the research, and time spent reading and correcting drafts of the thesis." For a summer program, the subcommittee estimated that the supervisor will need to spend between 5 and 15 hours per week per project for an 8- to 10-week session. Schwartz [13] researched on the impact on research-active STEM faculty as undergraduate research mentors. He found that a cross-case analysis indicates that the time consuming nature of undergraduate research mentoring is detrimental to career advancement. While the faculty wanted to participate in undergraduate research, they were conflicted by the impact on their own research and publication. Eagan et al. [3], noted additional barriers to faculty: "a heavy workload, a reward structure that does not incentivize mentoring students, limited funding, and the potentially daunting amount of time required to mentor and train undergraduate researchers." This pilot project aimed to relieve the faculty from developing introductory material by bringing in virtual speakers and pre-packaged reading and "research questions" for the students.

The OSU-HBCU pilot program was designed for a specific target audience, students from HBCUs. The majority of the HBCU faculty acting as mentors are not active researchers in applied mathematics, much less mathematical biology. The intent of the organizers was to encourage more HBCU students to enter the field of mathematical biology. We believe that this program may be introduced by many other groups in order to achieve varied goals. For example, faculty from liberal arts institutions, or even two-year college institutions, who wish to introduce students to undergraduate research may adapt this workshop to their audience, tailoring it to their students' backgrounds. If the faculty do not wish to use these exact talks and material presented in this paper, we suggest that institutions group together and follow this model of virtual colloquia and pre-prepared research topic questions from each presentation to create their own set of presentations. If each institution gives one presentation and shares material, then it will divide the preparation workload for the faculty. In addition, the students will benefit by having a larger group of students with whom to share the experience.

This paper seeks to introduce this program and aims to encourage others to follow this shared method of "research introduction." This paper has two main objectives: 1) to present three different talks and their activities, and 2) to present how faculty members can use the materials to discuss with students. This article includes the recordings of the talks and activities. The creators of these materials have given their permission for others to freely use them. We offer this material also as a guide for other faculty members to use as they develop their own shared material and talks.

We will first start with a brief history of the pilot program and the goals of the program itself. Then we will discuss the resources we are providing for the readers to explore and use for their research programs. The logistical details on how the program was arranged and funded are in the conclusion.

1.1. A Brief History

The OSU-HBCU Mathematical Biology Workshop was a project envisioned by Dr. Michael Reed at Duke University and the late Dr. Aziz Yakubu from Howard University. The program has proceeded with Dr. Janet Best at the Ohio State University (OSU) as the pilot project lead.

The project was developed (1) to encourage undergraduate students at the HBCUs to pursue graduate school in the STEM disciplines; (2) to connect math faculty at the HBCUs to faculty and research programs at OSU and other large research institutions; (3) to experiment with virtual colloquia and virtual teaching techniques in mathematics.

The year-long program concluded with a 5-day workshop at OSU that provided a research experience for the students accentuated by daily colloquium talks and discussions, organized sessions to help students use computer software for mathematical problems, a graduate student panel, a virtual industrial career panel, and an outing to see a local podcast with a black poet as host. Faculty advisors were also in attendance to work with each other in addition to participating in mathematical discussions with the daily colloquia speakers from OSU.

There were six HBCUs selected for this pilot program: Howard University, Xavier University of Louisiana, Clark Atlanta University, Norfolk State University, Texas Southern University, and Morgan State University. Faculty members at these institutions each recruited two to three students to participate in a year-long virtual colloquia program. These colloquia were tailored to be introductory for the material to be digestible to students who were in their early undergraduate mathematics courses. Materials prepared by the speakers provided follow-up activities that were explored with the local faculty members.

Although the virtual colloquia were geared toward early college undergraduates, Professors Best, Reed, and Yakubu aimed to impact both the students and the faculty members at HBCUs. Many of the faculty members at the HBCUs were new to mathematical biology research, so the program was designed to guide the faculty mentors. The speakers provided pre-talk reading material for the students and advisors to read and discuss pre-lecture. Then after the talk, there were post-talk mini-research assignments that could be used to start open-ended projects.

By engaging the HBCU faculty members as partners in the project, the goal was to create pipelines for the HBCU students to pursue graduate degrees. Additionally, the project sought to establish and enhance the working relationships and networking between HBCU faculty members and faculty members at large research institutions. This project emphasized mathematical biology, but the same ideas would work when applied to other branches of pure and applied mathematics, as well as other scientific disciplines. By making the virtual colloquia as interactive as a classroom, the project created a template for faculty at other research universities to participate in and enhance the training of undergraduates at HBCUs, and that, in and of itself, created pipelines from the HBCUs to the research institutions, both for students and for faculty.

1.2. Motivation and Goals

In this section, we will describe the motivations and goals of this paper.

Depending on the nature of the institution, different approaches are needed to engage math/applied math majors in the desire for research. Unlike many sciences, math as a concept in traditional college entry-level courses provides few "hands-on" lab activities or applications outside of course homework. However, there have been efforts in the community to find meaningful connections for calculus students [2,17]. Faculty mentors used the talks from this project to fill the gap we see in our undergraduate curricula between courses and students' desires to find applications in their future careers. The authors hope this paper can provide readers with guidance and resources to engage students and begin their journey in mathematical research as many of our students have this past year.

