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Understanding Preservice Elementary Teachers as Mathematical Modelers and Their Perceptions of the Process

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A growing consensus holds that preservice K–8 teachers (PSTs) need to experience the modeling process as learners to understand it and envision teaching modeling in their future classrooms. We examine this recommendation by exploring how PSTs construct models and how collaborative learning practices influence them in revising and refining their models. We also explore their reflections on modeling as a pedagogical experience. We introduce Modeling Decision Maps as a tool to examine how PSTs construct and refine mathematical models, and we draw on reflective journal entries to capture PSTs' perspectives on the process. Our findings indicate that realistic modeling tasks provide opportunities to foster PSTs' understanding of modeling, grow their mathematical modeling skills, and attune them to important pedagogical practices.

Keywords: Mathematical modeling; Preservice K–8 teacher education; Modeling decision maps

Mathematical modeling, the process of using mathematics to solve real-world problems, has gained increasing attention in the United States and abroad in recent years (Consortium for Mathematics and Its Applications [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2016). Modeling provides powerful learning opportunities for students to understand the role of mathematics in their world. K–8 teachers are expected to possess tools to effectively integrate modeling into their practices (Association of Mathematics Teacher Educators [AMTE], 2017), but few have knowledge of modeling or have experienced it firsthand. To prepare preservice K–8 teachers (PSTs), researchers emphasize the need to provide them with opportunities to experience modeling as learners before exploring pedagogical practices (Anhalt & Cortez, 2016; Niss et al., 2007). More research is needed to understand how PSTs model, what practices productively support PSTs as modelers, and what they glean from engaging in the modeling process.

PSTs are unique mathematical learners. When they engage in mathematics, they are not just learning content. They are reflecting on pedagogical practices with respect to their existing belief system and envisioning whether and how they feel able to incorporate these experiences into their future classrooms (Pajares, 1992; Philipp, 2007). In this article, we intend to capture how PSTs engage in modeling as learners and how they reflect on and contextualize modeling as a pedagogical experience. The pedagogical experience includes both engaging in the modeling process as learners and also observing and reflecting on the process as future teachers. Many studies have shown that PSTs can model and improve their ability to do so across time (Çiltaş & Işık, 2013; Durandt & Lautenbauch, 2020; Karacı Yaşa & Karataş, 2018; Tidwell et al., 2023) but few have provided qualitative details on how PSTs' models evolve and how modelers interact with one another. Researchers have also found evidence through questionnaires following modeling tasks that PSTs want to share modeling tasks with their future students (Ikeda & Stephens, 2021; Stohlmann et al., 2015). However, what, specifically, they drew from the modeling process that affected their interest in incorporating modeling in their future classrooms remains unclear. This constitutes a gap in the literature documenting how PSTs develop models and conceptualize modeling as a pedagogical experience. In this article, our contribution lies in capturing how PSTs move through the modeling cycle using a qualitative tool we created, the Modeling Decision Map. Through these maps, we note how models grow and change and how discussion and feedback inform changes. Drawing on PSTs' reflections, we explore their perceptions of the process to highlight important connections they see between their experiences as learners and their perceptions of the modeling process. We address the following research questions:

1. How do PSTs construct, refine, and grow their mathematical models with respect to the modeling cycle?
2. What is the role of collaborative learning practices, specifically discussion and targeted feedback, in refining PSTs' models?
3. How do PSTs perceive modeling as a pedagogical experience?

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For clarification, we define the *modeling cycle* as the written or pictorial description of what occurs during modeling and the *modeling process* as the actions of the learner or modeler engaging in the modeling cycle.

Relevant Literature

We begin this section by discussing a modeling perspective to situate our work within the broader landscape of teaching and learning modeling. From there, we examine the literature on PSTs as modelers to understand what we know as a field and what remains to be investigated.

Perspectives on Mathematical Modeling

Different modeling perspectives are characterized by different takes on the modeling process, the nature of the task, and the intended purpose of the model (Abassian et al., 2020). Across perspectives, scholars emphasize using mathematics as a tool to solve problems arising in one's life (Blum & Borromeo Ferri, 2009; Lesh et al., 2000) and using mathematics in robust and meaningful ways (Zbiek & Conner, 2006). When considering the teaching and learning of modeling, Niss and Blum (2020) discussed "mathematics for the sake of modelling" versus "modelling for the sake of mathematics" (p. 28), also described in the literature as "modelling as content" versus "modelling as a vehicle" (Julie & Mudaly, 2007, p. 503). In describing these different pathways, Niss and Blum (2020) stated, "These two reasons are in no way contradictory to one another. . . . They are, however, analytically distinct, and they do give rise to different consequences in terms of priorities and activities" (p. 28). Some modeling perspectives focus primarily on understanding the modeling cycle and developing related competencies (e.g., realistic, as in Pollak, 2007, 2016), some focus on the development of a particular mathematical idea or related reasoning (e.g., epistemological, as in Gravemeijer & Doorman, 1999), and some aim to serve both purposes (e.g., educational, as in Blomhøj, 2009). Some perspectives have additional goals, such as focusing on the models elicited through solving a model-eliciting activity and applying it to a new related problem (e.g., contextual, as in Lesh & Doerr, 2003), empowering the modeler as a decision maker in society (e.g., sociocritical, as in Barbosa, 2009) or understanding the "cognitive and affective barriers to successful modeling" (e.g., cognitive, as in Kaiser, 2017, p. 274).

Among the multiple perspectives of mathematical modeling, we situate our study within the realistic perspective (Kaiser & Sriraman, 2006; Pollak, 2007, 2016), in which the emphasis is on the modeler making sense of "realistic, authentic, and messy tasks" through the modeling process (Abassian et al., 2020, p. 55). Although researchers taking the realistic perspective do not agree on a single modeling process, modeling cycles often consist of the modeler encountering a real-world situation and constructing a model to interpret it (Biccard & Wessels, 2011; Blum, 2011; Ludwig & Reit, 2013; Pollak, 2016). The modeler mathematizes the situation and examines "key aspects or variables to structure a real model" (Abassian et al., 2020, p. 55). The modeler also translates an authentic scenario into a mathematical representation and draws on the representation and solutions found to make decisions about the authentic scenario. The purpose is for the modeler to analyze and understand real-world situations while experiencing the phases of the modeling cycle.

In our work with PSTs, we draw on the realistic perspective (Kaiser & Sriraman, 2006; Pollak, 2007, 2016) for two reasons. First, evidence has shown that attention to modeling is not usually a part of teacher preparation programs, especially for elementary teachers (Anhalt & Cortez, 2016; Doerr, 2007). The goal of modeling, from the realistic perspective, is to help learners become more familiar with using modeling as a tool to interpret real-world situations. We wanted PSTs to experience and understand the value of modeling, the components of the modeling process, and its iterative nature.

Second, elementary PSTs are a particularly vulnerable population with respect to mathematical identity and self-efficacy (Bursal & Paznokas, 2006; Emenaker, 1996) compared with other undergraduate students. They enter college mathematics classrooms with feelings of uncertainty, irrational dread of the subject, and shame over their perceived lack of ability (Gresham, 2008). The realistic perspective focuses on using mathematics as a tool for life and provides a foothold to enter tasks through daily experiences. Through modeling tasks, we aim to bolster elementary PSTs' agency and allow them to "identify themselves as powerful mathematical thinkers who construct rigorous mathematical understandings, and who participate in mathematics in personally and socially meaningful ways" (Turner, 2003, p. iv). Through this lens, the primary goal of teaching modeling is not to find one exact solution to a problem but to support PSTs as they learn about and apply modeling as a skill for life.

PSTs as Modelers

When modeling with PSTs, we must consider the knowledge they will need as future teachers. The *Standards for Preparing Teachers of Mathematics* (AMTE, 2017) states that well-prepared beginning teachers of mathematics should be able to "apply their mathematical knowledge to real-world situations by using mathematical modeling to solve problems appropriate for the grade levels and students they will teach" (p. 9). Some evidence has suggested that attention to models and modeling is not usually part of teacher preparation programs (Anhalt & Cortez, 2016; Doerr, 2007). Although modeling is highlighted as an important competency, one that requires time to be understood in order to teach it, we are just beginning to grasp what the teacher education experiences that support elementary PSTs need to look like.

When integrating these recommendations into practice, researchers have suggested that PSTs must participate in the process as learners before delving into the exploration of teaching skills (Anhalt & Cortez, 2016; Niss et al., 2007). Engaging in the modeling process helps them understand the different phases and conceptualize related competencies (Anhalt & Cortez, 2016; Gould, 2013; Zbiek, 2016). Blomhøj and Jensen (2003) defined *modeling competency* as "being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context" (p. 126). Maaß (2006) delineated modeling competencies into five distinct categories:

1. "Competencies to understand the real problem and to set up a model based on reality" (p. 116)
2. "Competencies to set up a mathematical model from the real model" (p. 116)
3. "Competencies to solve mathematical questions within this mathematical model" (p. 116)
4. "Competencies to interpret mathematical results in a real situation" (p. 116)
5. "Competencies to validate the solution" (p. 116)

Kaiser (2017) expanded on these competencies to develop metacognitive subcompetencies that are essential to carrying out the modeling process. These include reflecting on the modeling process, seeing connections between mathematics and reality, developing insight into the subjectivity of models created, and honing the ability to discuss and communicate effectively.

