

Movable Antenna-Aided Broadcast Packet Erasure Channels: Capacity With Dynamic Position Plan

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Abstract—The emergence of new wireless technologies, such as movable antennas and reconfigurable intelligent surfaces, enable channel morphing. In this new paradigm, variations in channel statistics may be decided and thus known prior to communications, deviating from the conventional ergodic models in which the underlying processes that govern channel statistics are assumed to be temporally independent. To provide a fundamental understanding beyond the typical physical-layer studies, we study the capacity region of movable antenna-aided broadcast packet erasure channels, and provide new inner and outer bounds. The proposed linear network-coding protocol opportunistically benefits from the prior knowledge of future statistical changes, and achieves the outer-bounds for a wide range of parameters. Network capacity can be further maximized in a cross-layer fashion by determining optimal channel statistics controlled by the movable antenna position.

Index Terms—Movable antenna, broadcast packet erasure channels, controllable statistics, channel capacity, spatial DoF.

I. INTRODUCTION

RECENT advances in wireless communication technology provide us with the ability to control and alter channel statistics. This ability is essential in higher mmWave and (sub-)THz bands where large path-loss and rank-deficient channels may severely reduce the inherent attractiveness of these bands due to their large available bandwidth [1], [2], [3], [4]. The most recent technological advancement in this domain is the concept of movable antenna (MA), also known as fluid antenna, which can change its position and/or orientation to alter and improve channel statistics [5], [6]. Several recent results have incorporated MA in wireless networks to simplify hardware equipment, improve multi-user scaling, and security [7], [8], [9], [10], [11], [12]. In this context, one of the main tasks is to find the optimal MA position(s) throughout the communication block.

Most prior results focus on the physical-layer and aim to achieve objectives such as range extension or signal-to-interference-plus-noise (SINR) optimization. Further, prior

results for the most part aim to find some position/orientation for the movable antenna, and then leave the network in that configuration for the entire communication block. While these results provide valuable insights, they can be inherently far from optimal. First, it is well-known that the optimal multi-user network capacity (*i.e.*, Shannon capacity) may be significantly superior to SINR maximization methods [13], [14], [15]. Second, there is no guarantee that leaving the network in one fixed configuration would result in the best possible throughout. Further and from a theoretical standpoint, the ability to alter channel statistics during the communication block results in a non-ergodic channel model, as opposed to the commonly assumed ergodic setting [16] where statistical changes are independently distributed and, for instance, governed by an underlying Markov process. For non-ergodic settings, the theoretical foundation is yet to be developed.

To improve upon the shortcomings, we go beyond the typical physical-layer studies to shed light on the theoretical foundations of MA-aided communications. We study the well-known broadcast packet erasure channel (BPEC), which is well-suited to model the intermittent connectivity of higher frequency bands [17], [18], [19]. In this context, our focus is not on typical problems such as beamforming, but rather on finding the maximum network capacity given the channel statistics, or alternatively, finding the channel statistics that would maximize network capacity. We assume the transmitter is equipped with a movable antenna and consider on packet transmission as opposed to the physical-layer channel models. At the end of every channel use, each receiver will inform other nodes whether or not it received the transmitted packet via short-length ACK/NACK signaling. For this problem, we focus on network capacity and instead of looking for a static MA position, we search for the “optimal plan,” which may happen to be dynamic. Our contributions are then multi-fold:

- We show dynamic antenna position plans enable adaptive protocols that outperform static plans in capacity;
- We provide a new set of outer-bounds on the capacity region of a multi-user non-ergodic MA-aided BPEC;
- We present an opportunistic linear protocol that exploits the non-causal knowledge of statistical changes, and determine the resulting achievable region;
- We show the achievable region matches the outer-bounds for a wide range of channel parameters;
- Based on our results, we deduce the optimal position plan.

Recently, [20] considered a multiple-input single-output Gaussian broadcast channel with perfect channel knowledge with a reconfigurable intelligent surface, and concluded that the gain of dynamic beam allocation is marginal. This is not surprising given the perfect channel knowledge assumption. It is well known that the gap between the more practical no and delayed feedback is significant [14], [15], which is our focus.

