Lepton flavor violation by two units

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Charged lepton flavor violation arises in the Standard Model Effective Field Theory at mass dimension six. The operators that induce neutrinoless muon and tauon decays are among the best constrained and are sensitive to new-physics scales up to 10^7 GeV. An entirely different class of lepton-flavor-violating operators violates lepton flavors by two units rather than one and does not lead to such clean signatures. Even the well-known case of muonium–anti-muonium conversion that falls into this category is only sensitive to two out of the three $\Delta L_\mu = -\Delta L_e = 2$ dimension-six operators. We derive constraints on many of these operators from lepton flavor universality and show how to make further progress with future searches at Belle II and future experiments such as Z factories or muon colliders.

I. INTRODUCTION

The Standard Model (SM) has accidental symmetries that lead to the conservation of electron, muon, and tauon number [1, 2]. Neutrino oscillations are proof that these symmetries are ultimately violated in nature, but might be unobservably suppressed by the tiny m_ν^2 in the charged-lepton sector in the worst-case scenario [3]. Luckily, many SM extensions violate these flavor symmetries and can lead to testable flavor-violating processes, unsuppressed by the neutrino mass [1, 2].

To stay model agnostic, we can resort to the Standard Model Effective Field Theory (SMEFT, see Ref. [4] for a recent review), which extends the SM by higher-dimensional effective operators suppressed by powers of some high scale Λ [5, 6]. Integrating out heavy new particles produces exactly these kind of operators. Ordering the higher-dimensional operators by their mass dimension (i.e. powers of $1/\Lambda$), the leading one is Weinberg's $LLHH/\Lambda$ that generates Majorana neutrino masses [5], solving one of the SM's biggest problems. At $1/\Lambda^2$, or mass dimension $d = 6$, there are thousands of operators [7, 8], most of which just lead to small corrections to processes that are already allowed in the SM. But some of them violate the SM's accidental symmetries and thus lead to completely different processes that in principle have zero background.

Dimension-six operators with $\Delta L_\alpha = -\Delta L_\beta = 1 - \Delta L_\gamma = 1$, where α, β, and γ are distinct lepton flavors, can be probed in decays of an α or β lepton, leading to fully visible neutrinoless two-body signatures such as $\mu \to e\gamma$ or $\tau \to \mu\pi^0$. These are the most studied lepton-flavor-violating operators/signatures, both theoretically and experimentally, probing Λ scales up to 10^7 GeV in the muon sector and 10^4 GeV in the tauon sector [1, 2].

Alas, there are 21 $d = 6$ operators that violate lepton flavor by *two* units rather than one, and thus might not give rise to neutrinoless decays. For example, operators of the form $\mu\mu\bar{e}\bar{e}$ violate $\Delta L_\mu = -\Delta L_e = 2$ and do not lead to on-shell muon decays. In this particular example, muonium–anti-muonium conversion provides a good alternative signature for two out of three $\mu\mu\bar{e}\bar{e}$ operators. However, once we have a look at the tauon sector, e.g. $\tau\tau\bar{e}\bar{e}$, even the leptonium option is removed and the operators are seemingly unconstrained despite violating lepton flavor and being of low mass dimension.

Here, we will investigate such $\Delta L_\alpha = 2$ operators and identify possible ways to constrain them. Weinberg's $d = 5$ operator already contains $\Delta L_\alpha = 2$ pieces, but since they are suppressed by neutrino masses these operators are rendered unobservable, with the possible exception of neutrinoless double beta decay [9]. At $d = 6$, all $\Delta L_\alpha = 2$ operators are part of the four-lepton operators [8]

$$\mathcal{L} \supset \sum_{a,b,c,d=e,\mu,\tau} \left[y^{LL}_{abcd} \bar{L}_a \gamma^\alpha L_b \, \bar{L}_c \gamma_\alpha L_d + y^{LR}_{abcd} \bar{L}_a \gamma^\alpha L_b \, \bar{\ell}_c \gamma_\alpha \ell_d + y^{RR}_{abcd} \bar{\ell}_a \gamma^\alpha \ell_b \, \bar{\ell}_c \gamma_\alpha \ell_d \right] + \text{h.c.}\,, \tag{1}$$

where L_a is the left-handed lepton doublet of flavor a, ℓ_a the right-handed charged lepton of flavor a, and the $y = \mathcal{O}(\Lambda^{-2})$ are the Wilson coefficients of mass dimension -2. UV-complete realizations of these operators involve neutral or doubly-charged bosons and can for example be found in Refs. [10–13].

We stress that the $\Delta L_\alpha = 2$ operators under investigation here are fundamentally distinct from the more familiar $\Delta L_\alpha = 1$ operators; since they carry different lepton numbers it is easily possible to impose a symmetry in the charged-lepton sector that would forbid $\Delta L_\alpha = 1$ but allow for $\Delta L_\alpha = 2$. Examples are $U(1)$ flavor symmetries or their $\mathbb{Z}_N$ subgroups [3], and lepton flavor triality $\mathbb{Z}_3$ [14–16], which arises in many neutrino-mass models based on discrete symmetries such as A_4 [17–19] and only allows for operators of the form $\bar{\tau}\bar{\mu}ee$, $\bar{\tau}\bar{e}\mu\mu$, and $\bar{\tau}\bar{\tau}e\mu$. Renormalization-group running can turn $\Delta L_\alpha = 1$ operators into $\Delta L_\alpha = 2$, but not vice versa, rendering dedicated searches for $\Delta L_\alpha = 2$ absolutely necessary to cover these blind spots.