Many researchers have evaluated PSTs' knowledge of modeling by focusing on their performance across modeling competencies before and after a modeling task(s) (Çiltaş & Işık, 2013; Durandt & Lautenbauch, 2020; Karacı Yaşa & Karataş, 2018; Tidwell et al., 2023). To evaluate competencies, researchers often consider PSTs' performance across different phases of the modeling cycle and include competencies such as posing a mathematical problem, building a model, and validating a mathematical solution. Multiple researchers have shown that PSTs demonstrate broad improvement across competencies before and after completing modeling task(s) (Çiltaş & Işık, 2013; Karacı Yaşa & Karataş, 2018; Tidwell et al., 2023). Highlighting specific competencies, Durandt and Lautenbach (2020) analyzed 10 groups of high school PSTs as they engaged in two modeling tasks. They found that PSTs were successful early in the process with competencies like identifying relevant information, making assumptions, and linking mathematical results to the real world. Some competencies developed over time, such as acquiring new mathematical knowledge, considering the implications of decisions and results, and suggesting improvements to the model. Durandt and Lautenbach highlighted that after two modeling tasks, PSTs still found it challenging to realize that a problem could be approached in multiple ways and that multiple approaches could be valid.

PSTs develop additional subcompetencies as they engage in the modeling process. Govender (2020) asked PSTs to determine the height of a tree. Different groups of PSTs used different mathematical ideas to solve the task, and PSTs were able to analyze and critique different approaches to facilitate revision. Tidwell et al. (2023) noted that by revising and refining ideas in the modeling process, PSTs improved in their communication, mathematical language, and reasoning. Ikeda and Stephens (2021) surveyed PSTs after they had completed a modeling task and found that 97.3% reported that the task enriched their understanding of mathematics, and 60% acknowledged that the task helped them to see connections between mathematics and their lived experiences.

Researchers have also documented the challenges PSTs face during the modeling process. Zeytun et al. (2017) worked with five PSTs and found that sometimes PSTs used intuitive decision making when working through tasks, meaning they did not draw on mathematics. Sometimes, they ignored a relevant variable to make a task easier to solve. Few wanted to test their model, and they viewed their work as proof that the model was sufficient. Nuances arise when moving between the real world and mathematical spaces that often require exploration, refinement, and analysis from multiple perspectives. For example, when asking PSTs to design a parking lot and to determine the number of cars that could be safely parked, Widjaja (2013) found that the PSTs took the area of a parking spot relative to the area of the space to determine the number of cars. The PSTs needed additional discussion to revise and refine the model to determine how angled parking and driving space would affect the layout.

Researchers have also asked PSTs what they take away after engaging in the modeling process. Stohlman et al. (2015) found that PSTs indicated they were more likely to implement a modeling task after experiencing one. Ikeda and Stephens (2021) found that 54% of their PSTs wanted to use in their own classrooms a task like the one they had experienced. Son et al. (2017) found that PSTs could conceptualize an effective modeling lesson and identify ways in which modeling could be facilitated in the classroom.

Facilitating Modeling Tasks and Supporting PSTs as Modelers

Researchers have suggested that the ways PSTs experience modeling tasks matter and that the nature of instruction influences their perceptions of the process. Borromeo Ferri (2018) introduced the idea of teaching modeling to teachers as a "pedagogical double-decker" (p. 6), referring to a double-decker bus. That is, the instruction has two layers. First, mathematics teacher educators engage PSTs in the modeling process so they learn what modeling entails. Second, during or after the process, teacher educators provide opportunities for reflection on the enactment of the task and examination of the pedagogical practices involved in teaching modeling tasks. This allows PSTs the opportunity to reflect on the process and envision how they might carry out a task in their own classrooms. One pedagogical practice evident in the modeling cycle is revising and refining models through testing and feedback. Several researchers have advocated for a community-based approach (e.g., Aguirre et al., 2019) and a cooperative learning-based approach (e.g., Borromeo Ferri, 2018), in which focusing on student engagement allows for exploration of different perspectives and approaches as students model. Throughout the process, students have opportunities to share expertise, give constructive feedback, and develop knowledge collectively. We argue that these pedagogical practices matter in developing PSTs' understanding of what modeling is and their vision of how it might be incorporated in their classrooms.

From our perspective, pedagogical practices—and specifically discussion practices—drawn from the broader literature base have the potential to create a powerful learning space for developing modelers and making use of each of the phases of the modeling cycle to its full potential. In their chapter on core practices in mathematics teaching, Jacobs and Spangler (2017) described four goals that teachers can have when leading discussions: (a) engaging students with their peers' mathematical thinking, (b) pressing, (c) scaffolding, and (d) positioning all students as competent. Engaging students with their peers' mathematical thinking facilitates rich discussion in which "students are explaining their thinking and making sense of and critiquing the reasoning of others" (p. 779). Asking students to compare their thinking with another student's thinking or give feedback to another student helps students to be more engaged with one another and the mathematical task at hand (Webb et al., 2014).

The modeling cycle is filled with opportunities to engage with collaborative learning practices as a tool to foster understanding of the modeling cycle and develop the modelers' autonomy. In this study, we focus specifically on exploring and giving feedback to classmates' models as a collaborative learning practice. Research on discussions has suggested that it could help modelers see multiple perspectives to the model, provide more complete and justified explanations, and become more engaged in the process overall (Rodgers et al., 2015). In each phase of the modeling process, engaging in collaborative learning practices forces modelers to reconcile their own thoughts and approaches with the perspectives of their classmates to potentially build a better solution and experience the process in a more meaningful way.

Our survey of the literature reveals several questions that remain unanswered. Many have argued that PSTs must experience modeling directly as the first step to becoming teachers of modeling (Anhalt & Cortez, 2016; Borromeo Ferri, 2018; Gould, 2013; Zbiek, 2016), but we need to understand why this recommendation is warranted. This includes learning about the ways in which PSTs engage in the modeling process and what, if anything, PSTs glean from the process as learners because this will determine how they envision using modeling in their future classrooms.

Theoretical Perspectives

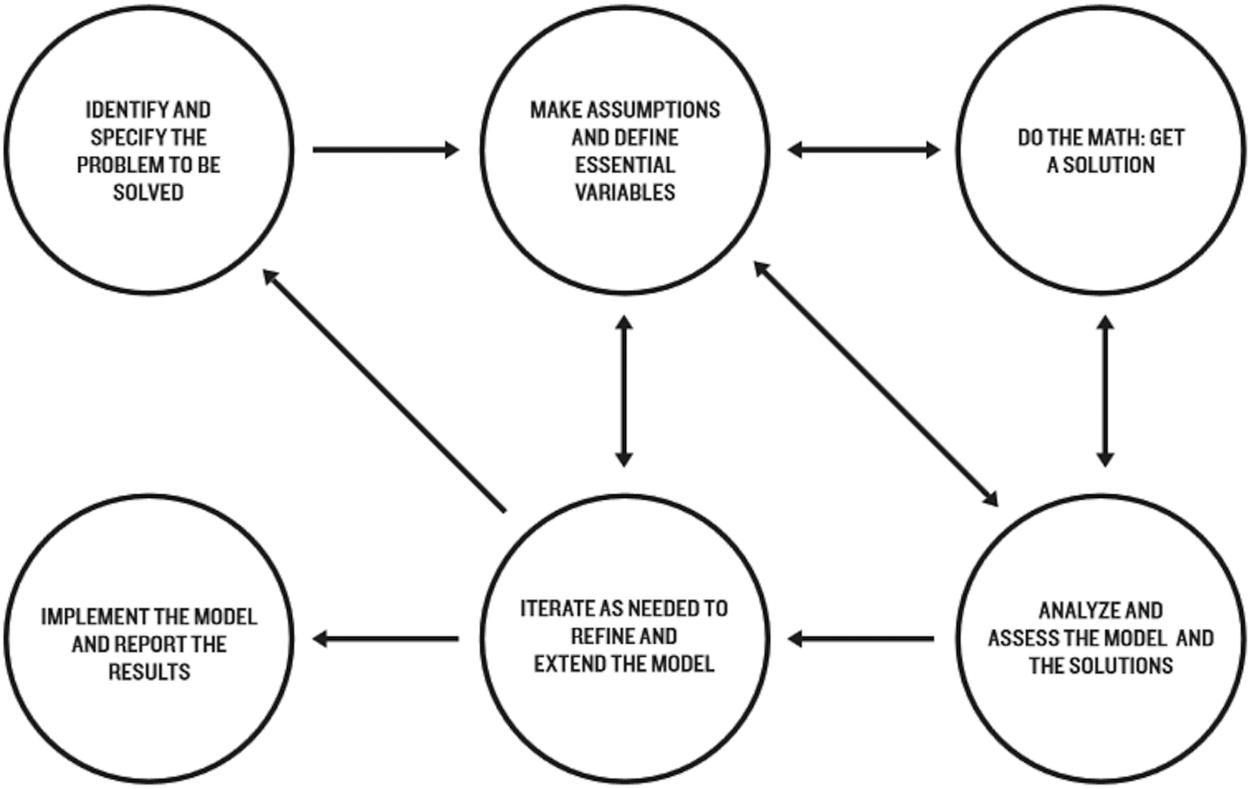
Drawing on Borromeo Ferri's (2018) conceptualization of modeling with teachers, we think about our work through two lenses. The first lens is understanding PSTs as modelers and how their models develop. The second lens is how they can draw on these experiences as learners to build an understanding of modeling. We believe that by drawing on these two lenses in tandem, we can identify concrete connections between the two spaces that allow us to better understand how PSTs develop as learners and how we can support them through the modeling process to foster their understanding of modeling for teaching.

