Exploring the Tradeoff between Age of Information and Synchronization over Broadcast Channels

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***Abstract*—We consider a scenario whereby the state of a common source is being updated at multiple distributed devices. We are particularly interested in the tradeoff that exists between the *freshness* of the updates at the distributed devices and the *synchrony* of the updates across them. In this paper, we explore this tradeoff in a wireless downlink setting whereby the transmitter can choose between unicast transmissions (with given success probabilities) to particular users and broadcast transmissions (with a smaller success probability) to all users. After discussing the Linear Programming (LP)-based optimal design and extreme choices of "always-unicasting" and "always-broadcasting" policies, we note that the optimal design is not scalable and the extreme policies are inefficient. This motivates us to develop two classes of policies, namely a "mixed randomized policy" and a "feature-based learning policy", which have desirable performance and computational-complexity characteristics. We perform extensive numerical studies to compare the performance of these designs over the benchmarks to reveal their gains.**

I. Introduction

In recent years, the exponential growth of connected devices for the next-generation wireless networks and the advent of latency-sensitive applications, such as industrial automation, vehicular networks, and the Internet of Things (IoT), have shifted the focus from traditional communication metrics like throughput to more nuanced performance indicators. Age of Information (AoI) is one such metric that quantifies the freshness of information by measuring the time elapsed since the generation of the most recent update received by a user (see, for example, [1]–[3]).

Since the introduction of the AoI metric, numerous related studies emerged in various networking scenarios, including wireless random access networks (e.g., [4], [5]), content distribution networks (e.g., [6], [7]), scheduling (e.g., [8]–[10]) and queuing networks (e.g., [11], [12]). More recently, various extensions and variants of the AoI metric have been proposed to address different aspects of information freshness. Peak Age of Information (PAoI in [13]) is one such metric that captures the worst-case AoI by considering the maximum value of AoI over a time window and is especially important in applications where information staleness could lead to severe consequences. The Weighted Age of Information (WAoI, see [14]) is another extension that assigns different weights to updates, reflecting their relative importance in the system. The AoI violation rate metric (see [15]) describes the time ratio of AoI violating a fixed level and is used in scenarios where the AoI for each source can tolerate occasional violations.

In this paper, we will introduce and study the measure of *age of synchronization* among distributed users in a wireless downlink system, which measures how similar the age (and hence the freshness) levels are at the users. In particular, we explore the trade-off between the freshness and the synchrony of the updates under different transmission policies. Synchronization is a critical aspect of future wireless communication systems since accurate synchronization is essential for coordinating time-sensitive operations among different users (see [16]). There are many scenarios where synchronization among users takes precedence over AoI, such as in distributed control systems, cooperative communication networks ([17]) and so on (e.g., Vehicular Networks [18], Wireless Sensor Networks [19] and Precision Agriculture [20]). For example, in industrial automation and process control applications, distributed control systems involve multiple sensors, actuators, and controllers that need to coordinate their actions in real-time. Accurate synchronization among these users is critical for maintaining the stability and efficiency of the system, while the AoI may be of secondary importance (see [21]).

Thus, achieving a balance between AoI and synchronization is therefore of paramount importance for the effective functioning of these systems. The remainder of this paper is organized as follows:

- In Section II, we build our system model in a discrete-time wireless downlink setting whereby the transmitter can choose between unicasting and broadcasting with different transmission success probabilities. We formulate our problem as minimizing the weighted sum of the AoI and age of synchronization (AoS) to study the trade-off between the freshness and the synchrony.
- In Section III, we study the optimal solutions via Linear programming for small number of users n due to the computational complexity of the optimal solution for large n. In section IV, we analyze the performance of two extreme policies: always unicasting and always broadcasting, and make comparisons. In section V and Section VI, we

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propose a mixed randomized policy and a feature-based learning policy, both with good scalability characteristics and non-negligible performance gains compared with extreme policies with meaningful success probabilities.
- In Section VII, we execute simulations and compare all the mentioned policies. We observe that, with different number of users, success probability and weights, we may prefer different policies for optimizing the tradeoff. Counter-intuitively, we note that unicasting can be more preferable to broadcasting when aiming to minimize the synchronization under an unreliable communication environment. And in Section VIII, we conclude the paper and mention the potential future works.

In related literature, many works (e.g., [22]–[24]) have studied different types of the clock synchronization in a decentralized system, such as reference-broadcast synchronization (RBS) and time-stamp synchronization (TSS), but they focus on the structure of the protocols instead of considering transmission successes and failures. In [25], [26] and many other works, the authors have aimed to decrease the synchronization and other metrics with time-sensitive 5G networks, but by the means of improving the transmission architecture and mechanisms to provide ultra-reliability and low-latency communications (URLLC). More recently, [27] have presented an efficient window-based resource allocation method for the end-to-end time-sensitive network scheduling problem under the uncertainty of the channel. This work aims to reduce large-scale fading correlation across the devices which is different from our scope. There are other works that aim at minimizing other AoI metrics under fading channels which is different from our focus. To our best knowledge, there is no prior work considering the trade-off between the freshness and the synchrony among the users in an unreliable communication environment under different transmission strategies.

