Non-Gaussian Quantum State Engineering with Squared-Quadrature Quantum Nondemolition Measurements

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Abstract: We present a method for generating squeezed Schrödinger cat states and cubic phase states via quantum nondemolition measurement of the squared-quadrature operator, offering a realistic route to fault-tolerant universal continuous-variable quantum computation.

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One-way quantum computation with continuous-variable (CV) photonic cluster states is one of the leading candidates for building large-scale quantum computers. While all-Gaussian quantum computation can be performed by homodyne measurements and linear feed-forward operations on cluster states, non-Gaussian resources are essential to achieve quantum computational advantages and fault tolerance. A leading approach to introducing non-Gaussianity is through photon-number-resolving (PNR) measurements [1], which allows one to perform a variety of non-Gaussian quantum state engineering schemes [2]. This approach provides access to non-Gaussian states, including cat states and cubic phase states [3], but the requirements for cryostats and the slow speed of conventional PNR detectors (e.g., superconducting nanowires and superconducting transition-edge sensors) critically limit the scalability and clock speeds.

In this work, we propose an all-optical route to generate cat state and cubic phase state using quadratic ($\chi^{(2)}$) nonlinearity and homodyne conditioning. We show that applying strong squeezing operations to the signal and pump fields of a $\chi^{(2)}$ nonlinear system can engineer the system Hamiltonian into a form that is capable of performing a quantum-nondemolition (QND) measurement [4] of squared quadrature of the signal mode. The QND measurement result can be read out from the conditional displacement of the pump mode via a homodyne measurement, which conditionally projects the signal mode to a squeezed Schödinger's cat state.

We consider a resonant, single-mode $\chi^{(2)}$ nonlinear system with a Hamiltonian

$$\hat{H} = -g(\hat{a}^2\hat{b}^\dagger + \hat{a}^{\dagger 2}\hat{b}) \tag{1}$$

where $g > 0$ is the nonlinear coupling constant, and $\hat{a}$ and $\hat{b}$ are the annihilation operators for the signal and the pump modes, respectively. For an initial system state of $|\varphi(0)\rangle = |\varphi_a(0)\rangle|\varphi_b(0)\rangle$, we apply a pair of opposite squeezing operations $\hat{S}_a\hat{S}_b$ and $\hat{S}_b^\dagger\hat{S}_a^\dagger$ before and after the state evolves under (1) (see Fig. 1). This leads to the overall system evolution given as

$$|\varphi(t)\rangle = \hat{S}_b^\dagger\hat{S}_a^\dagger e^{-\mathrm{i}\hat{H}t}\hat{S}_a\hat{S}_b|\varphi(0)\rangle = e^{-\mathrm{i}\hat{H}_{\mathrm{eff}}t}|\varphi(0)\rangle, \tag{2}$$

where an effective Hamiltonian $\hat{H}_{\mathrm{eff}}$ is obtained via substitutions $\hat{a} \mapsto \hat{S}_a^\dagger\hat{a}\hat{S}_a$ and $\hat{b} \mapsto \hat{S}_b^\dagger\hat{b}\hat{S}_b$ in $\hat{H}$ [5]. We take $\hat{S}_c^\dagger\hat{c}\hat{S}_c = r_c\hat{x}_c + \mathrm{i}r_c^{-1}\hat{p}_c$ with $\hat{x}_c = (\hat{c}+\hat{c}^\dagger)/2$, $\hat{p}_c = (\hat{c}-\hat{c}^\dagger)/2\mathrm{i}$, and field gain $r_c \geq 1$ for $c \in \{a,b\}$. As a result, the effective Hamiltonian is

$$\hat{H}_{\mathrm{eff}} = -2gr_b(r_a^2\hat{x}_a^2 - r_a^{-2}\hat{p}_a^2)\hat{x}_b - 2gr_b^{-1}(\hat{x}_a\hat{p}_a + \hat{p}_a\hat{x}_a)\hat{p}_b = -2\tilde{g}\hat{x}_a^2\hat{x}_b + \mathscr{O}(r_a^0 r_b^{-1}) + \mathscr{O}(r_a^{-2}r_b) \tag{3}$$

with $\tilde{g} = r_a^2 r_b g$. Assuming $r_c \gg 1$, the time evolution under $\hat{H}_{\mathrm{eff}}$ can be approximately solved in the Heisenberg picture to give

$$\hat{x}_a(t) = \hat{x}_a(0); \qquad \hat{p}_a(t) = \hat{p}_a(0) + 2\tau\hat{x}_a(0)\hat{x}_b(0) \tag{4}$$

$$\hat{x}_b(t) = \hat{x}_b(0); \qquad \hat{p}_b(t) = \hat{p}_b(0) + \tau\hat{x}_a^2(0) \tag{5}$$

with a normalized interaction time $\tau = \tilde{g}t$. Notice that (5) implies that the pump quadrature operator $\hat{p}_b$ experiences conditional displacement depending on the value of $\hat{x}_a^2$. Note that $[\hat{H}_{\mathrm{eff}}, \hat{x}_a^2] \approx 0$ ensures $\hat{x}_a^2$ remains constant during