The authors have also found that this OSU-HBCU research format significantly reduced the preparation time barrier for faculty with teaching loads as high as 5/5 to engage with students in research. As one module would take approximately a week to ten days to complete (presentation, presentation, post-activity), it is an efficient way to introduce the idea of research to students.

The chosen material presented in this paper are the pre-talk reading material, post-talk mini-research assignments, and the talks themselves. The mathematical background for the first talk is only precalculus, while the next two talks require calculus (differential calculus is sufficient but an integral calculus background is useful). The authors' intention in sharing this material was to provide a means for over-scheduled faculty members to encourage undergraduates to think about mathematical research. With a few short projects outside of the regular classroom, the students can be prepared for a similar exploratory research experience. The QR codes for the recordings of these three talks can be found in the appendix.

2. TALK: HEART AND CIRCULATION

On October 6, 2022, Dr. Mike Reed, Arts and Sciences Professor of Mathematics at Duke University, gave the first talk of the research series on "Understanding the Heart and Circulation." The mathematics required for this talk was precalculus, although knowledge of calculus is encouraged for the students to have a deeper understanding of the idea and significance of steady-state simplifications. Students are encouraged to code or be able to code for the activities listed but students should be able to explore the calculations by brute force.

2.1. Pre-lecture Material for Advisors

The pre-lecture material prepared by Dr. Reed had instructions for the advisors. This material is written for the advisor and is therefore not designed to be digestible for beginning research students. We present Dr. Reed's material (lightly edited) in this section. In the following Section 2.2, we incorporate his instructions and ideas along with our summary of the suggested reading material [5].

(1) Have the students spend 30 minutes looking at pages 106–117 of [5]. It is not necessary that the students read the literature and learn the material before the lecture, but it would help if they spent half an hour looking to see what is there.
(2) The Peskin model of the circulation is given in Figure 1 [5]. The right and left heart seem to be reversed but that is because in physiology one always imagines the patient is facing you, so the patient's right and left are opposite to yours.
(3) In the talk, we will refer to the model of the heart and circulation system shown in Figure 1 [5]. In the lecture, we will simplify these equations. The model will consist of nine linear equations with the variables four volumes, the $V_{pa}, V_{pv}, V_{sa}, V_{sv}$, four pressures, the $P_{pa}, P_{pv}, P_{sa}, P_{sv}$, and one Q. All those different Q_i's above become one Q when we move to a steady-state model. The diagram shows eight equations. The ninth equation is simply that the sum of all the different volumes has to equal the total blood volume. We leave this off the pre-reading so the students will "discover" it in the lecture. These are linear equations (no calculus here), and they are solved by hand on pages 115–117 [5]. One can get explicit expressions for the variables in terms of the constants in the equations. We can think of the constants K_R, K_L as the ratios of each Q_R, Q_L with P_{sv} and P_{pv}, respectively. Additionally, we can think of $C_{pa}, C_{pv}, C_{sa}, C_{sv}$ as the ratios of $V_{pa}, V_{pv}, V_{sa}, V_{sv}$ and $P_{pa}, P_{pv}, P_{sa}, P_{sv}$, respectively.
(4) In the lecture we will talk about modeling a heart attack on the left side. We change one constant, K_L, to half its normal value. We then have to plug this change into the equation solutions, and we get new values for all the variables. It is amazing, but there is very little change to the new values compared to the normal values. For instance, Q did not drop much. Why? It is because the less efficient left heart causes blood to back up into the pulmonary veins raising the pressure P_{pv} there. The flow Q is K_L times P_{pv} so the drop in K_L is mostly compensated for by the rise in P_{pv}. And this corresponds to what is seen in the clinic.
(5) Then we will ask what happens for a heart attack on the right side. Answer: A lot!!! We cannot have this compensation effect very much because we cannot increase the volume in the systemic veins V_{sv} much more, as the blood is already there. The calculation for heart attack on the right side will be left to you and your students. We will also suggest cutting all the $C_{pa}, C_{pv}, C_{sa}, C_{sv}$ in half to see what happens (blood pressure doubles). This is why older people have problems with blood pressure because their arteries and veins get stiffer (corresponding to lower $C_{pa}, C_{pv}, C_{sa}, C_{sv}$).

Table 1. Normal and heart attack values for variables.