II. System Model

In this paper, we will consider the operation of a discrete-time wireless communication system, whereby a Base Station(BS) sends information updates to n users at the beginning of every time slot $t \in \{1, 2, 3, \cdots\}$ either by broadcasting the information to all the users with a relatively lower individual success probability (that are generated independently for each user) or by unicasting the information to a specific user with a relatively higher success probability. We assume that the BS refreshes its status and creates a new packet at the beginning of every time slot t. Accordingly, the BS always sends the freshest status to all the users. This assumption is especially reasonable for the scenarios where the state of the source is observable or accessible at the BS. More complicated models, such as randomly generated new packets, add more complexity and can be considered in the future extensions. Our goal is to find an effective strategy that can keep the information at the users fresh as well as the age of information amongst the users as synchronized as possible. We describe the key terminology and the essential system dynamics in the rest of this section. Then, in the following sections we formulate the problem and propose different strategies with different performance and complexity characteristics for its solution.

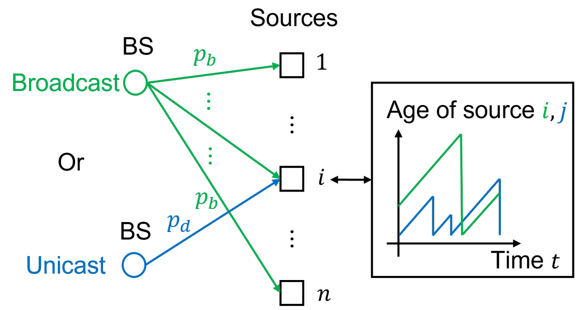


Figure 1. Base Station updates its status to n users by either broadcasting or unicasting to keep the age levels at users low and synchronized.

A. AoI AoS metrics

First, define the Age-of-Information(AoI) of user i as $U_i[t]$, which is the number of time slots elapsed at time t since the user i last received a successful update from the station. The AoI is updated as follows:

$$U_i[t+1] = \begin{cases} 0, & \text{if transmission of source } i \text{ succeeds} \\ U_i[t]+1, & \text{otherwise.} \end{cases}$$

Define $A[t]$ as the average AoI of all the users at time t, $A[t] = \frac{1}{n}\sum U_i[t]$. To study the information freshness difference between users, we will additionally define the Age-of-Synchronization metric $S^1[t]$ as the $1^{st}-$order average Age-of-Synchronization,

$$S^1[t] = \frac{1}{\binom{n}{2}} \sum_{i \neq j} |U_i[t] - U_j[t]| \, .$$

B. Broadcast/Unicast Model

We assume that in our model the base station will choose one of the actions $x[t] \in \mathcal{X}$ at every time slot t, where $\mathcal{X} = \{0, 1, \cdots, n\}$, $x[t] = 0$ represents that the station chooses to broadcast to all n users and $x[t] = i$ means that the station chooses to unicast with the i^{th} user.

Under the broadcasting model, we let $\mathbb{P}\{U_i[t+1] = 0\} = p_b$ be the probability of success, whereby the success/failure outcomes of each user is independently determined[1]. Under the unicasting model, when $x[t] = i$, $\mathbb{P}\{U_i[t+1] = 0\} = p_d$, $\mathbb{P}\{U_j[t+1] = 0\} = 0$ for $j \neq i$.

C. Objective

In this paper, we focus on minimizing the weighted sum of the long-term Age-of-Synchronization and Age-of-Information, which allows us to study the trade-off between the information freshness of the users and the information synchronization between users. Define the cost at t to be a function of the weight $\alpha \in [0, 1)$:

$$C_\alpha[t] \triangleq (1-\alpha)A[t] + \alpha S^1[t].$$

[1] In reality, users may have different success probabilities under the broadcasting model due to user locations, which can be discussed in future works.

Notice that α is not allowed to be 1 in our model, since only minimizing AoS can push the system into an unstable operating mode where none of the users wants to get updates when their AoS is small. We will study the optimal solution to the problem of minimizing the long-term average:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[C_\alpha[t]\right]$$

via Linear Programming in Section III, extreme policies with either always broadcasting or always unicasting with the oldest users in Section IV, a mixed randomized policy in Section V, and feature-based learning policies in Section VI. We will compare the theoretical and simulation performance of these designs in Section VII.

III. Optimal Design

In this section, we formulate the minimization problem under the Markov Decision Process(MDP) setup. Let the state be the current age of n users: $\mathbf{U} = [U_i[t]]_{i=1}^n \in [0, D]^n$ where D is an upper bound on the ages[2] and $P(\mathcal{X})$ is the probabilistic policy on set $\mathcal{X}$. Then, the MDP problem for n users can be formulated as:

$$\min_{x[t]\in P(\mathcal{X})} \quad \lim_{T\to\infty}(1-\alpha)\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[A[t]\right]+\alpha\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[S^1[t]\right]$$

Theoretically, this problem can be solved by transforming the MDP in an appropriate Linear Program (LP). However, this approach is not scalable due to the exponential growth of the problem size the n. Nevertheless, using this solution for small n values will allow us to use it as a benchmark for our designs to compare against. As such, for completeness, we provide the optimal solution LP for $n = 2$ users, which can be generalized to $n > 2$ with increasing notational complexity.