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II. $\bar{\mu}\bar{\mu}ee$

Let us first focus on the well-known $d = 6$ operators with $\Delta L_\mu = -\Delta L_e = 2$:

$$\begin{aligned}\mathcal{L} \supset\, & y^{LL}_{\mu e\mu e} \bar{L}_\mu \gamma^\alpha L_e\, \bar{L}_\mu \gamma_\alpha L_e + y^{LR}_{\mu e\mu e} \bar{L}_\mu \gamma^\alpha L_e\, \bar{\ell}_\mu \gamma_\alpha \ell_e \\ & + y^{RR}_{\mu e\mu e} \bar{\ell}_\mu \gamma^\alpha \ell_e\, \bar{\ell}_\mu \gamma_\alpha \ell_e + \text{h.c.}\end{aligned} \tag{2}$$

All three operators will contribute to muonium–antimuonium conversion [12, 24–27]. Using the experimental setup of the PSI experiment that provides the strongest limit to date [20], the conversion probability P takes the approximate form [12]

$$\begin{aligned}P \simeq\, & \frac{7.58 \times 10^{-7}}{G_F^2} |y^{LL}_{\mu e\mu e} + y^{RR}_{\mu e\mu e} - 1.68 y^{LR}_{\mu e\mu e}|^2 \\ & + \frac{4.27 \times 10^{-7}}{G_F^2} |y^{LL}_{\mu e\mu e} + y^{RR}_{\mu e\mu e} + 0.68 y^{LR}_{\mu e\mu e}|^2 .\end{aligned} \tag{3}$$

The PSI limit $P < 8.3 \times 10^{-11}$ [20] at 90% C.L. then puts strong upper limits of order $(3\,\text{TeV})^{-2}$ on the Wilson coefficients $|y^{LL}_{\mu e\mu e} + y^{RR}_{\mu e\mu e}|$ and $|y^{LR}_{\mu e\mu e}|$, but is insensitive to the linear combination $y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}$, which corresponds to the vector–axial-vector operator $\bar{\mu}\gamma_\alpha e\, \bar{\mu}\gamma^\alpha \gamma_5 e$. Keeping the other Wilson coefficients free, we obtain the limits $|y^{LL}_{\mu e\mu e} + y^{RR}_{\mu e\mu e}| < (2.9\,\text{TeV})^{-2}$ and $|y^{LR}_{\mu e\mu e}| < (3.4\,\text{TeV})^{-2}$. We can put stronger limits, shown in Tab. I, by setting one of the linear combinations to zero, i.e. forbidding interference between the contributions. Ref. [24] has shown that muonium conversion is actually also affected by $y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}$ through the muonium widths difference, but since these effects are further suppressed by the Fermi constant G_F, the resulting limits are only of order $(1.2\,\text{GeV})^{-2}$, probing scales far *below* the electroweak scale and thus not particularly relevant for the SMEFT. Future experiments such as the Muonium-to-Antimuonium Conversion Experiment (MACE) at the China Spallation Neutron Source (CSNS) [28] and a new setup at the Japan Proton Accelerator Research Complex (J-PARC) [29] are expected to improve the old PSI bounds by orders of magnitude.

Note that the three operators in Eq. (2) carry the same quantum numbers and thus mix via loops or renormalization group equations [30]. In the SMEFT energy region above the electroweak scale, this requires insertions of the lepton Yukawa couplings $y_\ell \equiv m_\ell/174\,\text{GeV}$, see Fig. 1. For example, the operator with coefficient $y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}$ then generates a $y^{LR}_{\mu e\mu e}$ operator of magnitude

$$y^{LR}_{\mu e\mu e} \simeq \frac{y_e y_\mu}{16\pi^2} \left(y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e} \right) . \tag{4}$$

The tiny prefactor $y_e y_\mu / 16\pi^2 \simeq 10^{-11}$ renders these effects small and gives irrelevant limits on $y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}$.

Better limits on the linear combination $y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}$ can be obtained by noticing that the underlying operator contains both neutrinos and charged leptons and thus leads

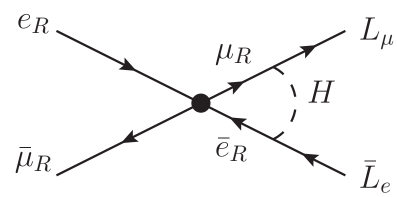

FIG. 1: Example Feynman diagram that shows the conversion of an $\bar{\ell}_\mu \gamma^\alpha \ell_e\, \bar{\ell}_\mu \gamma_\alpha \ell_e$ operator (filled circle) to a $\bar{L}_\mu \gamma^\alpha L_e\, \bar{\ell}_\mu \gamma_\alpha \ell_e$ operator at loop level using the SM Higgs interactions.

to the muon decay $\mu^- \to e^- \bar{\nu}_\mu \nu_e$. Since we are looking at $\Delta L_\mu = -\Delta L_e = 2$ processes, this decay does not interfere with the $\Delta L_\mu = \Delta L_e = 0$ SM decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$. We find that our operators from Eq. (2) generate exactly the same electron energy spectrum as the SM decay, so the Michel spectrum remains unperturbed; only the overall muon lifetime or decay rate is affected:

$$\Gamma_\mu = \Gamma_\mu^{\text{SM}} \left(1 + \frac{|2 y^{LL}_{\mu e\mu e}|^2 + |y^{LR}_{\mu e\mu e}|^2}{(2\sqrt{2} G_F)^2} \right) , \tag{5}$$

even including radiative QED corrections. Since $|y^{LR}_{\mu e\mu e}|$ is already constrained to be tiny from the muonium limits, we can safely neglect it here. To obtain limits on the Wilson coefficients, we employ lepton-flavor-universality tests [31],[1] i.e. we calculate $\Gamma_\mu / \Gamma_{\tau\to e\nu\nu}$,