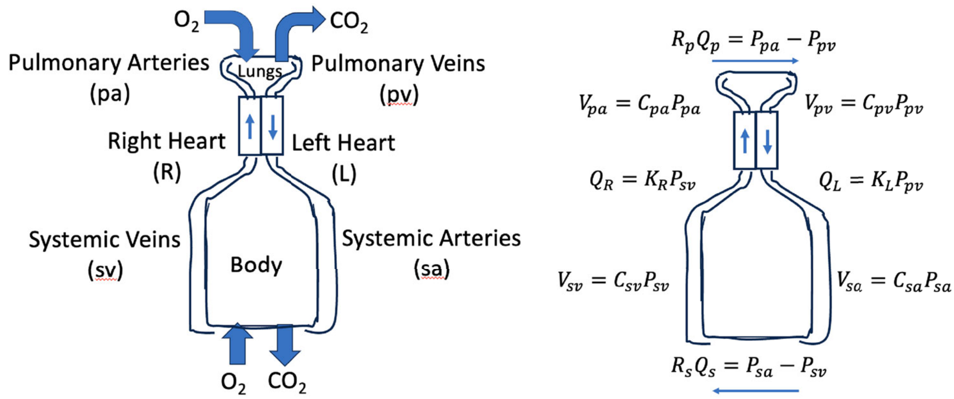
Variable	Normal	Heart attack (Left) $K_L \to K_L/2$	Heart attack (Right) $K_R \to K_R/2$
Q	5.6	5.15	3.286
P_{sa}	100	92	59
P_{sv}	2	1.84	2.35
P_{pa}	15	18.43	8.816
P_{pv}	5	9.2	2.93
V_{sa}	1.0	0.922	0.5985
V_{sv}	3.5	3.22	4.11
V_{pa}	0.1	0.123	0.058
V_{pv}	0.4	7.36	2.34

(6) The answers for the heart attack models are in Table 1. The right column of Table 1 is what you should calculate with your students. The model is so simple, yet contains lots of real physiology. We say "simple," as the equations are only linear. However, there are many variables with different subscripts, thus introducing a level of complexity (because we, specifically our human bodies, are inherently complicated) and you'll have to let your students be comfortable with that. The lesson the point that we do not just want to make models. We want to experiment with them to learn about how physiology works.

2.2. Pre-Lecture Material for Students

This material is our summary of Dr. Reed's suggested background reading from [5]. This summary omits the details of the heart and circulation not immediately used in the talk or activities. After the activities are concluded, interested student researchers and their research mentors can then read [5] for more details to bring more biological complications to the model.

(1) Let us start with a diagram of the Peskin [5] model of circulation. We start with the biology version as shown in Figure 1(a). The circulation of blood within the body forms a closed loop. The biology is overly simplified in the figure but still allows us to understand the circulatory system. The lungs inhale oxygen (O_2) and exhale carbon dioxide (CO_2) and the heart makes the blood circulate all with the goal of distributing oxygen and removing carbon dioxide. The oxygen-rich blood is pumped through the left chamber of the heart to the systemic arteries. Smaller arteries actually branch off, as in a tree, with systemic capillaries branching off these. The O_2 and CO_2 exchange happens in the capillaries. From the systemic capillaries, the blood enters the systemic veins. As compared to the extensive number of very small capillaries, the systemic veins are larger, following the tree analogy returning from the smallest branches toward to trunk of a tree. These systemic veins bring the blood back to the heart. The right chamber of the heart then pumps the blood through the pulmonary arteries to the lungs. Pulmonary capillaries branch off and this is where the CO_2 is exchanged for O_2. The oxygen-rich blood is collected by the pulmonary veins and brought to the left chamber of the heart.



(a) Biology Version: Heart and Lungs (b) Math Version: Heart and Lungs

Figure 1. The circulatory system based on figures from [5]. (a) Biology Version: Heart and Lungs and (b) Math Version: Heart and Lungs.

We take this simplified biology and create a correspondingly mathematical model describing the circulatory system. Although the systemic arteries and veins are not physically connected, we model the action of the systemic arteries and veins closing the loop. Similarly, the math model for pulmonary arteries and veins is made a closed loop by including the action of the pulmonary capillaries.

(2) Looking at Figure 1(b), we see there is a relationship between blood volume, V, and pressure, P. The subscripts indicate the "location," that is, P_{pv} represents the pressure in the pulmonary veins, and V_{sa} represents the volume in the systemic arteries. Blood is nearly incompressible, so we will assume the amount of blood in the systemic veins, for example, is given by the volume of the blood in the systemic veins, V_{sv}. Pressure is the force per unit area and can be measured by the height of a column of mercury. The pressure that is recorded in the pulmonary veins, P_{pv}, is the pressure of the blood in the pulmonary veins with respect to the atmosphere.

In Figure 1(b), we see an equation, ignoring subscripts, $Q = KP$. The symbol Q represents the flow and if we investigate the units, we can see that $K = 1/R$ where R is some constant to represent the "resistance" of the heart muscle (pipe). Let us simplify the heart chamber to be a pipe as in Figure 2. Q_{in}

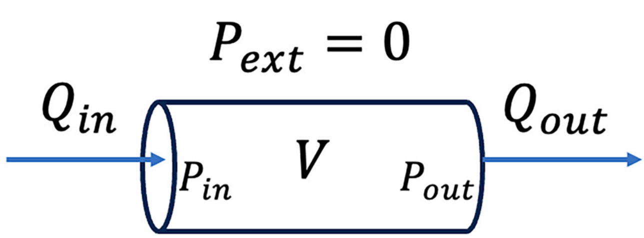


Figure 2. Fluid flow in a pipe.