Theorem 1: The solution to the 2 users minimization problem can be obtained by solving the following linear programming problem:

$$\begin{aligned}
\min_{y^k_{a_1,a_2}} \quad & \sum_{a_1,a_2=0}^{D}\sum_{k=0}^{2}\left[\frac{1-\alpha}{2}\left(a_1+a_2\right)+\alpha|a_1-a_2|\right]y^k_{a_1,a_2}\\
\text{s.t:} \quad & 0\le y^k_{a_1,a_2}\le 1 \quad \forall 0\le a_1,a_2\le D, 0\le k\le 2,\\
& \textstyle\sum_{a_1,a_2=0}^{D}\sum_{k=0}^{2} y^k_{a_1,a_2}=1,\\
& \mathbf{Q}\boldsymbol{y}=\mathbf{0},
\end{aligned}$$

where $\boldsymbol{y}$ is a column vector of size $3(D+1)^2$ with $\boldsymbol{y} = (y^0_{0,0}, y^1_{0,0}, y^2_{0,0}, \cdots, y^0_{D,D}, y^1_{D,D}, y^2_{D,D})^T$ as its components, D is an upper bound on the age state in the system which can be set sufficiently large so that the probability of reaching D is vanishing. And $\mathbf{Q}\boldsymbol{y} = \mathbf{0}$ is the matrix representation of the (Markov balance) equations in Appendix. A. If this LP is feasible and $\boldsymbol{y}$ is an optimal solution, then the optimal policy is a probabilistic policy $P(\mathcal{X})$, whereby the probability $f^k_{a_1,a_2}$ of choosing $x[t] = k$ when the age is at state (a_1, a_2) equals:

$$f^k_{a_1,a_2}=\begin{cases}\dfrac{y^k_{a_1,a_2}}{\sum_{k=0}^{2}y^k_{a_1,a_2}}, & \text{if} \sum_{k=0}^{2}y^k_{a_1,a_2}\neq 0\\[2ex] \frac{1}{3}, & \text{if} \sum_{k=0}^{2}y^k_{a_1,a_2}=0\end{cases}$$

for $(a_1, a_2) \in [0, D]^n$.

Proof: The proof follows directly from the equivalency between MDP and LP problem and is omitted here (refer to [28]). ∎

Since the computational complexity is high for solving the Linear Programming problem in a high dimensional setup, we will study more policies with better scalability in following sections.

IV. Extreme Policies

To develop an understanding of the broadcasting and unicasting decisions, in this section, we study the performance of age and synchronization metrics for two extreme policies as a function of n for different p_b and p_d, and compare the theoretical results at the end of this section. The related simulation performance can be found in Section VII.

A. Always-Broadcasting Policy

In this section, we study the policy that selects $x[t] = 0$ for all t, i.e., the BS always chooses to broadcast the current information to n users. We will analyze the long-term average of $A[t]$ in Theorem 2 and the long-term average of $S^1[t]$ in Theorem 3.

Theorem 2: The long-term average of $A[t]$ for always-broadcasting equals to:

$$\frac{\sum_{k=0}^{\infty}(1-p_b)^k\cdot\frac{1}{2}k(k+1)}{\sum_{k=0}^{\infty}(1-p_b)^k(k+1)}=\frac{(1-p_b)}{p_b},$$

which remains constant as n increases.

Proof: Since each user is statistically identical with respect to age and user successes are independent, it is sufficient to calculate the long-term average of $U_1[t]$. The result follows from the characteristics of the associated geometric distribution. Details are omitted due to limited space. ∎

Theorem 3: The long-term average of $S^1[t]$ for always-broadcasting equals to:

$$\begin{aligned}&\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{S^1[t]\}=\lim_{t\to\infty}\mathbb{E}\{S^1[t]\}\\&=\sum_{k=0}^{\infty}\frac{2p_b(1-p_b)}{1-(1-p_b)^2}p_b(1-p_b)^k(k+1)=\frac{2(1-p_b)}{1-(1-p_b)^2}.\end{aligned}$$

Proof: Since each user pairs are identical, it is sufficient to calculate the long-term average of the absolute age gap between any two users. Details are omitted due to space. ∎

[2] In reality, D can be viewed as an upper bound where ages older than D make no difference to the system. Theoretically, as D approaches infinity, the solution approaches the solution of the infinite CMDP where ages are unbounded.

B. Always-Unicasting Policy

In this section, we study the policy that selects $x[t] = \arg\max_i U_i[t]$ for all time slots t, i.e., the BS always chooses to unicast information to the user with the highest age. We will analyze the long-term average of $A[t]$ in Theorem 4 and the long-term average of $S^1[t]$ in Theorem 5. This policy will be more complex than the previous always-broadcasting policy since this is a state-dependent policy.

Theorem 4: The long-term average of $A[t]$ for always-unicasting equals to:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{A[t]\} = \lim_{t\to\infty}\mathbb{E}\{A[t]\} =$$

$$\frac{\sum_{k=0}^{\infty} p_d^n(1-p_d)^k \begin{pmatrix} n+k-1 \\ n-1 \end{pmatrix} \frac{1}{2}(n+k)(n+k-1)}{\sum_{k=0}^{\infty} p_d^n(1-p_d)^k \begin{pmatrix} n+k-1 \\ n-1 \end{pmatrix} (n+k)}$$

$$= \frac{\frac{1}{2}\cdot -np_d^{-n-2}(-n+2p_d-1)}{np_d^{-n-1}} = \frac{n-2p_d+1}{2p_d}.$$

Note that the average AoI is linear in n for a fixed success probability p_d.

