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Teachers creating mathematical models to fairly distribute school funding

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ABSTRACT

Our study aims to investigate what teachers do as they draw on their mathematical understanding and personal experiences to engage in social justice-oriented mathematical modeling. We analyze what ideas were expressed by teachers regarding their mathematical identities while they explore, wrestle with, and reconcile the underlying societal values that support mathematical models. We invited groups of teachers to make mathematical models for distributing school funding given real data from diverse, anonymized schools. Our results show that teachers created and refined diverse mathematical models to connect the mathematical world and societal space and these models reflected different societal values. Drawing on their own experiences, teachers expressed a sense of agency and critical consciousness while making decisions about school funding. This study delineates mathematical contents and processes necessary for advancing a societal goal of fairly distributing funds and we explore how teachers connect to this context as learners and members of society.

1. Introduction

There has been a growing consensus that mathematical modeling can be used to invite students and teachers to use mathematics as an analytical tool for challenging inequities and exploring societal issues (Aguirre, Anhalt, Cortez, Turner, & Simic-Muller, 2019; Barbosa, 2006; Cirillo, Bartell, & Wager, 2016). Mathematical modeling is a process of designing a representational system that can be used to interpret and solve a real-world problem (Lesh & Doerr, 2003). Researchers have presented the importance of teachers experiencing modeling and its processes as learners (Biembengut & Hein, 2013). By experiencing the modeling cycle, they come to understand what modeling entails and see the value of modeling as a tool to empower their students as knowers and doers of mathematics (Anhalt & Cortez, 2016; Cai et al., 2014). Teachers are one of the main agents for changes in the classroom (Biembengut & Hein, 2013); in fact, after experiencing modeling tasks, teachers often want to share this powerful learning experience with their students (Ikeda & Stephens, 2021; Son et al., 2017; Stohlmann et al., 2015).

Many of the processes involved in mathematical modeling overlap with teaching math for social justice such as wrestling with complex, real-world problems, exploring different solutions to a problem with respect to students' perspectives, and supporting students as competent knowers and doers of mathematics (Cirillo et al., 2016). There is often an inherent tension for teachers in enacting mathematical tasks focused on societal issues (Bartell, 2013; Turner et al., 2012; Wager & Stinson, 2012) including visualizing social justice issues as related to the teaching and learning of mathematics (Rodriguez, 2005; Simic-Mueller, Fernandes, & Felton-Koestler,

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2015) and determining how to create and facilitate appropriate tasks (Aguirre, 2009; Aguirre, Zavala, & Kantanyoutant, 2012). Built on these previous studies, an important step in advancing societal objectives through mathematical modeling in schools is to provide opportunities for teachers to engage in social justice-oriented mathematical modeling. This allows room for them to learn about and explore the powerful role that mathematics has in exploring societal issues and making decisions about social justice problems and the role of modeling as a vehicle to accomplish these goals. Regardless of such needs, researchers recognize that much work is needed to provide teachers with appropriate learning experiences and relevant resources for engaging students in mathematical modeling and social justice (e.g., Anhalt et al., 2017; Jung & Magiera, 2021; Tate, 1994; Turner & Strawhun, 2007).

In our project, we addressed this issue by exploring what teachers do to connect mathematics and societal challenges through a newly designed social justice-oriented mathematical modeling (SJMM) task. A task is classified as an SJMM task when its problem is situated in a context that addresses micro- and macro-level social justice issues, requires the development of a mathematical model, and leads to the results that can be shared with a broader audience who cares about the issue (Jung & Magiera, 2021). When we developed an SJMM task, we were curious as to how teachers would address the societal challenges with mathematical models and express their beliefs and dispositions toward their ability to address the societal challenges. We hypothesize that when teachers engage in an SJMM task, they are able to share facets of their mathematical identity, develop critical consciousness through the task, and understand why these matter in the teaching and learning of mathematics. Mathematical identity is the dispositions that learners construct about their ability to participate in mathematics and use it across their lives (Aguirre et al., 2013). Critical consciousness is one's ability to understand the macro-level social justice systems (e.g., the social structure that created inequity in the first place) and to develop a sense of agency (Freire, 1970). Numerous studies focus on students' development of mathematical identity and critical consciousness (e.g., Aguirre et al., 2013; Cabrera et al., 2014; Ginwright, 2009; O'Connor, 1997), but to the best of our knowledge, there is no study around teachers' development of these dispositions and its connection to mathematical models they create. Thus, we explored the following research questions:

- What do teachers do to connect the mathematical space and societal challenges through social justice-oriented mathematical modeling?
- What ideas were expressed by teachers regarding their mathematical identities and critical consciousness while they explore, wrestle with, and reconcile the underlying societal values that support mathematical models?

2. Equity and social justice in mathematics education and modeling

The Standards for Preparing Teachers of Mathematics proposed five primary assumptions about mathematics teacher preparation based on the emerging consensus of mathematics teacher educators (Association of Mathematics Teacher Educators, 2017). The first assumption is that a focus on equity, diversity and social justice should be addressed in its own right and integrated into all standards. This assumption is built on multiple recommendations that every student regardless of their cultural and linguistic backgrounds should receive access to quality mathematics instruction; and equity requires taking into consideration diverse contexts and the needs of each learner, rather than offering identical learning opportunities to all (NCTM, 2000, 2014). Researchers also found that equity goes beyond instructional quality and there are multiple aspects of equity (Gutiérrez, 2012; Nasir & Cobb, 2002). One of the aspects is access to "the tangible resources that students have available to them to participate in mathematics" (Gutiérrez, 2012, p. 19). This access includes high-quality teachers, supplies in the classroom, and curriculum resources. Gutiérrez argues that even if students have access to high-quality instruction, resources, and support, equity is not achieved if society does not change the views of mathematical fields and their connection to the mathematical access to marginalized students. This perspective is built on the idea that mathematical learning is a practice that reflects the priorities and values that decision makers bring to the table in society. This notion of power and domination is closely related to the role of mathematics in addressing social and political issues in the world.

Mathematics educators are showing a growing interest in supporting mathematics learners' development of critical consciousness (Kokka, 2020) - the ability to analyze the systems that create inequity and to develop a sense of agency (Freire, 1970). In the context of working with adult workers in Brazil, Freire (1970) found that inequity is sustained when the individuals most affected by it are unable to interpret the social structure that creates and sustains the injustice. Researchers also studied critical consciousness of younger learners and reported that critical consciousness expanded students' commitment to challenge unfairness (Ginwright, 2009) and increased their academic engagement and achievement (Cabrera et al., 2014; O'Connor, 1997). Gutstein (2016), one of the key contributors who connected Freire's humanizing pedagogy with mathematics education, built on students' life experiences to support their development of critical consciousness in his mathematics classroom. In the context of teacher education, an increasing number of mathematics teacher educators are working with teachers to explore issues of equity, fairness, and justice through mathematical tasks (e.g., Bartell, 2013; Felton-Koestler, 2020; Simic-Muller et al., 2015) and, specifically, through mathematical modeling tasks (e.g., Aguirre et al., 2019; Cirillo et al., 2016; Jung & Magiera, 2021). The goal is that, through modeling, teachers and students grow to understand that they are capable of using mathematics as a tool to make sense of and act on issues that arise in their world.

3. Social justice-oriented mathematical modeling with teachers

Few studies have explored engaging teachers in mathematical modeling tasks with an emphasis on social justice (e.g., Aguirre et al., 2019; Felton-Koestler, 2020; Jung & Magiera, 2021; Seegmiller, 2020). The most notable example is the work done by Aguirre et al. (2019) who created a figure that depicts a conceptual interaction between modeling and social justice. This conceptual interaction shows the process in which *learners* engage when they participate in a social-justice-oriented mathematical modeling task. Another conceptualization illustrates what *problem posers* consider when they design a social justice-oriented modeling task for teachers (Jung & Magiera, 2021). In this section we synthesize these two conceptualizations and explain how they guided our study.

Aguirre et al. (2019) introduced a social-justice-oriented modeling task, the Flint Water Task, and utilized it to build the conceptual framework shown in Fig. 1 below.

The figure connects two domains, one with social justice and another with modeling process. During the process, the modelers are engaged in learning about the specific situation and determining a way to utilize mathematics to investigate claims. As they engage in the modeling cycle and formulate a response, they come back to their situation and are able to evaluate it with their model in mind, building greater awareness of the issue. Regarding social justice, the Flint Water Task is grounded in broad environmental justice issues, especially around how to provide drinking water to the people of Flint, Michigan who experienced a water crisis related to an aging plumbing infrastructure. Several national beverage companies suggested sending 6.5 million bottles of water for local school children to use throughout the school year. Teachers were asked to consider if this plan was good and if the needs of the children would be met. The teachers generated models to investigate how much water a child might use during the day and for what purposes. As they created this model, they were able to circle back and evaluate the beverage companies' plans and also gain greater awareness of water issues by examining their own needs.