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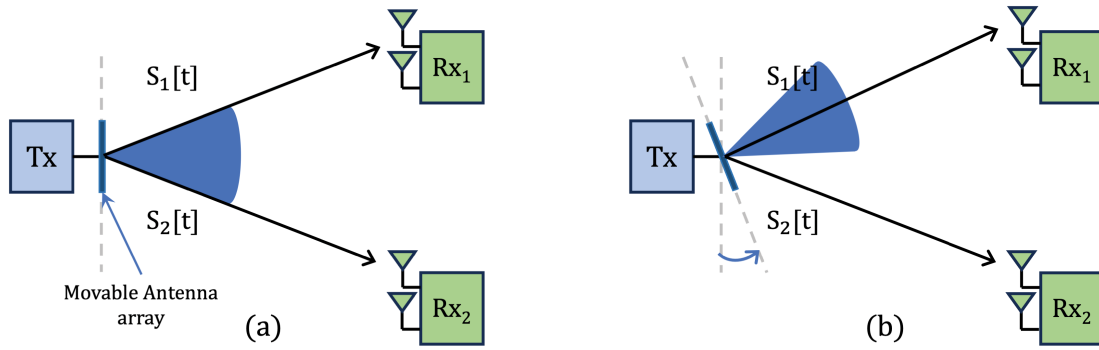


Fig. 1. MA-aided BPEC: (a) MA array in neutral position, (b) MA array positioned to enhance the reception at Rx₁.

II. SYSTEM SETUP

To better understand how movable antennas would impact the capacity of wireless networks, we adopt the celebrated broadcast packet erasure channel (BPEC) framework where each communication link is a memoryless erasure channel [17]. Typically, the transmitter equipped with a movable antenna array wishes to form a beam pointed to each receiver, and the link erasure would capture whether this beam-pointing process was successful [21]. The BPEC model captures the broadcast nature of complicated physical-layer transmission in a tractable abstraction and has a better prediction over the conventional error-free model in a wireless downlink [17]. In our work, MA can control the erasure probability of each link over time via changing their position.

A. Files

The transmitter, Tx, has two independent files, W_1 and W_2 , and wishes to deliver them to the two receivers over n channel uses. Each file, W_i , contains $|W_i| = m_i = nR_i$ data packets in $\mathbb{F}_q$ where R_i is the rate for user i , $i = 1, 2$. The unit for rates is then packets per channel use.

B. Input-Output Relationship

We focus on packet networks where the channel from the Tx to Rx _{i} is described by $S_i[t] \in \{0, 1\}$, $t = 1, 2, \dots, n$, and $i = 1, 2$. At time instant t , the messages are mapped to channel input $X[t] \in \mathbb{F}_q$, and the corresponding received signals are:

$$Y_1[t] = S_1[t]X[t] \quad \text{and} \quad Y_2[t] = S_2[t]X[t]. \quad (1)$$

In this context, $S_i[t] = 1$ means the transmitted packet at time t is delivered successfully to Rx _{i} , while $S_i[t] = 0$ implies the packet did not arrive at that terminal due to collision or Rx _{i} 's link being in deep fade [22].

C. Channel Statistics and Role of MA

We assume each channel $S_i[t]$, $i = 1, 2$ is distributed according to a Bernoulli distribution affected by the MA position at the transmitter. We assume the erasures occur independently across users and time, but the distribution at each time is determined by the MA position. If the movable antenna is in "neutral position" as in Figure 1(a), it would be enhancing the reception at both receivers equally. However, if the MA is positioned to enhance the reception at one of the two receiver terminals, Rx₁ in Figure 1(b), then the erasure probability at the enhanced user would decrease, while it would increase at the other user compared to when the MA is in neutral position. Mathematically, if the MA is in neutral position, then, $S_i[t] \sim \mathcal{B}(1 - \delta_{\text{both}})$, and if the MA is positioned to enhance the reception of Rx _{i} , then, $S_i[t] \sim \mathcal{B}(1 - \delta_{\text{assist}})$, while erasure at

the other user is governed by a Bernoulli $\mathcal{B}(1 - \delta_{\text{base}})$ process. Naturally, we assume $0 \leq \delta_{\text{assist}} < \delta_{\text{both}} < \delta_{\text{base}} \leq 1$.