To understand PSTs as modelers, we sought to understand the modeling process in action and document how PSTs engage in each phase of the modeling cycle. Researchers have acknowledged that mathematical modeling involves "translating between mathematics and reality in both directions" (Blum, 2011, p. 17). Blum and Leiß (2007) and subsequently Blum (2011) highlighted that the first phase of the modeling cycle is to construct a situation model or make sense of the real situation. Second, the modeler creates a real model, often defining and exploring ambiguous terms (i.e., best, worthwhile, efficient, fair) and what they mean in relation to the situation Then, the modeler creates a mathematical problem and model to address the situation, which often includes expressing ideas through mathematical notation, such as variables or equations. The fourth step is doing the calculations and getting mathematical results, which are then interpreted as real results. Last, the modeler determines whether the solution best fits the needs of the situation and whether it needs to be refined and further validated before sharing the solution. Kaiser (2017), citing Borromeo-Ferri (2011), discussed that most modeling cycles "idealize[] the modeling process" as "linear sequential steps," but, in reality, most "include frequent switching between the different steps of the modeling cycle[]" (p. 276).

To communicate the modeling cycle to PSTs and articulate what we expected in the process, we drew on the modeling cycle represented in the *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME) report (COMAP & SIAM, 2016) in Figure 1 because it was written from a practitioner perspective but captures the theoretical underpinnings we valued from Blum (2011).

Figure 1

The Mathematics Modeling Process From the GAIMME Report



Note. Source: COMAP & SIAM (2016, p. 13).

Drawing on these modeling cycles, for each phase, we wanted to capture the variety of ways PSTs engage in related competencies, such as identifying the problem to be solved, making assumptions, defining variables, getting a solution, and analyzing their model. These experiences provide the foundation and concrete examples in helping us to understand both how PSTs' models evolve across time and the instances they may refer to as they reflect on the process.

To contextualize PSTs as both teachers and learners, we draw on the work of Borromeo Ferri (2018), who captured the dual role of teacher as learner and the ways teachers might draw on experiences as learners of modeling to inform the ways they understand modeling for teaching. From her perspective, teacher educators must facilitate tasks in ways that align with how they envision PSTs' teaching tasks. PSTs must have opportunities for reflection and connections to teaching as they engage in modeling tasks. Drawing from her previous work, Borromeo Ferri (2018) outlined four important dimensions for teachers to develop for teaching modeling: theoretical, task, instruction, and diagnostic. The *theoretical dimension* refers to teachers' understanding of the modeling cycle. It might include their awareness of the cyclic nature of modeling, different types of models, and reasons to model. The *task dimension* refers to the development of modeling tasks, which might include determining a modeling task, reflecting on the cognitive demand of the task, and mapping out potential solution strategies. The *instructional dimension* refers to the act of planning the modeling lesson and carrying it out in a classroom. This might include considering what support and feedback students may need during the task and how to respond in real time. Last, the *diagnostic dimension* refers to evaluating students' work. This may include evaluating how students have engaged in each phase of the modeling process and determining if their work is sufficient or needs to be revised.

We consider modeling as a broader pedagogical experience that PSTs could draw on to inform their understanding of modeling both as learners and as teachers. Teaching practices embedded in the modeling cycle have the potential to be meaningful for PSTs and shape their understanding of the process. When engaging in modeling tasks, PSTs learn about the theoretical dimension firsthand, including the cyclical nature and phases of the modeling cycle. They learn about the task's aims and perspectives from their instructor and colleagues. They may also encounter components of the task dimension by exploring different approaches or reflecting on what makes a task meaningful or cognitively challenging. Stepping away from modeling as mathematical content, PSTs may engage in the instruction dimension by reflecting on pedagogical practices embedded in the modeling cycle, like collaborative learning or opportunities to revise their thinking. They may also follow attributes of the diagnostic dimension by observing how the instructor supports them through challenges and gives feedback.

Methods

This study uses a qualitative method, case study. We consider the focal case to be a descriptive, paradigmatic case in that we gain insights into the ways PSTs create models, engage in the modeling process collectively, and experience it pedagogically, especially given that they are in a designated modeling course designed and enacted to support them as learners and teachers. Generalizability is not the intent of this type of work (Yin, 2018). Instead, we provide rich descriptions of PSTs' models and their perceptions of the modeling process.

Setting and Participants

This study is part of a larger NSF-funded project in which both authors created modeling tasks, implemented them in their respective universities, and investigated PSTs' understanding of modeling attributes developed throughout the implementation of the tasks. While one author implemented the task, the other author participated in some of the sessions to take notes, and afterward, the two authors debriefed together. This collaborative approach facilitated the iterative process of designing and refining the tasks (Cobb et al., 2003). For this article, we focus on data collected from the first author's 12-week modeling course at a public university in the Mountain West region of the U.S. The course was offered to elementary education majors as a special topics elective. To register for the course, PSTs needed to have completed two mathematics content courses, one focused on number and operations and the other on geometry and measurement. The primary goal was to engage PSTs in the modeling process and explore how modeling can be used to help people answer relevant real-life questions. All 11 of the enrolled PSTs agreed to participate in this study. Ten PSTs identified as women, and one as a man. All the PSTs were between the ages of 19 and 24 and were sophomores and juniors at the university.

Development and Implementation of Mathematical Modeling Tasks

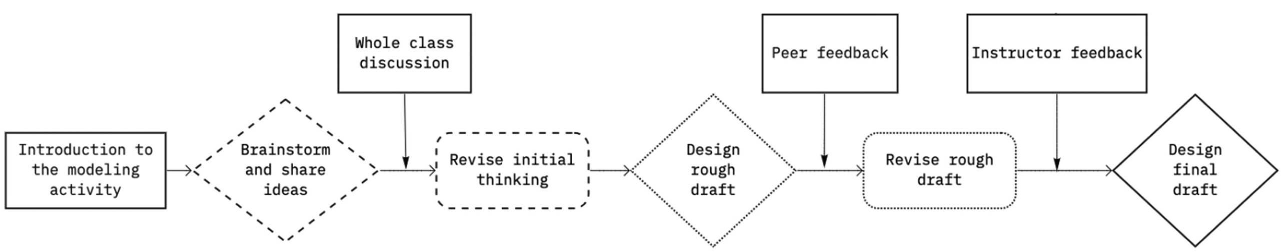
Drawing from relevant literature (e.g., Arnold et al., 2021; Blum, 2011; COMAP & SIAM, 2016; Lesh et al., 2000), we developed four tasks that would support PSTs in understanding modeling attributes, including task design and relevance, the modeling process, mathematical practices involved in modeling, and types of models. We designed the tasks such that the mathematical content related to common coursework PSTs typically take. We engaged in multiple rounds of teaching to test and refine the tasks. To productively engage PSTs in revising their models, we drew on Jansen's (2020) work by asking the PSTs to make rough drafts. This created opportunities for PSTs to engage with classmates' thinking and allowed us to frame revision as an opportunity to be more precise, develop ideas, and make connections. During instruction at each site, we followed the structure shown in Figure 2.

We introduced a modeling activity and guided small-group and whole-class discussions for PSTs to generate ideas. Building from these conversations, PSTs worked in small groups to design their initial models. They received peer and instructor feedback and then designed a final model. They publicly presented their proposed solution, discussed modifications made throughout the process, and explained the reasons behind their decisions.

This course was taught in a hybrid format during the fall of 2020 amid the COVID-19 pandemic. The course met twice a week for 75 min. In a typical week, class was held virtually 1 day so that PSTs could work in groups in designated breakout rooms. On the 2nd day, class was held in person for presentations and whole-group discussions. The first author was the lead instructor, but because much of our coursework was virtual, the second author was able to observe live remotely and take notes in preparation for debriefing. When coursework was held in person, it was recorded and shared with the

Figure 2

The Instructional Structure of Implementing Modeling Tasks



second author and PSTs through online storage space. The first author kept a written journal (shared with the second author) summarizing what happened during class and recording instructional choices that she made. During the entire semester, the authors met virtually after each class to debrief the session and discuss next steps.

Campus Tour Modeling Task

This article focuses on the implementation of and PST work on the Campus Tour Task. We chose this task because it was the first full-scale task the PSTs worked with. Because of the absence of a specialized modeling course for elementary PSTs in many universities, we thought this task would best inform the field of what is possible when incorporating one or two modeling tasks into an existing content course. This task began the third week of the semester. Before this task, PSTs had participated in two shorter modeling tasks. After each task, we showed them the simplified modeling cycle created by Arnold et al. (2021) and asked them to consider what steps occurred along the way as they translated an authentic experience into the mathematical world (see Figure 3). To us, *authentic* meant a situation that PSTs were probably familiar with and could relate to.

During the discussion, PSTs were able to reflect on and highlight phases of the modeling cycle without being explicitly introduced to them. From these discussions, the authors felt that the PSTs had a foundation of what modeling and the related competencies were.

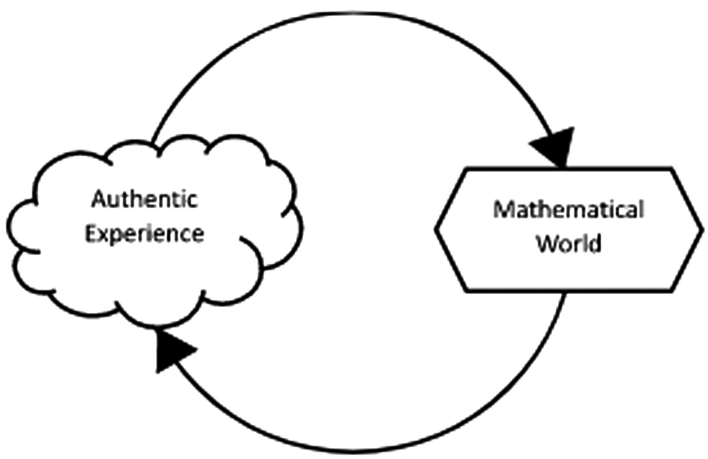
In the Campus Tour Task, PSTs were broken into three groups and challenged to think about the optimal way to organize time and distance to design a tour. They were given the following prompt:

> Every week the university provides campus tours to prospective students to introduce them to campus. The Office of Admissions has requested your help in designing a campus tour. They want you to design the tour so that prospective students will be motivated to come to our school. They would like the tour to last about an hour.
>
> - What is important to consider when planning a campus tour?
> - What tools or information would be helpful to have in mapping out your tour?
> - What information or data might be important to convey to the office of admissions?
> - How will you know if your tour will work?