From their work with teachers, Aguirre et al. (2019) discussed three evidenced-based themes related to teacher learning. First, the teachers recognized that modeling elicited rich and diverse mathematical explorations and allowed them to use mathematics as a tool to understand the specific water crisis. Second, modeling "increased and yielded opportunities for future mathematical modeling problems" (p.15). Once the teachers were engaged in the process of determining a solution they became invested in the situation and created additional modeling questions like "What will be done to help the people who are ill from drinking the water?" Third, the task increased teachers' awareness of systemic injustices as contexts for modeling problems and most were able to identify and envision a modeling task related to a justice-related issue in their local communities. To engage teachers in this important work, Aguirre et al. (2019) also provided recommendations for teacher educators looking to create social justice modeling tasks. Mathematics teacher educators must be aware of current events to understand what might be important issues for teachers to explore.

Drawing on Aguirre et al. (2019) work that focused on the cycle that problem solvers engaged in, Jung and Magiera (2021) proposed a conceptual framework that *problem posers* consider when creating a social-justice-oriented mathematical modeling task. Fig. 2 shows a conceptual interaction that connects posing a modeling task with the macro- and micro-level social justice issues.

We noticed that Fig. 2 centers social justice in the center of the three outer circles with each circle focusing on the key features of mathematical modeling, whereas Fig. 1 showed two separate domains (i.e., social justice and modeling) connected through four components (i.e., specific situation, make sense of the situation, validate outcome, and report out). These two frameworks guided our current work in two ways: (a) posing a social justice-oriented mathematical modeling task for teachers (i.e., School Funding Task

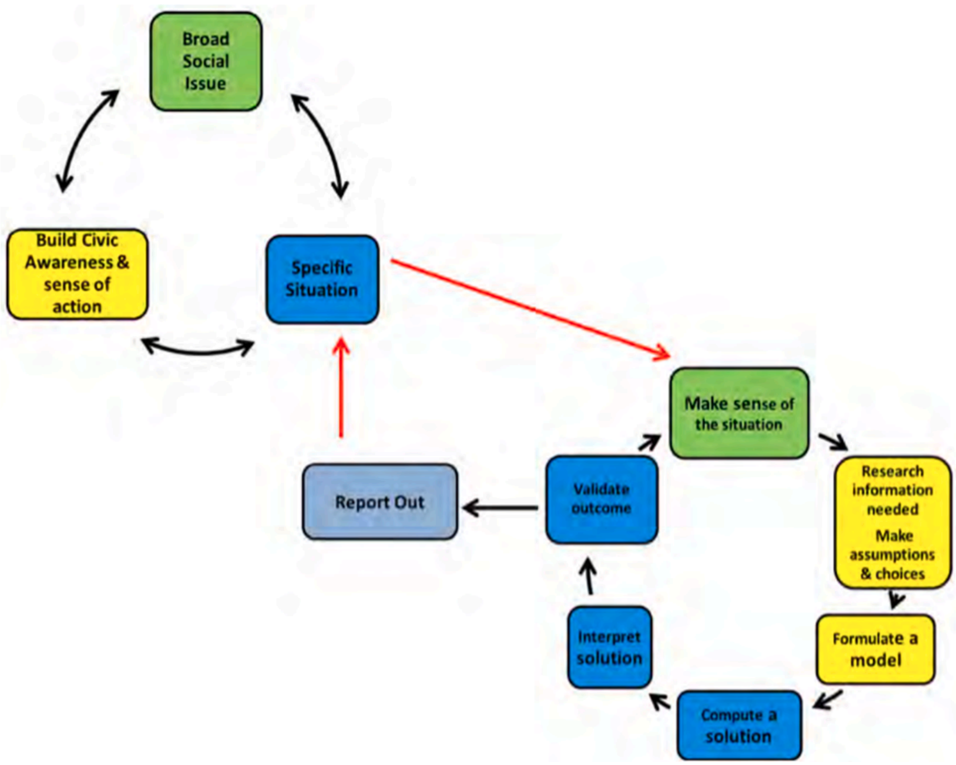

Fig. 1. A conceptual interaction between modeling and social justice (Aguirre et al., 2019, p. 10).

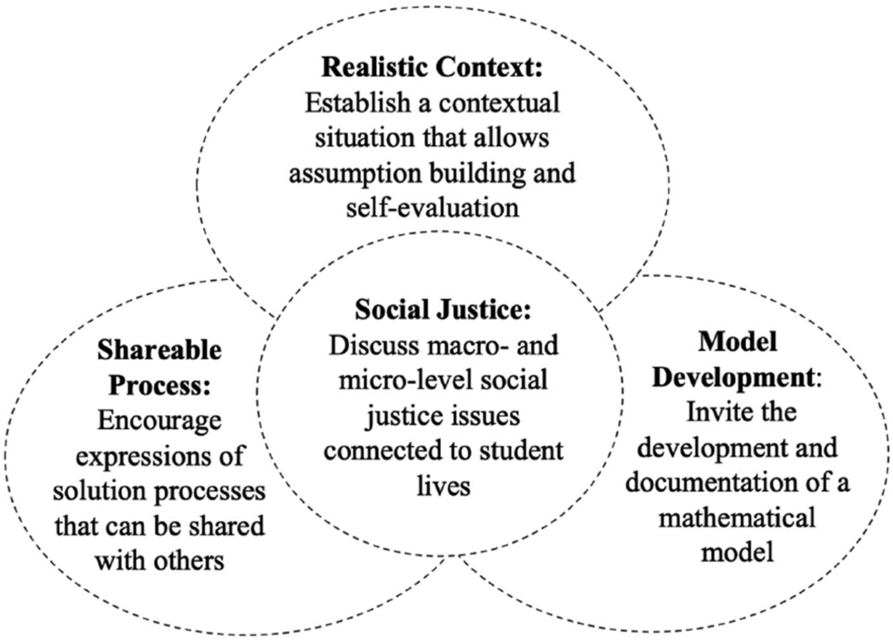

Fig. 2. A conceptual framework that guides problem posing around social justice-oriented mathematical modeling (Jung & Magiera, 2021, p. 4).

Module) and (b) analyzing teachers' modeling process connected to social justice issues. When we developed the School Funding Task, we thought deeply about the components in both Fig. 1 and Fig. 2. In particular, we wanted a task that would draw teachers in. School funding is something that US teachers recognize as problematic but often feels out of their control. By working through the task, our goal was to help them build an understanding of this societal issue and empower them as agents of change. The first figure helped us to organize how the teachers constructed models, the values these models reflect, and also how the teachers connected their models to societal issues. In our study we build on and extend these studies by exploring what ideas teachers express regarding their critical consciousness and mathematical identity while engaging in social-justice-oriented mathematical modeling that invites them to fairly distribute funding to schools with diverse societal factors.

4. Conceptual framework: socio-critical modeling and modeling process

Our work is situated within the socio-critical modeling perspective that focuses on the role of mathematical modeling as a tool to investigate everyday life and societal problems (Barbosa, 2006). The foundation of the socio-critical modeling perspective is built on Skovsmose's (1994) critical mathematics education and Freire's (1970) work that supports humanizing pedagogy. In this perspective, educators listen to their learners and build on their experiences and knowledge to engage in contextualized situations that confront social issues and develop critical consciousness for social transformation (Salazar, 2013). The socio-critical modeling perspective serves dual goals, one within mathematics and the other in the societal space. *Within mathematics*, the perspective supports how mathematical models can be used to make decisions. In the *societal space*, the context involves social issues, including social inequalities and injustices. Through critical mathematics and humanizing pedagogy, learners can recognize the role of mathematics as a tool to critique unfair aspects of society and propose methods to create a more just world (Gutstein, 2016). Because mathematical space and societal space are closely intersected in this socio-critical modeling perspective (Barbosa, 2006; Jung & Magiera, 2021; Jung & Brand, 2021), we adapted a modeling cycle (Bliss et al., 2014) that has a potential to delineate such connection.

Bliss et al. (2014) described the components of the modeling process, including (a) defining the problem, (b) making assumptions, (c) defining variables, (d) research and brainstorming, (e) building the mathematical model, and (f) getting a solution (pp. 6–7). We hypothesized that each of these components could be connected to social justice-oriented issues when teachers engaged in a social-justice mathematical modeling task.

(a) Defining the problem: When learners define the problem, they would define both mathematical problem and social justice problem;
(b) Making assumptions: Assumptions would be made based on their societal knowledge and understanding of mathematical data presented in the problem;
(c) Defining variables: Learners will define variables based on the assumptions connected to societal and mathematical spaces;
(d) Research and brainstorming: When learners research and brainstorm ideas, they would critique existing ideas in relation to their own understanding and values;
(e) Building the mathematical model: The mathematical model that they develop will be used to solve societal issues;

(f) Getting a solution: The solution integrates both mathematical and social justice concepts.

In Fig. 3 below, we depicted the connections between the modeling process and social-justice issues through one of the modeling modules, titled School Funding Modeling task, which we describe in detail in the methods section.