Proof: By symmetry, we only need to calculate the long-term average of user 1. Since the Markov Chain $U_1[t]$ is positive recurrent, thus the sequence of entry times to state $U_1[t] = 0$ can be viewed as the arrival epochs of a renewal process. Define T_N as the time of the N^{th} entries to state 0 with $T_0 = 0$, $N \in \mathcal{N}$ and define $\Delta_N = T_{N+1} - T_N$ to be the time interval between two entries.

Since we always perform direct transmission, after a success at user 1, $U_1[t]$ will keep increasing by one until another success happens at user 1, and since we always choose the user with the largest age to transmit, between two successes at user 1, all the other $n-1$ users must succeed once. Based on the above description, independent of the starting points, the steady-state probability distribution of the interarrival time $P(\Delta_N = k) = 0$ when $k = 0, 1, \cdots, n-1$; $P(\Delta_N = n+k) = p_d^n(1-p_d)^k \begin{pmatrix} n+k-1 \\ n-1 \end{pmatrix}$, when $k = 0, 1, \cdots$. And when $\Delta_N = n+k$, $U_1[t+\tau] = \tau$ for $\tau = 0, \cdots, n+k-1$ in this renewal, so by the Wald's identity ([29]), we get the claimed formula of the average age. ■

Theorem 5: The long-term average of $S^1[t]$ equals to: $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{S^1[t]\} = \lim_{t\to\infty}\mathbb{E}\{S^1[t]\} = \frac{n+1}{3p_d}$, which is linear in n for a fixed probability p_d.

Proof: The proof is more involved and is moved to Appendix B to avoid disrupting the flow of the main text. ■

C. Discussion on the Performance of Extreme Policies

By comparing the long-term average of the average AoI $A[t]$ of always-broadcasting and always-unicasting policies, we can see that when the broadcasting success probability $p_b > \frac{2p_d}{n+1}$, always-broadcasting policy provides better average AoI performance. Assume that the expected number of successes is unchanged for always-broadcasting when n increases, i.e., assume that $p_b = \frac{\mu}{n} \in [0,1]$ where μ is a positive constant, then the average AoS under the broadcasting policy will become:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{S^1[t]\} = \frac{2(1-p_b)}{1-(1-p_b)^2} = \frac{2(n-\mu)n}{(2n-\mu)\mu}.$$

Recall that for always unicasting policy, the average AoS equals $\frac{n+1}{3p_d}$. Therefore, asymptotically speaking, average AoS approaches $\frac{n}{\mu}$ and $\frac{n}{3p_d}$ respectively for always-broadcasting and always-unicasting. Combining both metrics together to minimize the long-term average of $C(\alpha) = (1-\alpha)A[t] + \alpha S[t]$ for a given α, we get for always-broadcasting policy:

$$\lim_{n\to\infty}\frac{1}{n}\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{C_\alpha[t]\} = (1-\alpha)\frac{1}{\mu} + \alpha\frac{1}{\mu} = \frac{1}{\mu},$$

and for always-unicasting policy:

$$\lim_{n\to\infty}\frac{1}{n}\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{C_\alpha[t]\} = \frac{(1-\alpha)}{2p_d} + \frac{\alpha}{3p_d}. \quad (1)$$

Hence, when 1 is larger than $\frac{1}{\mu}$ or p_b decays slower than $\frac{\mu}{n}$, we would prefer always-broadcasting eventually, otherwise, always-unicasting policy eventually becomes better, see Section VII for simulation results. We also observe that, when one of the success probabilities p_b and p_b is large enough so that the average performance of one extreme policies is much better than the other one, extreme policies perform good enough compared to LP solutions. It is because this case implies that broadcasting or unicasting dominates the other on most of the age states. However, when $2p_d \le np_b \le 3p_d$ (from 1), the average performance of the extreme policies are comparable, in which case we need to find other policies to achieve better performance. This motivates us to develop new policies in the following sections.

V. Mixed Randomized Policies

In this section, we introduce a mixed randomized policy that employs a combination of randomization and the current age state to make broadcasting or unicasting decisions. In particular, in each time slot t, suppose the policy decides to broadcast, i.e., sets $x[t] = 0$, with probability $p \in [0,1]$; and otherwise unicasts the update with the largest age, i.e., sets $x[t] = \arg\max_i U_i[t]$. As such, this policy randomizes between broadcasting to all users and unicasting to the user with the oldest age. This policy, being state-dependent, is far more difficult to analyze when compared to the extreme designs of the previous section. Next, we provide a detailed analysis of the average age performance of the two-user case.

Theorem 6: The long-term average AoI under $n = 2$ will be,

$$\frac{\sum_{i=1}^{\infty}\frac{i(i-1)}{2}\rho(i)}{\sum_{i=1}^{\infty}\rho(i)\cdot i}, \quad (2)$$

where $\rho(1) = P_{0|0}P_{0,:|\text{Eq}} + P_{+|0}P_{0,:|\text{Sm}}$, and for $i \geq 2$,

$$\rho(i) = P_{0|0}(P^{i-1}_{+,+}P_{0,:|\text{Eq}} + P_{0,+|\text{Eq}}P_{0,:|\text{La}}\frac{P^{i-1}_{+,:|\text{La}} - P^{i-1}_{+,+}}{p \cdot p_b(1-p_b)}) + P_{+|0}(P^{i-1}_{+,+}P_{0,:|\text{Sm}} + P_{0,+|\text{La}}P_{0,:|\text{La}}\frac{P^{i-1}_{+,:|\text{La}} - P^{i-1}_{+,+}}{p \cdot p_b(1-p_b)}).$$

The definitions of the notations can be seen in Appendix C.

