Non-Hermitian Topologically Enhanced Sensing

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Abstract: We experimentally demonstrate enhanced sensitivities within non-Hermitian topological lattices realized in a dissipatively-coupled network of time-multiplexed resonators. Our demonstration paves the way for realizing optical sensors with unprecedented sensitivities using notions of non-Hermiticity and topology.

Sensors are widely used in various settings ranging from smartphones and autonomous vehicles to the healthcare industry and space technologies. Using multiple sensors that collectively interact with the signal to be measured could lead to enhanced signal-to-noise ratios (SNR) compared to individual elements. In quantum settings, for instance, such distributed sensing techniques have been implemented to achieve a linear increase in the SNR via using entangled states [1]. In non-Hermitian systems, coupled resonator arrays have provided yet additional degrees of freedom to obtain higher sensitivities via higher-order exceptional points [2]. Quite recently, a new class of sensor arrays, known as non-Hermitian topological sensors (NTOS) have been theoretically proposed [3]. Remarkably, the synergistic interplay between non-Hermiticity and topology can bestow such sensors with an enhanced sensitivity that grows exponentially with the size of the sensor network.

Here, we experimentally realize NTOS using a network of time-multiplexed photonic resonators. By using delay lines with appropriate lengths and controllable throughputs, we implement the prototypical Hatano-Nelson model [4] for our non-Hermitian topological sensing scheme. Our experiments involving arrays with different numbers of elements confirm the characteristic exponential enhancement of the sensitivity that is expected from this class of sensor arrays. This unique response arises from the synergy between non-Hermiticity and topology — something that is unattainable in Hermitian topological lattices.

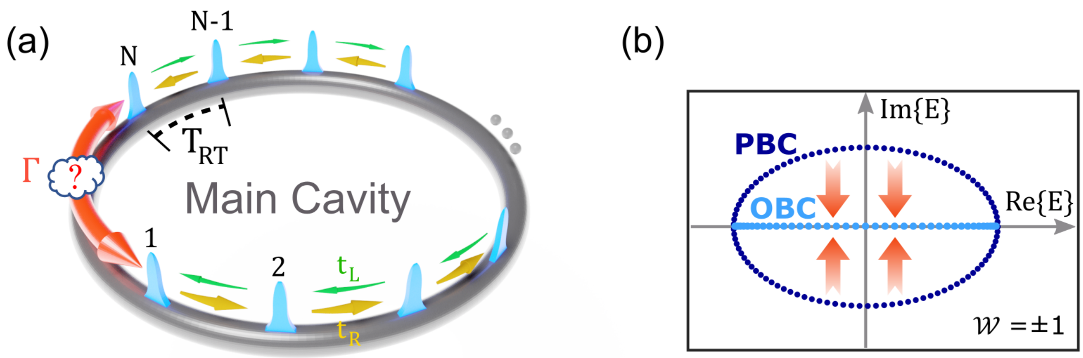

Fig. 1: **NTOS based on the Hatano-Nelson model. (a)** Schematic diagram of the NTOS demonstrated here based on the Hatano-Nelson model featuring nonreciprocal couplings between the adjacent array elements. **(b)** Depending on the boundary conditions, this lattice exhibits different eigenvalue spectra. In particular, when Γ is maximum, i.e. under periodic boundary conditions (PBC), the eigenvalues form an ellipse around the origin in the complex plane, corresponding to a nonzero winding number $\mathcal{W}$. In contrast, when $\Gamma = 0$, i.e. under open boundary conditions (OBC), all the eigenvalues reside on the real axis.

For our realization of NTOS, we consider the Hatano-Nelson model [4] shown schematically in Fig. 1(a) described by the Hamiltonian:

$$\hat{H}_{HN} = \sum_n t_R \hat{a}^\dagger_{n+1}\hat{a}_n + t_L \hat{a}^\dagger_n \hat{a}_{n+1}, \tag{1}$$

where $\hat{a}_n^{(\dagger)}$ is the annihilation (creation) operator associated with site n while t_R, t_L represent the nonreciprocal right and left nearest-neighbor couplings within the lattice. When the structure is terminated with open boundary conditions (OBC), the eigenvalues of this Hamiltonian are entirely real (Fig. 1(b)). Under such OBC conditions, when the number of elements in the lattice is odd $N = 2k+1$, the Hamiltonian $\hat{H}_{HN}$ possesses an eigenstate $|\psi_0\rangle_R$