$$|2 y^{LL}_{\mu e\mu e}|^2 = 8 G_F^2 \left(\frac{\Gamma^{\text{exp}}_{\mu\to e\nu\nu} m_\tau^5 f\left[\frac{m_e^2}{m_\tau^2}\right] R_W^{e\tau} R_\gamma^\tau}{\Gamma^{\text{exp}}_{\tau\to e\nu\nu} m_\mu^5 f\left[\frac{m_e^2}{m_\mu^2}\right] R_W^{e\mu} R_\gamma^\mu} - 1 \right) , \tag{6}$$

using the definitions and experimental values from Ref. [21], which gives

$$|2 y^{LL}_{\mu e\mu e}|^2 = (-2.42 \pm 3.10)\,\text{TeV}^{-4}. \tag{7}$$

Using instead the ratio $\Gamma_\mu / \Gamma^{\text{SM}}(\tau \to \mu\nu\nu)$ puts a better limit,

$$|2 y^{LL}_{\mu e\mu e}|^2 = (-6.10 \pm 3.13)\,\text{TeV}^{-4} , \tag{8}$$

because the two-decades old measured $\tau \to \mu\nu\nu$ rate shows a 2σ increase with respect to the SM. This is likely a statistical fluctuation or systematic effect since a recent preliminary measurement of $\Gamma(\tau \to \mu\nu\nu)/\Gamma(\tau \to e\nu\nu)$ at Belle II

[1] Alternatively, one could track the impact of the so-modified Fermi constant on electroweak precision data [16], which is however currently difficult given the recent anomalous W-mass measurement by CDF-II [32].

Wilson coefficient	Upper limit	Process	Violated quantum numbers
$\lvert y^{LL}_{\mu e\mu e} + y^{RR}_{\mu e\mu e}\rvert$	$(3.2\,\text{TeV})^{-2}$ [90% C.L.]	Mu-to-$\overline{\text{Mu}}$ [20]	$\Delta L_\mu = -\Delta L_e = 2$
$\lvert y^{LR}_{\mu e\mu e}\rvert$	$(3.8\,\text{TeV})^{-2}$ [90% C.L.]	Mu-to-$\overline{\text{Mu}}$ [20]	$\Delta L_\mu = -\Delta L_e = 2$
$\lvert y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}\rvert$	$(0.74\,\text{TeV})^{-2}$ [95% C.L.]	$\Gamma(\mu\to e\nu\bar\nu)/\Gamma(\tau\to\mu\nu\bar\nu)$ [21]	$\Delta L_\mu = -\Delta L_e = 2$
$\lvert 2y^{LL}_{\tau e\tau e}\rvert, \lvert y^{LR}_{\tau e\tau e}\rvert$	$(0.67\,\text{TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau\to e\nu\bar\nu)/\Gamma(\tau\to\mu\nu\bar\nu)$ [21] [22]	$\Delta L_\tau = -\Delta L_e = 2$
$\lvert y^{RR}_{\tau e\tau e}\rvert$	$(1.2\,\text{GeV})^{-2}$ [95% C.L.]	$Z\to\tau^\pm\tau^\pm e^\mp e^\mp$	$\Delta L_\tau = -\Delta L_e = 2$
$\lvert 2y^{LL}_{\tau\mu\tau\mu}\rvert, \lvert y^{LR}_{\tau\mu\tau\mu}\rvert$	$(0.63\,\text{TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau\to\mu\nu\bar\nu)/\Gamma(\tau\to e\nu\bar\nu)$ [21] [22]	$\Delta L_\tau = -\Delta L_\mu = 2$
$\lvert y^{RR}_{\tau\mu\tau\mu}\rvert$	$(1.2\,\text{GeV})^{-2}$ [95% C.L.]	$Z\to\tau^\pm\tau^\pm\mu^\mp\mu^\mp$	$\Delta L_\tau = -\Delta L_\mu = 2$
$\lvert y^{LL}_{e\tau\mu\tau}\rvert, \lvert y^{LR}_{\mu\tau e\tau}\rvert$	$(0.60\,\text{TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau\to e\nu\bar\nu)/\Gamma(\mu\to e\nu\bar\nu)$ [21]	$\Delta L_\tau = -2\Delta L_\mu = -2\Delta L_e = 2$
$\lvert y^{LR}_{e\tau\mu\tau}\rvert$	$(0.55\,\text{TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau\to e\nu\bar\nu)/\Gamma(\tau\to\mu\nu\bar\nu)$ [21] [22]	$\Delta L_\tau = -2\Delta L_\mu = -2\Delta L_e = 2$
$\lvert y^{RR}_{e\tau\mu\tau}\rvert$	$(1\,\text{GeV})^{-2}$ [95% C.L.]	$Z\to\tau^\pm\tau^\pm e^\mp\mu^\mp$	$\Delta L_\tau = -2\Delta L_\mu = -2\Delta L_e = 2$
$\lvert y^{LL}_{\mu e\tau e}\rvert, \lvert y^{LR}_{\mu e\tau e}\rvert, \lvert y^{LR}_{\tau e\mu e}\rvert, \lvert y^{RR}_{\mu e\tau e}\rvert$	$(10\,\text{TeV})^{-2}$ [90% C.L.]	$\tau\to\bar\mu ee$ [23]	$\Delta L_e = -2\Delta L_\tau = -2\Delta L_\mu = 2$
$\lvert y^{LL}_{e\mu\tau\mu}\rvert, \lvert y^{LR}_{e\mu\tau\mu}\rvert, \lvert y^{LR}_{\tau\mu e\mu}\rvert, \lvert y^{RR}_{e\mu\tau\mu}\rvert$	$(8.8\,\text{TeV})^{-2}$ [90% C.L.]	$\tau\to\bar e\mu\mu$ [23]	$\Delta L_\mu = -2\Delta L_\tau = -2\Delta L_e = 2$

TABLE I: Current limits on the magnitudes of the 21 $\Delta L_\alpha = 2$ dimension-six Wilson coefficients as well as the corresponding processes and the violated quantum numbers. Details are given in the text.