Proof: Same as in theorem 4, the sequence of entry times to state 0 for $U_2[t]$ can be viewed as the arrival epochs of a renewal process. In this section, define T_N as the time of the N^{th} entries to state 0 with $T_0 = 0$, $N \in \mathcal{N}$ for user 2 and define $\Delta_N = T_{N+1} - T_N$ to be the time interval between two entries. For simplicity, use Δ to denote the interarrival times under the steady-state distribution.

Under the event where $\Delta = i$, there are two cases, $U_1[t] = 0$ and $U_1[t] \neq 0$. Then $P(\Delta = i) = P_{0|0}P(\Delta = i|U_1[t] = 0) + P_{+|0}P(\Delta = i|U_1[t] \neq 0)$. Since the relationship between the age of the two users will affect the success probability of each user, in each case, there are two sub-cases: user 1 never succeeds in time slots $t+1$ to $t+\Delta-1$ and user 1 succeeds at least once in time slots $t+1$ to $t+\Delta-1$ (the probability of the second sub-case equals 0 when $i = 1$). So, $P(\Delta = 1) = P_{0|0}P_{0,:|\text{Eq}} + P_{+|0}P_{0,:|\text{Sm}}$. And for $i \geq 2$, $P(\Delta = i|U_1[t] = 0) = P^{i-1}_{+,+}P_{0,:|\text{Eq}} + P_{0,+|\text{Eq}}P_{0,:|\text{La}}\sum_{j=0}^{i-2}P^j_{+,+}P^{i-2-j}_{+,:|La}$, where $\sum_{j=0}^{i-2}P^j_{+,+}P^{i-2-j}_{+,:|La} = \frac{P^{i-1}_{+,:|La} - P^{i-1}_{+,+}}{P_{+,:|La} - P_{+,+}} = \frac{P^{i-1}_{+,:|La} - P^{i-1}_{+,+}}{p \cdot p_b(1-p_b)}$. Similarly, we can calculate $P(\Delta = i|U_1[t] \neq 0) = P^{i-1}_{+,+}P_{0,:|\text{Sm}} + P_{0,+|\text{La}}P_{0,:|\text{La}}\sum_{j=0}^{i-2}P^j_{+,+}P^{i-2-j}_{+,:|La}$ for $i \geq 2$. And finally, since when $\Delta = i, i = 1, 2, \cdots$, we will have $U_1[t+\tau] = \tau$ for $\tau = 0, 1, \cdots, i-1$ in this renewal, so the long-term average AoI equals to 2 followed by the Wald's identity ([29]), where $\rho(i)$ denotes $P(\Delta = i)$.

For higher dimensional cases, we will explain how the above two-user case approach can be extended with increasing notational complexity. Take $n = 3$ as an example to explain the difficulty and why the same method can be applied to the higher dimensional cases. The difficulty for higher dimensions comes from the fact that, conditioned on the case where $U_1[t] = 0$ and $U_2[t], U_3[t] \neq 0$, we cannot easily calculate the probability of the sub-cases of $U_2[t] \neq U_3[t]$ or $U_2[t] = U_3[t]$. And whether $U_2[t] = U_3[t]$ or not will affect the success probability of user 2 and 3 under the unicasting model and further affects the success probability of user 1. However, by carefully calculating the conditional probabilities, we find that whether $U_2[t] = U_3[t]$ or not will not affect the probability distribution of the slots until user 1 succeeds next time, so the same methods can be applied to higher dimensions as well. ■

The performance of the policy can be seen in Fig. 2 and 5. Notice that all the terms in Eqn 2 are in the form of arithmetico-geometric series ([30]), so the result can be simplified as an explicit expression without summations. For higher dimensional cases, the result is much more complicated but is still an explicit expression, unlike what would be obtained from solving the steady state of the Markov chains.

For the long-term average of synchronization when $n = 2$, we have to utilize the steady-state distribution of $S^1[T_N]$. Through the steady-state balance equation, it is only possible to find a recursive formula of $P(S^1[t] = i), i \in \mathcal{N}$. This is omitted here due to page limitation.

VI. Feature-Based Learning Policy

Since formulating the Linear Programming problem in Theorem 1 is very complicated even for $n > 2$, in Figure 3 and 4, we perform Monte Carlo tabular learning on state space $\mathbf{U} = [U_i[t]]_{i=1}^n \in [0, D]^n$ for $n = 3, 4$ cases to compute the performance of the optimal solution that minimizes $C(\alpha)$. However, Monte Carlo tabular learning algorithm becomes much slower with higher $n > 4$, so instead of seeking the exact solutions, we next introduce a feature-based learning algorithm which is based on the $(A[t], S^1[t])$ state and compare the performance with other policies.

A. Feature-based learning policy

Intuitively speaking, when one of the users $U_i[t]$ is much higher, and all the other users are at a much lower age level, the Age of Synchronization is relatively high and the average AoI is relatively low. Then, to reduce the synchronization metric, we would intuitively unicast with the user i. In contrast, when all the users have relatively high age levels, but their age is closer to each other, we would prefer broadcasting to all of the users. Based on this, we will perform the following feature-based learning algorithm where the feature state $Z[t] = ([A[t], S^1[t]])$ is used to decide $x[t]$. In the next section, we will see that when n is small, the feature-based learning policy provides near-optimal performance, for moderate n, the feature-based learning policy still performs better than the mixed randomized policy.

