Age-Optimal Multi-Channel-Scheduling under Energy and Tolerance Constraints

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***Abstract*—We study the optimal scheduling problem where n source nodes attempt to transmit updates over L shared wireless on/off fading channels to optimize their age performance under energy and age-violation tolerance constraints. Specifically, we provide a generic formulation of age-optimization in the form of a constrained Markov Decision Processes (CMDP), and obtain the optimal scheduler as the solution of an associated Linear Programming problem. We investigate the characteristics of the optimal single-user multi-channel scheduler for the important special cases of average-age and violation-rate minimization. This leads to several key insights on the nature of the optimal allocation of the limited energy, where a usual threshold-based policy does not apply and will be useful in guiding scheduler designers. We then investigate the stability region of the optimal scheduler for the multi-user case. We also develop an online scheduler using Lyapunov-drift-minimization methods that do not require the knowledge of channel statistics. Our numerical studies compare the stability region of our online scheduler to the optimal scheduler to reveal that it performs closely with unknown channel statistics.**

I. Introduction

In recent years, the Internet of Things (IoT) has become one of the most important frameworks of the next-generation wireless networks, whereby a large number of mobile devices need to be supported over an ultra-wide frequency spectrum (see, for example, [1]). In particular, for many real-time IoT applications, it is necessary for the devices to send *fresh* updates over the shared spectrum. To measure the freshness of data, the concept of *Age of Information (AoI)* has been introduced over the last decade (see, for example, [2]–[4]), which is defined concisely as the elapsed time since the generation time of the last received status update. Since the introduction of the AoI metric, numerous related studies emerged in various networking scenarios, including wireless random access networks (e.g., [5], [6]), content distribution networks (e.g., [7], [8]), scheduling (e.g., [9]–[11]), queuing networks (e.g., [12], [13]).

Recently, other AoI related metrics have been developed in order to address more generalized or different forms of ageing, such as: non-linear AoI (e.g., [4], [14]), peak AoI (e.g., [15]), time-since-last-service (e.g., [16]), age upon decisions (e.g., [17]), to name a few. Among them, the metric, called the *age-violation-rate* (see [13], [18], [19]) is of particular interest for real-time IoT services that have hard age-deadline constraints and a limited tolerance to violating this deadline (see [20], [21] for further motivation of this metric).

In view of its significance for next generation IoT networks, in this paper, we study the general optimal multi-channel scheduling problem to optimize varying forms of age performances under energy and age-violation tolerance constraints. Our contributions can be listed as:

- We provide a generic formulation of age-optimization as a Constrained Markov Decision Problem (CMDP) (see [22]–[24]) and obtain the age-optimal multi-source multi-channel scheduler as the solution of an associated Linear Programming problem. We discuss some important applications for both single-source scenario and multi-source scenario and reveal the key insights behind the solutions.
- For the single-source multi-channel scenario, we investigate the characteristics of the optimal schedulers under energy constraints for two age metrics that are important for IoT applications: (i) *average-age* minimization; and (ii) *age-violation-rate* minimization, a non-convex/concave metric (in Section III-C). Our investigations reveal various insights on different energy allocation structures, as well as the common properties of the optimal schedulers for minimizing these two metrics.
- For the multi-source age optimal scheduling problem, we also study the feasibility region of the average-age-optimal scheduler under age-violation-rate tolerance constraints to contrast its results with those of related earlier works that are developed for the single-channel multi-user scenario (see Section III-D and Section V).
- Moreover, we develop (in Section IV) an *online* scheduler using Lyapunov-drift-minimization methods (e.g., [25]) that does not require the knowledge of channel statistics, and compare its performance to the optimal and earlier designs to reveal how much the knowledge of channel statistics affects the feasibility region (see Section V).

Our work relates to, but also differs from several other related works in this domain. Many early works (e.g., [9], [11], [26]) aim to minimize AoI under power constraints but with the assumption of reliable channels as opposed to the fading channels that we consider. More recent works (e.g., [10], [27]) aim to minimize AoI-related costs based on max-age matching, while other works (e.g., [26], [28]) proposed AoI minimization schedulers based on *Whittle Index* approach. However, to the best of our knowledge, prior works

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predominantly assume that one source can choose at most one channel, which is an important factor in proving the *Whittle Indexability* of the corresponding problems they solve. In contrast, one of the key features our setting is the possibility of each user to transmit over multiple channels as enabled by new wireless technologies. Furthermore, most of the above mentioned works have average or peak AoI as the objective function, while we consider more general age-based objective functions, which for example allows the objective function to be a non-convex metric such as the age-violation-rate. In this multi-channel setting with general objectives, we observe (cf. Section III-C) that the optimal solution can in fact possess *non-monotone* characteristics, which make the Whittle Indexability approach infeasible in general. The work in [18] has considered the multi-source single-channel scheduling problem under tolerance constraints, which is a special case of our setting. We would like to note that this interesting work [18] has been a primary motivation for our current work in exploring a different approach based on the CMDP framework that guarantees optimality and applies to more general multi-channel scenarios with additional energy constraints. There are also works (e.g., [29], [30]) that focus on learning-based approaches which can be considered as complementary to the focus of this work.

II. System Model

We consider the operation of a discrete-time wireless access system, whereby n source nodes share L on/off fading wireless channels to update their ageing status at a receiver under energy and violation tolerance constraints (see Figure 1).