We selected this context because most of our students have pride in their campus and have previously experienced this type of tour. We designed the task so PSTs would draw on their understanding of geometry and measurement—including unit rate, length measurement, conversions, and scaling—while also learning about prescriptive models and optimization. *Prescriptive models* seek to “create or organise” reality (Niss & Blum, 2020, p. 20), in this case organizing a campus tour that fits certain constraints. We also anticipated that PSTs would consider optimization as they planned their route and examined minimizing the distance students would be traveling to maximize the time they would have visiting specific spots on campus. We anticipated that the open nature of this question could lead down different paths, including (a) finding unit rates to make sense of distance and time and (b) investigating ranking and rating to determine which spots on campus are ideal to visit.

Figure 3

Simplified Modeling Cycle



Note. Source: Arnold et al. (2021, p. 5).

Data Collection

Data for this study include video recordings of the classroom during implementation, PSTs' written solutions to the modeling task (both initial and final drafts submitted by each group), peer and instructor written feedback, and PSTs' individual journal reflections following implementation.

Video recordings were captured using a SWIVL camera (http://www.swivl.com) in person and web recording software online. The SWIVL camera has audio markers that capture discussion and follow the lead presenter as they speak. We captured three days of classroom instruction and had these videos transcribed verbatim. For PSTs' written solutions to the modeling task, we provided them with a detailed rubric (see Appendix A) of what should be included in their write-up of the modeling process, which resulted in rich written descriptions of their thought processes across modeling phases. When asked to give peer feedback, they were given prompts, detailed later. Last, PSTs kept a journal across the semester in which they were asked to respond to several prompts related to utility, emotions/self-efficacy, and social/group work (Middleton et al., 2017) and to reflect on the modeling process overall (see Appendix B). In Table 1, we offer a description of each research question matched with the data sources we used to address that question.

Data Analysis

PSTs' Written Solutions to the Modeling Task

We analyzed the data chronologically, beginning with the PSTs' initial models. For each group's model, we read through their written reflection and related presentation slides with two goals in mind. The first was to record details for each group for each phase of the modeling cycle. This would allow us to look for similarities and differences across different groups' approaches. The second goal was to look for interconnections across phases or how each phase of the modeling process informed the next.

PSTs make decisions as they engage in the modeling process, and often, a decision in one phase informs decisions made across the process. To capture the decisions PSTs make when modeling, we created a *decision model*, which is "a visual network graphic that outlines the thoughts, plans, and choices/decisions made during a flow of actions embedded in a range of conditions" (Miles et al., 2019, p. 202). To analyze each group's work, we thought about the modeling cycle as the flow of action and each phase of the cycle as a place for PSTs to make decisions. We drew on the decision model as an analytical tool to illustrate and make sense of the PSTs' models and their evolution over time. For clarity, we renamed "decision model" as Modeling Decision Map. Each map is an overall illustration of the model created and shows the PSTs' decisions across the modeling process.

We used dashed vertical lines between phases because modelers often bounce back and forth between phases, which are often not distinct. The first column, given/constraints, contains the two stipulations given to all groups: The tour must take place on campus and last at most an hour. The second column represents the problem(s) the PSTs identified that they needed to solve. The third column represents the assumptions they were making and the variables they considered about the problems to be solved. The fourth column represents the mathematics they completed to get a solution, and the fifth column is the generalization and analysis of their mathematical work. In their write-ups, PSTs were asked questions about each phase of the modeling cycle, including important factors and choices to consider, assumptions they were making, variables in the context of the problem, resources they were drawing on, and the mathematics they had decided to employ and their reasons for it. Attending to each prompt in turn allowed us to glean and flesh out each phase of the modeling cycle for each group. The example in Figure 4 shows the assumptions PSTs in Group 2 noted in their presentation (shown on the left) and how we represented these assumptions in the Modeling Decision Map (on the right).

We used arrows to show how the decisions groups made in one phase of the modeling cycle informed the next. For example, we show Group 1's initial model in Figure 5. In their description of who would take the tour, they described that they could have people with disabilities, so we drew an arrow connecting the question "who will take the tour?" to

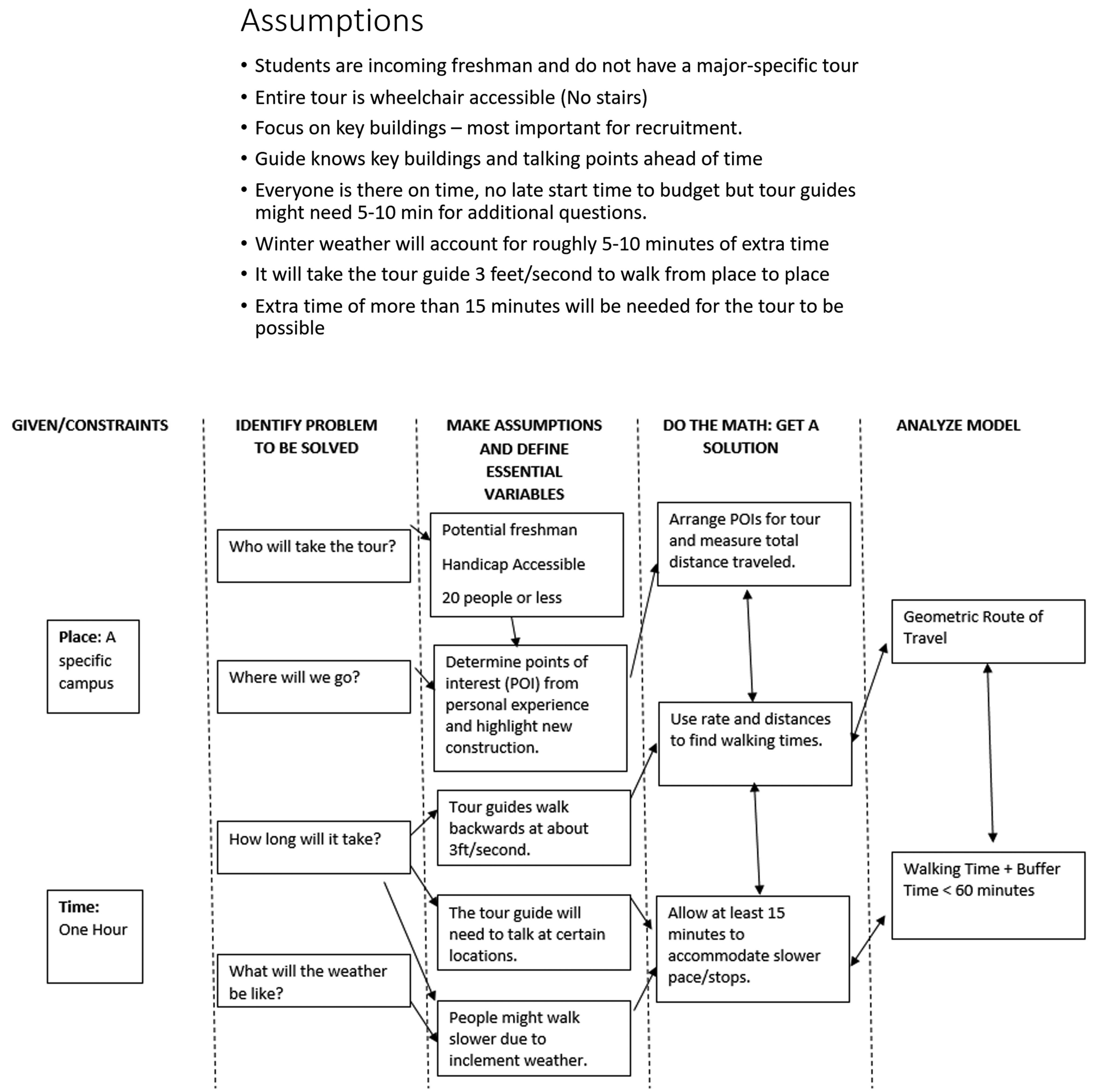
Table 1

Data Sources Used for Each Research Question

Research question	Data sources
1. PSTs' constructing, refining, and growing mathematical models across time	Primary: PSTs' written solution to modeling task—collected by the group Secondary: classroom video data
2. Role of discussion and targeted feedback	Primary: PSTs' and instructor feedback Secondary: classroom video
3. PSTs' perceptions of the modeling process	Primary: PST written journal reflections—collected individually