with zero eigenvalue. We use a time-multiplexed photonic resonator network [5] with a main fiber loop that supports N resonant pulses that are separated by a repetition period T_{RT}. Each pulse represents an individual resonator associated with the annihilation (creation) operator $\hat{a}_j^{(\dagger)}$ in Eq. 1. To implement the non-reciprocal couplings t_R and t_L we use optical delay lines to dissipatively couple nearest-neighbor pulses. Each delay line is equipped with an intensity modulator to control the strength of the associated coupling. For our sensing scheme, we consider a change in the coupling between the first and the last pulse described by the perturbation $\Delta\hat{H} = \Gamma\hat{a}_1^{\dagger}\hat{a}_N$ corresponding to a small deviation from the OBC configuration. To realize this experimentally, we use a third delay line which couples the first pulse to the last one in a non-reciprocal fashion with controllable strength that translates into different values of Γ. The unperturbed eigenstate $|\psi_0\rangle_R$ will respond to this change by converting to a new eigenstate $|\psi(\Gamma)\rangle_R$ associated with a perturbed eigenvalue that shifts from zero by ΔE. To perform the experiment, we intitialize the system by shaping the amplitudes and phases of the input pulses to represent the zero eigenstate $|\psi_0\rangle_R$. Followed by this, the input to the cavity is blocked so that the pulses start to circulate through the cavity and the delay lines. At each time step defined by the integer multiples of the cavity round-trip time, we project the state of the network on the left eigenstate associated with the unperturbed Hamiltonian $|\psi_0\rangle_L$. The sensor response ΔE can now be estimated from the decay rate of this projection under both unperturbed and perturbed conditions.

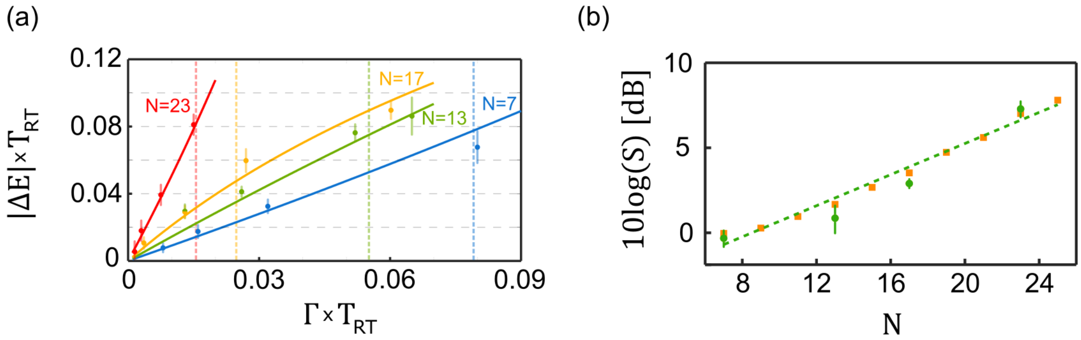

Fig. 2: **Experimental demonstration of NTOS.** **(a)** Experimentally measured shifts in the eigenvalue ΔE as the coupling strength Γ changes in lattices with different number of elements $N = 7, 13, 17$ and 23, together with theoretically expected values shown as solid curves. As long as Γ is small enough, our NTOS responds linearly to the induced perturbations. For larger perturbations, NTOS exhibits a transition to the nonlinear regime as marked by vertical dashed lines for different lattices. **(b)** Experimentally obtained sensitivity S of the NTOS for different lattice sizes N shown as green circles together with a linear fit depicted as the green dashed line. This confirms an exponential increase in the sensitivity as the lattice size grows. The corresponding theoretically predicted values are also displayed as orange squares.

Figure 2(a) shows experimentally measured values corresponding to different lattices with various number of elements together with simulated results when $t_R/t_L = \sqrt{2}$. We note that for perturbations well below a critical value $\Gamma \ll \Gamma_C$, the NTOS responds linearly to the input perturbation. However, for larger inputs, the perturbed eigenvalue associated with $|\psi(\Gamma)\rangle_R$ is no longer real, marking a transition to the PBC where the sensor response is no longer linear. The onset of this non-Hermitian phase transition which is also displayed in Fig. 2(a) with vertical dashed lines, ultimately sets the dynamic range of our demonstrated NTOS. To evaluate the performance of NTOS, we calculated the sensitivity parameter defined as $S \equiv \partial E/\partial\Gamma$ using our measurement data in the small parameter regime $\Gamma \ll \Gamma_C$. Figure 2(b) shows theoretically expected values along with experimental results for different lattice sizes N. It is evident from here that our demonstrated NTOS becomes exponentially more sensitive as the number of array elements N grows.

In summary, we have experimentally demonstrated enhanced sensitivity in non-Hermitian topological sensors. For various lattices with different number of elements, we characterized the sensitivity of the NTOS in terms of the shift in the zero eigenvalue as the lattice boundary condition is perturbed. Experimentally measured sensitivities confirm an exponential growth with the lattice size N, in agreement with theoretical predictions. Our results represent new opportunities for realizing optical sensors with unprecedented sensitivities by combining the concepts of non-Hermiticity and topology.

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