



A physics-informed impact model refined by multi-fidelity transfer learning

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ABSTRACT

Impact performance is a key consideration when designing objects to be encountered in everyday life. Unfortunately, how a structure absorbs energy during an impact event is difficult to predict using traditional methods, such as finite element analysis, because of the complex interactions during high strain-rate compression. Here, we employ a physics-based model to predict impact performance of structures using a single quasistatic experiment and refine that model using intermediate strain rate and impact testing to account for strain-rate dependent strengthening. This model is trained and evaluated using experiments on additively manufactured generalized cylindrical shells. Using transfer learning, the trained model can predict the performance of a new design using data from a single quasistatic test. To validate the transfer learning model, we extrapolate to new impactor masses, new designs, and a new material. The accuracy of this model allows researchers to quickly screen new designs or leverage pre-existing databases of quasistatic test data. Furthermore, when impact tests are necessary to validate design selection, fewer impact tests are necessary to identify optimal performance.

1. Introduction

Structures that absorb energy in impact events are critical for everyday life. They are used in bicycle helmets [1,2], the crumple zones of cars [3], packaging for shipping [4,5], and even ballistic and blast protection [6]. Despite these critical needs, development of protective impact structures is challenging because performance depends on the properties of the object to be protected, the energy of the impact, and the height and contact area of the protective structure, requiring thousands of time-consuming physical tests to optimize [10,11,7,8,9]. Further complicating the design process is that it is extremely challenging to obtain precise agreement between experiment and traditional physics-based approaches, namely finite element analysis. [12,13] Disagreement often arises due to the difficulty associated with capturing the complex interactions that occur at high strain rate, at high strain, and with self-self-contacts of the structures.

To overcome these challenges, many impact structures make use of simple bulk materials such as foams [7,14]. Such materials are simple to work with because their density can be tuned in a manner that

predictably changes their performance [15]. This scaling leads to the common use of empirically measured cushion curves, which compare the peak acceleration of an impactor versus the static pressure exerted by that impactor at rest. Such cushion curves are helpful in selecting the ideal foam height and density for a given application, but assume that the height can be changed without affecting performance, limiting their applicability when applied to non-homogenous materials. Despite the widespread use of cushion curves for foams, significant research focuses on developing materials and structures that outperform foams in weight, volume, and safety metrics.

The increasing reliability and decreasing cost of additive manufacturing (AM) has opened the opportunity to manufacture structures that are more complex than traditional foams. [16,17,5] One common approach is to use AM to produce uniform lattice structures, once again depending on density scaling or repeated unit cells to predict performance [18–21]. Some researchers have investigated how structure affects performance for designs with consistent density, such as functionally grading strut thickness along an axis [22,23], auxetic structures [24,25], or using tapered beams [26]. Our previous work showed that

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Gaussian process regression (GPR) models could be used to predict peak acceleration of impact tests for lattice structures where the mass of the lattice is shifted between the joints and different types of struts [27]. Two promising developments are the innovation of automated testing of bulk materials to optimize impact properties [28] and the use of autonomous experiments to optimize structure for quasistatic mechanics [29,30]. Despite their promise, optimizing novel structured materials for impact performance can quickly become impractical due to the immense number of possible design choices and the burdensome testing requirements, both in researcher time and in the number of repeat samples needed for optimization.

Many efforts have been made to decrease the burden of impact testing. Early attempts focused on predicting the amount of energy that foam samples could efficiently absorb [31,32]. To account for strain-rate effects, a common strategy is to treat the material as viscoelastic or scale quasistatic curves by multiplying them by empirical constants or dynamic factors that include both strain and strain rate [33-36,9]. Recently, more numerical models have appeared to model the dynamics of impact [8,37]. Despite these efforts, little work has been done to extend these predictive models beyond homogenous foams. Facile models that accurately predict impact performance remain an open challenge.

To speed up development of new impact structures, we develop a physics-based transfer learning model to predict the ideal impact velocity V_0^* for polymer structures. To explore this concept, we experimentally study generalized cylindrical shells (GCS), which have superlative energy absorbing capacity in quasistatic (QS) compression tests and are inherently easy to print using fused filament fabrication (FFF) [30]. The development of this model involves three key steps, (1) the extraction of key metrics from QS testing, (2) the use of intermediate strain rate (ISR) tests to determine strain-rate dependent strengthening, and (3) the refinement of the model using impact tests. After these initial training steps, the model is able to predict impact performance on previously unseen samples with a single QS test and even extrapolate to test data using different impactor mass, designs, and materials with an RMSE of 0.23 m/s.

2. Theory

An impact test consists of an impactor of mass m dropping on a test component and hitting it with initial impact velocity V_0 . The initial kinetic energy KE of the impact is given by,

$$KE = \frac{1}{2} m V_0^2. \quad (1)$$

During an impact experiment, the acceleration a of the impactor is tracked and used to assess the performance of the component. Typically, the impact performance of a component is defined by the maximum acceleration a_m observed during the test, which should be minimized to prevent damage. However, a_m is not always the clearest way to compare the performance of components as a_m generally increases monotonically with V_0 and m . For ease of comparison, impact performance is often described using the non-dimensional Janssen factor J , which is defined as $J = a_m/a_t$ where a_t is the theoretical minimum acceleration that could stop the impactor given by,

$$a_t = \frac{V_0^2}{2h}, \quad (2)$$

where h is the height of the test component. While J depends on the details of the impact test, namely V_0 and m , there exists a critical initial velocity V_0^* for which the minimum Janssen factor J^* is observed for a given m .