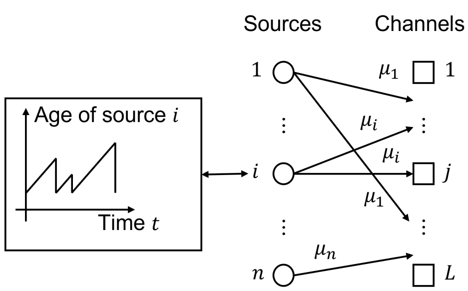

Figure 1. n sources share L on-off fading channels to update their status to a receiver under energy and tolerance constraints to keep their age levels low.

Our goal is to develop generic solution strategies to find optimal schedulers that can optimize diverse age-based metrics while meeting certain requirements on energy consumption and tolerance levels.

Scheduling policy and age-violation-tolerance: We assume that each source node $i \in \{1, \cdots, n\}$ refreshes its status and creates a new packet at the beginning of every time slot $t \in \{1, 2, 3, \cdots\}$. Source nodes attempt to transmit their freshest packet to the receiver, for example a base station (BS), whenever they get a chance to transmit. Every time the BS successfully receives a new status from source node i, it saves the current status and discards all previous packets received from that node. As such, the BS keeps only one packet from each source node, namely the freshest one. We use $X_i[t]$ to denote the generation time of the packet stored at the BS from source i at time t. We define the *age* $A_i[t]$ of source node i at time t as the time that has elapsed since the generation of its last received packet[1]: $A_i[t] \triangleq t - X_i[t]$. We use[2] $\boldsymbol{A}[t] \triangleq (A_1[t], \cdots, A_n[t])$ to denote the ages of all sources at time slot t.

At the beginning of each time slot, the centralized scheduler decides which channels each of the source nodes will use to transmit to the base station based on the ages $\boldsymbol{A}[t]$ of all source nodes. Let $u_i(\boldsymbol{A}[t])$ be the number of channels source node i uses to transmit at time t. Each transmission attempt can resolve in success or failure which we will describe below as part of the channel success model. If the base station successfully receives the packet from source i at time t, then its age at time $t+1$ will reset to 1, otherwise its age will increase by one, i.e.,

$$A_i[t+1] = \begin{cases} 1, & \text{if transmission of source } i \text{ succeeds} \\ A_i[t]+1, & \text{otherwise.} \end{cases}$$

We allow each source i to have a desired age threshold/deadline τ_i. The information of source i is up-to-date if its age is less than or equal to this threshold τ_i. Otherwise, we speak of an age violation in that slot. In particular, we define the *age-violation-rate* of source i as the long-term average fraction of time slots when the source's age $A_i[t]$ exceeds its threshold τ_i, i.e., $\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}\{A_i[t] > \tau_i\}$. We use $\epsilon_i \in [0,1]$ to indicate the tolerance of source i that measures the maximum allowed age-violation-rate for its updates. ($\epsilon_i = 1$ indicates that there is no violation rate constraint, and $\epsilon_i = 0$ indicates that we do not allow any deadline violation.) When the age violation rate is no greater than the tolerance rate, the age violation tolerance constraint is satisfied.

Channel success model and energy constraints: The n source nodes share L wireless on/off fading channels, each of which can accommodate at most one packet transmission. However, even when there is a single transmission over a channel, a successful transmission is not guaranteed. In particular, source node i has a channel success probability of μ_i when transmitting over each of its assigned channels[3].

We call the update of source i in a slot to be a success if any one of its transmissions over its assigned channels is successful. Since the channel is a collision channel, for an optimal scheduler we always have $\sum_{i=1}^{n} u_i(\boldsymbol{A}[t]) \le L$. Once the value of $u_i(\boldsymbol{A}[t])$ is decided for all i, the scheduler will assign different channels to different sources, so that no two sources transmit over the same channel. Also, note that under

[1] This metric is also referred to as *Age-of-Information (AoI)* and *Time-Since-Last-Service (TSLS)* in different contexts. In the rest of the paper, we will refer to it as AoI or simple as age, interchangeably.

[2] We will consistently use bold symbols to represent vectors.

[3] All our development can be generalized to the case when the success probability between source i and channel j is allowed to be different as μ_{ij}. However, this is omitted here as it increases the complexity of the exposition without adding to the substance.

the described channel success model, the probability for the BS to successfully receive an update from source node i when the node uses l channels is $1-(1-\mu_i)^l$.

We assume that each transmission over a channel comes with an energy cost of 1 unit[4]. We require that the aggregate time-average energy cost for source i is not greater than a given constraint b_i channels per slot, i.e., we require

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}u_i\left(\boldsymbol{A}[t]\right)\le b_i,\quad b_i\in\mathbb{R}^+.$$

It is obvious that transmitting over more channels will increase the success probability of a source, but increase energy consumption. We are interested in finding the number of channels that when allocated to sources optimize the desired age performance given the current age, as well as energy and and tolerance constraints. In the next section, we attack this problem within the constrained Markov Decision Process (MDP) framework and then discuss several important applications where other methods are not directly applicable.

III. Generic Age-Optimal Multi-Channel Scheduling

In this section, we first formulate a general constrained age-optimal scheduling problem for multi-user multi-channel scheduling in Section III-A, and then provide a solution to it in Section III-B, using the framework of Constrained MDP (CMDP). In Section III-C, we apply the general solution methodology to develop insights for particular cases of interest for single-user multi-channel scheduling, namely, those aimed at minimizing the *average-age* and minimizing the *age-violation-rate* cases. Finally, in Section III-D, we investigate the feasibility and stability region of the optimal policy along with alternatives from related literature associated with multi-user settings.