represents the flow into the pipe (heart chamber) and Q_{out} is the flow out of the pipe (heart chamber). Assuming that the patient does not have a hole in a heart chamber, then the amount of blood that flows into the heart chamber must equal the amount of blood flowing out of the heart chamber, that is $Q_{in} = Q_{out} = Q$. Therefore, we only need to talk about the flow in the left chamber, Q_L, and the flow in the right chamber, Q_R, as marked in Figure 1(b). Similarly, we have P_{in} the pressure of the blood flowing into the pipe and P_{out} the pressure of the blood flowing out of the pipe. If the pipe is rigid, then the volume of the pipe is fixed, and all we need is a relationship between Q and the pressures. And the important pressure is the difference in the pressures, $P_{out} - P_{in}$. So we have the relationship:

$$Q = K(P_{out} - P_{in}). \tag{1}$$

But what happens if the pipe were not rigid? An artery or vein is elastic and can distend if the pressure is really high, just as a balloon expands as it is filled with air. When the balloon expands, the volume expands. So a simple equation describing this relationship is

$$V = CP, \tag{2}$$

where C is a constant called the "compliance" of the pipe. These two relationships describe six of the equations in Figure 1(b).

Last, we turn to the system closure mimicking the capillaries between the systemic arteries and systemic veins and between the pulmonary arteries and pulmonary veins. Blood is not flowing across a pipe here, but we can still use this idea. Look at the systemic arteries and veins. We know the blood pressures in the systemic arteries, P_{sa}, and veins, P_{sv}. Again, the body is a closed system, and blood is not leaking out of the capillaries. Only O_2 and CO_2 are exchanged in the capillaries with the rest of the body. So the flow in the systemic arteries and systemic veins must be equal. So we have Q_{sa}=Q_{sv}=Q_s. Now we go back to the analogy with the pipe and Equation (1), $P_{out} - P_{in} = Q/K$. We said K is $1/R$, so we rewrite the "resistance" of the capillary system of the systemic arteries and veins as R_s. We define R_p similarly. This explains the last two equations in Figure 1(b).

This will give the students enough to get to the "heart" of the talk (pun intended), without being overwhelmed by the biology and fluid dynamics.

2.3. Material for Activities Post-Lecture

The activities for this talk are straightforward in the sense that students can confirm the calculations of the second column in Table 2. After these confirmations, they can fill in the third and fourth columns. Next, the students can make conjectures as to why the values are quite different compared to each other. Other calculations that students can compute are what would happen if the blood pressure was doubled

Table 2. Chart for the heart and circulation activities.

Variable	Normal	Heart attack $K_L \to K_L/2$	Heart attack $K_R \to K_R/2$	Increase Pressure	Decrease Pressure
Q	5.6				
P_{sa}	100				
P_{sv}	2				
P_{pa}	15				
P_{pv}	5				
V_{sa}	1.0				
V_{sv}	3.5				
V_{pa}	0.1				
V_{pv}	0.4				

(column 5)? Or halved (column 6)? Why would the blood pressure change? Can we describe physically what is happening to make the blood pressure change within this simple system? After answering these questions, if the students are interested and have a calculus background, then they can read [5] and answer questions with a more complex mathematical model.

2.4. Student Reflections

Several students had strong positive reactions to Dr. Reed's Heart and Circulation talk. One student wrote "(i)t was simple, yet interesting, and I was proud to be able to do most of the calculations on my own. I learned that with simple calculations, we can look at changes in heart circulation that result in complications such as heart attacks…From observing the variables that were changed, I was able to see that a heart attack on the right side is more dangerous than a heart attack on the left side." Students were able to use various tools in this talk that they would not normally use in a math or biology course. Some even expressed that "(t)his talk was also one of my favorites because it resulted in my first introduction to using Octave. It was cool to have this tool show me how changes in one variable in the equations can drastically change the outcome of the results."

In the second year of this program, we shared this talk with a group of students ready to choose a research problem. One of the students was very interested in space research. She wondered if it the heart and circulation model would work for reduced or no gravity. This talk was the stepping stone to developing a mathematical model using differential equations for an artificial heart experiment performed on parabolic flight tests aboard the NASA KC-135 aircraft [10].

3. TALK: CANCER

On November 10, 2022, Dr. Trachette Jackson, Professor of Mathematics and University Diversity and Social Transformation Professor at the University of Michigan gave a talk on "Cancer Summed Up." The mathematical background required for this talk is differential calculus although an understanding of integrals and differential equations is preferred. Knowledge of coding is encouraged to help students visualize solutions to the differential equations models presented in the talk but is not necessary.

3.1. Lecture Material for Advisors and Students

The materials that Dr. Jackson provided were a brief summary of mathematics and its history with cancer as well as exercises in conjunction with her talk. Within her talk, she established the history of tumor growth, how many researchers initially gained insight on how to treat cancer, how experimentation changed the perception of tumor growth, and what is being used today to capture the decelerating growth from treatment (see [6] as a reference). In this section, we present Dr. Jackson's material. We have added explanations to make her questions and comments better understood before viewing her talk.