Figure 4

Capturing the PSTs' Assumptions in the Modeling Decision Map

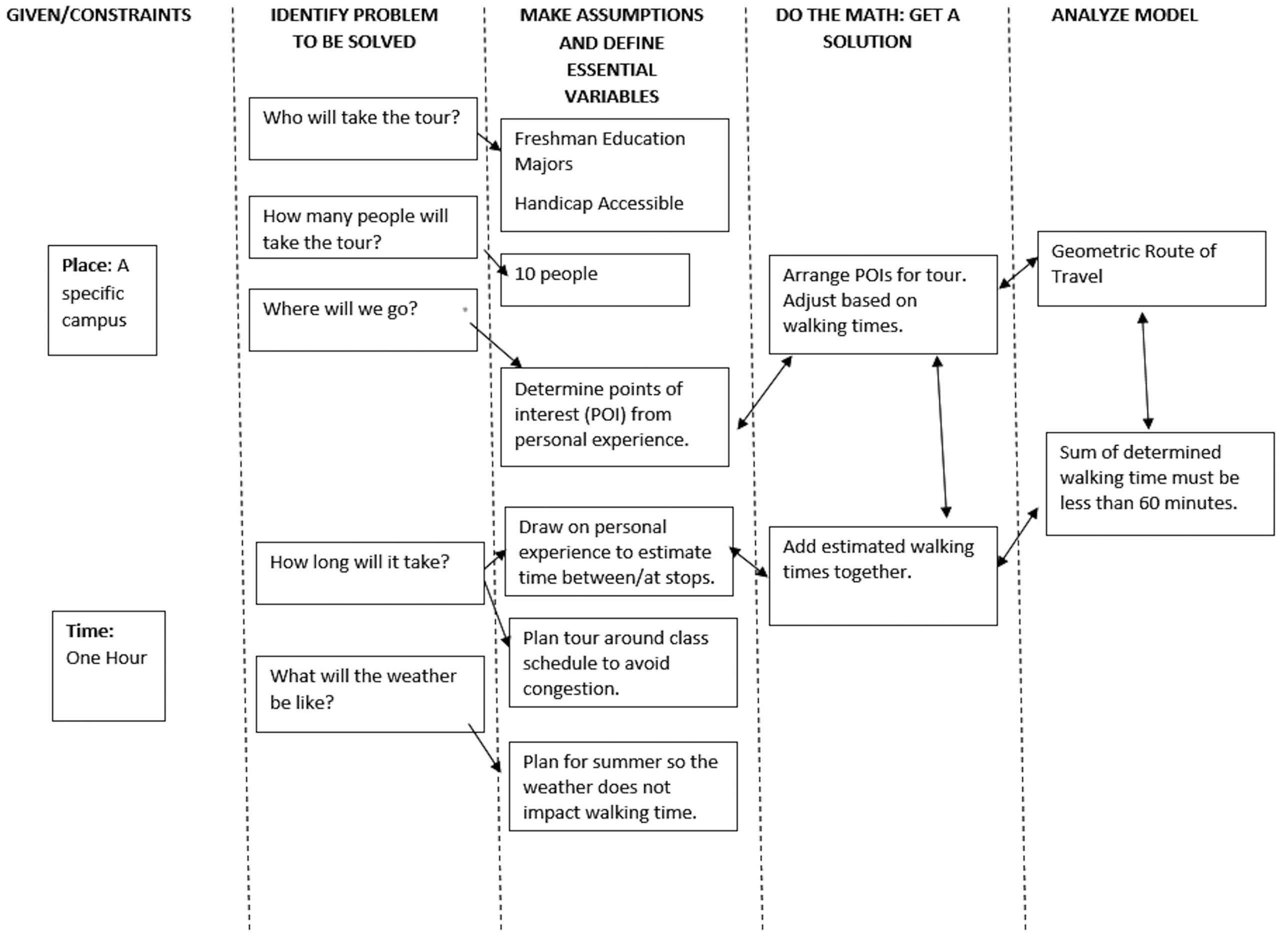


"accessibility." The group did not draw on this assumption when doing the mathematics or creating their model, so it did not connect to other phases of the cycle. The group also identified that people on the tour will be freshman education majors. They drew on their knowledge as education majors to determine relevant points of interest and then arranged those points of interest for a tour. These three items were connected because they informed one another.

We used bidirectional arrows to indicate places where modelers moved back and forth, which occurred primarily in the "doing mathematics" component of the modeling cycle. As shown in Figure 5, Group 1 chose essential places on campus to visit. However, after connecting the stops, measuring distances, and calculating times, they realized that their tour was not feasible and had to start the process again.

Figure 5

Flow Between Phases in Group 1's Modeling Decision Map



We constructed two Modeling Decision Maps for each group, one for the initial and one for the final (six maps in total for the three groups). We began with one group, and both authors individually read through the group's initial write-ups, presentation slides, and transcripts of classroom discussions. We each created a Modeling Decision Map, compared what we had created, and discussed any discrepancies to ensure we accurately captured the data. Discrepancies initially occurred when we discussed problems to be solved and assumptions/variables. For example, when we discussed assumptions/variables, one author may have noticed an assumption that the other author overlooked and would then bring it to the discussion for consideration.

We continued data analysis with each author individually, creating a Modeling Decision Map for each remaining group's initial model. We met to compare and discuss our individual maps to reach a consensus for a Modeling Decision Map for each group's initial models. Next, we overlaid each decision map on top of one another, combining all three groups' data in a relatively messy composite sequence analysis, which "integrates multiple participants' journeys into a single diagram" (Miles et al., 2019, p. 202). We merged the models to compare and contrast different group's processes, which allowed us to identify specific contributions from each group to whole-class learning.

We next engaged in the decision modeling process again for each group's final model. Once we had maps created for the initial model and final model for each group, we compared the two in relation to the feedback tables—described next—to understand how the groups addressed each component of the modeling cycle and how their models evolved from the initial model to the final model. We were also interested to see which mathematical ideas introduced during classroom discussion and feedback were taken up across groups and why, and which ideas were abandoned. Through this process, we were able to map the evolution of each group's models across time and understand them in relation to one another.

Peer and Instructor Feedback

Our next step in the analysis process was to understand how the groups' models changed from the initial to their final model and whether or how feedback and discussion informed these changes. To engage PSTs with their peers' thinking, a team member from each group was asked to evaluate another group's model, and they were given a set of prompts that included the following:

- Is the proposed tour feasible? (i.e., Can it be completed in an hour?) Use mathematics to test the tour.
- How did you come to your decision?
- What are the strengths of the tour?
- What are some changes that you might suggest to make the tour better? Why?
- Are there situations where you see the proposed tour falling short?
- What, if anything, will you take away from your classmates' tour to make your tour better?

Each group had feedback from two groups (six documents total). The goal of the instructor's feedback was to press students' thinking and position each group as competent by highlighting unique contributions from each model. For each group, we compiled peer and instructor feedback by question into tables to understand what feedback was given and why. We sought to understand what type of feedback each group received, noting similarities and differences across groups. We also watched the classroom video, noting any additional feedback that was given verbally. Next, we compared the final decision map with the feedback each group received to see whether and how they incorporated that feedback into their model. We also observed the classroom videos, looking for feedback made public across groups, specifically by the instructor, to see how instructor feedback may have influenced revisions across the final models.

PSTs' Journal Responses

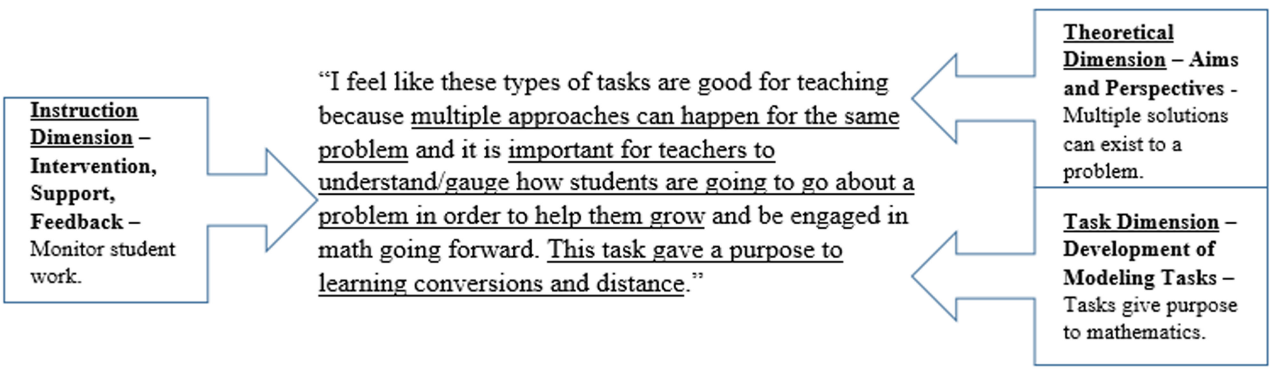
To understand PSTs' perceptions of the modeling process, we drew on the journals that they each completed. We identified the following question to help us understand what they found meaningful in the modeling process and why:

- What have you learned from working through this task (about modeling, your mathematical self, modeling in relation to teaching)?

PSTs typically wrote a paragraph for each of the prompts. We began our analysis of the responses by independently reading each entry sentence by sentence and sorting their responses by what they referred to (i.e., modeling, mathematical self, modeling in relation to teaching). We considered only sentences in which a PST referred to their understanding of modeling, the task, or teaching modeling. Using a descriptive coding process (Miles et al., 2019), we drew on Borromeo Ferri's (2018) work to develop provisional codes and subcodes as they related to her four dimensions (theoretical, task, instruction, and diagnostic). We coded independently twice and met to collectively revise and refine our codebook. As new codes emerged, the data were reanalyzed. The first level of coding concerned the dimension, the second was competencies noted by Borromeo Ferri (2018) within a particular dimension, and the third-level code was a specific idea or takeaway that the PST shared about that competency. In Figure 6, we provide a snippet from one of the PSTs' journal entries in which we identified three codes. For example, in the PST's first sentence, she stated, "Multiple approaches can exist for the same problem."

Figure 6

Qualitative Coding Example for PSTs' Journals



First, we coded this as a statement related to the nature of mathematical modeling. Next, we looked at what dimension this could connect to and identified the theoretical dimension (Borromeo Ferri, 2018). From there, we identified a subtheme related to the competency of aims and perspectives of modeling and a code that the PST understood multiple approaches could happen for the same problem.

