# Complex Permittivities of Ultra-Low-Loss 4H-SiC from 55 GHz to 330 GHz

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Abstract — Hexagonal semiconductors such as GaN and SiC have important power applications at radio and millimeter-wave (mmW) frequencies. They are characterized by both ordinary and extraordinary permittivities, parallel and perpendicular to the densest packed c plane, respectively. However, due to the challenges of high-frequency measurements, little reliable data exist for these permittivities especially at mmW frequencies. Recently, for the first time, we reported the extraordinary permittivity of 4H SiC at mmW frequencies using substrateintegrated waveguides. We now report the ordinary permittivity of the same material using several Fabry-Perot resonators to cover most mmW frequencies. The resulted relative ordinary permittivity of  $9.76 \pm 0.01$  exhibits little dispersion and is significantly lower than the previously reported extraordinary permittivity of  $10.2 \pm 0.1$ . This confirms that both ordinary and extraordinary permittivities are needed for accurate design and model of devices fabricated on 4H SiC. By contrast, the measured loss tangent increases linearly from  $3 \times 10^{-5}$  to  $1.6 \times 10^{-4}$  from 55 GHz to 330 GHz and can be fitted with  $(4.9 \pm 0.1) \times 10^{-16}$  f, where f is the frequency in Hz. In fact, 4H SiC is the lowest-loss solid we have ever measured. The present approaches for permittivity characterization can be extended to other solids.

Index Terms — Fabry-Perot resonator, loss tangent, millimeter wave, permittivity, silicon carbide

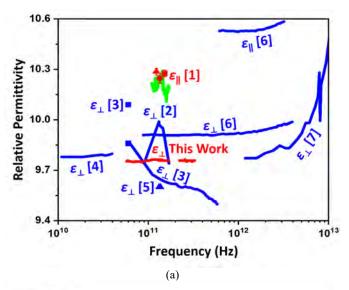
#### I. INTRODUCTION

Millimeter-wave (mmW) frequencies are increasingly important due to spectrum congestion at microwave frequencies and ever-increasing demand for high data rate, low latency, and precise beam forming/steering. For example, 5G wireless communications are pushing the operating frequency f to above 30 GHz and 6G communications will push it further above 100 GHz. Current-generation automobile radars are around 77 GHz and will be around 120 GHz for the next generation. Fiber-optic communications have long exceeded 100 Gbps with commensurate high-frequency electronics. Satellite communications, Internet of Space, security systems, home-care systems, and many other applications are using or planning to use mmW frequencies.

For accurate design and modeling of mmW devices, precise knowledge of material properties such as the electrical permittivity  $\varepsilon$  are needed. In general,  $\varepsilon$  is a complex number:

$$\varepsilon = \varepsilon_0 \left( \varepsilon'_R - j \varepsilon''_R \right) = \varepsilon_0 \varepsilon'_R \left( 1 - j \tan \delta \right), \tag{1}$$

where  $\varepsilon_0$  is the vacuum permittivity,  $\varepsilon'_R$  is often referred to as the dielectric constant Dk or K, and  $\tan \delta$  as loss tangent or dissipation factor Df. As an electromagnetic wave propagates in the material, its delay is characterized by  $\varepsilon'_R$  while its attenuation (loss) is characterized by  $\tan \delta$ .



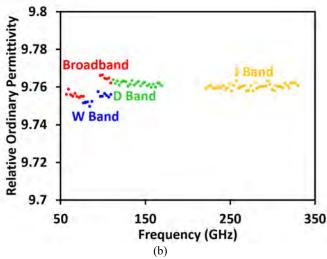
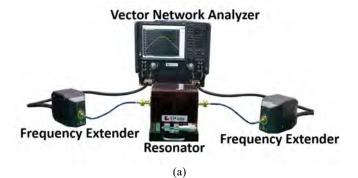


Fig. 1. (a) Relative ordinary permittivity  $\varepsilon_{\perp}$  vs. relative extraordinary permittivity  $\varepsilon_{\parallel}$  of 4H SiC [1] –[7]. (b) Details of  $\varepsilon_{\perp}$  from this work.

To date, little permittivity data exist at mmW frequencies and, when they exist, data obtained by different labs and techniques on the same material such as 4H SiC can differ by as much as 10% (Fig. 1) [1]–[7]. Moreover, lacking more precise measurement,  $\tan \delta$  is often reported as 0.01. The lack of data, consistency, or precision can be attributed to optical or quasi-optical techniques used in most permittivity measurements [8].