D. MA Position Plan

A position plan, $\mathcal{P}^n$, determines the position/orientation of the movable antennas, and therefore channel statistics, for any given time instant during the communication blocklength of n . We further assume the MA position plan is decided ahead of time and known to all nodes.

E. Feedback

At each time t , the receivers estimate the channel strength knowing whether the receive signal-to-noise-ratio was above the threshold to enable successful decoding of a packet [22]. Then, they will inform other nodes regarding the status of the transmitted packets via short-length ACK/NACK signaling. We note that the feedback mechanism results in a non-degraded channel setup [18], [23].

F. Transmit Signal

The constraint imposed at time index t on the encoding function $f_t(\cdot)$ at the transmitter is:

$$X[t] = f_t(W_1, W_2, S^{t-1}, \mathcal{P}^n), \quad (2)$$

where $S^{t-1} = (S_1^{t-1}, S_2^{t-1})$.

G. Decoding

Then, the decoding function for Rx _{i} , $i = 1, 2$ is $\varphi_{i,n}(Y_i^n, S^n, \text{SI})$. An error occurs whenever $\widehat{W}_i \neq W_i$. The average probability of error is given by

$$\lambda_{i,n} = \mathbb{E}[P(\widehat{W}_i \neq W_i)], \quad (3)$$

where the expectation is taken with respect to the random choice of the transmitted messages.

H. Capacity Region

For a given MA position plan, $\mathcal{P}^n$, over the communication block, we say that a rate-pair (R_1, R_2) is achievable, if there exist a block encoder at the transmitter and a block decoder at each receiver, such that $\lambda_{i,n}$ goes to zero as the block length n goes to infinity. The capacity region, $\mathcal{C}_{\mathcal{P}}$, is the closure of the set of the achievable rate-pairs.

III. NETWORK CAPACITY BOUNDS

One of our key contributions is a new set of network capacity bounds for the non-ergodic MA-aided BPEC that define the boundaries on the region within which asymptotically reliable communication is feasible. We first define a balanced MA position plan, and then present the outer-bounds.

A. Balanced Plan

For the balanced MA position plan, during the first ηn channel uses ($\eta \leq 1/2$), the MA will be positioned to enhance only the reception at R_{x_1} , meaning that $S_1[t] \sim \mathcal{B}(1 - \delta_{\text{assist}})$ and $S_2[t] \sim \mathcal{B}(1 - \delta_{\text{base}})$ for $t = 1, 2, \dots, \eta n$. Without loss of generality, we assume $\eta n \in \mathbb{Z}^+$. During the following ηn channel uses, the MA will be used to enhance only the reception at R_{x_2} ; and during the remaining $(1 - 2\eta)n$ channel uses, the MA will be in neutral position.

Theorem 1: For the two-user MA-aided BPEC described in Section II with the balanced movable-antenna position plan defined above, we have:

$$\mathcal{C}_{\mathcal{P}} \subseteq \mathcal{C}_{\mathcal{P}}^{\text{out}} \equiv \begin{cases} 0 & \leq R_1 + \beta R_2 \leq \beta(1 - \bar{\delta}), \\ 0 & \leq \beta R_1 + R_2 \leq \beta(1 - \bar{\delta}), \end{cases} \quad (4)$$

where

$$\beta = \begin{cases} \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}, & \text{if } \delta_{\text{both}} \leq \frac{\delta_{\text{base}}(1 - \delta_{\text{assist}})}{1 - \delta_{\text{base}}} \\ 1 + \delta_{\text{both}}, & \text{if } \delta_{\text{both}} > \frac{\delta_{\text{base}}(1 - \delta_{\text{assist}})}{1 - \delta_{\text{base}}} \end{cases} \quad (5)$$

and

$$\bar{\delta} = \eta\delta_{\text{assist}} + \eta\delta_{\text{base}} + (1 - 2\eta)\delta_{\text{both}}. \quad (6)$$

The following section provides detailed examples to: (1) prove these outer-bounds can be indeed achieved in non-trivial instances, and (2) to compare the resulting region with several benchmarks and demonstrate its superiority. The proof of Theorem 1 is presented in Section V.