The first equation is given in a form for students with calculus to be able to graph and observe as a function of N. The differential equation in parentheses is given for more advanced students who wish to incorporate coding in their solution. The presentation format of this model is repeated for all three models discussed in this pre-lecture material. The first model for the rate of change of the growth of the tumor cells is the Gompertz growth, $g(N)$, model [1]:

$$\text{Gompertz model:} \quad g(N) = (b - a\ln N)N, \quad \left(\frac{\mathrm{d}N}{\mathrm{d}t} = g(N)\right), \tag{3}$$

where N is the number or density of tumor cells, ln is the natural logarithm, and a and b are constant parameters that we can adjust to the data. As we can see from the equations, $g(N)$ measures the rate of change of the N tumor cells. The speaker then has several exercises for the students:

(1) What is $\lim_{N\to 0^+} g(N)$?
(2) Plot the Gompertz growth function, $g(N)$ versus N, with $a = 1$ and $b = 0$ and describe what you think it means for tumor growth.
(3) The constant tumor size that the cancer cell population approaches is called a steady state or equilibrium of the model. Steady states are found by setting the time derivatives equal to zero and finding the roots of the growth function. Find the steady states for the Gompertz model by finding the roots of $g(N)$.
(4) Several mathematical models have similar features. Two very popular models, the logistic and von Bertalanffy models, are given:

$$\text{Logistic model:} \quad l(N) = rN\left(1 - \frac{N}{K}\right), \left(\frac{\mathrm{d}N}{\mathrm{d}t} = l(N)\right), \tag{4}$$

$$\text{von Bertalanffy model:} \quad v(N) = \alpha N^{2/3} - \beta N, \left(\frac{\mathrm{d}N}{\mathrm{d}t} = v(N)\right). \tag{5}$$

Plot the Logistic and von Bertalanffy growth functions, $l(N)$ and $v(N)$, along with the Gompertz growth function on the same set of axes. Use $r = K = \alpha = \beta = a = 1$, $b = 0$. Describe the similarities and differences you see.

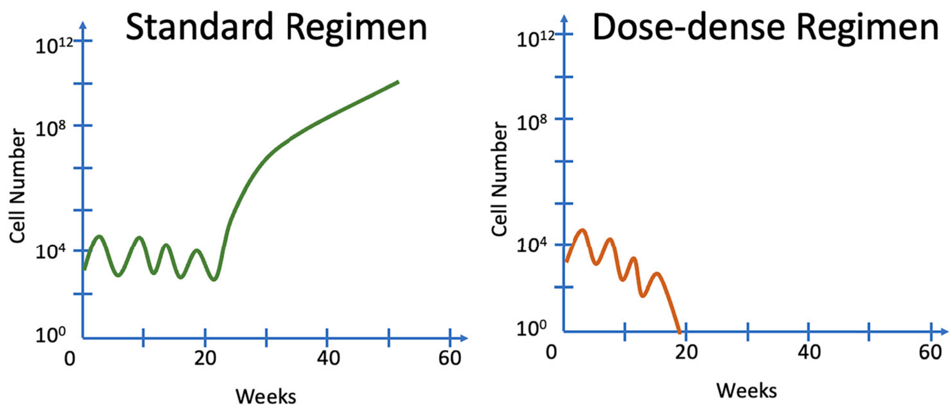


Figure 3. Cancer treatment given every three weeks (left) versus every two weeks (right), in what's called a dose-dense fashion. Figure created by Dr. Jackson for post-lecture materials and approved for publication in this form.

(5) Using your plot of $g(N)$ from exercise 3, estimate when the tumor growth rate is at a maximum.
(6) Using a bit of calculus, determine the size at which the tumor has its maximum growth rate. That is, at what value of N does the maximum growth rate occur? Also, what is the maximum growth rate? That is, what is the max of $g(N)$?

The speaker then touched on the impact of treating at shorter time intervals, which is demonstrated in Figure 3. In this figure, chemotherapy is given every three weeks in the left graph and every two weeks on the right, in what is called a dose-dense fashion. This is what led to the Norton-Simon Hypothesis [14]. This hypothesis states that the rate of cancer cell death in response to treatment is directly proportional to the tumor growth rate at the time of treatment. The hypothesis is a direct consequence of the mathematical implications of the Gompertz tumor growth model.

3.2. Activities Post-talk

Given the nature of the talk, this would be a prime opportunity for students to practice their ability to computationally simulate these models to confirm the calculations of the exercises from above. Students were given the choice to use MATLAB (if purchasing a license was available), Octave, Mathematica, Python, or any other platform to write elementary code to run an ODE solver. Students were asked to plot the Gompertz growth function $g(N)$ versus N with the parameters given. Students were also asked to explore what happens when using different parameter values and make conclusions based on what these parameters represent. Students can confirm the maximum growth rate and size of the tumor at the maximum growth rate through these simulations. Overall, these activities are an opportune way for students to apply their knowledge of calculus and learn about basic programming.