A. Problem Formulation

The problem of minimizing generic age-based objectives under constraints can be formulated as the following constrained Markov decision problem:

$$\begin{aligned}\min_{\boldsymbol{u}(\boldsymbol{A})}\quad &\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[\omega_0(\boldsymbol{A}[t])\right] \qquad (1)\\ \text{s.t}\ :\quad &\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[u_i\left(\boldsymbol{A}[t]\right)\right]\le b_i,\ i=1,\cdots,n,\\ &\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[\omega_k\left(\boldsymbol{A}[t]\right)\right]\le c_k,\ k=1,\cdots,K,\\ &u_i(\boldsymbol{A}[t])\in\{0,1,\cdots,L\},\ i=1,\cdots,n,\\ &\sum_{i=1}^{n}u_i(\boldsymbol{A}[t])\le L\end{aligned}$$

where $\boldsymbol{u}=(u_1(\boldsymbol{A}),\cdots,u_n(\boldsymbol{A}))$ denotes the scheduling policy at state $\boldsymbol{A}$ with $u_i(\boldsymbol{A})$ as the number of channels allocated to source i. The optimization is performed over Markovian policies described by a function $\boldsymbol{u}$ since such Markovian policies are sufficient for optimal operation [22].

The first group of constraints capture the heterogeneous energy constraints discussed in the system model, which means node i can transmit over at most b_i channels per slot on average. The weight functions $\omega_k(\cdot),k=0,1,\cdots,K$, map the age states to cost values that capture age-related objectives and constraints[5]. By setting different mappings for the weight function $\omega_0(\boldsymbol{A}[t])$, the objective can be changed into different commonly used age-related objectives: letting $\omega_0(\boldsymbol{a})=-\sum_i\mathbb{1}\{a_i=1\}$ transform the objective to maximizing the average throughput; letting $\omega_0(\boldsymbol{a})=\sum_i a_i$ makes the objective minimize the average AoI; letting $\omega_0(\boldsymbol{a})=\sum_i\mathbb{1}\{a_i\ge d_i\}$ make the objective minimize the average age-violation rate. Note that this allows the objective function to be a non-convex nor-concave function.

B. Optimal Solution to the Generic Problem

Next, we establish the equivalence of the multi-user problem formulation to a linear programming (LP) problem. To enable a more compact notation, we will use $\boldsymbol{a}\triangleq(a_1,a_2,\cdots,a_n)$ and $\boldsymbol{l}\triangleq(l_1,l_2,\cdots,l_n)$ to denote values of $\boldsymbol{A}[t]$ and $\boldsymbol{u}(\boldsymbol{A})$, respectively. We further define sets $\mathcal{A}\triangleq\{1,\cdots,D\}^n$, where D is an upper bound on the age state in the system which can be set sufficiently large so that the probability of reaching D is vanishing.[6] $\mathcal{L}\triangleq\{1,\cdots,L\}^n$, and $\mathcal{L}_1\triangleq\{\boldsymbol{l}:l_\Sigma\le L\}$ where $l_\Sigma\triangleq\sum_{i=1}^{n}l_i$.

Theorem 1: The solution of the multi-user age-optimization problem (1) can be obtained by solving the following linear programming problem:

$$\begin{aligned}\min_{y_{\boldsymbol{a}}^{\boldsymbol{l}}}\ &\sum_{\boldsymbol{a}\in\mathcal{A}}\sum_{\boldsymbol{l}\in\mathcal{L}_1}y_{\boldsymbol{a}}^{\boldsymbol{l}}\omega_0(\boldsymbol{a})\\ \text{s.t:}\ &\sum_{\boldsymbol{a}\in\mathcal{A}}\sum_{\boldsymbol{l}\in\mathcal{L}_1}y_{\boldsymbol{a}}^{\boldsymbol{l}}l_i\le b_i,i=1,2,\cdots,n\\ &0\le y_{\boldsymbol{a}}^{\boldsymbol{l}}\le 1\quad\forall\boldsymbol{l}\in\mathcal{L},\boldsymbol{a}\in\mathcal{A}\\ &y_{\boldsymbol{a}}^{\boldsymbol{l}}=0\quad\forall\boldsymbol{l}\in\mathcal{L}/\mathcal{L}_1\\ &\sum_{\boldsymbol{a}\in\mathcal{A}}\sum_{\boldsymbol{l}\in\mathcal{L}_1}y_{\boldsymbol{a}}^{\boldsymbol{l}}=1\\ &\sum_{\boldsymbol{a}\in\mathcal{A}}\sum_{\boldsymbol{l}\in\mathcal{L}_1}y_{\boldsymbol{a}}^{\boldsymbol{l}}\omega_k(\boldsymbol{a})\le c_k,k=1,\cdots,K \qquad (2)\\ &\mathbf{Q}\boldsymbol{y}=\mathbf{0}\end{aligned}$$

where $\boldsymbol{y}=[[y_{\boldsymbol{a}}^{\boldsymbol{l}}]_{\boldsymbol{l}\in\mathcal{L}}]_{\boldsymbol{a}\in\mathcal{A}}$ is a column vector of size L^nD^n. $\mathbf{Q}$ is a matrix of size $D^n\times L^nD^n$ with each row representing the global balance equation of one state $\boldsymbol{a}\in\mathcal{A}$, and its $j=L^ni+k^{th},0\le i\le D^n-1,1\le k\le L^n$ column equaling

[4]This can also be generalized to non-uniform energy costs over different channels, but omitted to avoid cumbersome notation.

[5]We note that the problem can also solved with the same approach (but heavier notation) by more generally defining $\omega_k(\boldsymbol{A}[t],\boldsymbol{u}(\boldsymbol{A}[t]))$ to be functions of both the age and the action.