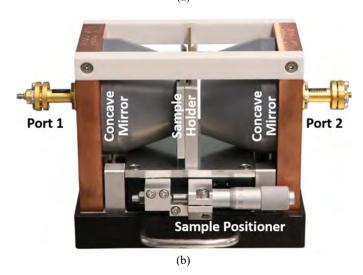


Fig. 2. (a) Permittivity measurement setup. (b) Details of a Fabry-Perot open resonator with ultra-low loss.

Hexagonal semiconductors such as 4H SiC have important power applications at radio frequencies. They are characterized by both ordinary permittivity  $\varepsilon_{\perp}$  and extraordinary permittivity  $\varepsilon_{\parallel}$ , parallel and perpendicular to the densest packed c plane. Recently, using transverse-electric (TE) waves in substrate-integrated waveguides, we reported  $\varepsilon_{\parallel}$  of 4H SiC across the D band (110–170 GHz) [Fig. 1(a)] [1]. In this paper, we report  $\varepsilon_{\perp}$  of the same material using transverse-electromagnetic (TEM) waves in Fabry-Perot resonators to cover most mmW frequencies.

## II. MEASUREMENTS

## A. Measurement Setup

Fig. 2 shows the measurement setup consisting of a vector network analyzer (Keysight N5290A PNA 10 MHz–110 GHz) in conjunction with a Fabry-Perot resonator (EM labs) and two frequency extenders (Virginia Diodes) to cover different mmW bands. Four different sets of resonators and frequency extenders are used: broadband (25–110 GHz), W band (75–110 GHz), D band, and J band (220–330 GHz). The broadband and W-band sets, with their overlapping frequencies, are used to check the consistency of the measurements.

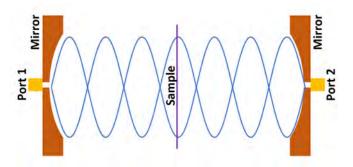


Fig. 3. Schematics of a 12-cm-long Fabry-Perot resonator showing a standing wave with 7 anti-nodes.

Fig. 3 shows the schematics of a resonator inside the air cavity bound by two concave mirrors with a radius of curvature R = 9.6 cm. To excite TEM waves and detect resonances, at the center of each mirror is a loop antenna for broadband or a coupling hole for W, D and J bands. With the distance between the mirrors D = 12 cm and the vacuum speed of light  $c = 3 \times 10^{10}$  cm/s, resonances occur at

$$f_N = Nf_0 + (1/\pi)\arccos(D/R - 1), \tag{2}$$

where  $N = 1, 2, 3 \dots, f_0 = c/2D = 1.25$  GHz, and the last term of (2) is approximately 1.45 GHz. The quality factor of the resonances  $Q_N = 1-3 \times 10^5$  for all mmW frequencies, allowing  $\tan \delta$  to be measured down to the  $10^{-5}$  range. For measurements of 55–330 GHz, standing waves are formed with 45 to 263 antinodes, where the electric field is at the maximum. Only  $f_N$  of odd N are used to ensure the sample, positioned at the center of the cavity, is subject to the strongest electric field.

## B. Measurement Procedure

From the measured magnitude of the transmission coefficient  $S_{21}$  across a resonator,  $f_N$  and  $Q_N$  are extracted with  $Q_N = f_N/BW$ , where BW is the 3-dB bandwidth of the resonance (Fig. 4). Usually,  $Q_N$  is so high that it is not affected by the raw frequency response of the PNA without a 2-port calibration. The bandwidth of the intermediate frequency and the number of averaging used by the PNA are optimized to efficiently achieve sufficient signal-to-noise ratio. When a sample is inserted in the middle of the air cavity of the resonator,  $f_N$  and  $Q_N$  shift lower to  $f_N'$  and  $Q_N'$ . Using the micrometer of the sample positioner shown in Fig. 2(b), the sample position is finetuned to maximize the shift. All measurements are performed under  $21 \pm 0.5$  °C and  $27 \pm 1\%$  relative humidity.

The present sample is from the same batch as in [1]. It is a high-purity, semi-insulating ( $\geq 10^7~\Omega \cdot \mathrm{cm}$ ) 4H SiC 100-cm-diameter wafer supplied by PAM-Xiamen and thinned by Rokko Denshi to a thickness  $T=98~\mu\mathrm{m}$  with an average surface roughness of 2 nm. The surface is within 0.1° of the c plane. The micropipe density is less than 10 cm<sup>-2</sup>. The bow and warp are both less than 10  $\mu\mathrm{m}$ . The wafer is divided into four quarters: two of them are measured in this work, the remaining quarters are saved for other uses. The quarters are too small for

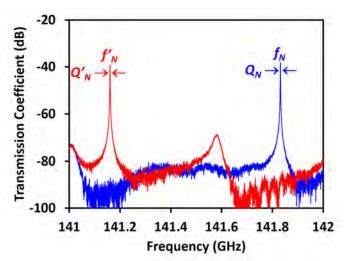


Fig. 4. Typical transmission coefficients measured on a D-band resonator before  $(f_N, Q_N)$  and after  $(f'_N, Q'_N)$  98- $\mu$ m-thick SiC is inserted in the middle.

measurement below 55 GHz. In the future, round wafers of the proper size can be used to extend the frequency range below 55 GHz.