In contrast with impact tests in which the strain rate varies during the experiment, fixed-velocity tests feature a constant strain rate. Under these conditions, the useful non-dimensional metric to consider is the

energy absorbing efficiency K_s of a component, which describes how efficiently it absorbs energy below a force threshold F_t and is given by,

$$K_s(F_t) = \frac{\int_0^{D_t} F(D) dD}{F_t h}, \quad (3)$$

with the displacement threshold D_t taken as the largest D for which $F(D) \leq F_t$ for all $D \leq D_t$. Practically speaking, this equation quantifies the amount of energy absorbed before exceeding F_t and normalizes it by $F_t h$, which is the total amount of energy that could be absorbed if $F = F_t$ for $0 \leq D \leq h$. The maximum energy absorbing efficiency K_s^* occurs at the critical force threshold F_t^* . Maximizing K_s^* requires a long, flat plateau region, which is why most structures designed to absorb energy take advantage of productive buckling phenomenon. While there are similarities between K_s^* and J^* , it is worth emphasizing that K_s^* can be found using a single QS test while finding J^* requires empirically searching for the minimum value in V_0 at the relevant m .

3. Results and Discussion

In order to examine the behavior of a component when impacted under various conditions, a specific GCS design was selected and fabricated out of thermoplastic polyurethane (TPU, Ninjatek – Cheetah) using FFF (Fig. 1a) and subsequently tested in impact (Fig. 1b). Representative a -displacement D curves illustrate the outcome of testing this component with too little KE , an ideal amount of KE , and too much KE , all relative to the ideal conditions for this component (Fig. 1c). At low V_0 , a_m increases slowly with increasing V_0 due to strain-rate dependent material properties increasing the yielding force (Fig. 1d). However, once V_0 increases to the point where densification is reached, a_m increases at a much faster rate with increasing V_0 as the component is unable to absorb KE without densifying further. This transition point coincides with (V_0^*, J^*) , which is the condition under which the design is most efficient (Fig. 1e). Because of this, (V_0^*, J^*) is a highly useful way to quantify impact performance. However, determining this value required a large number of impact tests on independently prepared samples and is only valid for one value of m .

We hypothesize that fixed-velocity tests (Fig. 1f) could be used to predict V_0^* . While there are substantial differences between impact and fixed-velocity testing, the move to fixed-velocity would be a tremendous reduction in the experimental burden as fixed-velocity tests use relatively common universal testing machines (UTMs), are amenable to automation, and feature fewer variables than impact tests. The result of a fixed-velocity test is a plot of force F vs. D (Fig. 1g). One key question in exploring our hypothesis is determining a way to extract information from this QS curve that can be useful for predicting impact performance. In analogy with the non-dimensional J , we consider K_s (defined in Eq. (3)) as an important non-dimensional metric of a component's ability to absorb mechanical energy as previously explored by Gruenbaum, G. & Miltz [32]. Graphically, K_s can be calculated by dividing the amount of energy absorbed (light blue area in Fig. 1g) while $F \leq F_t$ by the maximum amount that could be absorbed at that F_t (light red area in Fig. 1g). While F_t is an independent parameter that is most often chosen with consideration of the operational conditions of the component, there often exists an optimal force threshold F_t^* where the component exhibits its maximum energy absorption efficiency K_s^* . Based on these metrics, a simple prediction of the ideal impact conditions can be found by equating impact KE with the amount of energy most efficiently absorbed during QS testing,