(combined with older measurements) is perfectly compatible with the SM [22]:

$$\frac{\Gamma(\tau\to\mu\nu\nu)}{\Gamma(\tau\to e\nu\nu)} = (1.0009\pm0.0027)\,\frac{\Gamma(\tau\to\mu\nu\nu)_{\text{SM}}}{\Gamma(\tau\to e\nu\nu)_{\text{SM}}}\,. \tag{9}$$

The difference in the limits from $\tau\to e\nu\nu$ and $\tau\to\mu\nu\nu$ will therefore probably decrease with dedicated measurements of the tauon branching ratios at Belle II. In the meantime, we construct one-sided 95% C.L. confidence intervals from the above to find a limit on $|2y^{LL}_{\mu e\mu e}|$ of $1.8/\text{TeV}^2$, which can also be written as

$$|(y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}) + (y^{LL}_{\mu e\mu e} + y^{RR}_{\mu e\mu e})| < 1.8/\text{TeV}^2. \tag{10}$$

As $y^{LL}_{\mu e\mu e} + y^{LL}_{\mu e\mu e}$ is already constrained to be ~ 20 times smaller than the first term we obtain an upper bound on the Wilson-coefficient linear combination that is unconstrained by the muonium of $|y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}| < 1.8/\text{TeV}^2$ (Tab. I). This is the strongest limit on the remaining Wilson coefficients, corresponding to new-physics scales of 0.74 TeV, above the electroweak scale and thus perfectly applicable to our SMEFT ansatz. Since the uncertainties on the flavor-universality ratios are dominated by the tau lifetime and branching ratios, these are the quantities that need to be measured more precisely in order to improve the bound on $|y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}|$; Belle II will likely achieve this in the near future [22].

Although not currently relevant, let us mention some other experiments and signatures that could play a future role in constraining the $y_{\mu e\mu e}$ Wilson coefficients. $\bar\mu\bar\mu ee$ operators involving neutrinos can induce mixed-flavor neutrino-trident effects, e.g. $\nu_\mu X\to\nu_e\mu^+e^-X$ or $\nu_e X\to\nu_\mu e^+\mu^-X$, which could be probed in future neutrino detectors such as DUNE [33–36]. Due to the non-interference with SM amplitudes the effects are expected to be small, roughly

$$\frac{\sigma(\nu_\mu X\to\nu_e\mu^+e^-X)}{\sigma(\nu_\mu X\to\nu_e\mu^-e^+X)_{\text{SM}}} \sim \frac{|2y^{LL}_{\mu e\mu e}|^2 + |y^{LR}_{\mu e\mu e}|^2}{(2\sqrt{2}G_F)^2}, \tag{11}$$

which is at most 10^{-2} given the above-derived constraints. Because of this, tridents will be at most useful at constraining the weakest linear combination, $|y^{LL}_{\mu e\mu e} - y^{RR}_{\mu e\mu e}|$. For a pure ν_μ beam, the μ^+e^- appearance would be an unambiguous sign of lepton flavor violation and thus a background-free signature, but realistic neutrino beams will have admixtures of ν_e and $\bar\nu_\mu$ that induce indistinguishable SM processes such as $\nu_e X\to\nu_\mu\mu^+e^-X$, rendering trident searches for $\Delta L_\mu = 2$ difficult. Still, tridents might eventually become a probe competitive with lepton flavor universality violation.

The $\bar\mu\bar\mu ee$ operators could also be probed at future lepton colliders via the background-free $e^-e^-\to\mu^-\mu^-$, $\mu^+e^-\to\mu^-e^+$, or $\mu^+\mu^+\to e^+e^+$. A setup for the latter two initial states was recently proposed as μTRISTAN [37], a high-energy lepton collider using the ultra-cold antimuon technology developed at J-PARC [38] that could run in the μ^+e^- mode with $\sqrt{s} = 346\,\text{GeV}$, and later in the $\mu^+\mu^+$ mode [39] with $\sqrt{s} = 2\,\text{TeV}$ or even higher. Judging by the analyses of similar four-lepton operators in Refs. [40, 41], we can expect μTRISTAN to probe all $|y_{\mu e\mu e}|$ down to $(\mathcal{O}(10)\,\text{TeV})^{-2}$, superseding all current limits. The high centre-of-mass energy might even allow for a direct production of the mediators underlying our $d = 6$ operators, see Ref. [42–45] for such studies. While we have focused on μTRISTAN here, other collider designs could also provide good reach for $\bar\mu\bar\mu ee$ operators as long as they collide $\mu^\pm\mu^\pm$, $e^\pm e^\pm$, or $\mu^\pm e^\mp$. μ^-–e^- scattering, e.g. at MuonE [46], has much weaker sensitivity to our $\Delta L_\mu = -\Delta L_e = 2$ opera-

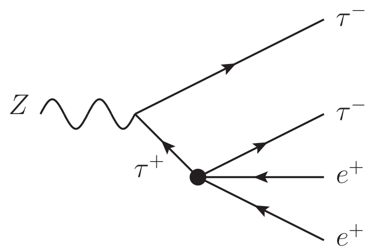

FIG. 2: Example Feynman diagram for $Z \to \tau^-\tau^-e^+e^+$ via the $y_{\tau e\tau e}$ operator (filled circle).

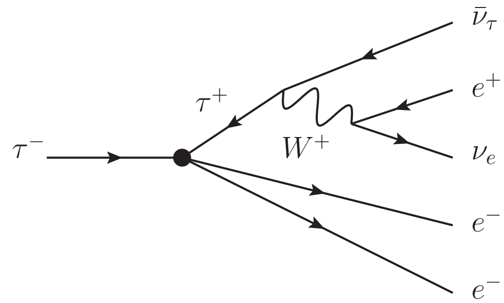

FIG. 3: Example Feynman diagram for $\tau^- \to e^-e^-e^+\nu_e\bar{\nu}_\tau$ via the $y_{\tau e\tau e}$ operator (filled circle).