After analyzing all the responses in this manner, we discussed how these codes were related to one another and whether codes could be collapsed. We counted the frequency with which each code emerged across participants and displayed this in a results table.

Last, we looked across data sources (PSTs' written solutions, feedback, and journal entries) to identify themes across the process related to the PSTs' perceptions of modeling with respect to the four dimensions of teaching and experiences as learners. The different sources acted in tandem to help us understand how PSTs engage in modeling and what they draw from the process as learners. The Modeling Decision Maps and videos of the classroom provided specific evidence of how PSTs engaged in the process. The journals illuminated the experience from the PSTs' perspectives and allowed us to interpret whether and how PSTs were drawing on these experiences.

Trustworthiness and Validity

Within this study, we identified potential threats to trustworthiness and validity, such as teacher/researcher bias, and we implemented several procedures to help establish trustworthiness. The first author is also the instructor of record for the enactment of the modeling task. To mediate teacher/researcher bias, the second author acted as an observer while the first author enacted the task so that we had an additional set of observations of PSTs engaging in the process. Second, the two authors coded the data separately and shared analysis in process, making sure to discuss any discrepancies until reaching consensus. This allowed for two different perspectives of the data to help mediate biases the first author might introduce.

To accurately capture and share PSTs' work and perceptions of the modeling process, we incorporated triangulation through multiple sources of data collection—written work, video recordings, and journal entries. We could not member-check our findings with the PSTs because we did not have the means to contact them so long after implementation. They did understand the journals were there for them to share their thoughts and feelings openly. To minimize bias, we did not review the journals until all grades were submitted.

Results

In the following sections, we analyze each group's initial and final model to understand how they engaged in the modeling process and how their models grew and changed over time. We summarize feedback and discuss how collaborative learning practices informed models. Finally, we examine PSTs' journal entries to determine how they perceived this experience in relation to the four dimensions of modeling for teaching (Borromeo Ferri, 2018) and their experiences as learners.

Initial Models

When Group 1 began the task, they had five problems to solve, including who would take the tour, where they would go, how long it would take, how many people would take the tour, and what the weather would be like (Figure 7). Two of the problems led to assumptions that did not affect mathematical work. For example, when considering how many people would take the tour, the PSTs assumed 10 people but did not explore whether having more or fewer would affect the tour. They also assumed that the tour would occur in the summer, so walking speed would not vary.

Two of the assumptions and variables led to mathematical work to find a solution, with the primary variable being walking time. To narrow their tour, they drew on their experiences as education majors and identified relevant points of interest (POIs) to future education majors. They also drew on their personal experiences walking on campus to estimate the amount of time needed to move between buildings and make stops. The mathematical work began when they arranged the POIs into a tour and added their estimated walking times together. The PSTs guessed and checked routes to ensure that when they summed the time between routes, it was less than an hour.

When Group 2 began the modeling task, they had three problems to solve and all led to mathematical work (Figure 8). They considered who would take the tour, where to go, and how long it would take. When considering assumptions and essential variables, they identified talking and walking times as things that could vary. They assumed that tour guides walk backward at a slower rate (3 ft/s) than a typical person walking forward and decided that this needed to be accounted for. They also noted that certain places on campus, like the student union and athletic complex, would require time to stop and explore. When considering who would take the tour, this group assumed freshmen and drew on important POIs for all students across campus.

When doing the mathematics to get a solution, the group identified a potential route and measured distances. They calculated walking times using their walking rate to estimate the time it would take to walk the route. Once walking time

Figure 7

Decision Map Capturing Group 1's Initial Model

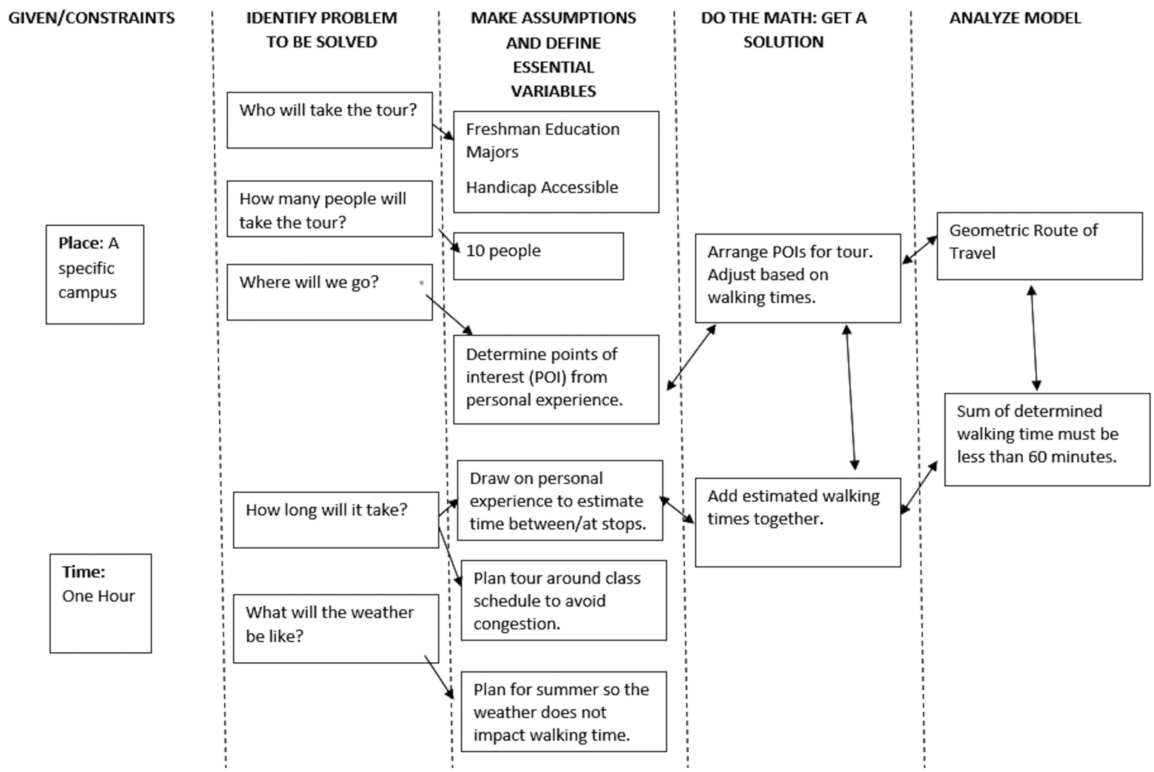


Figure 8

Decision Map Capturing Group 2's Initial Model

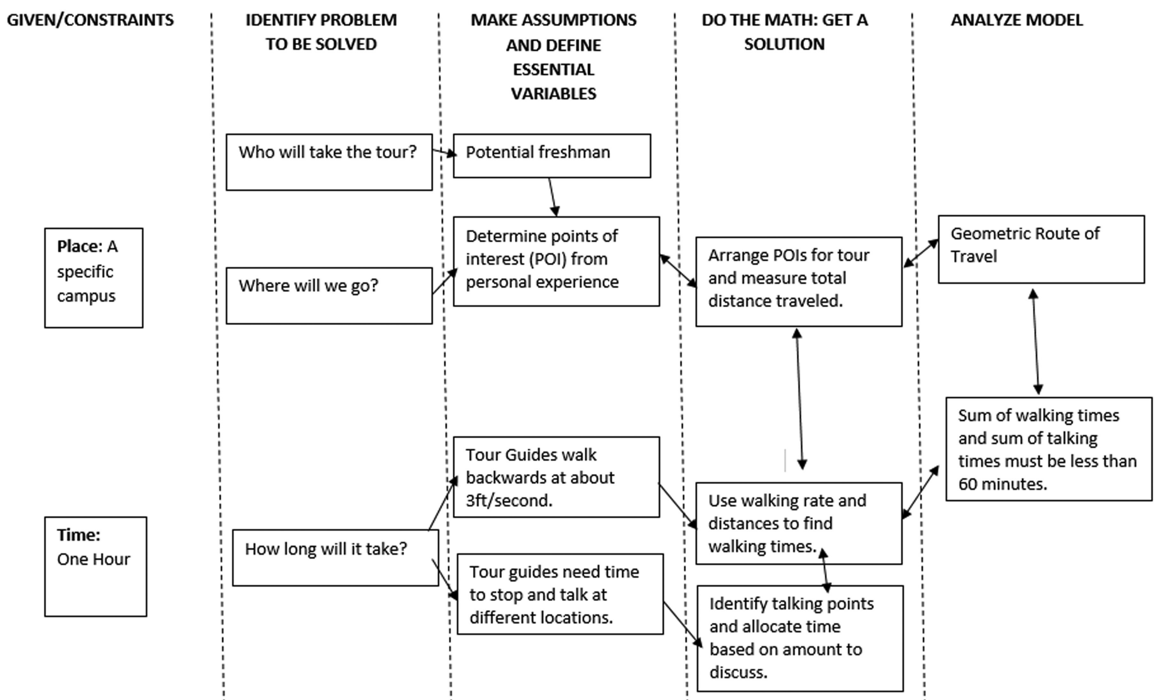


Figure 9

Group 2's Mathematical Work

Map #	Action of Tour Guide	Talking Points	Distance Covered	Time Allotted
1	Gather group at the Student Union Building	Introduce yourself, include brief description of your experience at MSU	0 ft	5 minutes (Talking)
2	Walk to the library and gather inside the first floor	Tell the group the library hours and explain what each floor has to offer. Point out the coffee shop as well.	150 ft	1 min (Walking) 3 mins (Talking)
3	Walk past Montana Hall	Explain what Montana Hall is and tell the classic bell tower ft. cow story	200 ft	1 min (Walking) 2 mins (Talking)
4	Walk to Rendezvous Dining Hall and show them around inside.	Explain which stations are there every day versus which stations change day to day. (all meals)	500 ft	2.5 mins (Walking) 2 mins (Talking)

was accounted for, they identified potential stopping points and allocated additional time for the tour guide to allow participants to stop and explore. Figure 9 highlights an example showing their mathematical work in the form of a table. Their final model was composed of two variables, walking time and talking time, and these two needed to sum to under 60 min.