Ladson-Billings (1998) stated that "inequality in school funding is a function of institutional and structural racism" (pp. 20). In addressing this systemic racism, providing all schools with equality of inputs, such as the same amount of funding and the same instruction of learning, is different from providing all students with accommodations that promote access to overcome opportunity gaps (Flores, 2007; Gutiérrez, 2013; NCTM, 2000). In our school funding modeling module, we provided an opportunity for our learners to utilize mathematics to analyze inequality in school contexts and construct an alternative model to fairly distribute school funding. Using our framework (shown in Fig. 3), we share more details about how the participating teachers drew on their mathematical knowledge to address a societal issue when they engaged in the school funding modeling module.

We also hypothesized that our teachers would express mathematical identities and critical consciousness when they explore, wrestle with, and reconcile the underlying societal values that support mathematical models. Aguirre et al. (2013) defined mathematical identity as the dispositions that learners "develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives" (p. 14). Identity addresses not only how learners connect mathematics to the real-world defined by textbooks, but also to contexts that are meaningful to their lives (Gutiérrez, 2012). Learners often bring their cultural capital to classrooms; and it is a teacher's role whether they use it to stimulate mathematics learning or ignore their cultural capital (Averill et al., 2009). As teacher educators, we believe that teachers bring their cultural capital to classrooms as their students do. In the School Funding tasks, teachers bring a diverse set of cultural capital to the task in that they currently work in a school and are usually products of the school system. We hypothesize that when teachers engage in social justice-oriented mathematical modeling, they will connect their personal experiences with the societal context and use mathematics to solve a problem. This will lead to the development of their critical consciousness and mathematical identity.

The ability to participate in both mathematical and real-world contexts can be captured by the term agency (Aguirre et al., 2013). One of the key aspects of the agency is the critical mathematical agency, which is learners' capacity to identify themselves as mathematical thinkers who develop meaningful mathematical understanding in connections to their personal and social lives (Turner, 2003). This conceptualization of agency emphasizes that learners are active participants in their mathematics learning, rather than passive recipients of knowledge. They can develop these forms of agency when collaboratively constructing mathematical strategies while utilizing mathematics as a tool to investigate the real world (Aguirre et al., 2013). Mathematical modeling can be used to engage students in the collaborative development of mathematical strategies (e.g., mathematical models) that can be used to interpret real-life events. Through the socio-critical modeling perspective, we aimed to investigate how teachers utilize mathematics to address societal challenges, as well as the critical moments they used mathematics to consider societal values and make decisions about the social justice problem.

5. Methods

This study is qualitative and we consider it a paradigm case in that we can gain insights into how development of identity and critical consciousness are intertwined with the ways teachers create mathematical models and apply this knowledge to teaching. Generalizability is not the intent of this type of work. Instead, we consider this work a case of what is possible when teachers are enrolled in a designated modeling course designed and enacted based on the conceptual framework that connects socio-critical modeling and the modeling process described in our previous section.

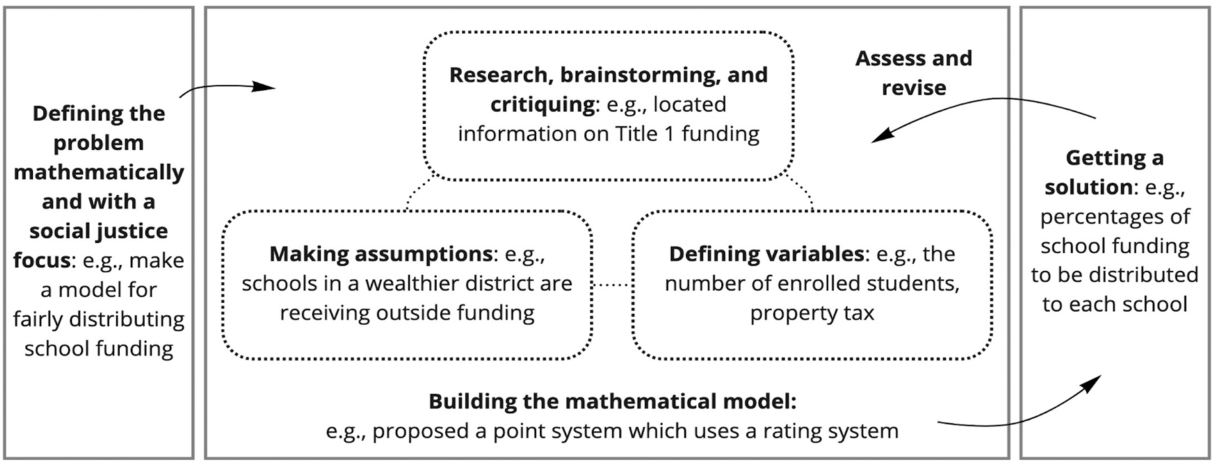

Fig. 3. Potential modeling process in social-justice-oriented mathematical modeling: the case of the School Funding Modeling task. Adapted from Bliss et al. (2014, p. 6).

5.1. Setting and Participants

Data was collected from an NSF-funded project that aimed to design, implement, and refine mathematical modeling modules in teacher education programs at two institutions. Both authors in this study are modeling researchers with multiple years of experience working alongside K-12 students and teachers. As part of the project, the two authors designed and enacted four mathematical modeling modules that they tested and refined, as instructors, in designated modeling courses. Participants include two groups of 21 K-12 preservice and in-service teachers at universities located in the Rocky Mountain West and Southeastern regions of the U.S. The modeling task was enacted in the Rocky Mountain West and in the Southeast. All participants were enrolled in the designated courses on mathematical modeling for teaching and had experienced several modeling tasks prior to engaging with this one. A total of 21 participants were enrolled in the courses and all consented to be in the study. Eleven of the participants were from the Rocky Mountain West and were pre-service K-8 teachers who had completed all required mathematics coursework and were taking the course as an elective. Most of the pre-service teachers were from the same state where the university is located. The remaining nine participants were a group of in-service mathematics teachers and pre-service teachers. A teacher lived in South Carolina, another lived in Alaska, two lived in East Asia, and five lived in Florida. In each class, the participants worked on the task in groups of two or three resulting in 9 groups total for analysis.

5.2. School Funding Task Module and Data Collection

The purpose of the school funding model is for teachers to think about how to distribute funds based on many different factors. The task typically takes about one or two weeks of instruction (e.g., three 50-minute class periods) so that teachers have enough time, both during and outside of class, to propose, discuss, and make revisions to their models. The authors designed the task to help teachers connect mathematical ideas about fractions and division to a real world situation to answer the question "Does equitably dividing the funds mean equally dividing the funds?".

To begin the task, teachers watched a video on school funding in the United States and were asked to reflect on the following questions:

- Why does school funding matter to students?
- In your opinion, how should the school funding system be changed?

This allowed teachers to share their experiences as both students and teachers and how funding has affected their learning.

Next, teachers were given the following table (see Table 1) and prompt:

Imagine you are working for your state's Department of Education and that you are in charge of formulating a procedure for distributing funding to these schools. Data which can be considered for this procedure has been provided by the department.

We asked teachers to examine the data and describe what they noticed and wondered about it. The data in the table was drawn from actual data from diverse, anonymized schools.

Next, we invited the teachers to make a mathematical model for distributing school funding. We asked them to reflect on the following prompts as they constructed their models:

- What assumptions are you making in order to create your model?
- What do you think are important factors to consider? Why?
- What factors are you not considering? Why?
- What are important variables and numbers to consider? Which ones do you think will change and which will remain constant?
- What choices are you making? Why do they seem appropriate in this situation?
- What resources will you use to inform your model? Why?
- What visual representations (e.g., pictures, graphs) might help the department of public instruction understand your model?
- What mathematics does your model rely on? How did you use mathematics to describe the situation and solve your problem?

Table 1
School Information.

School	Apple Tree	Blue Mountain	Cool Valley	Deer Creek
Grades	PK-8	PK-6	PK-8	3–8
Number of Students Enrolled	969	516	504	320
Estimated Fraction of Total Students which are Emergent Bilingual Students	1/3	1/6	1/7	1/20
Estimated Fraction of Total Students with Special Needs	1/7	1/25	1/6	1/26
Percentage of Students Meeting Standards in Math	13%	65%	30%	59%
Percentage of Students Meeting Standards in English	16%	61%	34%	57%
Percentage of Students Meeting Standards in Science	18%	71%	21%	68%
Percentage of Students which are Low Income	90%	22%	75%	12%
Student-to-Teacher Ratio	14:1	16:1	16:1	11:1
Median Home Cost	$123,100	$365,400	$140,400	$330,330
Property Tax Rate per $1000	$14.3	$25.1	$15.1	$22.8

We chose this task for the paper because teachers wrestled with and were passionate about the underlying value of "fairness" and if and how different models fairly distributed school funding. School funding models in the United States are varied by state and funding is typically a mix of local, state, and federal dollars. Many states allocate a minimum dollar value for each student (i.e, $10,000 per student per year) and determine that a certain percentage of this dollar value will come from the state with the remaining value to be covered by local taxes. In areas where property values are more expensive, the same percentage of taxes yields different dollar values, so it is often easier for these areas to meet the minimum dollar value needed for each student. Other states might use a centralized model where taxes are collected across districts and the state is able to allocate money based on need so students in different areas receive similar funds. Determining school funding is challenging and often contentious in that taxpayers do not always agree with how they are taxed and how the funding is utilized by state and local governments.