Algorithm 1 Feature-based Monte Carlo learning

1: Initialize policy π_0 randomly
2: **for** $i = 0, 1, \ldots,$ number of episode **do**
3: Run policy π_i and observe sequence of states $\{\mathbf{U}\}_{0:T}$, actions $\{x\}_{0:T}$, costs $\{C_\alpha\}_{0:T}$
4: Calculate the feature $\{Z\}_{0:T}$ with $\{\mathbf{U}\}_{0:T}$
5: Run Monte Carlo with $(\{Z\}_{0:T}, \{x\}_{0:T}, \{C_\alpha\}_{0:T}$ and get the state-action value function $Q_i(Z, x)$
6: Update policy by $\pi_{i+1}(Z) = \arg\min Q_i(Z, x)$
7: **end for**

VII. Simulation

First of all, we compare the AoI and AoS performance trade-off of all policies (optimal solutions via LP, two extreme policies, mixed randomized policy, and the feature-based learning policy) under a two-user scenario in Figure 2. In this simulation, we set the broadcasting success probability to $p_b = 0.32$ and the unicasting success probability to $p_d = 0.4$.

The ends of the mixed randomized policy represent the performance of two extreme policies (the left end is always-broadcasting). The black dots are the feature-based learning results with different α values, since the feature state $\mathbf{Z}[t]$ and age state $\mathbf{U}[t]$ is a one-to-one mapping in the two-user case, the black dots are very close to the optimal solutions. Through the figure, the minimum long-term average of $C(\alpha)$ can be easily found by finding the lowest line intersects with the policy curve with the slope being $-\frac{1-\alpha}{\alpha}$.

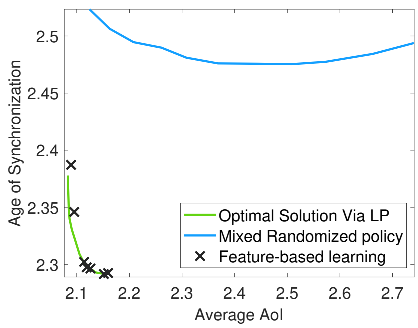


Figure 2. The trade-off between average age and synchronization for all policies when $n = 2, p_b = 0.32, p_d = 0.4$.

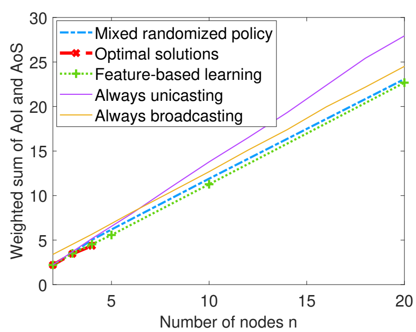


Figure 3. Performance comparison against increasing n when $p_d = 0.3$, $p_b = 0.7/n, \alpha = 0.9$.

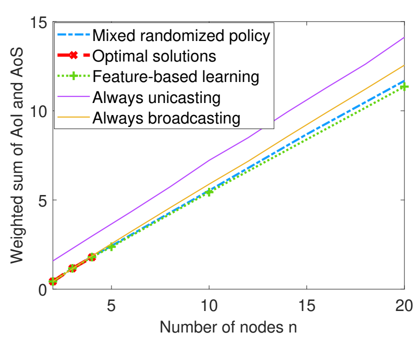
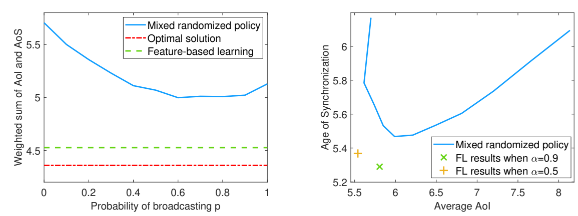


Figure 4. Performance comparison against increasing n when $p_d = 0.6$, $p_b = 1.5/n, \alpha = 0.5$.

Secondly, we plot $C(\alpha)$ the weighted sum of long-term average AoI and AoS under different policies against an increasing number of users n. In Figure. 3, we set $\alpha = 0.9$, $p_d = 0.3$ and $p_b = 0.7/n$ with different n. In Figure. 4, we set $\alpha = 0.5$, $p_d = 0.6$ and $p_b = 1.5/n$ with different n. In figures, the performance of the mixed randomized policy is with the best choice of p for different n, and the optimal solutions are solved via LP for $n = 2$, and from the Monte Carlo Tabular learning algorithm for $n = 3, 4$.

Before making observations about these two figures, notice that in both cases, we set np_b in the range of $2p_d$ to $3p_d$ in order to make a meaningful comparison for large n as explained in Section IV-C after Eqn 1. For this range of values, the performances of optimal solutions, mixed randomized policy, and feature-based learning policy overlap with the broadcasting policy when n is small. This choice of probabilities also makes the performances of policies to be comparable, because in those cases the performance of two extreme policies does not differ significantly from each other. See Figure 5 as a reference. 5(a) shows that the feature-based learning policy outperforms the mixed randomized policy by 10% and is much closer to the optimal level while mixed randomized policy outperforms always-broadcasting by 2.5% when $n = 4$. 5(b) shows that the feature-learning based policy can outperform the mixed randomized policy (left ends of blue curve is $p = 1$) for $n = 10$ where the optimal solution is unknown.