3.3. Student Reflections

From the participant feedback we received, Dr. Jackson's talk was a highlight to many. Students wrote that "(t)he most important reason was that there were plenty of visual aids and in-depth descriptions that made the information both interesting and clear to understand. The mathematical component was thoroughly detailed, while the biological component was short and sweet." In addition to the mathematical portion of the talk, Dr. Jackson spent a significant time talking about her education which resonated the most with students: "It was especially inspiring to see someone with the same racial and gender identities as me finding success in mathematics. One of the areas that I am particularly interested in doing research in is cancer, therefore, I was especially connected to this talk." Another student wrote, "(c)ompared to earlier talks, Dr. Jackson's seemed more intimate. I believe she did a wonderful job of making her lecture more than just numbers, making the environment more intimate and interesting, and holding my interest long enough for me to properly comprehend her topic. In addition to being a black woman, Dr. Jackson has the same major and minor as I do. I adored her to the point of tears because she attended the same school I was interested in attending and had similar professional experiences in domains I could only dream of experiencing. Her talk motivated me to work hard in college and to keep in mind why I enjoy mathematics." In addition to the students' positive feedback of the speaker and the experience, we have also noted that students expressed positive experience working with technology and their applications to calculus. Some students worked with their mentors to code the system of differential equations in MATLAB to observe the graphs in the talk, how changing the parameters affects the graphs, as well as use their knowledge in calculus to understand various properties of the system.

4. TALK: CALORIES

On February 17, 2023, Dr. Kevin Hall, Laboratory Head, National Institute of Diabetes and Digestive and Kidney Diseases gave a talk on "The calculus of Calories: Quantifying Human Body Weight Regulation." The mathematics involved in this talk is mainly calculus.

The talk described how mathematical models can be used to help understand and predict how the body will respond to various diet and physical activity programs. The talk also will describe why it is important to use mathematical models as opposed to popular rules of thumb to lose weight that previously led the field astray with their grossly inaccurate predictions. One such example is that if people were to decrease their calorie intake by 3500 calories, then they would lose 1 pound of body weight [4,16]. In the lecture, Dr. Hall illustrated useful tools that have been developed based on mathematical models, such as the NIH Body Weight Planner which has been used by millions of people to help predict how diet and physical activity dynamically interact to affect human body weight. While the other talks above do not require much preparation for faculty, one of the themes of this talk is to illustrate how a simple model can be used to describe an observation that the model itself might not be ideal. It would be imperative for the reader to spend some time thinking of answers to the questions below to address before the presentation, particularly question 7.

4.1. Lecture Material for Advisors and Students

The following questions can be addressed before students watch the presentation. This talk will start with issues about body weight and energy conservation.

(1) Although the mathematics behind Figure 4 is the most complicated model, it is a good place to begin the discussion with students. The idea of calories in and calories out can be related back to the pipe analogy in Equation (2) from the heart and circulation talk and activities.

(2) Consider W as body weight, I as calorie intake, E as calorie expenditure with ρ as the energy density of weight (with an approximate value of 3500 kcal/lb or 7700 kcal/kg). In this way, $\rho \Delta W$ would represent either a surplus or deficit of energy, represented in units of calories per lbs or calories per kg as weight, W, is represented in either lbs or kg.
Discuss how one might use a differential equation to describe the change in weight, W, using a simple approach (e.g. intake (I) - expenditure (E) = change in weight). The speaker will describe this as:

$$\rho \frac{dW}{dt} = I - E. \tag{6}$$

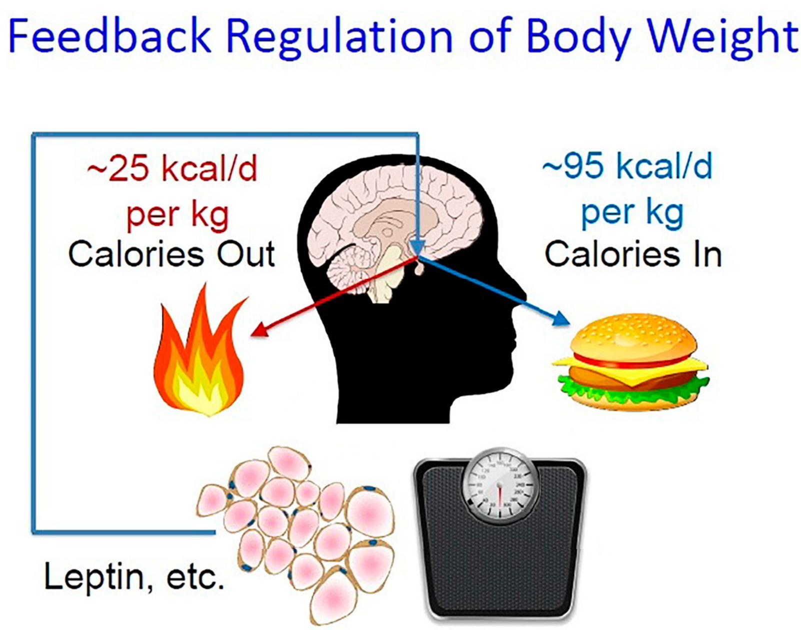


Figure 4. Feedback mechanism for reduction of intake and increase in appetite. Figure created by Dr. Hall for post-lecture materials and approved for publication in this form.

The variable, ρ is the energy density of W with a value of approximately 3500 kcal/lb or 7700 kcal/kg.

(3) What is the physical description of $\mathrm{d}W/\mathrm{d}t = 0$?