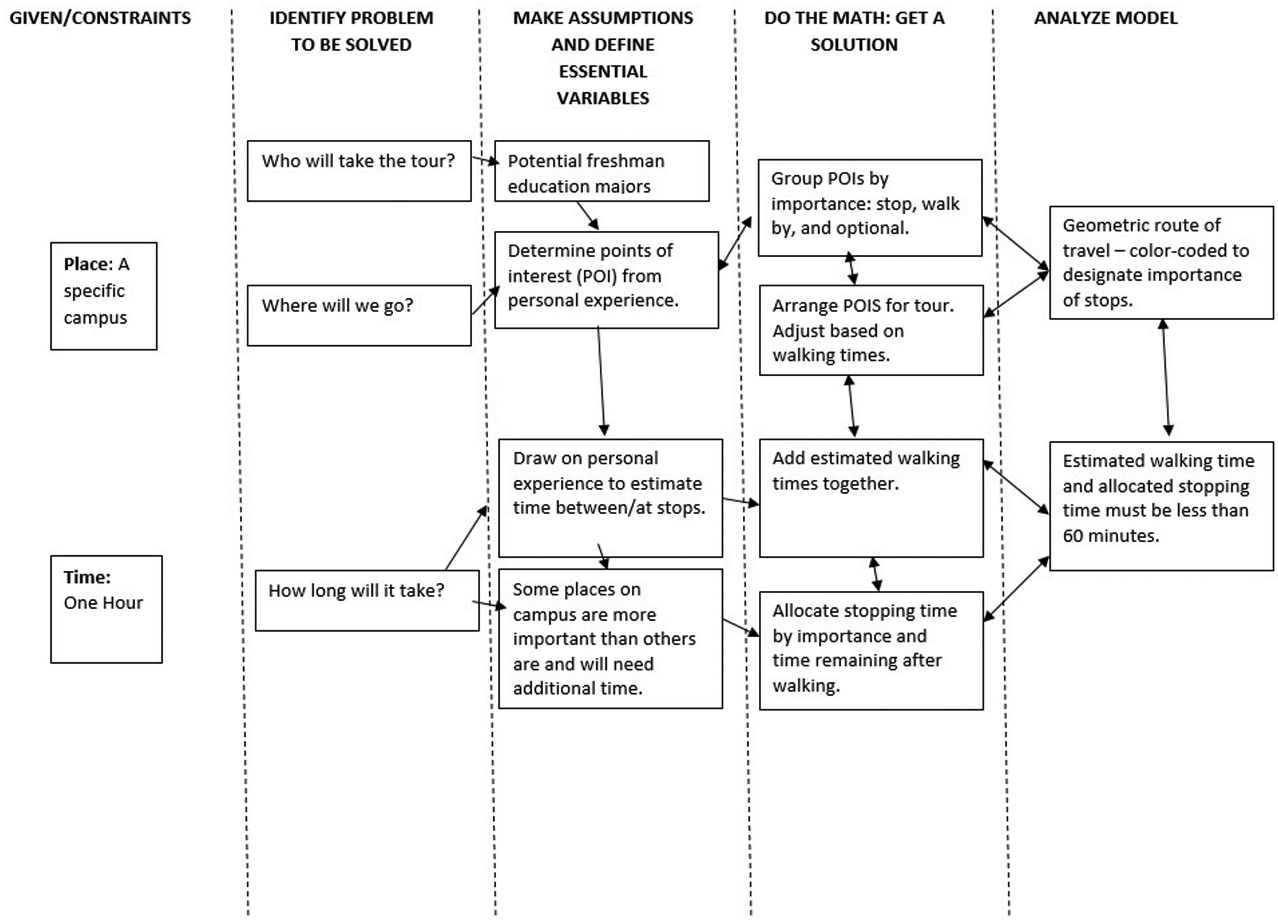
Group 3 considered similar aspects of the problem to those examined by Groups 1 and 2 but also took a unique approach. In their initial model, Group 3 had three problems: who would take the tour, where they would go, and how long it would take (see Figure 10). These problems led to assumptions that the students on the tour would be education majors and would want to visit the same POIs that the PSTs found important. Similarly to Group 1, they drew on their experiences walking on campus to estimate the time needed to move between buildings and make stops at places that they considered more important than other places.

The mathematical work began when they separated the POIs by importance (stop, walk by, and optional) and added their estimated walking times on the basis of their personal experiences. They assigned optional buildings as ones that participants could go to if time allowed. They allocated stopping time for each place according to importance, and time remaining after walking times were determined. They checked to make sure their walking times summed to less than 60 min, given an identified route of travel, and color-coded the POIs to indicate the route of travel. Figure 11 shows their proposed tour and a map of selected buildings on campus.

When looking across the three initial models, all the groups were able to identify problems that they could solve in relation to the campus tour. They could all make assumptions and define essential variables, but to different degrees. Much of their mathematical work was based on assumptions from their own experiences and needed refinement to develop into shareable and reliable models. From their initial models, we can see several mathematical pathways forward. Two groups, 2 and 3, separated their time into two variables—walking time and talking time—and Group 2 included a walking rate to help them determine whether their walking estimates were accurate. One mathematical pathway forward might be determing whether their walking rates are plausible and to specify time further. A second mathematical pathway forward is to consider maximizing time. All the groups discussed arranging POIs to maximize the time but did so by guessing and checking. Group 3 was the only group that discussed a method of sorting and prioritizing different stops so visitors could maximize time through choice.

Figure 10

Decision Map Capturing Group 3's Initial Model



Analyzing and Assessing Models Through Peer and Instructor Feedback

Looking across reviewer feedback and classroom interactions, each of the PSTs was able to successfully assess and propose changes that would improve the models for campus tours, as well as highlight the strengths of each group's approach. Table 2 highlights the contributions identified by both peers and the instructor that were taken up for whole-class consideration. Across groups, reviewers focused on maximizing the number of stops according to time and distance, considering other uses of time, such as talking or stopping, and determining whether the walking rate might change under varying circumstances, like weather and accessibility.

PST reviewers proposed that Group 1's tour was feasible but that they could change the geometry of their travel route to add additional POIs. Reviewers appreciated that Group 1 considered handicap accessibility and discussed weather as a consideration. For Group 2, reviewers proposed that they should take into account bad weather, specify who their tour is for, and review stopping times and why they were warranted. They determined that, with stopping times included, the tour was too long. The reviewers appreciated that Group 2 had considered the relationship between distance and time through a mathematical lens. For Group 3, the reviewers determined the tour was too long. They discussed separating the tour into different tours to provide optional stops for participants to visit at their convenience. They also proposed fleshing out discussion topics to anticipate whether stops were warranted and estimating how long they would take. Reviewers appreciated that Group 3 used color-coding to group stops by importance.

After discussing contributions, the instructor drew the PSTs' attention to three considerations they had to account for: weather, accessibility, and feasibility. Because many PSTs had estimated timing, the instructor focused on feasibility to draw their attention to purposely using mathematics to determine whether the tour would work and why. Weather and accessibility were tied to walking rate, and the instructor wanted them to consider how these two variables might have an

Figure 11

Group 2's Proposed Tour and Map

1. Strand Union Building (~5 minutes)
   - At the Gold Star; Meet and start here, pass through bookstore
2. Barnard/Cobleigh Hall (~3 minutes)
   - Only connected buildings on campus with a tunnel
   - Core classes could be held in Barnard
3. Library (~2 minutes)
   - Pass the library to show students where it is
4. Wilson Hall (~2 minutes)
   - Stop by to show where mathematics and writing classes are held
5. Jabs (~3 minutes)
   - Stop in business building talk about this is one of the newest buildings on campus
6. Rendezvous Dining Hall (~8 minutes)
   - Newest dining hall on campus
7. Bobcat Statue (~3 minutes)
8. Montana Hall (~2 minutes)
   - Oldest building on Campus

Key:
RED: places we decided we wanted to stop at and/or go into
GREEN: places we chose to pass by and talk about
BLUE: places we would make optional for students/families

9. Reid Hall (~5 minutes)
   - Main education building and where a lot of core classes are taken
10. North Hedges (~15-20 minutes)
    - One of the main and central dorms on campus
    - Look at a dorm room
11. Gym (if possible ~5 minutes)
    - Step into gym to show students where they could work out and participate in extracurriculars
12. Fieldhouse - if possible
    - Walk through to show students where they could spend their time supporting Bobcat Athletics
13. 7th street - if possible
    - Show families where the campus police station is in case of an emergency or any other needs
14. Norm Absjorbson Hall (~5 minutes)
    - Newest building on campus, home to the engineering department and Honors College
15. Strand Union Building (~1-2 minutes)
    - End here for question time