[6]In practice, moderate level of D is enough so that the dimension of LP won't be large. Also, when there is only age violation related objective and constraints, it's enough to set $D=\max_i\{d_i\}+1$ (see III-C and III-D).

the transition rate from the $(i+1)^{th}$ state to state $\boldsymbol{a}$ given the action $\boldsymbol{u}=\boldsymbol{l}_k$, the k^{th} action in $[y_{\boldsymbol{a}}^{\boldsymbol{l}}]_{\boldsymbol{l}\in\mathcal{L}}$ [7].

If this LP is feasible and $\boldsymbol{y}$ is an optimal solution, then the optimal policy $u_i^*(\boldsymbol{a})$ is a probabilistic policy, whereby the probability $f_{\boldsymbol{a}}^{\boldsymbol{l}}$ of choosing $\boldsymbol{l}\in\mathcal{L}$ channels for source nodes $i=1,\cdots,n$ when the AoI is at state $\boldsymbol{a}\in\mathcal{A}$ equals:

$$f_{\boldsymbol{a}}^{\boldsymbol{l}}=\begin{cases}\frac{y_{\boldsymbol{a}}^{\boldsymbol{l}}}{\sum\limits_{\boldsymbol{l}\in\mathcal{L}}y_{\boldsymbol{a}}^{\boldsymbol{l}}}, & \text{if}\sum\limits_{\boldsymbol{l}\in\mathcal{L}}y_{\boldsymbol{a}}^{\boldsymbol{l}}\neq 0\\ \frac{1}{|\mathcal{L}|}, & \text{if}\sum\limits_{\boldsymbol{l}\in\mathcal{L}}y_{\boldsymbol{a}}^{\boldsymbol{l}}=0\end{cases}\tag{3}$$

Proof: As shown in [22], it is enough for us to optimize over the Markovian policies for Problem 1. Since the process is not affected by a shift in time, we will use $f_{\boldsymbol{a}}^{\boldsymbol{l}}$ to denote the probability of choosing $\boldsymbol{l}=(l_1,\cdots,l_n)$ channels for source nodes $(1,\cdots,n)$ when the AoI is at state $\boldsymbol{a}$. Thus $\sum\limits_{\boldsymbol{l}\in\mathcal{L}}f_{\boldsymbol{a}}^{\boldsymbol{l}}=1$, and $f_{\boldsymbol{a}}^{\boldsymbol{l}}\geq 0$ for all $\boldsymbol{a}$.

Notice that the system state can be fully characterized by a $n-$dimensional Markov Chain with age $\boldsymbol{A}[t]$ as state. Given $\boldsymbol{A}[t]$, the system state at the next time slot $\boldsymbol{A}[t+1]$ depends only on the current state $\boldsymbol{A}[t]$ and the current action $\boldsymbol{u}[t]$. In addition, the objective and constraints only depend on the current state and action. So an equivalent MDP problem can be formulated. Since there are finitely many states, there exists a stationary distribution $\pi(\boldsymbol{a})$ for every $\boldsymbol{a}$. Let $\mathcal{C}$ be the set of all recurrent states, then $\mathcal{C}$ is irreducible and closed, thus $\mathcal{C}$ is positive recurrent. When $\boldsymbol{a}\in\mathcal{C}$, the stationary distribution $\pi(\boldsymbol{a})$ is equal to the long term average $\lim\limits_{T\to\infty}\frac{1}{T}\sum\limits_{t=1}^{T}\mathbb{1}\{\boldsymbol{A}[t]=\boldsymbol{a}\}$ independent of the starting point. When state $\boldsymbol{a}\notin\mathcal{C}$, then both the stationary distribution and the long term average are equal to zero. So the optimization problem is equivalent to the following constraint MDP problem:

$$\begin{aligned}\min\quad & \sum_{\boldsymbol{a}}\pi(\boldsymbol{a})\omega_0(\boldsymbol{a})\\ \text{s.t:}\quad & \sum_{\boldsymbol{a}}\sum_{\boldsymbol{l}}\pi(\boldsymbol{a})f_{\boldsymbol{a}}^{\boldsymbol{l}}l_i\leq b_i, i=1,2,\cdots,n \qquad (4)\\ & f_{\boldsymbol{a}}^{\boldsymbol{l}}=0\quad\forall\boldsymbol{l}\in\mathcal{L}/\mathcal{L}_1\\ & \sum_{\boldsymbol{a}}\pi(\boldsymbol{a})\omega_k(\boldsymbol{a})\leq c_k\quad k=1,\cdots,K\\ & H\cdot\Pi=\Pi,\quad\mathbf{1}\cdot\Pi=1, \qquad (5)\end{aligned}$$

where the indices range over $\boldsymbol{a}\in\mathcal{A}$ and $\boldsymbol{l}\in\mathcal{L}$; $\pi(\boldsymbol{a})$ is the stationary distribution of state $\boldsymbol{a}$; and $\omega_k(\boldsymbol{a}), k=0,1,\cdots,K$, are age related objective and cost functions. The constraints 4 bound the average energy of nodes i by b_i for $i=1,\cdots,n$. In the constraint 5, Π is a $D^n\times 1$ stationary distribution vector with $\pi(\boldsymbol{a}),\boldsymbol{a}\in\mathcal{A}$ as entries. H represents the $D^n\times D^n$ transaction matrix with $h_{i,j}$ equals the probability of transaction from the j^{th} state in Π to the i^{th} state in Π, which can be detailed by using the age evolution and channel success probability equations similarly as regular. We then define

$$y_{\boldsymbol{a}}^{\boldsymbol{l}}\triangleq y_{a_1,a_2,\cdots,a_n}^{l_1,l_2,\cdots,l_n}=\pi(\boldsymbol{a})f_{\boldsymbol{a}}^{\boldsymbol{l}}.$$