The teachers engaged in the following process for the modeling module:

- brainstorm ideas for the problem;
- design rough draft of solutions;
- receive instructor feedback;
- design final draft of solutions;
- wrote reflections on the solution process; and
- present their processes and solutions.

We first introduced a modeling activity and provided the opportunity for teachers to brainstorm and share ideas through small group and whole class discussions. The whole class discussion at the beginning enabled teachers to revise their initial ideas about how they would proceed with the problem. In small groups, teachers designed their rough draft (Jansen, 2020), received peer feedback, and revised their rough draft based on peer feedback. They also received instructor feedback and designed a final draft based on the collective ideas gained from the whole class. In the final draft, they proposed their solution and discussed changes they made and why. Following task implementation, the teachers were given journal prompts to reflect on the modeling process, individually, that addressed the context of the problem, utility of the task, emotions/self-efficacy, social/group work, and reflection on the process overall. Throughout the process, we found that the teachers had to justify the mathematics they employed through descriptions of fairness and what it means to be fair. In this paper, we focus primarily on the last three processes that teachers engaged in, specifically, their final drafts of solutions, written reflections of the process, and video recordings of their presentations. Our intent is not to explore how teachers refined their models but to analyze how the finalized models connect to social justice issues.

6. Data analysis

Driven by the mathematical modeling process in the context of social justice issues (Aguirre et al., 2019; Bliss et al., 2014; Jung & Magiera, 2021) and the concepts of mathematical identity and agency (Aguirre et al., 2013; Turner, 2003), we began by analyzing the teachers' final models and journal reflections in conjunction with classroom videos of the final presentations. We intended to make sense of our first research question through understanding the assumptions, their rationales, and the mathematics the teachers used to make sense of the problem and develop a model. For each of the nine groups we analyzed their models (drawn from their written work and transcripts of their oral presentations) by asking the following questions (See Table 2):

After we completed this analysis across all groups, we individually sorted the groups by types of models that each group of teachers developed. To establish validity, we discussed our coding until we reached a consensus on each of the parts of the modeling process, the

Table 2
Questions to guide analyzing the modeling process and rationales.

Modeling Process	Guiding Questions for Data Analysis
Defining the Problem Mathematically and with a Social Justice Focus	• How did teachers conceptualize the problem mathematically and with a social justice focus? • How did their personal experiences and values inform these conceptualizations?
Research, Brainstorming, and Critiquing	• What resources did the teachers consult to better understand the problem and conceptualize their model? • How did they critique existing resources?
Making Assumptions	• What assumptions did they make about the data (schools, teachers, community)? • Which data did they include/not include and why? • How did their personal experiences inform these assumptions?
Defining Variables	• How did they define variables in relation to the data? • How did their personal experiences inform their choice of variables?
Building the Mathematical Model	• What type of model did they use to understand the social-justice situation? • How did the model reflect the assumptions they made about the situation? • What ideas about fairness did this model elicit?
Getting a Solution	• How did the teachers award funding? • What reasoning, both mathematical and social-justice oriented, did the teachers use to support their decision-making?

types of models created, and their rationales. Teachers created the following types of models to understand the situation and often drew on more than one to make sense of the situation (Table 3).

Two groups used a rating and ranking system, four groups used a base rate per student, and three utilized redistribution paired with a secondary model to determine how to distribute funds. For this study, we narrowed our data down to three cases (Group 2, Group 7, and Group 8) to represent the types of models the teachers, overall, created. We chose these cases because, of the models created, they contained the clearest description of the process utilized.

In teachers' journals they were given prompts related to their perceptions of the modeling process, the utility of the modeling task, and their development of knowers and doers of mathematics. Their written journal entries gave us insights into how they connected the task to their lived identities and developed critical consciousness. We used prior studies that discuss frameworks to guide analyses of teachers' journal reflections (Aguirre et al., 2013; Borromeo Ferri, 2017; Jung & Magiera, 2021). Specifically, we looked for dispositions that teachers engage in as they are building critical consciousness including reflection, agency, analysis, and action. Each of us examined the teachers' journals, looked for instances of each of these, and made a consensus on our interpretations (See Table 4).

In the paper, we share examples of these dispositions in each of the cases we chose and, at the end of the results section, we share parts of the modeling cycle that were important in eliciting conversations about and developing critical consciousness.

7. Results

In this section we illustrate what teachers do to connect the mathematical space with societal challenges through mathematical modeling. We found that each group of teachers created a model that involves their own unique approaches to interpreting and representing the social justice situation. Although some models had similar approaches, no two models were identical. Each of the models reflects different societal questions and values that the groups had to wrestle with and address. In the rating system, money is not given to individual students. It is up to the school to determine how funding is distributed once it is received. Teachers wrestled with whether schools would use the money appropriately without allocating it to a particular student. In the second model, base rate per student, money is allocated per student. Teachers wrestled with whether smaller classrooms, with fewer students, would have equitable opportunities and if schools with less local funding would be able to meet the base rate. In the last model, redistribution, excess money, primarily from taxes, is redistributed across schools based on need. Teachers wrestled with being taxpayers and if taxpayers would support money leaving their local area to benefit students in another area. Across models, the teachers had to wrestle with the idea of need and how to allocate funding fairly based on need. We illustrate the three major types of models that integrate mathematical and societal problems. We also describe how teachers went through the modeling process, using the theoretical lens described in Fig. 1 and their reflections that reveal their critical consciousness and mathematical identity.

8. First model: rating system

We start with the rating system model because several teachers used the rating system as a basis for their solution approaches. We use Group 2's work to illustrate the modeling process associated with the rating system. The teachers in Group 2 identified the problem as creating "a model or procedure for their district that distributes funds to the schools fairly." The teachers defined fairness in a way that stemmed from their understanding that "the schools that are doing poorly should receive more funding than the schools that are excelling." Their interpretations of the problems show that the teachers considered the problem both mathematically (e.g., creating a model for distribution) and with a social justice focus (e.g., what does it mean to be fair?) (*defining the problem mathematically and with a social justice focus*).

They made assumptions based on their understandings of society and personal experiences that include: "Schools in a wealthier district are receiving outside funding and may need less than schools in a less wealthy district," "Schools that are underperforming need more money for materials to help students perform" (*making assumptions*). The teachers also considered emergent bilingual students, students with special needs, and low-income students, and assumed that schools with a higher volume of these students require more funding for their needs. As shown in these teachers' rationales, assumptions they make about the schools and students influence the mathematical decision they make to distribute the funding.

The assumptions teachers made especially affected what variables they chose to consider: the number of emergent bilingual students, students receiving special education services, test scores, low income student percentage, and the number of students (*defining variables*). They stated that they selected test scores because they wanted to distribute more funding to those who are underperforming; they considered low income student percentages because "they will receive less outside funding." The teachers considered the number of students that are meeting standards. As pre-service teachers, this group had no experience as homeowners or paying property taxes,

Table 3
Types of key models used to solve the school funding modeling task.

Types of Models	Sample Approaches
Ranking/Rating System	Teachers either ranked or rated schools in relation to one another based on specified criteria from the data table. Sometimes they weighted parts of the data table to indicate importance.
Base Rate Per Student	Teachers started with a base rate per student and allocated additional funds based on varying criteria.
Redistribution	Teachers considered property tax income to understand how much money each school was allocated. Schools with excess funds were either not considered or their funds were redistributed to other schools.

Table 4
Description of Critical Consciousness Dispositions.

Dispositions	Description
Critical Reflection (Borromeo Ferri, 2017)	The teacher reflects on his/her awareness of and ways to address broader social and political contexts.
Critical Agency (Aguirre et al., 2013)	The teacher identifies himself/herself as a mathematical thinker capable of developing meaningful mathematical understanding and constructing powerful representations in relation to their lived experiences.
Critical Analysis (Aguirre et al., 2013)	The teacher seeks knowledge about the systems and structures that create and sustain inequities.
Critical Action (Aguirre et al., 2013)	The teacher discusses ways they could take action against oppressive conditions.

so they chose not to consider what criteria might mean in terms of the model.

As they brainstormed ideas while exploring the given data (*brainstorming*), the teachers found that Apple Tree and Cool Valley have significantly less students meeting standards than Blue Mountain and Deer Creek. At first, the teachers started by distributing 25% to each school and then redistributing funds as they seemed warranted (*building an initial mathematical model*). They said, "If we were to distribute each school with 25% initially and then took some money to fund programs at Apple Tree and Cool Valley each could have 30% and then Blue Mountain and Deer Creek would have 20%" (*assess*). They also researched the salaries of special education teachers and teachers of emergent bilingual learners, and average teacher salaries to see if they would need to allocate more funding for schools that have more students receiving special education services and emergent bilingual students (*research and brainstorming*). With the information, the teachers decided to allocate 10% more to Apple Tree by taking 5% from Blue Mountain, 4% Deer Creek, and 1% from Cool Valley (*building a revised mathematical model*) (see Fig. 4 below).