(a) Weighted sum of AoI and AoS when $\alpha = 0.9$, $n = 4$, $p_d = 0.3$, $p_b = 0.7/n$.

(b) AoI and AoS performance trade-off when $n = 10$, $p_d = 0.6$, $p_b = 1.5/n$.

Figure 5. Performance comparisons between mixed randomized policy, feature-based learning and Optimal solution for given n.

With the comparison of the weighted sum against n, we find: (i) Matched with our finding in Section IV-C, when $\alpha = 0.9$, $p_d = 0.3$ and $p_b = 0.7/n$, $(1) = 7/6 < 10/7 = 1/\mu$, we see in Figure 3 that unicasting eventually performs better than broadcasting. In constrast, when $\alpha = 0.5$, $p_d = 0.6$ and $p_b = 1.5/n$, $(1) = 25/36 > 2/3 = 1/\mu$, Figure 4 illustrates that broadcasting eventually performs better than unicasting. (ii) From the theoretical results of extreme policies, the AoI and AoS trade-off comparison for mixed randomized policy, the action learned from feature-based learning policy for different α values, and Figure 3, 4, we can see that unicasting is more preferable than broadcasting if we are more focused on minimizing the AoS (i.e., α is closer to 1) when np_b is in the range of $2p_d$ to $3p_d$. This is somewhat counter-intuitive as without careful thinking we may expect broadcasting to help synchronization more than unicasting. (iii) For a moderate number of users n in the system, the feature-based learning policy performs non-negligibly better than the mixed randomized policy. However, the advantage of feature-based learning will be diminishing with n since when the number of users is large, two features $A[t]$ and $S^1[t]$ cannot accurately distinguish which action is better anymore. So, for moderate n and under the case that the computational power is enough, we can apply a feature-based learning policy to achieve better performance while for larger n, the mixed randomized policy has good scalability as well as a gain compared with extreme policies that grow linearly with n. (iv) We also notice that when the success probabilities of broadcasting and unicasting are both small, the gains for mixed randomized policy and feature-learning policy are more obvious, which implies that in a bad communication environment, we should act more carefully to benefit more.

VIII. Conclusions

In this paper, we consider a time-sensitive scenario whereby the state of a common source is being updated at n distributed devices over unreliable channels. We study the tradeoff between the Age of Information (AoI) and the Age of Synchronization (AoS) whereby the transmitter can choose between unicast transmissions and broadcast transmissions for the updates.

We first pose and solve the optimal solution of the associated constrained MDP problem via Linear Programming, which is tractable only for small n values. Then, we analyze the AoI and AoS performance under two extreme policies (i.e., always-unicasting and always-broadcasting), where we point out how the success probabilities for unicasting and broadcasting along with n would affect the performance of both extreme policies. Motivated by the observations from the extreme policies, we propose a mixed randomized policy and a feature-based learning policy, both with good scalability characteristics and non-negligible performance gains compared with the extreme policies. Subsequently, we perform extensive numerical studies and observe that, for different number of users, success probabilities, and weights, we prefer different policies for optimizing the AoI-AoS tradeoff. Counter-intuitively, we note that unicasting is preferable to broadcasting when aiming to minimize the synchronization under an unreliable communication environment.

Throughout the study, we notice that the synchronization between users in an unreliable communication environment is a very complicated but interesting metric. We propose several different classes of policies with desired properties. Yet, the optimal structure of state-dependent choice to minimize the synchronization and stabilize the system with increasingly large n is still unknown and requires further investigation.

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APPENDIX A
THE BALANCE EQUATION OF LP IN THM 1

For $i = 1, \cdots, D-1$,

$$\sum_{k=0}^{2} y_{0,i}^{k} = \sum_{a_1=0}^{D} y_{a_1,i-1}^{0} p_b (1-p_b) + y_{a_1,i-1}^{1} p_d,$$

$$\sum_{k=0}^{2} y_{i,0}^{k} = \sum_{a_2=0}^{D} y_{i-1,a_2}^{0} p_b (1-p_b) + y_{i-1,a_2}^{2} p_d,$$

$$\sum_{k=0}^{2} y_{i,D}^{k} = (y_{i-1,D}^{0} + y_{i-1,D-1}^{0})(1-p_b)^2$$
$$+(y_{i-1,D}^{1} + y_{i-1,D-1}^{1} + y_{i-1,D}^{2} + y_{i-1,D-1}^{2})(1-p_d),$$

$$\sum_{k=0}^{2} y_{D,i}^{k} = (y_{D,i-1}^{0} + y_{D-1,i-1}^{0})(1-p_b)^2$$
$$+(y_{D-1,i}^{1} + y_{D-1,i-1}^{1} + y_{D-1,i}^{2} + y_{D-1,i-1}^{2})(1-p_d),$$

for $i, j = 1, \cdots, D-1$,

$$\sum_{k=0}^{2} y_{i,j}^{k} = y_{i-1,j-1}^{0}(1-p_b)^2 + \sum_{k=1}^{2} y_{i-1,j-1}^{k}(1-p_d),$$

$$\sum_{k=0}^{2} y_{D,D}^{k}$$
$$= \left(y_{D-1,D-1}^{0} + y_{D-1,D}^{0} + y_{D,D-1}^{0} + y_{D,D}^{0}\right)(1-p_b)^2$$
$$+ \sum_{k=1}^{2} \left(y_{D-1,D-1}^{1} + y_{D-1,D}^{1} + y_{D,D-1}^{1} + y_{D,D}^{1}\right)(1-p_d).$$