The measured  $f'_N$  can be used to extract  $\varepsilon'_R$  by numerically solving [9]

$$\cot\left(nkT/2 - \Phi_T\right) = n \tan\left[k\left(D - T\right)/2 - \Phi_D\right],\tag{3}$$

where W,  $\phi_r$  and  $\phi_D$  are the waist size and phase shifts of the Gaussian beam and

$$n^2 = \varepsilon'_R, \tag{4}$$

$$k = 2\pi f'_{N}/c , \qquad (5)$$

$$\Phi_T = \arctan\left(T/nkW^2\right),\tag{6}$$

$$\Phi_{D} = \arctan\left[\left(D - T + T/n^{2}\right)/kW^{2}\right] - \arctan\left[T/n^{2}kW^{2}\right], \quad (7)$$

$$(kW^2)^2 = (D - T + T/n^2)(2R - D + T - T/n^2).$$
 (8)

The measured  $Q'_N$  can be used to extract  $\tan \delta$  by solving [9]

$$\tan \delta = \left(\frac{1}{Q_N} - \frac{1}{Q_N}\right) \left\{ \frac{D + (\Delta - 1)T}{\Delta T + (1/k)\sin[k(D - T) - 2\Phi_D]} \right\}, \quad (9)$$

where

$$\Delta = \frac{1}{\sin^2(2nkT - \Phi_T) + (1/n^2)\cos^2(2nkT - \Phi_T)}.$$
 (10)

## III. RESULTS AND DISCUSSION

The  $\varepsilon'_R$  measured in this work have been included in Fig. 1 as the relative ordinary permittivity because the measurement is based on the perpendicular incidence of a TEM wave on the c-

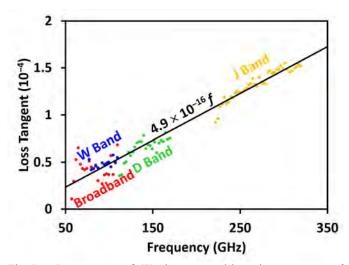


Fig. 5. Loss tangent of 4H SiC measured by using resonators of different frequency bands.

plane. It can be seen that  $\varepsilon'_R = 9.76 \pm 0.01$  with little dispersion between 55 and 330 GHz unlike previous reports [2], [3] at the same frequencies. The present  $\varepsilon'_R$  is significantly lower than our previously reported extraordinary permittivity of  $10.2 \pm 0.1$  [1], with the ratio between ordinary and extraordinary permittivities comparable to that reported at terahertz frequencies [6]. The standard deviation is 0.03%, which may be attributed to sample misplacement or human breath. By contrast, values of  $\varepsilon'_R$  measured on the same quarter wafer rotated around the c axis and on different quarters agree within 0.03%. This indicates that the anisotropy is significant for tilt with respect to the c axis, but insignificant for rotation around the c axis.

Fig. 5 shows the presently measured  $\tan \delta$ . It can be seen that, unlike  $\varepsilon'_R$ ,  $\tan \delta$  increases linearly with frequency so that, with the coefficient of determination  $R^2 > 0.96$ ,  $\tan \delta = (4.9 \pm 0.1) \times 10^{-16} f$ , where f is the frequency in Hz. The value of  $3 \times 10^{-5}$  at 60 GHz is the lowest we have measured on any solid. Despite increased uncertainty for such a low value, it is significantly lower than the values of our previous low-loss standards such as sapphire or alumina, which are all greater than  $> 1 \times 10^{-4}$  between 55 and 330 GHz.

#### IV. CONCLUSION

The ultra-low-loss Fabry-Perot resonators are proven to be capable of permittivity measurement on ultra-low-loss 4H SiC. The measured relative ordinary permittivity, at  $9.76 \pm 0.01$  exhibits little dispersion between 55 and 330 GHz. It is significantly lower than the extraordinary permittivity we previously reported, which shows that both ordinary and extraordinary permittivities are needed for accurate design and model of devices fabricated on 4H SiC. The measured loss tangent, as low as  $3 \times 10^{-5}$ , is the lowest of all solids we have measured. It increases linearly with frequency and can be fitted with  $(4.9 \pm 0.1) \times 10^{-16} f$ . The present techniques advance the state of the art of permittivity measurements and can be used on other solids.

#### ACKNOWLEDGEMENT

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