Remark 1 (Ergodicity): Traditionally, it is assumed that statistical channel variations are temporally independent [24], a setting known as ergodic [16]. In MA-aided BPEC, future changes are known a priori, resulting in a non-ergodic model. This non-ergodic model differs from the spatiotemporal correlation assumed in prior results [25], [26], [27].

IV. MOTIVATING EXAMPLE & NUMERICAL ANALYSIS

In this section, we present an adaptive protocol to harness the non-causal statistical knowledge and provide numerical analysis to compare the results to various benchmarks. We assume $\delta_{\text{base}} = 0.75$ (high erasure), $\delta_{\text{both}} = 0.5$ (moderate erasure), and $\delta_{\text{assist}} = 0.25$ (low erasure). Our goal is two-fold. First, to decide which MA position plan, $\mathcal{P}^n$, would result in the highest capacity; and second, how to devise a communication protocol to achieve the corresponding $\mathcal{C}_{\mathcal{P}}$. We evaluate a few different MA position plans.

A. Adaptive Transmission for the Balanced Plan

Here, we present a new adaptive linear transmission protocol that exploits the non-causal statistical knowledge of the channel to improve network capacity. Fix $\eta = 8/17$. The goal is to successfully deliver $m = 13/34n$ packets to each receiver terminal, which would result in a sum-rate of $2m/n = 13/17$. The communication block is divided into three modes based on the MA position plan: mode A in which R_{x_1} has low erasure; mode B in which R_{x_2} has low erasure; mode C in which receivers have moderate erasure.

B. Transmission in Mode A

At each time instant, the transmitter will send out a packet intended for R_{x_1} . The packet may get delivered to the intended user, to the unintended user but not the intended one, or get erased at both terminals. In the latter case, the packet will

be re-transmitted. In the first case, the packet is successfully delivered and no further action is required. In the second case, the packet is still needed at the intended terminal but is available at the other terminal as side-information. Thus, future retransmissions of this packet will not cause any interference at R_{x_1} . The transmitter will track such packets in a virtual queue $v_{1|2}$ to indicate their availability at R_{x_2} . The communication will last an average of:

$$\frac{1}{1 - \delta_{\text{assist}}\delta_{\text{base}}}m = \frac{8}{17}n = \eta n. \quad (7)$$

Further, the average number of packets in $v_{1|2}$ will be:

$$\frac{\delta_{\text{assist}}(1 - \delta_{\text{base}})}{1 - \delta_{\text{assist}}\delta_{\text{base}}}m = \frac{1}{13}n. \quad (8)$$

Remark 2 (Using Expected Values): To simplify the description, we use the expected values of the different random variables. In practice, the actual value may deviate from the mean, but using concentration-type analysis, it can be shown that the asymptotic results will not be affected [18].

C. Transmission in Mode B

Similar to mode A, but with interchanging user indices.