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III. $\bar{\tau}\bar{\tau}ee$

Next we consider the $d = 6$ operators with $\Delta L_\tau = -\Delta L_e = 2$:

$$\begin{aligned}\mathcal{L} \supset\, & y^{LL}_{\tau e\tau e}\bar{L}_\tau\gamma^\alpha L_e\,\bar{L}_\tau\gamma_\alpha L_e + y^{LR}_{\tau e\tau e}\bar{L}_\tau\gamma^\alpha L_e\,\bar{\ell}_\tau\gamma_\alpha\ell_e \\ & + y^{RR}_{\tau e\tau e}\bar{\ell}_\tau\gamma^\alpha\ell_e\,\bar{\ell}_\tau\gamma_\alpha\ell_e + \text{h.c.}\,. \end{aligned} \tag{12}$$

These operators do not immediately lead to neutrinoless tau decays, nor do we have any tauonium-antitauonium conversion experiments at our disposal. For y^{LL} and y^{LR}, we can once again calculate the decay rates and electron spectra of $\tau^- \to e^-\bar{\nu}_\tau\nu_e$ and compare them to experimental data. Just like for the muon decay, these operators generate the same electron spectrum as SM tauon decays, so the partial width is simply rescaled compared to the SM:

$$\Gamma_{\tau\to e\nu\nu} = \Gamma^{\text{SM}}_{\tau\to e\nu\nu}\left(1 + \frac{|2y^{LL}_{\tau e\tau e}|^2 + |y^{LR}_{\tau e\tau e}|^2}{(2\sqrt{2}G_F)^2}\right). \tag{13}$$

Again, we use lepton-flavor-universality tests to obtain limits. We calculate $\Gamma_{\tau\to e\nu\nu}/\Gamma_{\mu\to e\nu\nu}$ and compare to the experimental values [21]:

$$|2y^{LL}_{\tau e\tau e}|^2 + |y^{LR}_{\tau e\tau e}|^2 = (2.43 \pm 3.11)\,\text{TeV}^{-4}\,. \tag{14}$$

A stronger limit can be achieved using preliminary Belle II data [22] (combined with older measurements) for $\Gamma_{\tau\to e\nu\nu}/\Gamma_{\tau\to\mu\nu\nu}$:

$$|2y^{LL}_{\tau e\tau e}|^2 + |y^{LR}_{\tau e\tau e}|^2 = (-1.05 \pm 2.90)\,\text{TeV}^{-4}\,. \tag{15}$$

Using one-sided confidence intervals, we obtain a 95% C.L. limit on $|2y^{LL}_{\tau e\tau e}|$ and $|y^{LR}_{\tau e\tau e}|$ of $2.2/\text{TeV}^2$ (Tab. I). These are viable SMEFT limits that can be improved with future Belle II data.

$y^{RR}_{\tau e\tau e}$ remains unconstrained here since it does not involve any neutrinos. Closing SM loops to generate neutrinos – equivalent to mixing the three operators, see Eq. (4) and Fig. 1 – requires chirality flips and is thus heavily suppressed. To avoid such suppressions we can consider $Z \to \tau\tau\bar{e}\bar{e}$ decays, see Fig. 2. Neglecting lepton masses, the branching ratio for this process is

$$\begin{aligned}\text{BR}(Z \to \tau^\pm\tau^\pm e^\mp e^\mp) &\simeq 1.4\,\frac{M_Z^5}{49152\pi^5}\frac{e^2 s_W^2}{c_W^2\Gamma_Z}|y^{RR}_{\tau e\tau e}|^2 \\ &\simeq 4.18\times 10^{-11}\left|\frac{y^{RR}_{\tau e\tau e}}{(0.1\,\text{TeV})^{-2}}\right|^2, \end{aligned} \tag{16}$$

where M_Z (Γ_Z) is the Z mass (width), and s_W (c_W) the sine (cosine) of the weak mixing angle. Currently, we do not have any experimental constraints on this decay channel; demanding the branching ratio to be less than one gives the weak bound $|y^{RR}_{\tau e\tau e}| < 15.5/\text{GeV}^2$. The total Z width agrees very well with the SM prediction [47], which can be translated into a 2σ upper bound of 2×10^{-3} on any non-SM Z branching ratio.[2] This improves the bound to $|y^{RR}_{\tau e\tau e}| < 0.7/\text{GeV}^2$ (Tab. I). A dedicated LHC search for this decay could realistically reach branching ratios of order 10^{-5}, corresponding to a limit $|y^{RR}_{\tau e\tau e}| < 4.89\times 10^4/\text{TeV}^2$. Z decays could conceivably be measured more extensively in the future at a so-called Z factory [48], producing trillions of Z bosons. Assuming an optimistic reach of 10^{-12} for the above branching ratios would probe $|y^{RR}_{\tau e\tau e}| < 15.5/\text{TeV}^2$, just barely above the electroweak scale. This is likely our best shot at providing EFT limits on $y^{RR}_{\tau e\tau e}$. Better limits on explicit UV completions of this operator are of course possible and probe complementary parameter space, see Ref. [11, 16, 49]. Notice that four-lepton Z decays are also useful for many other SMEFT coefficients [50].