(4) Now consider what happens if $I - E$ is constant with respect to time. Then solve the ODE from Equation (6) with $I - E$ equal to a constant.

Solution: $\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{(I-E)}{\rho}$, which integrates to: $W(t) = W(0) + \frac{I-E}{\rho}t$.

(5) An example plot of the linear model of weight loss as a function of time is given in Figure 5. Figure 6 is a plot of intake and expenditure as a function of body

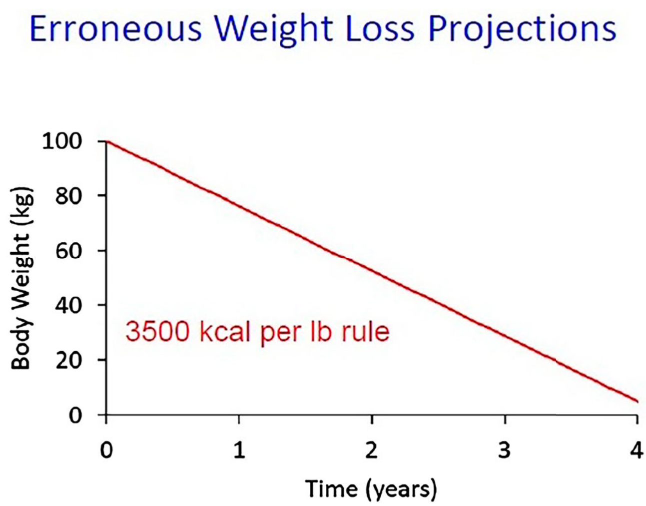


Figure 5. Weight loss with calorie intake – calorie expenditure, $I - E$, constant with respect to time. Figure created by Dr. Hall for post-lecture materials and approved for publication in this form.

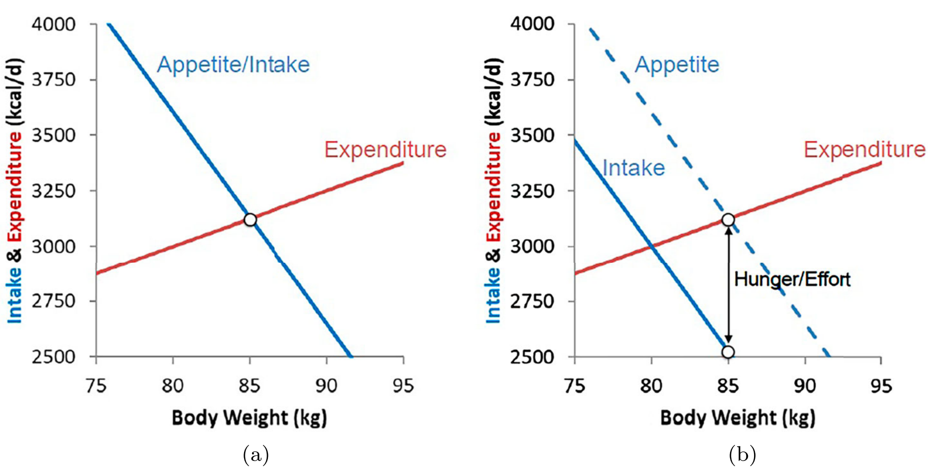


Figure 6. Intake and Expenditure vs Body Weight. (a) Appetite vs. Intake BEFORE caloric restriction and weight equilibrium. (b) Appetite vs. Intake AFTER caloric restriction and weight equilibrium. Figure created by Dr. Hall for post-lecture materials and approved for publication in this form. (a) No calorie restriction, Appetite = Intake and (b) Calorie restriction, Appetite $\neq$ Intake.

weight. Figure 6(a) represents the case of no calorie restriction. Figure 6(b) represents the case of calorie restriction.

(6) Discuss how the linear model of weight loss (see Figure 5) is not reasonable or realistic. Sketch a more realistic graph of weight loss.

(7) Consider a feedback model and discuss your own experiences with weight loss and weight gain to understand the role real-life connections and science have with any mathematical model. Consider changing expenditure, E, to reflect the current weight of the individual. This leads to the following:

$$\rho \frac{\mathrm{d}W}{\mathrm{d}t} = I - \epsilon W, \quad \text{where } \epsilon = 25 \text{ kcal/d per kg and } W(\infty) = \frac{1}{\epsilon}. \tag{7}$$

(8) Discuss how to solve Equation (7) to arrive at a new model

$$W(t) = W(0) - [W(0) - W(\infty)](1 - e^{-\frac{\epsilon}{\rho}t}). \tag{8}$$

(9) Using MATLAB or Mathematica, plot this model. Does the graph of this model fit weight loss better?

4.2. Material for Activities Post-lecture

(1) Discuss what the presentation presented about modeling and why the first linear model was not adequate.

(2) Discuss why the second model was an improvement.

(3) Consider the linear representations for appetite versus intake and discuss Figure 6 from the talk and what this information tells us about weight loss and what we might include for modeling weight loss.

(4) Discuss the *SGLT*2 inhibitor and how that data impacts the model adjustments with respect to appetite and discipline in the diet.