After feedback from peers and their instructors, the teachers determined they needed a structured system to help them justify how they allocated percentages. To do this, the teachers used a rating system to update their initial distribution (*assess and revise*). For example, the teachers assigned points for the variables they identified as important (e.g., emergent bilingual students, test scores, low income, the number of students enrolled) as shown in the table below. (Fig. 5).

They assigned more points to the schools that have higher percentages of emergent bilingual students, students who receive special education services, students enrolled, students with low income, and students with low test scores. Based on the points allocated for each school, the teachers provided the final version of the distribution (*building an updated mathematical model*) (Fig. 6).

After creating this model, the teachers reflected that they considered their model to be simpler than many of the models presented by their peers. With this opinion, it was simpler to add additional schools to the model, if needed; however, they recognized that once the funds were distributed, it might be more challenging for each school to determine how to distribute the funds within. As shown in this case, the rating system was not the first model that the teachers considered - they noticed a need for a structured system to justify choices made as they were considering social justice issues embedded in the task.

In reflecting on the task, the group discussed that when they entered the task they drew on their own values to identify certain criteria as more important than others, but it was challenging to determine if and how this could be translated into a mathematical model. One PST, Mary discussed that the ranking and rating system honored her values of what was important in the situation. She stated,

> It's hard when you consider multiple variables to find a process that honors those accurately, especially when you consider a variable more important than another. I think that when we started to get into the ranking and rating system was a pivotal moment… it forces students to use their emotional reasoning in conjunction with math reasoning. When we rank and rate we are picking things that are more important to us than others in order to accomplish this. Finding a mathematical process to justify this is super important to the success of this modeling task, and I think that it empowers students to make choices.

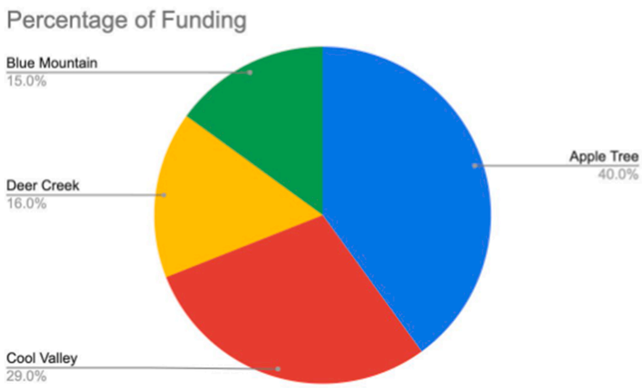

Fig. 4. Initial percentage of funding distributed to each school.

School	Apple Tree	Cool Valley	Deer Creek	Blue Mountain
ELL and SPED	8	6	2	4
Number of Students	4	2	1	3
Low Income Students	8	6	2	4
Test Scores	4	3	2	1
Total Points out of 60	24	17	7	12

Fig. 5. An example of a rating system.

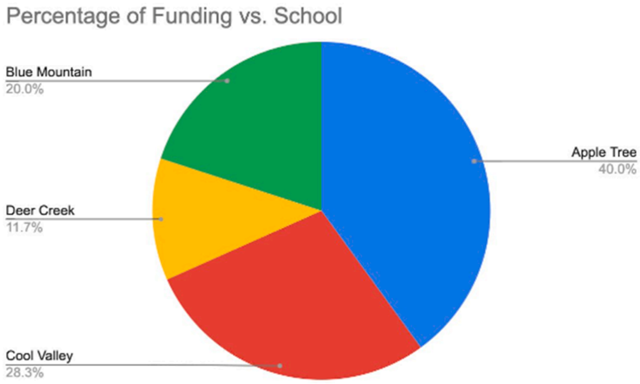

Fig. 6. The final percentage of funding distributed to each school.

Following the modeling task, both she and her partner felt more comfortable tackling modeling tasks and felt empowered to try out ideas and make choices that aligned with their knowledge and experiences. In this example, we can see that the student was building a *sense of agency* in that they are seeing mathematics as something they can use to help them make choices.

9. Second model: base rate per student and redistribution

Another model the teachers created was determining the base rate per student first and redistributing the remaining funding. We use Group 7's work to describe this model. Group 7 created a visual model that showcased a unique approach to representing multiple data sets. The teachers defined the problem as to "lay out justifications for a method by which this district can equitably allocate finite resources to help meet our educational goals.present visualizations of the data to support the conclusions that are made" This group had the district in mind considering it as an audience to present their conclusions. To present the visual models of data to the audience, they decided to lay out justifications for a method (*define the problem mathematically*) that allows an equal allocation (*define the problem with a social justice focus*).

They described three assumptions they made (*making assumptions*). First, based on their current residency, they assumed their funding sources were coming from their state's Education Finance Program. Second, they assumed that the federal Department of Education will continue to fund students who meet the income requirements. Third, they have built their assumptions on comparable property density for the school zones associated with the four schools in the district. As revealed from these three assumptions, the teachers made them as they were researching data from the state's Education Finance Program and property density for the school zones (*research*).

While looking into the given data table, the teachers made choices about which variables to consider (*defining variables*). They considered all the variables provided in the table including the number of students enrolled, estimated fraction of total emergent bilingual students, percentage of students meeting standards in math, English, and science, percentage of students who are low income, median home cost, and property tax rate per $1000. The teachers decided to use all except for the student-to-teacher ratios and the grades. They did not use student-to-teacher ratios because "the schools were found to have comparable student-to-teacher ratios." The grades were not also considered because "the grades served for Apple Tree and Cool Valley schools were identical and there were limited differences in the grades served between them." These mathematical decisions of defining variables were influenced by their interpretations of the societal issues, considering whether or not the student-to-teacher ratios and grades served for each school should be included when their values are similar across the schools.

As they were defining the problem, making assumptions, and defining variables, they looked into relevant resources. For example, teachers stated that they located information on the Individuals with Disabilities Education Act and Title 1 funding from the federal Department of Education websites and school grades and accountability from the state's Department of Education website. They utilized the data that was provided to generate their own school funding model instead of using an existing one (*research, brainstorming, and critiquing*). This process led to the mathematical model as shown below (*building the mathematical model*) (Fig. 7).

The visualization above illustrates the relationship between school enrollment, median property values, and the portion of home cost allotted to school funding, all of which concern societal issues. The teachers explained the model:

> Each school is allotted a rectangle proportion to enrollment. The largest rectangle corresponds to the highest enrollment school and the smallest rectangle corresponds to the smallest enrollment. Proportional circles were drawn on each school representing the median home cost and the colored circles inside those circles display the portion of the median home cost that schools can draw from. The larger circles indicate a higher median home cost and the smaller circles indicate a lower median home cost. Note that no information was given relating to home density so the total income from property taxes cannot be determined.

This initial model helped them make a conclusion that leads to an equation as described below (*getting a solution*).

> We concluded from the data that steps must be taken to help stabilize and equitably distribute property tax income between the four schools. We recommend setting a standard payment for each student and adjusting the property tax income to allocate the funding in this manner. Residual income for each school zone can remain in that school zone to avoid destabilizing the housing market. The most at-risk schools can then further their school funding from the federal sources mentioned above. Each school zone should calculate the property tax income based on median home cost multiplied by the property tax rate. Then the student enrollment for the entire district will be multiplied by the per student allocation. Subtract this sum from the property tax income to determine if the per student allocation is feasible. Any remaining funds can be placed in an account for the school in that zone to access through whatever process the district decides.

This process resulted in the percentage of school funding to be distributed to each school as Apple Tree (42%), Blue Mountain (22%), Cool Valley (22%), and Deer Creek (14%).

In reflecting on this task, one of the teachers in this group described that she was able to draw on her identity as a mathematics coach to help inform her colleagues of how Title 1 schools are funded. She described that she felt she contributed to her group by researching school funding and sharing her own perspectives working in a Title 1 school. She stated:

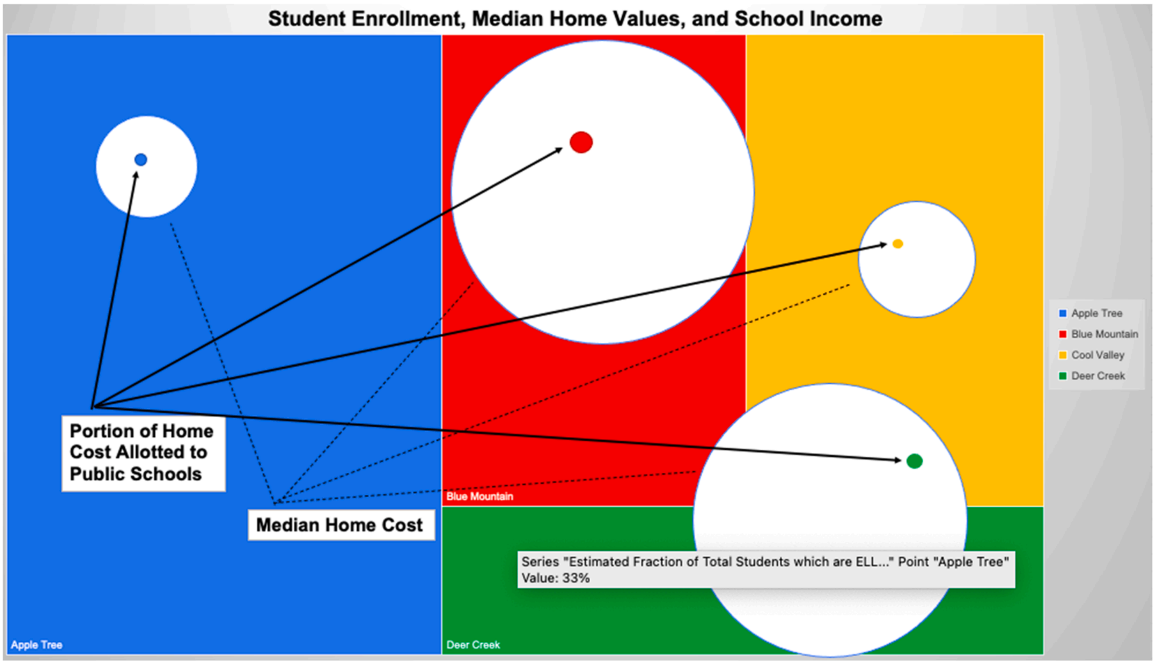

Fig. 7. Group 2's model describing school enrollment, median home values, and school income.