Yet another probe of the RR coupling can be found in tauon decays involving an off-shell tauon in the final state, see Fig. 3. These are additionally suppressed by G_F and

[2] The recent W-mass measurement by CDF-II [32] is ignored here.

phase space, but can be competitive due to the large number of collected tauon decays compared to Z. For massless final states we find

$$\begin{aligned}\mathrm{BR}(\tau^-\to e^-e^-\ell^+\nu_\ell\bar\nu_\tau) &\simeq 1.1\,\frac{m_\tau^9 G_F^2}{4718592\pi^7\Gamma_\tau}|y^{RR}_{\tau e\tau e}|^2\\ &\simeq 8.2\times10^{-15}\left|\frac{y^{RR}_{\tau e\tau e}}{(0.1\,\mathrm{TeV})^{-2}}\right|^2,\end{aligned}\tag{17}$$

where ℓ is an electron or muon; $\tau^-\to e^-e^-\pi^+\bar\nu_\tau$ has a similar rate. Probing scales above the electroweak scale is clearly out of the question even at Belle II, but at least we can confirm the current Z-decay limit from above. For this we notice that CLEO has long ago observed the SM decay $\tau^-\to e^-e^-e^+\nu_\tau\bar\nu_e$ [51] with branching ratio $(2.8\pm1.5)\times10^{-5}$ [51], compatible with the SM prediction 4.2×10^{-5} [51–53]. This yields a 95% C.L. limit of $4.4/\mathrm{GeV}^2$ on $|y^{RR}_{\tau e\tau e}|$, slightly worse than the Z-decay limit but with entirely different assumptions. Belle II can significantly improve this limit.

IV. $\bar\tau\bar\tau\mu\mu$

In this section we consider the $d=6$ operators with $\Delta L_\tau=-\Delta L_\mu=2$,

$$\begin{aligned}\mathcal{L}\supset\, & y^{LL}_{\tau\mu\tau\mu}\bar L_\tau\gamma^\alpha L_\mu\,\bar L_\tau\gamma_\alpha L_\mu + y^{LR}_{\tau\mu\tau\mu}\bar L_\tau\gamma^\alpha L_\mu\,\bar\ell_\tau\gamma_\alpha\ell_\mu\\ &+ y^{RR}_{\tau\mu\tau\mu}\bar\ell_\tau\gamma^\alpha\ell_\mu\,\bar\ell_\tau\gamma_\alpha\ell_\mu+\mathrm{h.c.}\,,\end{aligned}\tag{18}$$

which have a similar phenomenology as the $\bar\tau\bar\tau ee$ operators from the previous section. $y^{LL}_{\tau\mu\tau\mu}$ and $y^{LR}_{\tau\mu\tau\mu}$ give rise to $\tau^-\to\mu^-\nu_\mu\bar\nu_\tau$, once again just rescaling the SM decay rate. We compute $\Gamma_{\tau\to\mu\nu\nu}/\Gamma_{\mu\to e\nu\nu}$ and compare it to the experimental results, obtaining

$$|2y^{LL}_{\tau\mu\tau\mu}|^2+|y^{LR}_{\tau\mu\tau\mu}|^2=(6.14\pm3.17)\ \mathrm{TeV}^{-4}\,.\tag{19}$$

A better limit can be achieved using preliminary Belle II data (combined with older measurements) [22] for $\Gamma_{\tau\to\mu\nu\nu}/\Gamma_{\tau\to e\nu\nu}$:

$$|2y^{LL}_{\tau\mu\tau\mu}|^2+|y^{LR}_{\tau\mu\tau\mu}|^2=(1.05\pm2.91)\,\mathrm{TeV}^{-4}\,.\tag{20}$$

Using one-sided confidence intervals, one can get 95% C.L. limits on $|2y^{LL}_{\tau\mu\tau\mu}|$ and $|y^{LR}_{\tau\mu\tau\mu}|$ of $2.5/\mathrm{TeV}^2$ (Tab. I).

Just like in the previous section, the last Wilson coefficient $y^{RR}_{\tau\mu\tau\mu}$ can be constrained using $Z\to\tau\tau\bar\mu\bar\mu$ decays (Tab. I). Since we are considering tauons, muons, and electrons to be massless we obtain the same branching ratio as in Eq. (16) and the same current and future limits. The $\bar\tau\bar\tau\mu\mu$ operators could also be probed at μTRISTAN via $\mu^+\mu^+\to\tau^+\tau^+$, with sensitivity to all $|y_{\tau\mu\tau\mu}|$ down to $(\mathcal{O}(10)\,\mathrm{TeV})^{-2}$.

Five-body tauon decays $\tau^-\to\mu^-\mu^-\ell^+\nu_\ell\bar\nu_\tau$ can be obtained from Eq. (17) with $e\to\mu$, but in this case there is no experimental data to compare to.

V. $\bar\tau\bar\tau e\mu$

The last three sections deal with lepton-triality-allowed operators, first the ones with $\Delta L_\tau=-2\Delta L_\mu=-2\Delta L_e=2$:

$$\begin{aligned}\mathcal{L}\supset\, & y^{LL}_{e\tau\mu\tau}\bar L_e\gamma^\alpha L_\tau\,\bar L_\mu\gamma_\alpha L_\tau\\ &+y^{LR}_{e\tau\mu\tau}\bar L_e\gamma^\alpha L_\tau\,\bar\ell_\mu\gamma_\alpha\ell_\tau+y^{LR}_{\mu\tau e\tau}\bar L_\mu\gamma^\alpha L_\tau\,\bar\ell_e\gamma_\alpha\ell_\tau\\ &+y^{RR}_{e\tau\mu\tau}\bar\ell_e\gamma^\alpha\ell_\tau\,\bar\ell_\mu\gamma_\alpha\ell_\tau+\mathrm{h.c.}\end{aligned}\tag{21}$$