APPENDIX B
PROOF OF THM 5

Based on the symmetry,

$$\lim_{t\to\infty} \mathbb{E}\{S^1[t]\} = \lim_{t\to\infty} \mathbb{E}\left[\frac{1}{n-1}\sum_{j=2}^{n} |U_1[t] - U_j[t]|\right].$$

Define function $f(i, p_d)$ as the expectation of the number of slots until the next i^{th} successes happen among all users under steady state when the success probability for unicasting is p_d. In the following analysis, we use $f(i)$ instead of $f(i, p_d)$ for simplification. Similarly as in Theorem 4, define T_i as the time of the i^{th} successes among all the users with $T_0 = 0$, $i \in \mathbb{N}$ and define $\Delta_i = T_i - T_{i-1}$ to be the time interval between two successes. Then,

$$f(i) = \mathbb{E}[T_i] = \mathbb{E}\left[\sum_{j=1}^{i} \Delta_j\right] = \sum_{j=1}^{i} \mathbb{E}[\Delta_j] = i\mathbb{E}[\Delta_1] = \frac{i}{p_d},$$

where the last step is by the Blackwell renewal theorem.

To calculate the age difference between user 1 and others, similarly as in theorem 4, the sequence of entry times to state 0 for user 1 can be viewed as the arrival epochs of a renewal process. Then,

$$\lim_{t\to\infty} \mathbb{E}\{S^1[t]\} = \mathbb{E}\frac{1}{n-1}\sum_{j=2}^{n} |U_1 - U_j|$$
$$= \frac{1}{n-1}\sum_{j=1}^{n-1} \frac{f(j)\times f(n-j) + f(n-j)\times f(j)}{f(j)+f(n-j)}$$
$$= \frac{2}{n-1}\cdot\sum_{j=1}^{n-1}\frac{f(j)\times f(n-j)}{f(n)}$$
$$= \frac{2}{n-1}\times\frac{\sum_{j=1}^{n-1}\frac{j}{p_d}\times\frac{n-j}{p_d}}{\frac{n}{p_d}} = \frac{2}{n-1}\times\frac{\sum_{j=1}^{n-1} j(n-j)}{np_d}$$
$$= \frac{2}{n-1}\times\frac{1}{np_d}\times\frac{1}{6}n\cdot(n-1)(n+1) = \frac{n+1}{3p_d}$$

APPENDIX C
NOTATIONS IN THEOREM 6

Let us use $P_{0|0}$ for $P(U_1[t] = 0 | U_2[t] = 0)$ under the steady-state values of $U_i[t]$ under the above policy[3]. Similarly, all the probability notations below are under the steady state distribution of $(U_1[t], U_2[t])$. Let $P_{+|0}$ denotes $P(U_1[t] \neq 0 | U_2[t] = 0)$. Since $P(U_1[t] = 0, U_2[t] = 0) = p\cdot p_b^2$, $P(U_1[t]U_2[t] = 0, U_1[t] + U[t]_2 \neq 0) = 2p\cdot p_b(1-p_b) + (1-p)p_d$, by symmetry, $P(U_1[t]\neq 0, U_2[t] = 0) = p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d$. Then,

$$P_{0|0} = \frac{p\cdot p_b^2}{p\cdot p_b^2 + p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d};$$
$$P_{+|0} = \frac{p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d}{p\cdot p_b^2 + p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d}.$$

Additionally, denote $P_{+,+} = P(U_1[t] \neq 0, U_2[t] \neq 0)$, $P_{0,:|\text{Eq}} = P(U_1[t] = 0 | U_1[t-1] = U_2[t-1])$, $P_{0,:|\text{Sm}} = P(U_1[t] = 0 | U_1[t-1] < U_2[t-1]$, $P_{0,:|\text{La}} = P(U_1[t] = 0 | U_1[t-1] > U_2[t-1]$, then,

$$P_{+,+} = p(1-p_b)^2 + (1-p)(1-p_d);$$
$$P_{0,:|\text{Eq}} = p\cdot p_b + \frac{1}{2}(1-p)p_d;$$
$$P_{0,:|\text{Sm}} = p\cdot p_b;$$
$$P_{0,:|\text{La}} = p\cdot p_b + (1-p)p_d.$$

Denote $P_{0,+|\text{Eq}} = P(U_1[t] = 0, U_2[t] \neq 0 | U_1[t-1] = U_2[t-1])$, $P_{0,+|\text{La}} = P(U_1[t] = 0, U_2[t] \neq 0 | U_1[t-1] > U_2[t-1])$, $P_{:,+|\text{La}} = P(U_2[t] \neq 0 | U_2[t-1] > U_1[t-1])$, then,

$$P_{0,+|\text{Eq}} = p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d;$$
$$P_{0,+|\text{La}} = p\cdot p_b(1-p_b) + (1-p)p_d;$$
$$P_{:,+|\text{La}} = p\cdot p_b(1-p_b) + (1-p)(1-p_d).$$

[3]It can be shown that the system is stable under the proposed policy, which is omitted here.