D. Transmission in Mode C

The transmitter will place the MA in neutral position, and create the pairwise summations of the packets in $v_{1|2}$ and $v_{2|1}$. The new packets are of “common interest” to both receivers as each will have one part as side-information and will need the other part. Then, the resulting sequence will be sent to the terminals at an erasure rate of $(1 - \delta_{\text{both}})$. This mode will last an average of:

$$\frac{\delta_{\text{assist}}(1 - \delta_{\text{base}})}{(1 - \delta_{\text{both}})(1 - \delta_{\text{assist}}\delta_{\text{base}})}m = \frac{1}{17}n = (1 - 2\eta)n. \quad (9)$$

E. Achievable Region

The transmission described above achieves the following region:

$$\begin{aligned} \mathcal{R}_{\text{balanced}} &\equiv \begin{cases} 0 & \leq R_1 + \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}R_2 \leq \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}(1 - \bar{\delta}), \\ 0 & \leq \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}R_1 + R_2 \leq \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}(1 - \bar{\delta}), \end{cases} \end{aligned} \quad (10)$$

where $\bar{\delta} = \eta\delta_{\text{assist}} + \eta\delta_{\text{base}} + (1 - 2\eta)\delta_{\text{both}}$.

Comparing the region described in (4) and the one in (10), we have the following observation.

Corollary 1: For the balanced plan example described above, we have $\delta_{\text{both}} \leq \frac{\delta_{\text{base}}(1 - \delta_{\text{assist}})}{1 - \delta_{\text{base}}}$, which implies:

$$\mathcal{R}_{\text{balanced}} \equiv \mathcal{C}_{\mathcal{P}}^{\text{out}}. \quad (11)$$

In other words, the achievable region described in (10) is indeed optimal.

F. Benchmarks

We compare our results to several benchmarks.

G. Ergodic Model

The first natural question is what happens if we treat the problem as one where the erasure probability is constant

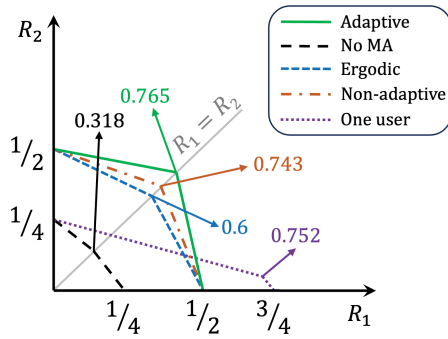


Fig. 2. Comparing the proposed design with different benchmarks.

and equal to the long-term average, *i.e.*, $\bar{\delta} = 0.5$ in this example. Then, the problem would fall under the well-known BPEC with ACK/NACK for which the capacity region is described by [28]:

$$\mathcal{C}_{\text{Ergodic}} \equiv \begin{cases} 0 \leq R_1 + (1 + \bar{\delta})R_2 \leq (1 - \bar{\delta}^2), \\ 0 \leq (1 + \bar{\delta})R_1 + R_2 \leq (1 - \bar{\delta}^2). \end{cases} \quad (12)$$

This would also be the region if the MA was left in the neutral position for the entire block ($\eta = 0$ in Theorem 1).

H. Assisting Only One User

If the MA is positioned to favor one user, say R_{x_1} , then the capacity region is described by [28]:

$$\mathcal{C}_{\text{user 1}} \equiv \begin{cases} 0 \leq R_1 + \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}R_2 \leq (1 - \delta_{\text{assist}}\delta_{\text{base}}), \\ 0 \leq \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{assist}}}R_1 + R_2 \leq (1 - \delta_{\text{assist}}\delta_{\text{base}}). \end{cases} \quad (13)$$

I. Non-Adaptive Protocol for the Balanced Plan

Finally, we could ask what would happen if for the balanced plan, we treated each mode separately and aimed for the maximum rate within that mode. In other words, we ask what would happen if the transmission protocol ignores the non-causal information. The region can be calculated as an average sum of the regions for the sub-problems and is included in our comparison.

Figure 2 provides the numerical comparison between the proposed adaptive protocol for the balanced MA position plan and other benchmarks including the case with no MA. As we can see, the balanced plan enables the adaptive protocol to achieve the highest overall sum-rate. Further, Corollary 1 shows this region is indeed optimal.

V. PROOF OF THEOREM 1

To prove Theorem 1, we focus on $\delta_{\text{both}} \leq \frac{\delta_{\text{base}}(1 - \delta_{\text{assist}})}{1 - \delta_{\text{base}}}$, thus based on (5), $\beta = \frac{1 - \delta_{\text{assist}}\delta_{\text{base}}}{1 - \delta_{\text{base}}}$. The proof for the other scenario would follow similar steps and thus omitted.