(5) Create a model that might take into account the information in Figure 4, more details of this figure are in [11], and consider solving this new equation.

(6) Plot the solution obtained and discuss.

(7) Discuss what we know about weight loss. Include in your discussion how a "feedback" mechanism can be incorporated. The feedback mechanism can describe the phenomenon where the more one loses weight, the harder it is to both lose more weight and over time, more difficult to keep the weight off.

4.3. Student Reflections

Students have expressed that they "appreciated that the biology component was simple enough to be able to learn something new while being able to bridge the gap between mathematical models and biological systems." In particular, Dr. Hall's talk, they enjoyed that "(he) gave us the opportunity to do some of the problems and explore their topics on our own. I really enjoyed this type of hands-on engagement.

It is one thing to watch someone talk about their mathematical model. However, being able to replicate their work brings a new perspective of why things work the way that they do."

5. CONCLUSION

The merit of what is being conveyed in this paper is similar to how a textbook benefits faculty in delivering a course, but in this case, for research and exploration. While many students can explore research topics in REUs by traveling to other institutions and dedicating weeks of their summer to explore research, this program can act as a different medium for students. The authors have learned a great deal from their initial experience in running this program, and would like to share their thoughts on what worked particularly well, what could be changed, and what faculty interested in organizing a similar program should focus on. Here are a few options for the readers to consider: (1) Use the talks and activities listed in this paper for your students; or (2) Use the talks as guidelines to invite speakers tailored to your students' backgrounds. The former is the most cost-effective for faculty as the talks and activities listed are presented for the readers to use freely with permission from the speakers. The latter is more work-intensive to establish and should be considered when creating proposals for funding and support. This benefits underrepresented students and institutions, as well as those institutions that are short on funds. This approach only requires dedication and time from faculty to establish interest from students.

While many HBCU faculty are experts in their own respective fields, this program was designed to encourage more HBCU students to enter the field of mathematical biology by providing the HBCU faculty/mentors access to materials in the mathematical biology field. In addition, because the teaching load remains high at many of these institutions, the inclusion of prepared introductory research material to complement the lectures was essential.

Due to the small cohort size, twelve students in total, it was difficult to quantify the feedback of the entire project as meaningful data. However, we gathered the comments from students that captured their overall perception of the speakers we featured above. While the feedback has been highly positive, we, as faculty mentors, have noted what changes the program could make to have a better overall experience for everyone involved while also maintaining the key factors that have made the program a positive experience for students.

The authors have observed that students' background and interests are among the key factors in the positive feedback from the students. When the students themselves found a connection to the speakers, they were inspired to be engaged. Should the readers feel inclined to invite their speakers for their respective colloquiums, it would be prudent to understand the types of students that will be your audience and have speakers that students can relate to in terms of background or education. For our speakers, the organizers first sought mathematical biologists as math biology was the primary subject of interest to the organizers to speak on, and then invited speakers who were women since many of our students were women. In the next iteration of the program, the organizers are inviting more African American mathematicians who have graduated from HBCUs as all of our student participants are in HBCUs.

The program's inaugural year had six colloquia spread throughout the academic year where students joined virtually to participate. Students then met with their faculty mentor or representative after each colloquium to discuss the exercises, and presentation, explore different possible questions to explore, and gauge interest from students of what they wanted to explore. The minimum time commitment from the faculty mentors was one hour after each colloquium. The organizers used their NSF pilot grant to provide $500 stipends to students who participated in the colloquium as well as attend the one-week in-person workshop. The NSF grant also provided the faculty mentors a small stipend for their efforts in mentoring students through the program. Lastly, organizers used NSF funding for travel expenses to hold the in-person workshop at the end of the program.

In the second year of this pilot program, the organizers are making some changes to the structure of the colloquium based on the comments received by the students. The organizers are most interested in finding speakers who are women and African American. The organizers are also seeking speakers who have reading material prepared for students as well as exercises as only a few speakers prepared these materials for students from the inaugural year. Comments from students were quite positive when speakers provided a repository of concepts to explore with their faculty mentors. Students and faculty found it difficult to come up with questions to explore from an hour-long presentation without guidance from the speakers.

The authors hope that faculty can find inspiration from our experience with this program and can adapt it for their students. We hope to gather more data, feedback, and responses from our students to inform and provide more guidance in the future.

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DISCLOSURE STATEMENT

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APPENDIX

We received permission in writing from all speakers presented in this paper to share their talks. Readers can access recordings via QR codes in Figure A1(a–c). We also received permission to use their materials (if provided) along with our extension and reframing of their materials.

(a) (b) (c)

Figure A1. QR codes to access the recordings of the talks. (a) Talk given by Dr. Reed. (b) Talk given by Dr. Jackson and (c) Talk given by Dr. Hall.

BIOGRAPHICAL SKETCHES

Anne Fernando received her PhD in Computational and Applied Mathematics from Old Dominion University. She holds an MS in Statistics, an MS in Applied Mathematics from the Georgia Institute of Technology. She is Professor and Chair at Norfolk State University. Her research areas include numerical analysis, epidemiological modeling and enjoy studying mathematics with students.

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