$$\frac{1}{2} m \tilde{V}_0^{*2} = F_t^* K_s^* h, \quad (4)$$

and then solving for predicted optimal impact velocity $\tilde{V}_0^*$

$$\tilde{V}_0^* = \sqrt{\frac{2F_t^* K_s^* h}{m}}. \quad (5)$$

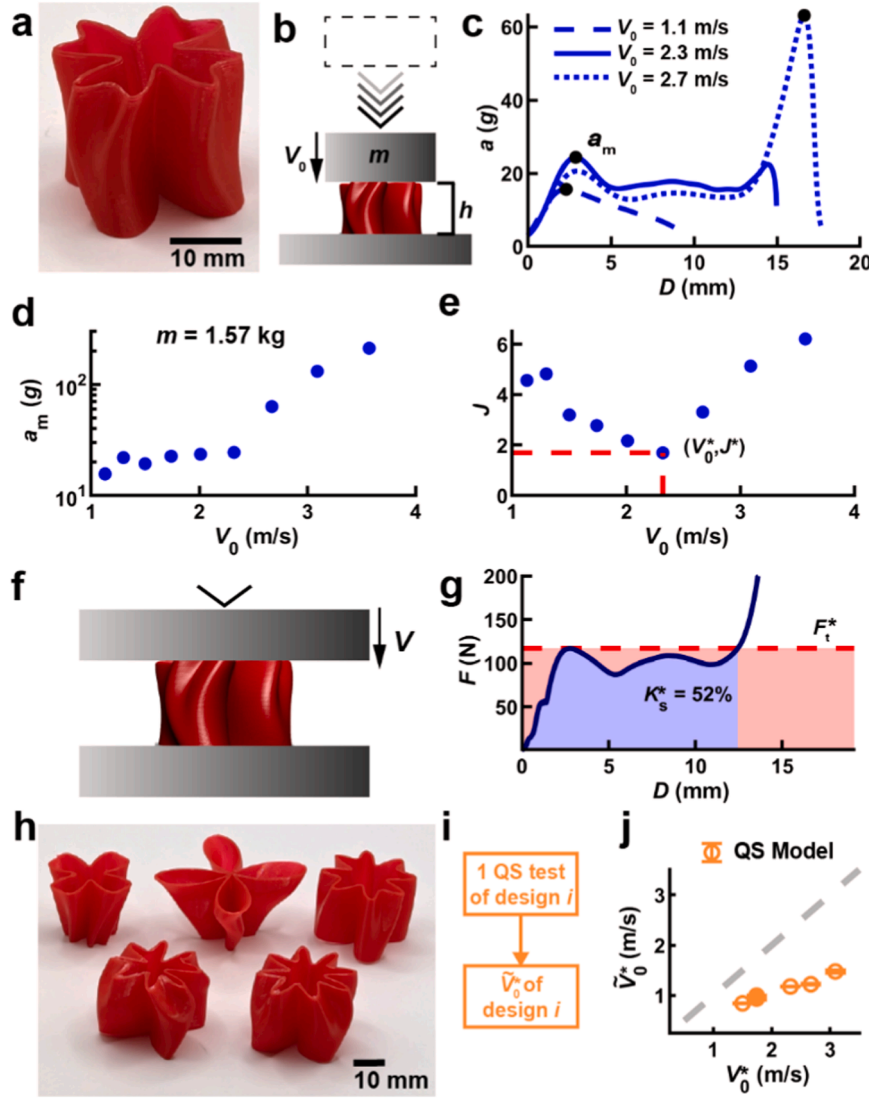


Fig. 1. Predicting impact performance from quasistatic tests. (a) Photograph showing a generalized cylindrical shell (GCS) component made from thermoplastic polyurethane (TPU). (b) Schematic of an impact test with impactor of mass m hitting the component of height h at an initial velocity V_0 . (c) Acceleration a vs. displacement D for impact tests of the same component at three different V_0 with $m = 1.57$ kg. The max acceleration a_m for each V_0 is denoted as a black dot. The units of a are the gravitational acceleration g . (d) The a_m at various V_0 with $m = 1.57$ kg. (e) Janssen factor J vs. V_0 . The test with the lowest J is designated as the point (V_0^*, J^*) . (f) Schematic of a fixed speed test in which a platen is lowered at constant velocity V . (g) Force F vs. D for quasistatic (QS) test ($V = 2$ mm/min). This data is used to compute the most efficient operating conditions in terms of the critical force threshold F_t^* and maximum mechanical energy absorbing efficiency K_s^* , which is defined by dividing the amount of energy absorbed (light blue area) by the maximum amount of energy that could be absorbed below F_t^* (light red area). (h) Photograph showing five additional GCS designs made from TPU. (i) QS model uses a single QS test to compute the predicted optimal impactor velocity $\tilde{V}_0^*$. (j) Parity plot showing $\tilde{V}_0^*$ vs. V_0^* for the six shown designs using the QS model for $m = 1.57$ kg. Error bars denote one standard deviation.

While the QS model is both simple and physics based, it neglects strain-rate dependent effects, which is expected to limit its predictive power.

To test the QS model, five additional designs were selected and fabricated from TPU (Fig. 1h). To apply the model, a single QS test is needed to determine F_t^* , K_s^* , and h for each design. With this data, $\tilde{V}_0^*$ for a given m can be predicted using Eq. (5) (Fig. 1i). For each of the six designs, components were fabricated and tested at 2 mm/min in triplicate (Fig. 1j). While the predictions preserve rank order, they consistently underestimate V_0^* because the QS model does not account for the strain-rate dependent strengthening. Despite this, because the prediction always underestimates V_0^* , it provides two key pieces of information. First, it provides a safe V_0 that will not reach densification. Second, if subsequent impact testing will be done to identify (V_0^*, J^*) more accurately, it provides guidance of the lower end of potential V_0^* values, potentially decreasing the number of experiments needed to identify the

point of maximum performance. Nevertheless, it is clear that more information is needed to further improve this model.