By changing the value of the weight functions, we can get different AoI related metrics, but all are linear with respect to $y_{\boldsymbol{a}}^{\boldsymbol{l}}$. Then,

$$\pi(\boldsymbol{a})=\sum_{\boldsymbol{l}}y_{\boldsymbol{a}}^{\boldsymbol{l}},$$

and the normalization constraint requires:

$$\sum_{\boldsymbol{a}}\sum_{\boldsymbol{l}}y_{\boldsymbol{a}}^{\boldsymbol{l}}=1.$$

Substituting $y_{\boldsymbol{a}}^{\boldsymbol{l}}$ into the CMDP problem, we obtain the equivalence of the LP problem. After obtaining the solution $\boldsymbol{y}$, we let $f_{\boldsymbol{a}}^{\boldsymbol{l}}=y_{\boldsymbol{a}}^{\boldsymbol{l}}/\sum\limits_{\boldsymbol{l}\in\mathcal{L}}y_{\boldsymbol{a}}^{\boldsymbol{l}}$ for $\pi(\boldsymbol{a})\neq 0$. For transient states with $\pi(\boldsymbol{a})=0$, the actions at these states do not affect the average results, so we adopt a simple policy as in (3), then the normalization constraint is also satisfied. ■

C. Characterization and Insights on Age-Optimal Schedulers for Single-User Energy Allocation Problem

Our general framework encompasses a wide range of objectives and constraints for different choices of $\omega_k(\cdot)$ functions. In this section, we focus on two important single-user energy allocation problems that can be expressed within our framework: *average age minimization* and *age-violation-rate minimization*. This effort will enable us to characterize their optimal schedulers and gain insights into their nature.

Optimal scheduler minimizing average age: Under single-user scenario, $i=1$ and $\boldsymbol{A}[t],\boldsymbol{u}[\boldsymbol{A}]$ simplifies to $A[t],u(A)$. When we set $\omega_0(a)=a$ in (1), the objective of the optimization problem becomes to minimize the average age

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{A[t]\}=\sum_{a=1}^{D}a\,\pi(a).$$

For this problem formulation, we retain the energy constraint $\lim\limits_{T\to\infty}\frac{1}{T}\sum\limits_{t=1}^{T}\mathbb{E}\left[u\left(A[t]\right)\right]\leq b$; but do not need additional age constraints. Hence, $\omega_k(a)=0$ and $c_k=0$, for all k and a.

Figure 2 depicts the average number of activated channels of the average-age optimal scheduler as a function of the age states under different channel success probabilities μ for the energy constraint $b=2$. We will further discuss these results at the end of this section in comparison with the next scheduler.

Optimal scheduler minimizing age-violation-rate: Setting $\omega_0(a)=\mathbb{1}\{a>\tau\}$ n (1), the objective becomes minimizing the average age-violation-rate

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\mathbb{1}\{A[t]>\tau\}\}=\sum_{a=\tau+1}^{D}\pi(a),$$

which is neither a convex nor a concave function. As before, we keep the energy constraint, but do not need additional age constraints, i.e., $\omega_k(a)=0$ and $c_k=0$, for all k and a.

[7]For brevity, we omit the detailed expression of the balance equation, which is provided in the extended technical report [31].

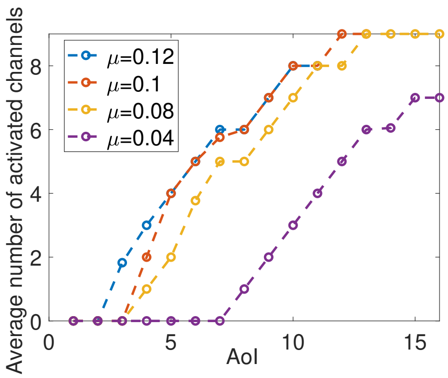

Figure 2. Optimal number of channels to choose to minimize average AoI when $b = 2$.

With this, the problem becomes minimizing the age-violation-rate under an energy constraint. Unlike in the previous problem, our goal is not to minimize the average age but to avoid age-violation events. In this scenario, we can view all the states with $a > \tau$ as state $\tau + 1$, so it's enough to set $D = \tau + 1$.

Figure 3 depicts the average number of activated channels of the violation-rate optimal scheduler as a function of the age states under different channel success probabilities μ for age threshold $\tau = 8$ and the same energy constraint $b = 2$. Next, we compare the optimal policies of these two schedulers and discuss the insights that can be gained from their study.

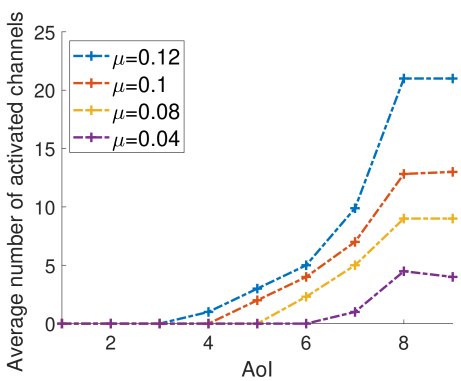

Figure 3. Optimal number of channels to choose to minimize AoI violation rate when $b = 2$ and $\tau = 8$.

Insights on the two optimal schedulers: We start by noting the similarities of the optimal policy under both scenarios:

(i) Each optimal policy is a probabilistic combination of at most two deterministic policies, which matches the result that the number of randomization is at most the number of constraints, as shown in [22].