Suppose rate-tuple (R_1, R_2) is achievable. We have:

$$\begin{aligned} n(R_1 + \beta R_2) &= H(W_1) + \beta H(W_2) \\ &\stackrel{(a)}{=} H(W_1|W_2, S^n, \mathcal{P}^n) + \beta H(W_2|S^n, \mathcal{P}^n) \\ &\stackrel{(\text{Fano})}{\leq} I(W_1; Y_1^n|W_2, S^n, \mathcal{P}^n) + \beta I(W_2; Y_2^n|S^n, \mathcal{P}^n) + n\xi_n \\ &= H(Y_1^n|W_2, S^n, \mathcal{P}^n) - \underbrace{H(Y_1^n|W_1, W_2, S^n, \mathcal{P}^n)}_{=0} \end{aligned}$$

$$\begin{aligned} &+ \beta H(Y_2^n|S^n, \mathcal{P}^n) - \beta H(Y_2^n|W_2, S^n, \mathcal{P}^n) + n\xi_n \\ &\stackrel{(b)}{\leq} \beta H(Y_2^n|S^n, \mathcal{P}^n) + n\xi_n \\ &\stackrel{(c)}{\leq} n\beta(1 - \eta\delta_{\text{assist}} - \eta\delta_{\text{base}} - (1 - 2\eta)\delta_{\text{both}}) + \xi_n \\ &\stackrel{(d)}{\leq} n\beta(1 - \bar{\delta}) + \xi_n, \end{aligned} \quad (14)$$

where $\xi_n \rightarrow 0$ as $n \rightarrow \infty$; (a) holds due to the independence of messages; (b) follows from the Entropy Leakage Lemma below (Lemma 1); (c) holds since the entropy of a binary random variable is at most 1 (or $\log_2(q)$ for packets in $\mathbb{F}_q$) and the channel statistics is governed by the movable antenna position plan; and (d) follows from (6). Dividing both sides by n and let $n \rightarrow \infty$, we get the first bound in (4) and the other bound follows by symmetry.

Lemma 1 (Entropy Leakage Lemma): For the two-user MA-aided BPEC with delayed feedback as described in Section II and for any encoding function satisfying (2), we have:

$$H(Y_1^n|W_2, S^n, \mathcal{P}^n) - \beta H(Y_2^n|W_2, S^n, \mathcal{P}^n) \leq 0, \quad (15)$$

Proof: We have:

$$\begin{aligned} &H(Y_2^n|W_2, S^n, \mathcal{P}^n) \\ &= \sum_{t=1}^n H(Y_2[t]|Y_2^{t-1}, W_2, S^n, \mathcal{P}^n) \\ &\stackrel{(a)}{=} \sum_{t=1}^{\eta n} (1 - \delta_{\text{base}}) H(X[t]|Y_2^{t-1}, W_2, S^n, \mathcal{P}^n) \\ &\quad + \sum_{t=\eta n+1}^{2\eta n} (1 - \delta_{\text{assist}}) H(X[t]|Y_2^{t-1}, W_2, S^n, \mathcal{P}^n) \\ &\quad + \sum_{t=2\eta n+1}^n (1 - \delta_{\text{both}}) H(X[t]|Y_2^{t-1}, W_2, S^n, \mathcal{P}^n) \\ &\geq \sum_{t=1}^{2\eta n} \frac{1}{\beta} H(Y_1[t], Y_2[t]|Y_1^{t-1}, Y_2^{t-1}, W_2, S^n, \mathcal{P}^n) \\ &\quad + \sum_{t=2\eta n+1}^n \frac{1}{(1 + \delta_{\text{both}})} H \\ &\quad \times (Y_1[t], Y_2[t]|Y_1^{t-1}, Y_2^{t-1}, W_2, S^n, \mathcal{P}^n) \\ &\stackrel{(b)}{\geq} \frac{H(Y_1^n, Y_2^n|W_2, S^n, \mathcal{P}^n)}{\beta} \stackrel{(c)}{\geq} \frac{H(Y_1^n|W_2, S^n, \mathcal{P}^n)}{\beta}, \end{aligned} \quad (16)$$