In order to improve predictions of V_0^* , it is necessary to model the strain-rate dependent effects on the components. Using our UTM, it is possible to test at V of up to 2000 mm/min (Fig. 2a). Although this is still more than an order of magnitude lower than V_0 during impact testing, it is several orders of magnitude higher than the 2 mm/min speed that was used in QS testing. Furthermore, when the F - D curves are compared for these ISR tests, strain-rate dependent strengthening is readily apparent (Fig. 2b). Specifically, the curves have similar shapes, but F increases with increasing V . Interestingly, higher V has a modest effect on K_s^* (Fig. 2c), which bears the advantage that QS tests can be used to estimate this value. In contrast, F_t^* monotonically increases with increasing V (Fig. 2d), making this parameter a clear quantification of strain-rate dependent strengthening. Empirically, we find that these experiments

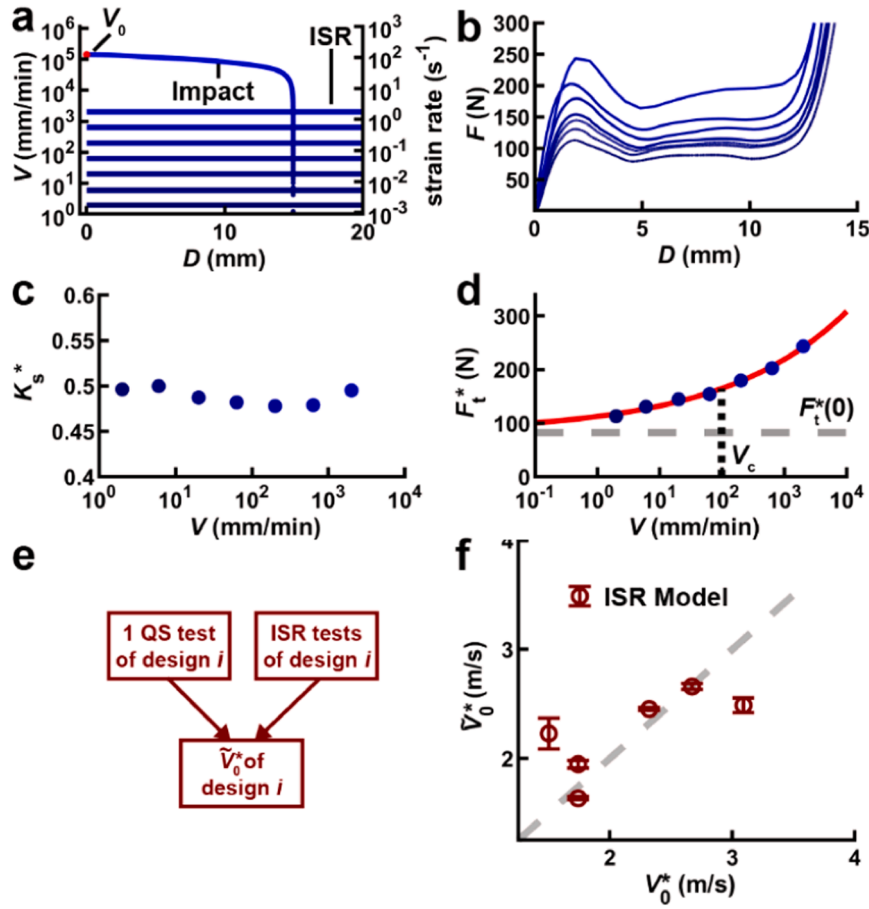


Fig. 2. Predicting impact performance from intermediate strain rate tests. (a) Velocity V vs. D during impact and intermediate strain rate (ISR) tests. While impact tests start at $V = V_0$ and decrease as energy is absorbed, ISR tests have a fixed V for the entire test. (b) F vs. D for ISR tests of copies of the component shown in Fig. 1a. (c) K_s^* vs. V for the ISR tests shown in b. (d) F_t^* vs. V for the ISR tests shown in b. The red line shows a fit to Eq. (6) with key fitting constants $F_t^*(0)$ and V_c marked by gray and black dashed lines, respectively. (e) ISR model uses both QS and ISR tests of the target design to predict $\tilde{V}_0^*$. (f) Parity plot showing $\tilde{V}_0^*$ vs. V_0^* for the six original designs using the ISR model for $m = 1.57$ kg. Error bars denote one standard deviation. In panels a, b, c, and d, shades of blue indicate V .