> I currently serve as a math content area specialist (math coach) and my position is funded by Title I to help close the achievement gaps for the low-income students at the school that I serve. I was able to help my group by providing information about Title I and ESSA funding for my group. They did not seem to be aware of the fact that low-income schools do receive additional funding to help close the achievement gaps. In addition, I was able to contribute to the group by providing information on how Florida distributes school funds and contributed to the various graphs available in the project.

She drew on her own knowledge of existing funding structures to help her colleagues in constructing a model. This teacher demonstrated *critical analysis* in that she drew on her own knowledge as a math coach and was motivated to learn more about how Florida allocates school funding. By understanding these structures, she could critique whether the existing system creates inequities.

10. Third model: base rate per student and rating

Next, we describe another model discussed by the teachers - determining the base rate per student and then using the rating system. We use Group 8's work to describe this model as they shared detailed calculations and processes in their work. They were the only group that did not allocate funds to one of the schools. The teachers identified the problem as thinking "of various ways funding could be distributed between the schools in the district in a 'fair' manner." They defined "fair" as each student receiving the same amount of funding ($8000) and they noticed that each school received varying amounts of money from property taxes (*defining the problem mathematically and with a social justice focus*). One of the teachers commented that she had attended a school that was poorly funded and realized, upon attending college, that her school experience was not universal. All students deserve fair funding.

When determining student needs in relation to property taxes, they made the assumption, which they acknowledged as false. There was only one child per household and all property taxes would go to education. When determining the points-based system, they assumed that the model is designed for the district in which they currently live, extracurricular activities at each school are being supported in some other way, and the model will follow the traditional distribution of a Foundation Grant (i.e., a non-profit organization created to support students and educators in the teachers' district) (*making assumptions while researching and brainstorming*).

They defined variables that included the number of enrolled students (transfers to and from occur throughout the school year), test scores from each school, median home value, and property tax. The teachers decided not to consider racial diversity because "more diverse schools may need other accommodations that may or may not fit into each budget". They also chose not to consider the physical location of each school because "if schools are part of different cultures, geographical location, or atmospheres in general, they may need more/less materials than others and it may or may not fit into the budget." Finally, they also decided not to consider "extreme circumstances that require funding (i.e., suicide rate, drug abuse, homelessness, etc.) As shown in their rationales, the teachers defined variables as they considered the various societal issues (*defining variables*).

The teachers shared the challenge of considering a large amount of information. They said, "At first, we weren't sure how to organize all of the information and numbers we came up with, but then we thought about using a points system for rating various categories that contributed to the amount of funding each school should get." They also researched how funding is distributed in other areas. A teacher mentioned, "Because we went to two different high schools and got two different experiences, we wondered how these different economies would receive "fair" funding based on our model. This is when we came up with the idea of setting a minimum per student. The property taxes would cover as much as they could to reach this minimum before the State stepped in with their funding (*research, brainstorming, and critiquing*)."

When calculating each of the school district's income from property tax, the group found that Apple Tree and Cool Valley had the largest gap between what was needed per student compared with revenue from property taxes. Blue Mountain had a surplus of funding per student, so the teachers made the decision to recapture that surplus and redistribute it to the other three schools based on need. They stated,

> Blue Mountain school was able to meet the needs of every student from property taxes alone. Any extra money that came from them got recaptured and redistributed to the other schools based on their needs found in the points breakdown. Schools like Apple Tree and Cool Valley have lower property tax rates and more of their students come from low income families. Their need for State funds or recaptured money from Blue Mountain was higher than Deer Creek's.

They proposed a points system that uses a rating system from 0 to 15 that establishes a weight for a given factor in relation to need and the evaluated factors to determine importance. Criteria like the number of students enrolled, the fraction of students with special needs, the fraction of English language learners, students who are low income were given higher priority (see Fig. 8 below). While factors like meeting content standards, student to teacher ratio, and the number of grades were weighted with less importance. For example, they provided 15 points (Max need amount) to 75–100% low income, ½ to ¼ fraction of emergent bilingual students, and 1/1 to ⅕ ratio of students with special needs while they provided 5 points (average need amount) to 25–49% low income, 1/11–1/15

Points Sytem			
15 (Max need amount)	75-100% low income	1/2 to 1/4 fraction of ELL students	1/1 to 1/4 SPED ratio
10 (Heavy need amount)	50-74% low income	1/5 to 1/10 fraction of ELL	1/5 to 1/10 SPED ratio
5 (average need amount)	25-49% low income	1/11 to 1/15 fraction of ELL	1/11 to 1/15 SPED ratio
0 (no need amount)	0-25% low income	1/16 of less fraction of ELL	1/16 or less SPED ratio

Fig. 8. Point System.

fraction of emergent bilingual students, and ⅕ to 1/10 ratio of students with special needs. These points were then added up to show the distribution of local funds (*building the mathematical model*). Fig. 9 below shows their model to determine state funding amounts based on local distribution.

When working on the first part of their model, the teachers also wondered if raising property taxes would help to balance the amount of funding per school. They noticed that when they raised the property taxes from 2.5% to 3%, Deer Creek came closer to meeting student needs via property taxes but Apple Tree and Cool Valley still had a large gap (*assess and revise*). The teachers mentioned that "our model is designed to be applicable for any school based on the property tax rate of their area. This was proven with our addition of what the distribution would look like if the property tax rate across the whole district was 3%. While our model is pretty much designed to be only for school funding distribution, a few changes could be made to make it applicable for other situations."

This process resulted in the percentage of school funding to be distributed to each school (See Fig. 10) (*getting a solution*).

This was the only group that did not allocate funds to Blue Mountain school so they had to think carefully about how this tied into their understanding of fairness and consider if it was fair to not qualify for any additional funds. Because all students at Blue Mountain were receiving appropriate funding, they justified it as a fair distribution of funds.

In reflecting on the task, both teachers commented that they were not very confident going into the modeling task because neither of them had much understanding of where school funding comes from. One of the groupmates, Kendall, informed the model by sharing her experiences attending a school that was poorly funded and this drove the group to make sure all students received the same base rate. Reflecting on the task, Kendall stated that the model helps to communicate what is needed and why. She discussed that each group's models had different strengths to fit various needs and the models they created could be leveraged to help voters understand why schools might need more funding. She stated,

> I wish my high school would have used a model like this, or even any of the models that were presented, because we all put in a lot of thought into how this funding would work. My high school never gets mill levies to pass because they don't know how to sell to the community what they want…If I were to ever go teach at my high school, I would definitely present all of the ideas everyone in this class came up with because they all had their own strengths to fit various needs. In this case, our model was

	A	B	C	D	E
1	School	Apple Tree	Blue Mtn	Cool Valley	Deer Creek
2	Grades	PK-8	PK-6	PK-8	3rd-8
3	Number of Grades	9	7	9	6
4	Number of Students Enrolled	969	516	504	320
5	Fraction of ELL students	1/3	1/6	1/7	1/20
6	Fraction of SPED students	1/7	1/25	1/6	1/26
7	Percentage of students meeting standards in Math	13%	65%	30%	59%
8	Percentage of students meeting standards in English	16%	61%	34%	57%
9	Percentage of students meeting standards in Science	18%	71%	21%	68%
10	Percentage of Students which are Low Income	90%	22%	75%	12%
11	Student to Teacher Ratio	14:01	16:01	16:01	11:01
12	Median Home Cost	$ 123,100.00	$ 365,400.00	$ 140,400.00	$ 330,330.00
13	Property Tax Rate	1.43%	2.51%	1.51%	2.28%
14					
15	Property Tax per Median Home Cost (Funding per student from property tax)	$ 1,760.33	$ 9,171.54	$ 2,120.04	$ 7,531.52
16	Funding from property tax (from all students)	$1,705,759.77	$4,732,514.64	$1,068,500.16	$2,410,087.68
17	Points System:				
18	Number of Grades	3	3	3	2
19	Number of Students Enrolled	10	8	8	5
20	Fraction of ELL students	15	5	5	3
21	Fraction of SPED students	10	2	10	2
22	Percentage of students meeting standards in Math	1	6	3	6
23	Percentage of students meeting standards in English	2	6	3	6
24	Percentage of students meeting standards in Science	2	7	2	7
25	Percentage of Students which are Low Income	15	0	10	0
26	Student to Teacher Ratio	3	6	6	3
27	Total	61	43	50	34
	Minimum Funding Total per Student	$8,000.00	$8,000.00	$8,000.00	$8,000.00
	Funding Gap between Local Funding and Minimum Funding	$6,239.67	-$1,171.54	$5,879.96	$468.48
	Recapture Amount	$ -	$ 1,171.54	$ -	$ -
	Recapture Distribution	$ 492.05	0	410.039	269.4542
	State Funding	$5,747.62	$0.00	$5,469.92	$199.02

Fig. 9. Model for Determining Funding in Relation to Student Needs.