(ii) For each scenario, as the channel success probability increases, the corresponding optimal policy starts transmitting at lower age levels, and also tends to choose more channels at the same age level. This is a somewhat counter-intuitive characteristic that indicates that the optimal policy should be more active and active earlier when the channels are more reliable.

(iii) The optimal policy in each scenario is idle when AoI is relatively small. This is meaningful once we observe that, when the age is relatively small, a successful transmission will not benefit the objective as much as when the age is large. Hence, the optimal scheduler saves energy for larger age states.

However, we also notice differences between the two sets of schedulers:

(i) The optimal policy in the average age minimization problem has an activation function $u^*(\cdot)$ that is monotone non-decreasing with increasing age state. On the other hand, the monotonicity does not hold in the age violation rate minimization problem. This difference comes from the non-convex nature of the the age violation rate function in the latter case. In [22] and many related works (e.g., [9], [32]), the authors exploit the monotone structure and threshold nature of the optimal scheduling policy for solving the CMDP, revealing insights as well as simplifying the algorithm by using the convexity or concavity of the objective functions. However, in our general treatment, the objective functions, such as age violation rate, are not necessarily convex or concave, which prevents us from using the same approach. Hence, to obtain the optimal policy, we use the generally applicable LP method despite the higher computational complexity that it may require in order to develop insights about the optimal solution.

(ii) In the average age minimization problem, the number of activated channels of the optimal policy experiences a sub-linear/concave like increase with respect to ages after the number of activated channels starts to be above zero. In contrast, the age violation rate minimizing schedulers experience a super-linear/convex like increasing with respect to age until the deadline level τ. This difference can be interpreted as follows: in the age violation rate minimization problem, the penalty happens only when the age is beyond the age deadline, and hence the optimal scheduler will be more aggressive as the threshold level is approached from below. In contrast, for the average age minimization problem, the number of activated channels increases more gradually to balance the tradeoff between consuming energy unnecessarily at very low age levels and waiting too long to consume the available energy, which yields an indefinitely increasing cost.

These insights on the structure of the allocation functions of the optimal schedulers can guide designers in restricting their search to classes of functions with sufficiently flexible but also tractable forms whenever the solution through the LP strategy is not possible due to lack of prior statistical information as well as computational resources.

To demonstrate how the age violation rate constraint effects the shape of the scheduler more clearly, in Figure 4 we set the objective function to be $\omega_0(a) = a$, the energy constraint to be $b = 3$, and the channel success probability to be $\mu = 0.2$. In addition, we set $\omega_1(a) = \mathbb{1}\{a > \tau\}$, where the age deadline $\tau = 5$. We set $c_1 = \epsilon$ and show how the number of activated channels changes over age states under different ϵ levels. By adding and tightening the tolerance constraint, we can see the transition from concave (or sublinear) to convex (or superlinear) form. As such, the optimal scheduler becomes

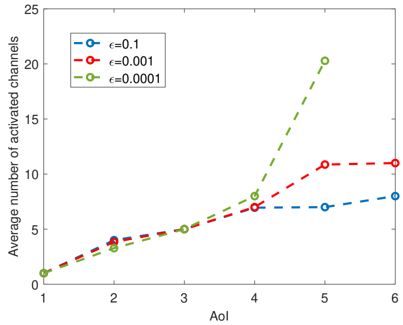

Figure 4. Optimal number of channels to choose to minimize average age under violation rate constraint when $\tau = 5, b = 3, \mu = 0.2$

more aggressive when the age increases. This reveals a trade-off between the average age and the age-violation-rate, namely that reducing the age violation rate calls for an increasingly more aggressive allocation function.

D. Characterization and insights on multi-user scheduling problem with violation tolerance Constraints

In this section, we will study the multi-user single-channel scheduling feasibility problem with age-violation tolerance constraint to investigate its performance and characteristics.

In particular, we will compare the stability region of the optimal scheduler with a previously developed algorithm that was developed for the special case of multi-user single-channel setting [18]. To that end, we set $L = 1$ and $b_i > 1$. Thus, all the energy constraints will be inactive, and we can focus on the tolerance constraint, as in [18]. Since we are only interested in feasibility, we set $\omega_0(\boldsymbol{a}) = 1$ for all $\boldsymbol{a}$. To express the age-violation rate constraints we define the weight functions

$$\omega_k(\boldsymbol{a}) = \begin{cases} 0, & \text{if } a_k \le \tau_k \\ 1, & \text{if } a_k \ge \tau_k + 1, \end{cases}$$

and set $c_k = \epsilon_k$ for $k = 1, 2, \cdots, K = n$, to represent the heterogeneous age-violation tolerance level for the k^{th} source.

Then the constraint $\sum_{\boldsymbol{a}} \pi(\boldsymbol{a})\omega_k(\boldsymbol{a}) \le c_k$ becomes

$$\pi_k(\tau_k + 1) \le \epsilon_k \quad \forall k = 1, \cdots, K = n,$$

where $\pi_k(\tau_k + 1)$ denotes the total probability (under the stationary distribution) that source k violates its age threshold τ_k. Since

$$\pi_k(\tau_k+1) = \sum_{j_1,\ldots,j_{k-1},j_{k+1},\ldots,j_n} \pi(j_1, \ldots j_{k-1}, \tau_k+1, j_{k+1} \ldots, j_n),$$

the constraint (2) in the linear programming problem becomes

$$\sum_{j_1,\ldots,j_{k-1},j_{k+1},\ldots,j_n} \sum_{\boldsymbol{l}} y^{\boldsymbol{l}}_{j_1,\ldots,j_{k-1},\tau_k+1,j_{k+1},\ldots,j_n} \le \epsilon_k.$$