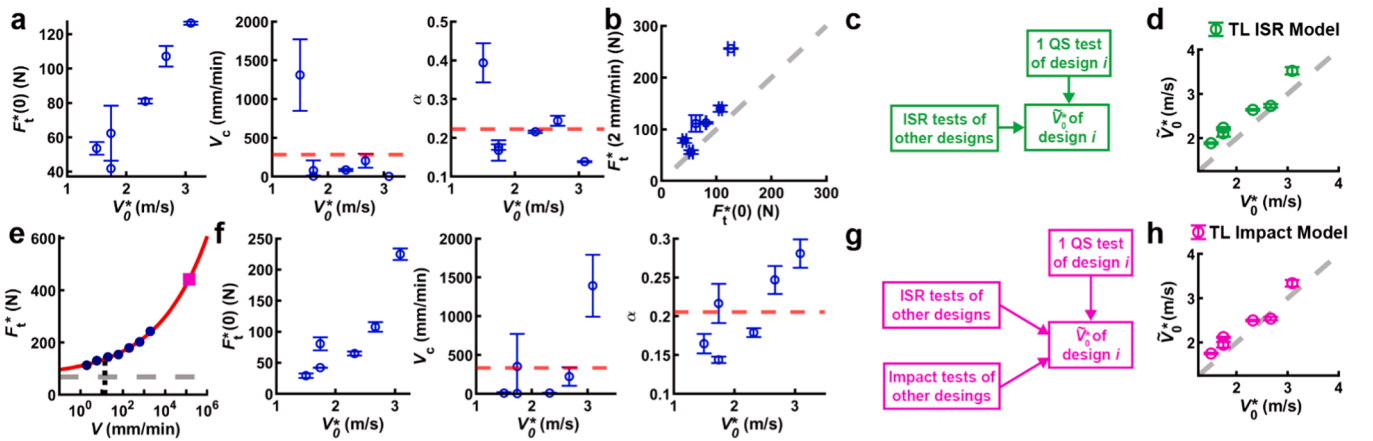


Fig. 3. Predicting impact performance using transfer learning. (a) Values from ISR fitting each of the six designs to Eq. (6) with the mean value shown as a dashed red line. (b) Plot of QS F_t^* (2 mm/min) vs. fitting constant $F_t^*(0)$. (c) Transfer learning (TL) ISR model that uses ISR tests of other designs to calculate V_c and α , allowing the prediction of $\tilde{V}_0^*$ with a single QS test for each target design. (d) Parity plot showing $\tilde{V}_0^*$ vs. V_0^* for the six designs using the TL ISR model for impactor mass $m = 1.57$ kg using leave-one-out cross validation (LOOCV) to calculate V_c and α for each prediction. (e) F_t^* vs. V for ISR tests with theoretical impact point (V_0^* , F_t^*) (pink square) calculated using Eq. (7). (f) Values from ISR and impact fitting each of the six designs to Eq. (6) with the mean value shown as dashed red line. (g) TL impact model uses ISR tests and impact F_t^* prediction from other designs to calculate parameters V_c and α , allowing the prediction of $\tilde{V}_0^*$ with just a single QS test for each new target design. (h) Parity plot of $\tilde{V}_0^*$ vs. V_0^* for the six original designs using the TL impact model for $m = 1.57$ kg using LOOCV. Error bars throughout represent one standard deviation.

can be fit using,

$$F_t^*(V) = F_t^*(0) \left(1 + \left(\frac{V}{V_c} \right)^\alpha \right), \quad (6)$$

with fitting constants $F_t^*(0)$, V_c , and α . These fitting constants have a physical basis: $F_t^*(0)$ represents the asymptotic limit of F_t^* for ISR testing when V approaches 0, V_c represents the characteristic velocity where $F_t^*(V)$ doubles, and α represents how quickly $F_t^*(V)$ increases as V increases. The ISR model is formed by combining Eq. (6) and Eq. (5) to obtain a transcendental equation that can be solved numerically to predict $\tilde{V}_0^*$ of a given design based upon QS and ISR tests for the design (Fig. 2e). The predictive performance of the ISR model is significantly improved, although the rank order is no longer correct (Fig. 2f). Despite this improvement, the ISR model only uses data from a given design and requires a large number of fixed velocity tests for each prediction.

Despite the improvements with the ISR model, taking a full sweep of ISR test data is much slower than taking a single QS test and high-speed tests have the potential to damage the load cells of UTMs. Therefore, we hypothesized that transfer learning could be employed to eliminate the need to take ISR tests for each new target design. By plotting the fitting constant values for $F_t^*(0)$, V_c , and α , some trends become clear (Fig. 3a). While $F_t^*(0)$ clearly varies for each design, both V_c and α have more closely grouped values for all tested designs, suggesting that their mean values could be used. Fortunately, $F_t^*(0)$ is correlated with $F_t^*(2 \text{ mm/min})$ determined by QS experiments of the target design (Fig. 3b). Therefore, a transfer learning model can be created that uses the mean value of V_c and α from other designs while establishing $F_t^*(0)$, K_s^* , and h from a single QS test (Fig. 3c) for each new target design. In this way, the time-consuming ISR tests can be done once for a representative set of designs to establish baseline values, and then subsequent designs can be predicted using a single QS test. Interestingly, when predicting impact performance using leave-one-out cross validation (LOOCV), the new TL ISR model performs significantly better than the ISR model trained on ISR data from each target point (Fig. 3d). This suggests that V_c and α are not design dependent, as averaging the values for several designs provides better predictive performance than using the value from the specific design.