where (a) holds since $X[t]$ is independent of $S[t]$ and the channel statistics are governed by the MA position plan; (b) holds since the omitted term is the product of a discrete entropy term with:

$$\left(\frac{1}{1 + \delta_{\text{both}}} - \frac{1 - \delta_{\text{base}}}{1 - \delta_{\text{assist}}\delta_{\text{base}}} \right), \quad (17)$$

which are both non-negative given that we assumed $\delta_{\text{both}} \leq \frac{\delta_{\text{base}}(1 - \delta_{\text{assist}})}{1 - \delta_{\text{base}}}$; and (c) follows from the non-negativity of the discrete entropy function. $\square$

Remark 3 (Changing MA Position): The position of a movable antenna can change in different ways; one simple way is to mount an antenna on a mechanical motor [6]. We ignored this “transition time”. From (16), we can show that any omitted term in step (b) that scales as $\mathcal{O}(n)$ would not alter our asymptotic analysis where we use the standard Landau notation. In other words, as long as the transition time scales as $\mathcal{O}(n)$ the same set of results would hold.

VI. OPTIMAL POSITION PLAN AND FURTHER DISCUSSIONS

Corollary 1 proves the optimality of the protocol proposed for the balanced plan of Section IV. In this part, we focus on finding the optimal MA position plan. Theorem 1 presents an outer-bound region on the capacity region, and in general, the achievability of these outer-bounds remains open. From Theorem 1, the maximum sum-rate cannot be larger than $\frac{2\beta(1-\delta)}{1+\beta}$ for β given in (5), which shows the only dependency on η comes from $\bar{\delta}$ given in (6). Three cases arise: (i) $2\delta_{\text{both}} > \delta_{\text{assist}} + \delta_{\text{base}}$: in this case and in terms of outer-bound, it is better to forgo the neutral position and alternate the focus to each user; (ii) $2\delta_{\text{both}} < \delta_{\text{assist}} + \delta_{\text{base}}$: in this case, it may appear it is best to allocate the majority of the communication block to the neutral plan, but this is misleading as if $\eta \rightarrow 0$, then the capacity is known from (12), which is not what the region in (4) converges to. This observation shows the answer in this case requires further investigation; finally, (iii) $2\delta_{\text{both}} = \delta_{\text{assist}} + \delta_{\text{base}}$: this is the case for the example of Section IV where there is no dependency on η . However, that is only concerning the maximum attainable rates and no other subtle issues such as complexity. The linear protocol of Section IV has rather low complexity. If instead we have $\eta = 1/2$, meaning that for the first half of the communication block, the MA is used to enhance the reception at Rx_1 , while for the second half, the MA is used to enhance the reception at Rx_2 , then the outer-bounds remain unchanged. On the other hand, the achievability would be more complex and would require an intricate mixture of the protocol presented earlier and its reverse implementation as introduced in [25]. In other words, while the capacity remains unchanged, the communication protocol will be noticeably more complicated. Finally, the discussion above based optimality on maximum sum-rate, which may not be the desired objective in some applications. In short, unlike the first question, the optimal position plan may be known in some specific cases but even then, it would require further considerations.

VII. CONCLUSION

We showed the significant benefits of dynamic movable antenna position plan over static assignments. To do so, we presented one of the very first capacity results for non-ergodic networks. In particular, we considered a MA-aided BPEC for which we presented a new set of inner and outer bounds. The outer-bounds reveal the dominating channel statistics, while our linear protocol exploits the knowledge of future statistical changes. We show the two regions match for non-trivial channel parameters and when that occurs, the optimal MA position plan may be inferred. The capacity region remains open in general and thus needs further investigation. Other future directions include extension to a larger number of users.

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