These terms lead to two different tau decay channels: $\tau^-\to e^-\nu_\mu\bar\nu_\tau$ from $y^{LL}_{e\tau\mu\tau}$ and $y^{LR}_{\mu\tau e\tau}$, and $\tau^-\to\mu^-\nu_e\bar\nu_\tau$ from $y^{LL}_{e\tau\mu\tau}$ and $y^{LR}_{e\tau\mu\tau}$:

$$\Gamma_{\tau\to e\nu\nu}=\Gamma^{\mathrm{SM}}_{\tau\to e\nu\nu}\left(1+\frac{|y^{LL}_{e\tau\mu\tau}|^2+|y^{LR}_{\mu\tau e\tau}|^2}{(2\sqrt2G_F)^2}\right),\tag{22}$$

$$\Gamma_{\tau\to\mu\nu\nu}=\Gamma^{\mathrm{SM}}_{\tau\to\mu\nu\nu}\left(1+\frac{|y^{LL}_{e\tau\mu\tau}|^2+|y^{LR}_{e\tau\mu\tau}|^2}{(2\sqrt2G_F)^2}\right).\tag{23}$$

We implement the by now familiar lepton-flavor-universality test and compare the above rates to $\Gamma(\mu\to e\nu\nu)$ to obtain:

$$|y^{LL}_{e\tau\mu\tau}|^2+|y^{LR}_{\mu\tau e\tau}|^2=(2.43\pm3.11)\mathrm{TeV}^{-4}\,,\tag{24}$$

$$|y^{LL}_{e\tau\mu\tau}|^2+|y^{LR}_{e\tau\mu\tau}|^2=(6.14\pm3.17)\mathrm{TeV}^{-4}\,.\tag{25}$$

Using one-sided confidence intervals, one can get 95% C.L. limits of $7.9/\mathrm{TeV}^4$ (Tab. I) and $11/\mathrm{TeV}^4$ on $|y^{LL}_{e\tau\mu\tau}|^2+|y^{LR}_{\mu\tau e\tau}|^2$ and $|y^{LL}_{e\tau\mu\tau}|^2+|y^{LR}_{e\tau\mu\tau}|^2$ respectively.

Finally, we compare Eq. (22) to Eq. (23):

$$\frac{\Gamma_{\tau\to e\nu\nu}}{\Gamma_{\tau\to\mu\nu\nu}}\simeq\frac{\Gamma^{\mathrm{SM}}_{\tau\to e\nu\nu}}{\Gamma^{\mathrm{SM}}_{\tau\to\mu\nu\nu}}\left(1+\frac{|y^{LR}_{\mu\tau e\tau}|^2-|y^{LR}_{e\tau\mu\tau}|^2}{(2\sqrt2G_F)^2}\right).\tag{26}$$

By comparing to the experimental data from Eq. (9) we get a limit on the following linear combination:

$$|y^{LR}_{\mu\tau e\tau}|^2-|y^{LR}_{e\tau\mu\tau}|^2=(-1.05\pm2.90)\,\mathrm{TeV}^{-4}.\tag{27}$$

This allows us to improve the limit on $|y^{LR}_{e\tau\mu\tau}|$. Combining equations (24) and (27) we have:

$$\begin{aligned}|y^{LR}_{e\tau\mu\tau}|^2&=|y^{LR}_{\mu\tau e\tau}|^2-\left(|y^{LR}_{\mu\tau e\tau}|^2-|y^{LR}_{e\tau\mu\tau}|^2\right)\\ &\le(3.48\pm4.25)\mathrm{TeV}^{-4}\,.\end{aligned}\tag{28}$$

Again, utilizing one-sided confidence intervals we recover a slightly improved limit on $|y^{LR}_{e\tau\mu\tau}|$ of $3.3/\mathrm{TeV}^2$ (Tab. I).

Three of the four Wilson coefficients are thus constrained from universality ratios. Similarly to sections III and IV one can obtain limits on $|y^{RR}_{e\tau\mu\tau}|$ from Z decays. The prefactor of the branching ratio is a factor of 2 smaller here since two final state particles are not the same anymore:

$$\mathrm{BR}(Z\to\tau^\pm\tau^\pm e^\mp\mu^\mp)\simeq2.09\times10^{-11}\left|\frac{y^{RR}_{e\tau\mu\tau}}{(0.1\,\mathrm{TeV})^{-2}}\right|^2.\tag{29}$$

This change makes the limit on $|y^{RR}_{e\tau\mu\tau}|$ a factor of $\sqrt{2}$ weaker compared to $|y^{RR}_{\tau e\tau e}|$ and $|y^{RR}_{\tau\mu\tau\mu}|$ (Tab. I).

Five-body tauon decays $\tau^- \to e^-\mu^-\ell^+\nu_\ell\bar{\nu}_\tau$ also have a factor-2 smaller branching ratio than Eq. (17):

$$\mathrm{BR}(\tau^- \to e^-\mu^-\ell^+\nu_\ell\bar{\nu}_\tau) \simeq 4.1\times 10^{-15}\left|\frac{y^{RR}_{e\tau\mu\tau}}{(0.1\,\mathrm{TeV})^{-2}}\right|^2 . \tag{30}$$

CLEO provides a 90% C.L. limit on the SM-allowed decay $\mathrm{BR}(\tau^- \to \mu^-e^-e^+\nu_\tau\bar{\nu}_\mu) < 3.2\times 10^{-5}$ [51] which translates to a bound on $|y^{RR}_{e\tau\mu\tau}| < 5.4/\mathrm{GeV}^2$.