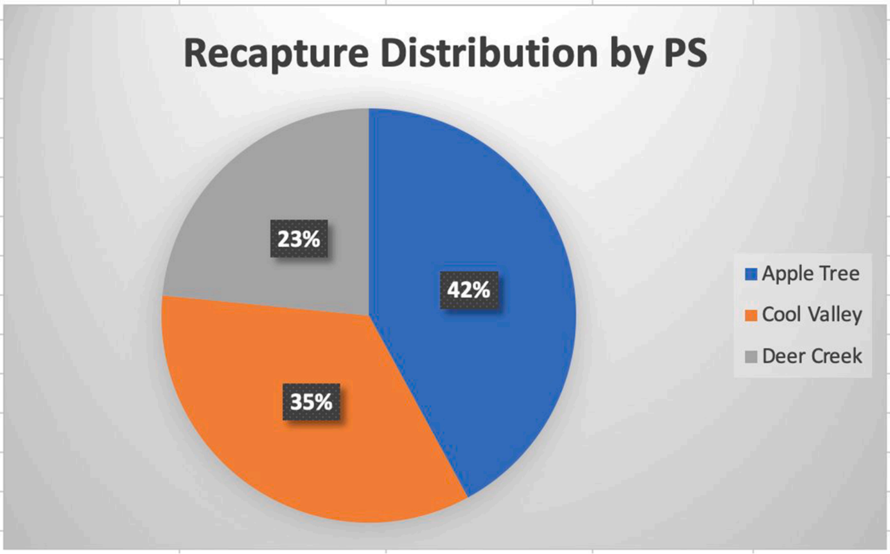

Fig. 10. Percentage of school funding to be distributed to each school.

accommodating to how property tax rates would affect school funding, which would be very useful when trying to get people in the community to agree with school proposals to get more money.

For this particular group, the modeling experience gave them a way to articulate and respond to inequities that they experienced in the educational system. They also saw the models as a way to convey ideas to society. Although they did not take action, they demonstrated *critical action* in that they were able to show how the model they created could be used to take action against oppressive conditions.

11. Summary of models and reflections across groups

Each group of teachers proposed a mathematical model that reveals their societal values or other aspects, such as practicality. In each of the three cases, the teachers initially drew on their knowledge and experiences to inform the modeling process, specifically in making assumptions and defining variables. This included their experiences as a student, teacher, and taxpayer. They were aware of the fact that equitable funding did not equate to equal funding. As they engaged in the modeling process, they learned from one another and were motivated to consult additional sources to research, brainstorm, and critique existing ideas and structures. For example, Group 1 began by allocating equal funding to each school and then adjusted based on need. They realized mathematics could be utilized as a structured system to help them make their decisions and designate some criteria as more important than others. They ranked each school from 1 to 4 across each of the criteria of their choice, including low-income students, students with special needs, students meeting standards, property tax, and enrollment rates. With further revisions, they could have considered more criteria, but had to limit themselves to ones that were available and accessible. Group 7 and Group 8 tackled the idea of redistributing funds and the possibility of taxes being shared across districts. Many of the groups found it challenging to acknowledge and grapple with the idea that before students enter the classroom they do not have access to the same resources and tools and it might be appropriate for some schools to receive little to no outside funding. Members of Group 8 shared their personal experiences of being students in schools with limited resources and leveraged them to help make decisions about how money should be distributed. We found it important that there were different models that reflect different values and assumptions and these different approaches led to important classroom conversations. They also allowed teachers to draw on their lived experiences and share them with one another. As teachers engaged in the modeling process, they were motivated to consult additional sources to research, brainstorm, and critique existing ideas, becoming more informed citizens.

For each of the cases above, we provided a glimpse into how the teachers engaged in critical reflection, developed mathematical agency, engaged in critical analysis, and engaged in critical action. In this section, we look across groups to highlight and reflect on other instances during the modeling process when we encountered teachers engaged in these practices.

When teachers attempted to *define the problem*, all of them wrestled with the idea of “fairness.” In this part of the modeling process, the teachers engaged in *critical reflection* and *critical analysis* to explore, understand, and critique what fair might mean. Some teachers connected to their lived identities and drew on and shared their own experiences in framing the problem. Others researched and critiqued existing models for funding school funding structures in their state to inform their work. Even though the schools were hypothetical, the teachers felt invested in the problem and wanted to make the best choices for the students involved. Similar to the students who constructed the First Model, another teacher, Bella, noted that while defining the problem she wrestled with what factors were most important to use in terms of fairness. Many teachers discussed that it was challenging to think about how to translate and explore their beliefs about equity into a mathematical formula. Similarly, Cooper mentioned, "The model required us to simply generate a 'fair' system of funding, and defining that and then using math to make that happen was difficult." Cooper continued,

> Our major pivot point was establishing the point system to put a quantitative scale on the very vague term "need". I think that when we chose to do that, it allowed us to really dive into the factors that describe equitable funding, rather than just going off of what is standard in the system.

Both Bella and Cooper demonstrated their awareness of the social context of defining fairness, need, and equitable funding (*critical reflection*). Their ways of defining the problem affected their decisions on determining the factors that describe the definitions. Through this process, the teachers showed how they took initiative in constructing the meaning of concepts in relation to mathematics.

We also witnessed teachers developing *critical reflection* and *critical agency* when *defining variables*. The teachers were presented with a table of information and they had to determine which variables they would use and why. It was challenging for them to determine which variables they considered most important and why. Teachers (Kali and Nora) mentioned this challenge in their reflections. Kali said, "I thought this task was a lot to take in. Between all of the numbers we had to deal with and decided how to incorporate all of those numbers, this task just felt like a lot." Similarly, Nora stated, "There were so many variables to consider that I got kind of overwhelmed on how I wanted to approach my problem and how the variables fit together." To overcome this challenge, the teachers found their own ways. Nora continued,

> Eventually, I decided to start with the easiest variable to work with, such as student enrollment and student/teacher ratio and then worked my way up from there until I eventually formed opinions on why or why not I included certain variables in my problem.

For many of the teachers, they had to wrestle with and reflect on each criterion to understand it within the context of the problem and society to determine if it should be included.

Once teachers created a model and saw how their choices came together, they felt empowered that they were able to identify variables and justify their choices in ways they felt were equitable. Nora related the process she experienced to posing a problem like this with her students. She anticipated that her students might also be overwhelmed but talking through variables, discussing experiences and options, and justifying ideas would give them opportunities to engage in reflection. She said,

> This task helped me get an understanding of how to approach a problem with students and help them not be so overwhelmed with all the avenues the problem could take. For example, I may start this problem out by asking my students to form opinions about what variables they want to use and why. Then have them reflect at the end, and after seeing peers' work, if they still hold onto those opinions or if they may change their model to better represent how they feel about the task at hand…This task also helped me see what it is like to work through a modeling task where you give students variables to consider and figure out what variables they want to consider and why or why not. It's the perfect task if you want students to justify their work…It's ok to leave certain variables out as long as you can justify why you didn't use them. This is a really awesome way of looking at math!

This reflection shows how Nora transformed her disposition toward mathematics and mathematics teaching. She expressed how her perspectives about dealing with multiple variables changed from a challenge to a moment of new insight into her ability to use mathematics in a powerful way across the context of her life (*critical mathematical agency*).

When the teachers brought all of their work together to *build the mathematical model* and *get a solution*, we saw evidence that they were reflecting on the entire process in relation to critical reflection, agency, critical analysis, and critical action. When asked about the usefulness of the task and developing their model, teachers (Cooper, Lucy and Kendall) expressed how they could use their models to become vocal in the school funding system (*critical action*) and the process, overall, helped to make them more informed members of society. Cooper mentioned,

> I thought that the task was super useful. It broke down where school funding comes from and allowed us to take a look at the system which many claim is flawed. I now have an opinion on the school funding system that is much more knowledgeable than my previous opinion…I really found that this task was super humbling to view from a teacher's perspective. I did not realize how much of my future was based on the government's decisions of what is fair. It is almost daunting. I found that as far as usefulness goes, this task has allowed me to grow in my knowledge of mathematics as well as my knowledge of government funding systems and equity-based practices.