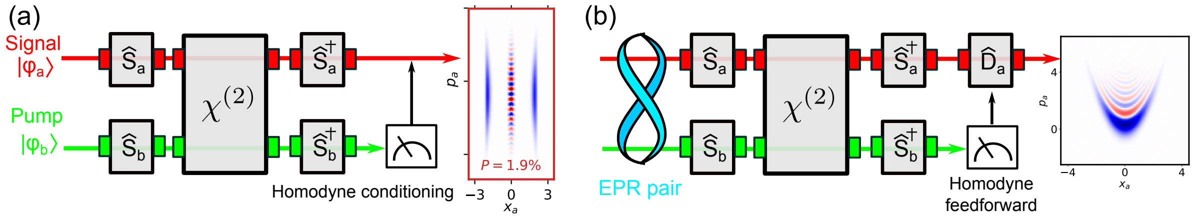

Fig. 1. (a) Conditional generation of a squeezed Schödinger's cat state and (b) deterministic generation of a cubic phase state using the nonlinear dynamics of a quadratic OPA.

the system evolution, enabling us to perform a QND measurement of squared quadrature $\hat{x}_a^2$ by measuring $\hat{p}_b$, e.g., via a p-homodyne measurement. The overview of the QND measurement protocol of squared quadrature $\hat{x}_a^2$ is illustrated in Fig. 1(a), where we show the efficacy of our scheme by generating a squeezed cat state with the success probability of 1.9%.

Next, we expand our scheme for the deterministic generation of cubic phase state. The overview of our protocol is illustrated in Fig. 1(b). As an initial state, we consider an EPR-pair with correlation $\hat{x}_a(0)-\hat{x}_b(0)\approx 0$ and $\hat{p}_a(0)+\hat{p}_b(0)\approx 0$. By virtue of (4) and (5), we can solve for the dynamics of the signal quadrature operator as

$$\hat{p}_a(t)=\underbrace{\tau(2\hat{x}_a(0)\hat{x}_b(0)+\hat{x}_a^2(0))}_{\approx 3\tau\hat{x}_a^2(t)}+\underbrace{(\hat{p}_a(0)+\hat{p}_b(0))}_{\approx 0}-\underbrace{\hat{p}_b(t)}_{\mapsto p_b},$$

where we can approximate the first term and the second term as $3\tau\hat{x}_a^2(0)\approx 3\tau\hat{x}_a^2(t)$ and 0, respectively. After propagating through the $\chi^{(2)}$ nonlinear medium, we perform the p-quadrature measurement on the pump mode, which collapses the third term to a real number p_b. As a result, applying signal p-displacement operation to compensate for this change, we can deterministically enforce $\hat{p}_a(t)=3\tau\hat{x}_a^2(t)$, which indicates that the final signal state becomes a cubic phase state.

Our scheme offers significant advantages over traditional quantum state engineering protocols. Generally, the purity of the resultant state in measurement-based schemes is critically limited by the overall quantum efficiency (QE) of the detectors. This issue is particularly severe in PNR-based schemes, where the imperfect QE of the PNR detectors degrades the purity of the heralded state. On the other hand, it is possible to mitigate the imperfect QE for quadrature measurements, e.g., homodyne measurements, by pre-amplifying the signal using optical parametric amplifiers [6]. In fact, our QND measurement scheme inherently involves such pre-amplification technique as the second-stage pump squeezing operation $\hat{S}_b^\dagger$ before the homodyne measurement, shown in Fig. 1. In addition to being loss-tolerant, OPA-assisted homodyne measurements can be performed with high speeds at room temperature, thereby circumventing the constraints inherent to PNR-based quantum state engineering protocols.

Recent experiments in $\chi^{(2)}$ nonlinear nanophotonics have made significant progress toward the strong photon-photon coupling regime. Using high-Q micro-ring resonators, $g/\kappa\sim 0.01$ has been achieved in nanophotonic platforms such as thin-film lithium niobate (TFLN) and indium gallium phosphide [7]. With further advances in the fabrication techniques that enable material-absorption-limited loss, $g/\kappa = 0.1-1$ could be envisaged. Beyond the conventional continuous-wave devices, $g/\kappa > 40$ might be possible by leveraging the three-dimensional confinement of optical fields using ultrashort pulses [8]. These numbers suggest that the experimental realization of our scheme might be within reach in the next-generation $\chi^{(2)}$ nanophotonics.

In conclusion, we proposed and analyzed the squared-quadrature QND measurement with optical parametric interactions for generating cat states and cubic phase states. Our scheme exploits significantly stronger quadratic nonlinearity compared to existing hybrid techniques that combine cubic nonlinearity and homodyne (heterodyne) measurements, offering an experimentally viable route to ultrafast quantum state engineering.

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