VI. $\bar{\tau}\bar{\mu}ee$

Now we investigate $\Delta L_e = -2\Delta L_\tau = -2\Delta L_\mu = 2$:

$$\begin{aligned}\mathcal{L} \supset\, & y^{LL}_{\mu e\tau e}\bar{L}_\mu\gamma^\alpha L_e\,\bar{L}_\tau\gamma_\alpha L_e \\ & + y^{LR}_{\mu e\tau e}\bar{L}_\mu\gamma^\alpha L_e\,\bar{\ell}_\tau\gamma_\alpha\ell_e + y^{LR}_{\tau e\mu e}\bar{L}_\tau\gamma^\alpha L_e\,\bar{\ell}_\mu\gamma_\alpha\ell_e \\ & + y^{RR}_{\mu e\tau e}\bar{\ell}_\mu\gamma^\alpha\ell_e\,\bar{\ell}_\tau\gamma_\alpha\ell_e + \mathrm{h.c.}\,,\end{aligned} \tag{31}$$

all of which give rise to the clean lepton-flavor-violating decay $\tau^+ \to e^+e^+\mu^-$ with rate

$$\Gamma \simeq \frac{m_\tau^5\left(|y^{LL}_{\mu e\tau e}|^2 + |y^{LR}_{\mu e\tau e}|^2 + |y^{LR}_{\tau e\mu e}|^2 + |y^{RR}_{\tau e\mu e}|^2\right)}{1536\pi^3}\,, \tag{32}$$

assuming vanishing electron and muon mass and hence no interference terms. Dedicated searches for this decay mode at Belle [23] yield the strong limits $|y^{LL}_{\mu e\tau e}|,\dots,|y^{RR}_{\tau e\mu e}| < 0.0096/\mathrm{TeV}^2$ (Tab. I). Belle II is expected to reach $\mathrm{BR}(\tau^- \to \mu^+e^-e^-) < 2.3\times 10^{-10}$ with $50\,\mathrm{ab}^{-1}$ [54, 55], which can probe the Wilson coefficients down to $(29.0\,\mathrm{TeV})^{-2}$.

The lepton-triality-allowed operators $\bar{\tau}\bar{\mu}ee$ have also recently been investigated in Ref. [44], where it was shown that $\mu^+e^- \to e^+\tau^-$ at μTRISTAN would likely provide weaker constraints on $|y_{\mu e\tau e,\tau e\mu e}|$ than Belle II through $\tau^+ \to e^+e^+\mu^-$.

VII. $\bar{\mu}\bar{\mu}e\tau$

Finally, we look at terms with $\Delta L_\mu = -2\Delta L_e = 2\Delta L_\tau = 2$:

$$\begin{aligned}\mathcal{L} \supset\, & y^{LL}_{e\mu\tau\mu}\bar{L}_e\gamma^\alpha L_\mu\,\bar{L}_\tau\gamma_\alpha L_\mu \\ & + y^{LR}_{e\mu\tau\mu}\bar{L}_e\gamma^\alpha L_\mu\,\bar{\ell}_\tau\gamma_\alpha\ell_\mu + y^{LR}_{\tau\mu e\mu}\bar{L}_\tau\gamma^\alpha L_\mu\,\bar{\ell}_e\gamma_\alpha\ell_\mu \\ & + y^{RR}_{e\mu\tau\mu}\bar{\ell}_e\gamma^\alpha\ell_\mu\,\bar{\ell}_\tau\gamma_\alpha\ell_\mu + \mathrm{h.c.}\,,\end{aligned} \tag{33}$$

which induce $\tau^+ \to \mu^+\mu^+e^-$ with rate

$$\Gamma \simeq \frac{m_\tau^5\left(|y^{LL}_{e\mu\tau\mu}|^2 + |y^{LR}_{e\mu\tau\mu}|^2 + |y^{LR}_{\tau\mu e\mu}|^2 + |y^{RR}_{e\mu\tau\mu}|^2\right)}{1536\pi^3}\,. \tag{34}$$

Comparison with Belle data [23] puts the following constraints on all four Wilson coefficients $|y^{LL}_{e\mu\tau\mu}|,\dots,|y^{RR}_{e\mu\tau\mu}| < 0.013/\mathrm{TeV}^2$ (Tab. I). Belle II should reach $\mathrm{BR}(\tau^- \to \mu^-\mu^-e^+) < 2.6\times 10^{-10}$ [54, 55], which translates to $|y| < (27.9\,\mathrm{TeV})^{-2}$.

Just like in the previous section, these lepton-triality-allowed operators $\bar{\mu}\bar{\mu}e\tau$ have recently been investigated in Ref. [44]. Here, $\mu^+\mu^+ \to e^+\tau^+$ at μTRISTAN can be competitive with Belle II through $\tau^+ \to \mu^+\mu^+e^-$ and reach $|y| < (\mathcal{O}(10)\,\mathrm{TeV})^{-2}$.

VIII. CONCLUSIONS

Tests of the SM's predicted lepton flavor conservation are among the best probes of new physics, notably in neutrinoless decays of muons and tauons. Not all lepton flavor violation comes with such clean signatures though: effective operators with $\Delta L_\alpha = 2$ are much harder to probe, even though they already arise at mass dimension $d = 6$ in the SMEFT and, of course, violate lepton flavor. 8 of these 21 operators give rise to the clean neutrinoless decays $\tau^- \to e^-e^-\mu^+, \mu^-\mu^-e^+$, while two more induce muonium–antimuonium conversion. The remaining 11 operators are rarely discussed, presumably because they are harder to detect. We have shown that 8 of them can be tested through lepton-flavor-universality violations, i.e. by comparing leptonic decay rates involving neutrinos, see Tab. I. This leaves only 3 operators that are currently unconstrained, or at least with such weak constraints that the use of effective field theory is questionable; these require future colliders for unambiguous tests, either in the form of a Z factory or like-sign electron or muon colliders. Most of the other operators will be tested more thoroughly at Belle II via searches for $\tau^- \to e^-e^-\mu^+, \mu^-\mu^-e^+$ as well as improved measurements of $\tau \to e\nu\nu$ and $\tau \to \mu\nu\nu$ that feed into lepton-flavor-universality tests. Upcoming muonium experiments such as MACE take care of two $\bar{\mu}\bar{\mu}ee$ operators. The present study serves as a reminder that lepton flavor violation could still be hidden at low scales in comparably murky observables.

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