Motivated by the success of incorporating ISR data from other designs, we hypothesized that using limited impact testing could further improve the prediction of $F_t^*(V)$. Unfortunately, impact tests do not produce an estimate of F_t^* that can be directly used to estimate strain-rate dependent strengthening at these higher strain rates. Despite this, the value of V_0^* determined by impact experiments for each design can be used to compute what the required $F_t^*(V)$ would need to have been to arrive at the correct prediction, which can be computed by rearranging Eq. (4) as,

$$F_t^*(V_0^*) = \frac{mV_0^{*2}}{2K_s^*h}. \quad (7)$$

This $(V_0^*, F_t^*(V_0^*))$ point can then be included in the $F_t^*(V)$ fit model to further improve its accuracy at high velocities (Fig. 3e). Fitting the ISR and impact data together to Eq. (6), we find that the additional point provided by impact testing barely changes $F_t^*(0)$, but slightly adjusts V_c and α (Fig. 3f). Importantly, the scaling parameter α decreases from 0.223 to 0.205, suggesting that the TL ISR model overestimates the strain-rate dependent strengthening at higher velocities. With this addition, the TL impact model can predict $\tilde{V}_0^*$ using a model initialized on a representative set of ISR and impact tests and a single QS test for each new design (Fig. 3g). The TL impact model shows slight improvements to predictive accuracy at the cost of including some impact data to train the initial model (Fig. 3h). The similarities between the results shown in Figs. 3d and 3h demonstrate that including the impact results merely refines the TL ISR model from what is possible when including ISR testing alone, which is further supported by the modest adjustments

to the fitting parameters V_c and α with the inclusion of the impact data.

To further explore the transferability of this model, we tested two new designs at $m = 3.1 \text{ kg}$, nearly twice the mass of the impactor used to generate all training data. The first design (Fig. 4a) had low stiffness and no clearly defined yield point (Fig. 4b). All models predict the very low $\tilde{V}_0^*$ for this soft design, consistent with the test data obtained from impact testing (Fig. 4c). In contrast, another design (Fig. 4d) was chosen that had a higher F_t^* than previous designs and a clear yielding region (Fig. 4e). Here, the models predict higher $\tilde{V}_0^*$ consistent with experimental results, with the QS model being the notable outlier because it fails to account for the strain-dependent strengthening of the material that is amplified by this stiff design (Fig. 4f).

As a final test of generalizability, we extended these models to a new material, a TPU filament mixed with a blowing agent (ColorFabb – VarioShore) such that when printed at 250°C , it forms a foam with a density half of the previously studied TPU. When testing solid cylinder samples fabricated with both the TPU and foaming TPU (Fig. 5a), the foaming TPU sample shows a significantly lower modulus and densifies at a much higher strain due to the added porosity from the blowing agent decreasing the relative density of the material at the microstructural scale. Furthermore, we switched to a more complex GCS design space (Fig. 5b), which adds a tapered region to provide an initial region with low stiffness. Some of the new foaming TPU designs had heights that included those much lower than the previously studied designs and some were up to twice the mass of any components in the training set (Fig. 5c). The new designs generally had much lower K_s^* due to the inefficiency of the tapered region (Fig. 5d), but they also had much higher F_t^* , which were designed to target higher V_0^* . All four models were applied to the 14 new foaming TPU designs, with both TL models' V_c and α parameter values being trained exclusively on data from the original six TPU designs. The TL impact model had an RMSE of 0.23 m/s, with the TL ISR model close behind and the ISR and QS models' errors significantly higher (Fig. 5e). The TL impact model trained exclusively on the original six TPU data shows excellent accuracy at predicting impact performance using only a single QS test for each new foaming TPU design (Fig. 5f), especially considering that the design space, h , component mass, and component material were all broadened from those contained in the training data. This suggests that the physics-informed transfer learning model may be able to extrapolate far beyond the limited training data used to train it.

4. Conclusion

This work reports a systematic exploration of how the impact performance of structures can be predicted using a set of models with varying complexity. In all cases, models are physics informed to meaningfully connect metrics from QS testing to impact performance. Our key finding is that the incorporation of related training data that includes testing in the ISR and impact regimes allows these models to powerfully extrapolate to new designs, materials, and impact conditions. From a design perspective, this work implies that once a representative set of data has been acquired, new designs can be rapidly evaluated based on a single QS test. Furthermore, the success of this model in extrapolating beyond the training materials suggests that models trained on large databases of polymeric structures may be useful with new materials and architectures. To that end, the TL impact model based on all data presented in this study is given by $V_c = 331 \text{ mm/min}$ and $\alpha = 0.205$, which, when combined with a single QS test of a new sample, can be used to estimate $\tilde{V}_0^*$.

The ability to predict performance with a single QS test allows researchers to screen candidate structures for impact performance quickly and easily. Additionally, it allows researchers to leverage databases of QS tests already published or that can be generated using high throughput experimentation or self-driving labs. Finally, researchers can take advantage of previously studied design motifs, such as how the