Cooper said the task was useful because it helped him gain knowledge about the system which is "flawed." (*critical analysis*). He recognized that he built an "opinion on the school funding system" (*sense of agency*). He also mentioned that this task had allowed him to grow in his knowledge of mathematics and equity-based practices. This reveals that he identified himself as a mathematical thinker who can develop meaningful mathematical understanding in connections to the broader social and political contexts (*critical reflection* and *critical mathematical agency*). Cooper was a member of group 8, the group that did not allocate any funding to Blue Mountain School. During classroom conversations, classmates critiqued this group's approach and argued it was not fair for a school not to receive any funding. Cooper and other members of his group had to wrestle with proposing their ideas when they were not widely shared by classmates. It allowed the class to see that different solutions exist and sometimes when we exert mathematical agency our perspectives may not be widely accepted or supported.

In developing the sense of critical mathematical agency, another teacher shared a similar perspective:

> I found this task to be personally useful. In [my state], when it comes to education there is an area of the state along I-95. It is often referred to as the "corridor of shame", due to the low level of academic achievement, which is directly related to the funding disparity in our state. Our funding model was established in 1977. In 1999, one of the counties sued the state, and the state Supreme Court ruled with the state. Based on the state constitution only providing for a free education, not an adequate one. This task has led me to become more vocal in reaching out to the state legislature to come up with a funding model that is better for the poorer districts in our state. (That was supposed to be the focus in 2020, however due to Covid it has not yet been addressed).

Kendall's response demonstrates an awareness of the broader political contexts, and how the funding disparity could cause an academic achievement gap, which could subsequently lead to deficit views that people may hold towards other people (e.g., people referring to an area as the "corridor of shame") (*critical reflection*). She also showed how she conceptualized the historical and political system that sustains inequity and critiqued the system (*critical analysis*). Kendall stated that the task has led her to become more vocal in reaching out to the state legislature to devise an alternative funding model. She showed her sense of power and capability (*sense of agency*) as well as her commitment to take action against the current state legislature (*critical action*).

12. Conclusion and implications

As we close our paper in this section, we revisit and discuss the following three areas of our study with the ideas for future implications: (a) the use of the existing modeling process framework and its extension for social justice; (b) teachers' reflections and their connections to existing studies; and (c) design principles and how they may guide future task designs. We then discuss how these highlights address the theme of this special issue - Mathematics in Society: Exploring the mathematics that underpins social issues.

First, we used the existing modeling process frameworks (e.g., Aguirre et al., 2019; Bliss et al., 2014; Jung & Magiera, 2021) as a starting point to support teachers' development of and reflections on their mathematical models. Specifically, we considered Jung and Magiera (2021) when creating our School Funding Modeling task, hypothesizing that the issue of fairly distributing funding (social justice issues) is intertwined across the modeling process, which involves assumption building, self-evaluation, model development, and shareable process. When we analyzed our data, we built our conceptual framework on prior studies (Aguirre et al., 2019; Bliss et al., 2014; Jung & Magiera, 2021). Our work extends these studies because we found that social justice and modeling are not just connected (as shown in Fig. 1) across this SMJJ task but inherently intertwined across the modeling process, as Jung and Mageira (2021) suggest in task design. Specifically, when teachers were defining the problem, they sought to make a model for fairly distributing school funding, considering both mathematical and societal spaces. The teachers researched and brainstormed, while some also critiqued the situation; for example, they located information on Title 1 funding in their state, critiqued the current model to distribute school funding, and brainstormed their ideas. The teachers also made assumptions, such as schools in a wealthier district are receiving outside funding. They then defined variables to consider, including the number of enrolled students and property tax. While they iteratively went through this process, they built and assessed the mathematical model, such as a point system which rates each school based on the assumptions and variables they considered. This process led them to get a solution that concerns both mathematical and societal values (e.g., percentages of school funding to be distributed to each school) and iteratively revise their process. Our work suggests that SJMM tasks might play out in different ways, depending on the nature of the task. For example, in the Flint Water Task (Aquirre et al., 2019), the teachers were using mathematics to evaluate a proposed solution to examine water usage in relation to the water crisis. Once they had an initial model of how water is used, they could begin to critique and respond to the proposed model in relation to the situation. In the School Funding Task, we found that teachers engaged in discussing social justice throughout the modeling process, rather than only at the beginning or/and at the end of the modeling task. Since defining the fair model was a major issue in this task, the teachers were wrestling with their understanding of the situation in society and notions of fairness as they worked through each part of the modeling process.

Second, we noticed that teachers' models and reflections revealed evidence of moments when they expressed their mathematical identities and critical consciousness while they wrestled with the problem of school funding. When asked to reflect on their modeling process, teachers expressed how they wrestled with defining "fairness" and "need," and how their definition of the problem affected their assumptions and choice of variables. They built diverse mathematical models to fairly distribute funding to schools and evaluated the solution to whether it reflected the definitions and assumptions they made earlier in the modeling process. Specifically, we found that some components of the modeling process (Bliss et al., 2014), such as defining the problem and making assumptions, are closely tied to societal challenges and issues proposed in the problem statement. In fact, we adapted a component (i.e., research and brainstorming) of the framework to "research, brainstorming, and critiquing" as we noticed that the teachers were critiquing the existing system as they were searching for resources and brainstorming ideas for developing the mathematical models. Some teachers explicitly mentioned that they built their models because they did not agree with the existing models implemented by their states. From these findings, we argue that the modeling process framework, which was designed to illustrate a learner's specific actions in solving a mathematical modeling problem (Bliss et al., 2014), can be also used as a guide for teachers to reflect on their ways to use mathematics to solve social-justice problems. Our results show that mathematical modeling allows teachers to connect the mathematical world and societal challenges by using mathematical models to organize data and provide solutions informed by data. Since there are multiple approaches to choosing and organizing data, each approach is often influenced by one's societal values throughout all these processes of making assumptions, defining variables, developing models, and drawing conclusions.

By engaging in the modeling process, the teachers felt more educated about and prepared to address issues in society. For example, after recognizing flaws in current school funding systems when designing their models, teachers expressed that they could become vocal about the school funding system for their state based on the new knowledge they gained. Teachers also recognized the historical and political systems that have perpetuated inequality, and how the current school funding systems might affect or are influenced by students' academic achievement. This result is aligned with other studies' findings that showcase learners' engagement in social justice-oriented mathematical modeling (e.g., Aguirre et al., 2019; Jung & Magiera, 2021). For example, Aguirre et al. (2019) reported that their SJMM task increased teachers' awareness of systemic injustice and helped strengthen their mathematical knowledge. To nurture more teachers' development of mathematical identities and critical consciousness, it seems beneficial to implement more SJMM tasks that connect macro- and micro-social justice issues that can be interpreted through a mathematical lens. Future mathematics educational research might investigate teachers' engagement in multiple SJMM tasks and document whether this tight connection between the modeling process and social justice occurs in any SJMM tasks or certain tasks depending on the specific features of the SJMM task.

Lastly, to design and implement more SJMM tasks, the design guidelines that we considered for this study might be useful in developing future tasks. When we designed this school funding task, we first considered situations that teachers might be passionate about or that they can develop cultural competency. We also explored oppressive conditions (e.g., unfair distribution of resources, power, and opportunities) that stem from the systems that create and sustain inequity (e.g., Aguirre et al., 2019; Berry III et al., 2020). In addition, we ensured that the problem requires the authentic use of mathematical models and involves a client that needs the solution to the problem (Lesh & Doerr, 2003). We hope these guidelines, along with an example of our developed task, diverse models designed by the teachers, and their reflections, might inform other teacher educators and researchers about ways to create or use SJMM tasks.

These three highlights above address the theme of this special issue that focuses on the exploration of the mathematics that underpins social issues. For example, in our work, we explored the diverse ways in which teachers interpreted and manipulated data, and created a model to solve a social justice problem. In their reflections, teachers shared how the opportunity to derive mathematical information is critical in interpreting the societal issues of fairly distributing school funding. In designing and reflecting on their models, teachers' personal experiences and values were considered in defining and addressing the social justice issue. When the teachers engage in this social justice-oriented mathematical modeling task, the connection between mathematical space and social justice space revealed from their models and reflections was closely entangled. We argue that such a task offers a new opportunity of developing the ability to integrate mathematical space and societal space in authentic ways - these abilities are rarely developed intasks that solely focus on one space or the other.

We also hope that our study benefits future researchers when they read our detailed illustrations of how teachers wrestle with fairness mathematically and model this societal issue. This works illustrates one of the ways mathematical modeling can be designed and utilized to advance social justice objectives. Our efforts build on numerous mathematics teacher educators who have supported teachers in connecting mathematical modeling and social justice (e.g., Aguirre et al., 2019; Cirillo et al., 2016; Felton-Koestler, 2020). We believe our study extends the prior research by delineating mathematical contents and processes that are necessary for advancing a societal goal and by articulating pathways of connecting critical mathematical understandings with one of the important social justice issues - how do we fairly distribute limited resources and what factors do we value the most in this process?

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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