For the sake of easy visualization, we study the case with $n = 2$ users. In this case, the LP problem is formulated as:

$$\begin{array}{ll} \min & 1 \\ \text{s.t:} & 0 \le y^{l_1,l_2}_{a_1,a_2} \le 1 \quad \forall l_1, l_2 = 0, 1 \\ & y^{l_1,l_2}_{a_1,a_2} = 0 \quad \forall l_1 + l_2 > 1 \\ & \sum_j \sum_{l_1,l_2} y^{l_1,l_2}_{\tau_1+1,j} \le \epsilon_1 \\ & \sum_j \sum_{l_1,l_2} y^{l_1,l_2}_{j,\tau_2+1} \le \epsilon_2 \end{array}$$

The numerical results can be seen in Figures 5 for different parameters where the upper right area of the solid blue line is the stability region of the optimal scheduler. These typical examples reveal the non-negligible gap between the performance of the optimal scheduler and the previously proposed design, even for a small two user setting.

This motivates the search for new algorithms that can perform closer to the optimal scheduler, even when the channel statistics are unknown a priori. This is performed in the next section along with further discussion about these numerical results after we discuss our online scheduling algorithm.

Before we proceed, we note even the above numerical results are for two-user single-channel scheduling problem under tolerance constraints for visualization purposes, our methods apply to the more general multi-user multi-channel scheduling problem under violation tolerance and energy constraints. Although the computational complexity may be relatively high for the LP solution compared to other solutions that exploit the special structure of particular problems, as we mentioned above, due to the non-convexity and non-concavity of the tolerance constraints, the monotone and threshold structure of the optimal policy does not hold. The Whittle Index approach (used, for example, in [26], [28]) which have relatively low complexity also does not apply to our multi-channel scheduling problems since each user in our setting is allowed to transmit over multiple channels simultaneously, whereby the Whittle's Indexability condition does not hold. Using the generally applicable LP-based approach reveals key insights that can guide the designers in developing efficient schedulers for future multi-channel wireless technologies.

IV. Online Scheduling under Unknown Channel Statistics

Until this point, we have assumed that the channel success probabilities are known when solving the optimization problems. In this section, we use a Lyapunov-drift-plus-penalty approach(see [25]) to solve the multi-user online age related optimization problem in the scenario when only the current channel states are known, but the channel statistics are unknown.

We will transfer all the energy and age-related constraints into the virtual queues and view the objective as a penalty term with parameter M. For the energy constraint of the source i, let us define the corresponding virtual queue as $Q_{1,i}[t]$, whose initial value is $Q_{1,i}[0] = 0$ and update equation is:

$$Q_{1,i}[t+1] = \left(Q_{1,i}[t] + u_i\left(\boldsymbol{A}[t]\right) - b_i\right)^+.$$

Similarly, we define the virtual queue $Q_{2,k}[t]$ for the k^{th} age-related constraint, whose initial value is $Q_{2,k}[0] = 0$ and update equation is:

$$Q_{2,k}[t+1] = \left(Q_{2,k}[t] + \omega_k\left(\boldsymbol{A}[t]\right) - c_k\right)^+.$$

Generically, if the virtual queue $Q_{1,i}[t]$ is stable, then its input rate $\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[u_i\left(\boldsymbol{A}[t]\right)\right]$ will be less than its output rate

b_i [25], so that the corresponding constraint can be satisfied. Define the state of both virtual queues and age at time t as $\boldsymbol{Q}[t] = (Q_{1,1}[t], \cdots, Q_{1,n}[t], Q_{2,1}[t], \cdots, Q_{2,K}[t], \boldsymbol{A}[t])$. Based on the virtual queues, we will define the quadratic Lyapunov function as:

$$V[t] = \frac{1}{2}(\sum_{i=1}^{n} Q_{1,i}^2[t] + \sum_{k=1}^{K} Q_{2,k}^2[t]),$$

and develop an online algorithm to greedily minimize the upper bound of the Lyapunov-drift-plus-penalty function $\Delta V(\mathbf{q}) + M\mathbb{E}[\omega_0(\boldsymbol{a})]$ given the current state $\boldsymbol{q} = (q_{1,1}, \cdots, q_{1,n}, q_{2,1}, \cdots, q_{2,K}, \boldsymbol{a})$, where:

$$\Delta V(\mathbf{q}) = \mathbb{E}[V[t] - V[t-1] | \boldsymbol{Q}[t] = \boldsymbol{q}].$$

We consider the multi-user single-channel scheduling problem under tolerance constraints as a specific example to present the design. Since there are no energy constraints, we do not need the set of virtual queues $\{Q_{1,i}[t]\}_i$. In order to express the k^{th} violation rate constraint for source $k = 1, \cdots, n$, we let $\omega_k(\boldsymbol{A}[t]) = \mathbb{1}(A_k[t+1] > \tau_k)$ and $c_k = \epsilon_k$. Then the virtual queue $Q_{2,k}[t]$, whose initial value is $Q_{2,k}[t] = 0$, updates as follows:

$$Q_{2,k}[t+1] = (Q_{2,k}[t] + \mathbb{1}(A_k[t+1] > \tau_k) - \epsilon_k)^+,$$

where $A_k[t+1] = 1 + A_k[t](1 - S_k[t]U_k[t])$; $S_k[t]$ represents the channel success; $U_k[t]$ represents whether the source is scheduled to transmit or not. If virtual queue $Q_{2,k}[t]$ is stable, its input rate, the threshold violation rate $\pi_k(\tau_k + 1) = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{1}(A_k[t+1] > \tau_k)$, will be less than its output rate ϵ_k.