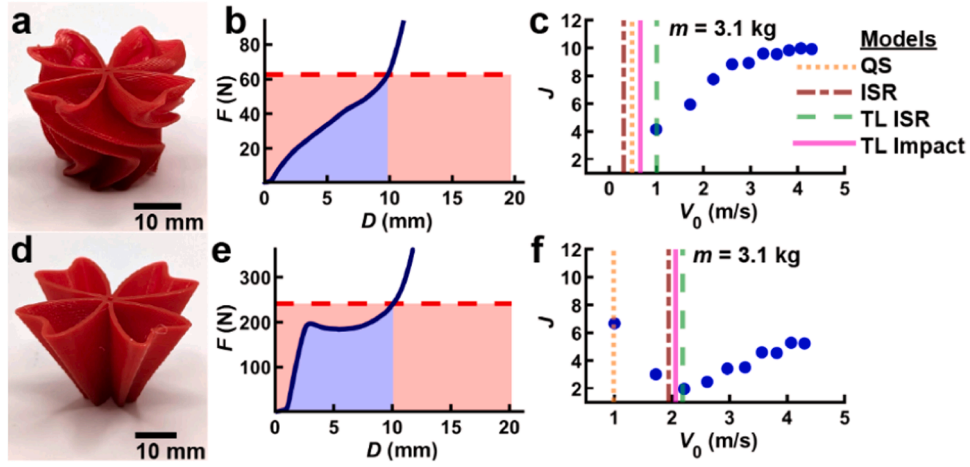


Fig. 4. Validation testing with new designs and a larger impactor mass. (a) New design printed from TPU. (b) QS test for new design showing F vs. D with red dashed line indicating F_t^* . (c) J vs. V_0 for the design shown in a with $m = 3.1$ kg. Dashed lines represent $\tilde{V}_0^*$ using the four models with color denoting the model. (d) Second new design printed from TPU. (e) QS test for second new design showing F vs. D with red dashed line indicating F_t^* . (f) Values of J vs. V_0 for design shown in d for $m = 3.1$ kg. Dashed lines represent $\tilde{V}_0^*$ using the four models with color denoting the model.

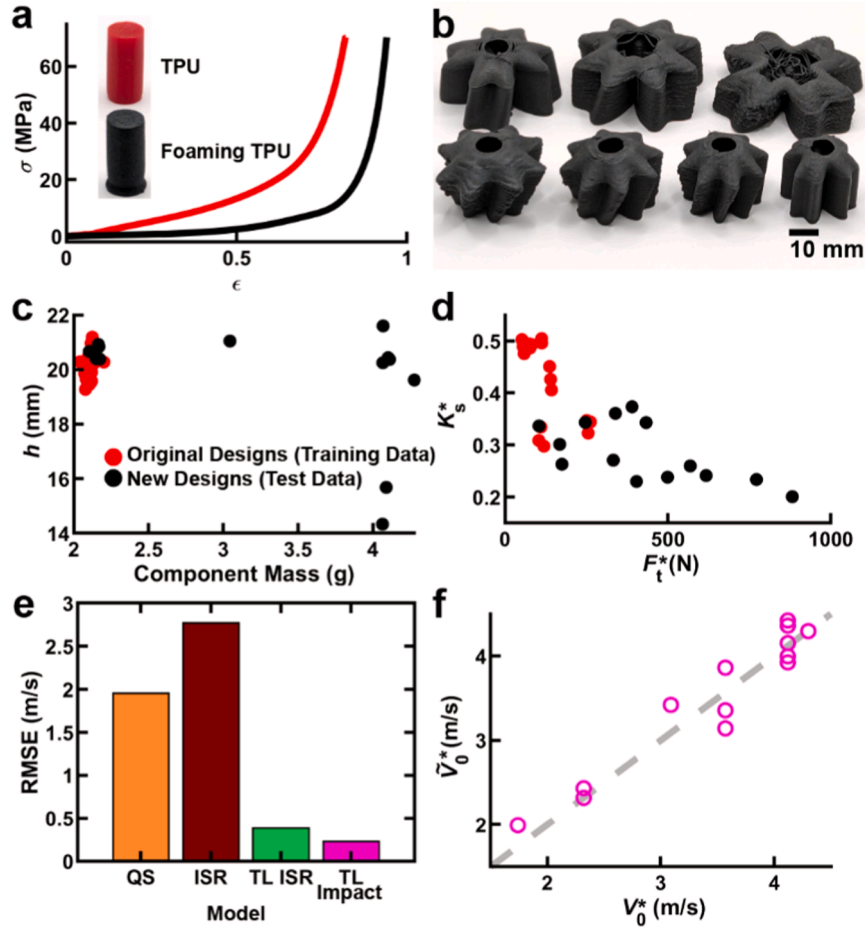


Fig. 5. Extending transfer learning models to an additional material. (a) Stress σ vs. strain ϵ for QS tests of solid cylinders 8 mm in diameter with $h = 16$ mm for material characterization. (b) Picture of seven of the 14 additional designs fabricated with a foaming TPU filament. (c) The h vs. component mass for original six designs and the new 14 foaming TPU designs. (d) The K_s vs. F_t^* for original six designs and new 14 foaming TPU designs. (e) Root mean squared error (RMSE) for predictions of V_0^* of new foaming TPU designs. Both TL models use fitting constants V_c and α trained solely on data from original six TPU designs. (f) Parity plot showing $\tilde{V}_0^*$ vs. V_0^* for the 14 foaming TPU designs using the TL Impact model for $m = 1.57$ kg.

angle, aspect ratio, or connectivity of lattices affect their force-displacement response, to predict their impact performance. [12,38,39] When impact testing is required to validate a chosen design, knowing the $\tilde{V}_0^*$ before impact testing can significantly decrease the number of tests needed to pinpoint (V_0^*, J^*). Furthermore, by using a physics-informed model instead of density or other approaches that depend on the specific structure of the individual design or material, it is more likely that the model will extrapolate to more disparate designs and structures.

By examining Eq. 5, it is possible to understand what properties are necessary for good impact performance. First, increasing the h of the component directly translates into improved impact performance, so the component should take advantage of as much vertical space as is feasible within design constraints. Second, the K_s^* is directly correlated with impact performance. Because K_s^* is found to be tolerant to strain-rate dependent effects, QS testing can be used to identify high performance designs with long, flat plateau regions. Finally, F_t^* affects impact performance. However, F_t^* is often limited by the fragility of the object to be protected and therefore usually must be optimized in coordination with the design objectives of the system as a whole.

5. Methods

GCS designs consist of 11 parameters that transform a standard cylindrical shell [30]. The STLs were generated using a previously published python script [40]. For foaming TPU designs, tapered regions were added to the GCS components by adding two distinct regions in the axial direction. First, there is a transition zone where the top shape of the GCS component increases in diameter while the lobes are decreased in size. The second region transforms the intermediate shape to a circular cross section with a small diameter, creating a reentrant region that can collapse into itself while offering low stiffness. These modifications retain the topological nature of the GCS space and can be considered a subset of GCS.

The STLs were converted into gcode using Slic3r using spiral vase mode and sent to the printers using Octoprint. Components were printed on one of five MakerGear M3-ID FFF printers on a glass bed covered with polyimide tape with 0.75 mm nozzles. Components were printed using either TPU (Ninjabot - Cheetah) or foaming TPU (ColorFabb - Vario-Shore) filament. Components were printed at 250 °C nozzle temperature and 80 °C bed temperature. The bed was heated to 100 °C before removal.

Components were removed from the printers using a UR5e (Universal Robotics) 6-axis robot arm and transferred to a scale (Sartorius CP225D) to record the mass. Afterwards, components for impact testing were placed on a table by the robot arm for temporary storage. Components for QS or ISR testing were moved by the robot to a UTM (Instron 5965). QS tests for each design were performed at 2 mm/min until the force reached 4.5 kN. A single ISR test for each design was performed at each speed (6, 20, 63, 200, 632, or 2000 mm/min). For higher testing speeds, the 4.5 kN threshold was lowered for some designs that densify quickly to protect the UTM from damage, but this did not prevent the proper calculation of K_s^* or F_t^* . For TPU, QS tests were performed in triplicate while single QS tests were performed for each foaming TPU design. Components tested in impact were transferred to another location for testing on a drop tower impact system (Ceast 9350) with a flat-ended steel impactor with a mass of either 1.57 or 3.1 kg. Because impact testing took place at an alternate site, there was an interval of several days between fabrication and testing of impact components.

QS and ISR tests record data as F - D data. The h of the component is calculated by measuring when the F first reaches 1 N. The K_s^* and F_t^* are then found by calculating the max and argmax of Eq. (3). For impact testing, each design is tested once at each V_0 . Impact data is recorded as a vs. time. Because of noise in the signal, a is smoothed using a Gaussian filter by using the MATLAB function “smoothdata” with a smoothing

factor of 0.13. The a_m is then taken as the max of this smoothed curve. Subsequently V_0^* and J^* are calculated by finding the minimum J for each design and its corresponding V_0 .

When calculating $\tilde{V}_0^*$, only a single QS test was used at a time. Therefore, to get standard deviation for the TPU models, the models were rerun with each of the three QS experiments. Likewise, when fitting Eq. (6), a single QS test was paired with all ISR tests for each of the three predictions using the MATLAB function “fit” with the restriction that of $F_t^*(0) \geq 0$, $V_c \geq 2$ mm/min, and $\alpha \geq 0$. For LOOCV predictions (Fig. 3), ISR data from the target design was excluded when calculating the mean of fitting constant V_c and α . However, fitting constant $F_t^*(0)$ was calculated from the F_t^* of the single QS test used for each of the three predictions for that design. When predicting $\tilde{V}_0^*$ for the new designs (Fig. 4c,f and Fig. 5), ISR data from all six original TPU designs were used when calculating the mean of fitting constants V_c and α . Only a single QS experiment was performed for each foaming TPU design, so no standard deviation was reported.

CRedit authorship contribution statement

Kelsey L Snapp: Writing – review & editing, Writing – original draft, Software, Investigation, Formal analysis, Data curation, Conceptualization. **Samuel Silverman:** Writing – review & editing, Software. **Thomas M Tiano:** Writing – review & editing, Conceptualization. **Richard Pang:** Writing – review & editing, Investigation. **Emily Whiting:** Writing – review & editing, Supervision, Conceptualization. **Timothy J. Lawton:** Writing – review & editing, Supervision, Investigation, Conceptualization. **Keith Brown:** Writing – review & editing, Writing – original draft, Supervision, Conceptualization.

Declaration of Competing Interest

The authors declare no competing financial interests.

Data availability

All data is available at <https://open.bu.edu/handle/2144/46509>.

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