The conditional Lyapunov drift can be bounded as follows:

$$\begin{aligned} &\Delta V(\mathbf{q}) \\ \leq\ & \sum_{k=1}^{n} q_{2,k}\mathbb{E}[R_k - \epsilon_k | q_{2,k}] + \sum_{k=1}^{n} \mathbb{E}\left[\frac{(R_k - \epsilon_k)^2}{2} | q_{2,k}\right], \end{aligned}$$

where $R_k \triangleq \mathbb{1}\{1 + A_k(1 - S_k C_k) > \tau_k\}$. At every time slot t, we can develop an online algorithm as summarized below to greedily minimize the upper bound of the Lyapunov drift given the queue lengths $\boldsymbol{Q}[t-1]$ and $\boldsymbol{A}[t-1]$ since there is no objective or penalty term in this case.

Algorithm 1 A Heuristic Scheduling Policy

1: Input current system state: $A_i[t]$,$Q_i[t]$.
2: Define available transmission decision set: only one $U_i[t]$ can be 1.
3: Choose $\boldsymbol{U}[t]$ to minimize the upper bound of Lyapunov drift function in the above inequality.
4: Update queue lengths for next time slot.

Again, for the sake of easy visualization, we will only present the simulation results for the two-user online scheduling problem under age tolerance constraints, but the online algorithm can be simply applied to any number of sources. The simulation results are illustrated in Fig 5 where the upper right area of the dash-dot purple line is the stability region of the online scheduler when the channel condition μ_i. The comparison will be in the next section.

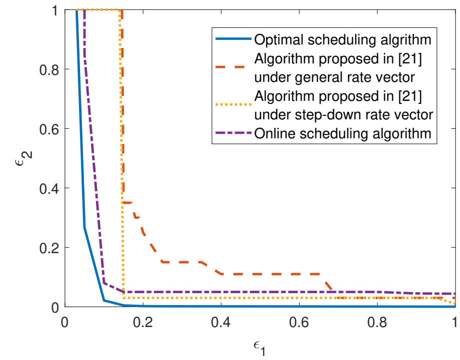

Figure 5. Stability region (upper-righter) comparison for asymmetric case.

V. Comparison of Stability Regions under Age Violation Constraints

In this section, we compare the performance of three different algorithms for the two-user single channel scheduling feasibility problem under age violation tolerance constraints. These are: the optimal scheduler from Section III; the prior design from [18] developed for a single-channel multi-user setting; and our online scheduler from Section IV that does not require channel statistics.

In Figure 5, we consider two source nodes with asymmetric age thresholds of $\tau_1 = 2, \tau_2 = 4$ and a common channel success probability of $\mu_1 = \mu_2 = 0.85$. Since the violation rate depends on both the age thresholds and the channel success probabilities, this is a non-homogeneous violation rate scenario. The upper right area of the blue line is the stability region for the optimal scheduling algorithm in Section III-D. The yellow and orange lines correspond to the algorithm in [18] and capture the two cases when the rate vector does or does not possess a special property (called step-down rate vector). The purple line marks the stability region for the online algorithm when the channel conditions μ_1, μ_2 are unknown. Several observations are in order from these simulation results:

(i) The optimum policy (blue line) outperforms other policies, with markedly better performance when one of the tolerance constraints is very strict, namely when ϵ_1 approaches 1. In this regime, the feasible tolerance level ϵ_2 of user 2 from other algorithms is bounded away from zero while the optimal algorithm decreases towards zero.
(ii) The online algorithm (purple line) performs very closely to the optimal policy, experiencing a small performance loss only at some extreme range of tolerance levels.
(iii) When compared with the algorithms from [18](yellow and red lines), the online algorithm performs particularly better when one of the tolerance rates is smaller than the corresponding channel loss probability, as observed by the vertical gap between purple and yellow lines.
(iv) The online and optimal policies are continuous with respect to the tolerance level, which eliminates the need to check if the tolerance rate vector satisfies certain properties, such as the step-down rate condition in [18].

These simulation results are typical of other circumstances, with the common observation that our online scheduler performs close to the optimal scheduler and typically non-negligibly better than the closely related state-of-art algorithm from [18], despite the fact that it operates without the knowledge of channel statistics that is assumed in the other designs.

VI. Conclusions

In this paper, we considered a general class of age-optimal scheduling problems for multi-source multi-channel communication. We formulated the generic age-optimization problem with flexible weight functions ω_k under energy and tolerance constraints in the form of a CMDP. We solved this generic problem, which a usual threshold-based structure policy does not apply, by relating it to the solution an associated linear programming problem using the powerful theory of CMDPs. Then, we focused on the special case of single-source multi-channel scenario to investigate the characteristics of optimal scheduler for the important special cases of average-age and violation-rate minimization.

Our investigations revealed several interesting insights, including the observation that age-violation-rate minimizing scheduler employs a super-linearly like growing energy allocation strategy with increasing age, as opposed to the sub-linearly like growing allocation for the average-age-minimizing scheduler. These insights may provide useful guidelines for IoT network designers in developing effective update strategies based on different sensitivities of applications to age performance.

We also studied the special case of multi-source single-channel scheduling problem with age violation rate constraints to investigate the feasibility region of the optimal scheduler together with that of most closely related prior works. Finally, we have developed an online scheduler that does not require the knowledge of channel statistics, and compared its performance to the optimal scheduler through simulations to observe that it performs closely to the optimal scheduler despite its lack of information on channel statistics.

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