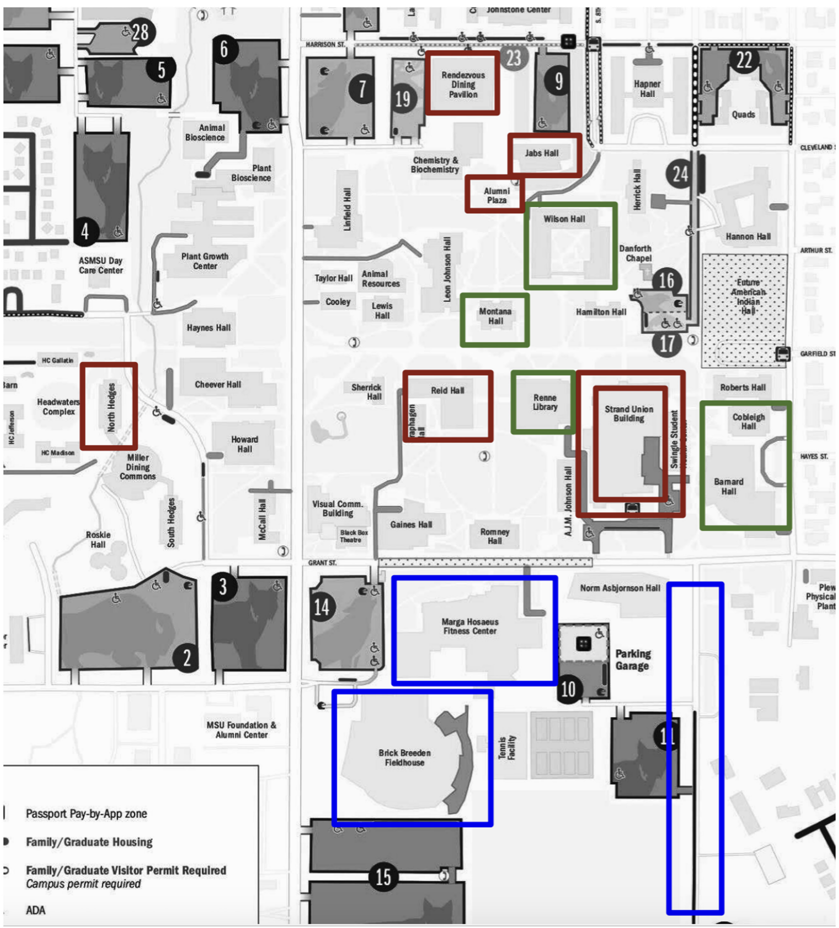


Table 2

Summary of Written Peer and Instructor Feedback

Group	Contribution to Consider	Feedback for Class Consideration
1	Weather	• Is it reasonable, living in the northern U.S., to only plan for summer conditions? • How will weather conditions affect our tour?
1	Number of people/accessibility	• How many people should we plan for on the tour? • Will the number of people affect our timing? • Is our tour accessible to all students?
2	Walking rate	• How do we know our tour will work? • Is there a way we can use mathematics to check?
2 & 3	Separating time into multiple variables (i.e., walking, talking, and stopping time).	• What are important time variables to consider? • How do they help us make sense of the situation?
3	Separating/color coding destinations by importance	• How will you make choices if you run out of time on the tour?

impact on timing and feasibility. For example, snow occurs on campus over half of the academic year and can significantly affect walking speed. Furthermore, all students should have an accessible tour. The instructor highlighted that not planning for either of these considerations was not realistic. For their final model, the PSTs had to justify how they knew their tour was feasible in an hour and accessible to all students, no matter the time of year.

Final Models

In this section, we share each group's final models and the refinement they incorporated after feedback. For all the figures, shaded areas of the Modeling Decision Map indicate where PSTs added or made changes to their model.

Following peer feedback, instructor feedback, and revision, Group 1 identified the same five problems to be solved, and four of them led to mathematical work (Figure 12). When considering how many people would take the tour, Group 1 did not consider varying the amount of people and capped the tour at 10. In considering where they would go, the PSTs drew on their experiences as education majors but revised their tour path to ensure the paths and buildings were accessible. In considering the length of the tour, PSTs broke their time into two variables: walking time and talking time. They determined walking time by assigning a walking rate relative to the distance traveled. When considering the weather, they acknowledged that in winter people walk slower and they should add buffer time. They determined talking time by considering the number of purposes the building served. For example, the student union serves several functions (i.e., student academic services, bookstore, meeting spaces) and would require more time than the dining hall, which serves one purpose.

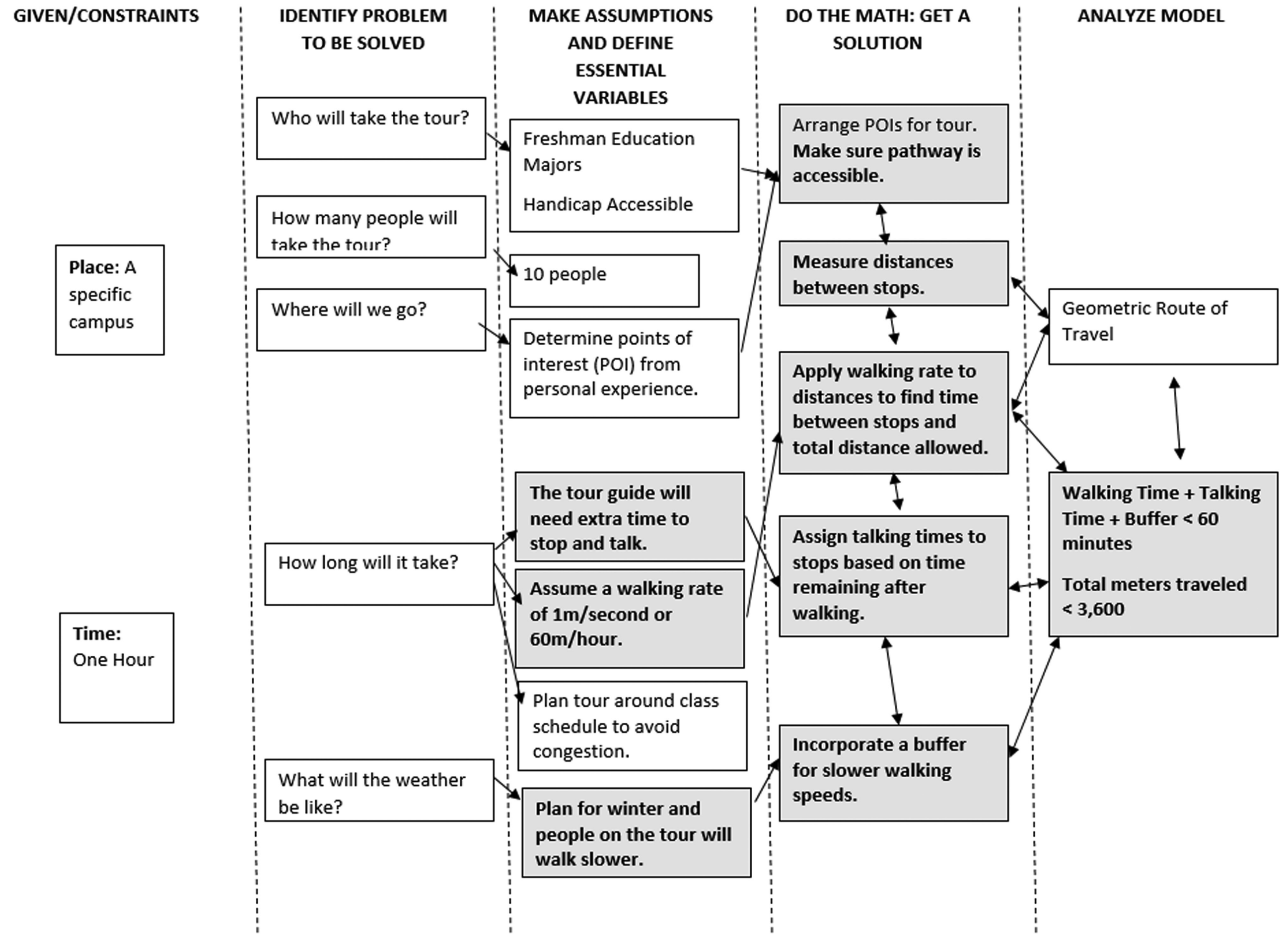
As they transitioned into mathematical work, they moved among arranging POIs, measuring distances, applying walking rates, assigning talking time, and assigning buffer times to determine an appropriate tour. In the end, they developed two models to understand travel time. The first model broke the time into three categories: walking time, talking time, and buffer time, and those three times needed to be under 60 min. They also quantified time as distance through the walking rate and determined the total distance traveled needed to be under 3,600 m.

When considering Group 1's initial model compared with their final model, we see that their mathematical work became more detailed and precise about distance and time. They drew on feedback to estimate walking time using a walking rate. They could envision time as a variable and allocate it for different purposes as it related to the tour. They also provided a more detailed way to check the walking time of the tour through walking rate and distance traveled. They took up the instructor's feedback and accounted for weather by incorporating a slower walking speed and a buffer time and for accessibility by modifying their travel route. They did not attend to peer feedback investigating how to mathematically maximize time through exploring different geometric travel routes. Figure 12 shows that the two suggestions that were taken up significantly changed the mathematical work and the resulting model. It allowed them to talk about their tour through two perspectives: time and distance traveled, and accommodation for slower travel speeds.

In their final model, Group 2 added an additional problem of considering the weather (Figure 13). Group 2 was satisfied with the tour they created and the mathematics they used to determine feasibility but needed to incorporate this additional consideration. By incorporating weather and accessibility, they identified walking/travel rate as an essential variable that could change. They also realized they might not have enough time to stop, especially considering a slower pace. The time they had initially allocated to stop might be consumed by travel time.

Figure 12

Decision Map Capturing Group 1's Final Model



As shown in Figure 14, when doing the mathematical work, the group accounted for variance in both travel speeds and time to stop by allocating a block of time (at least 15 min) to address weather and accessibility. They recognized that different tours would have different participants, so it was up to the tour guide to determine how they wanted to use this additional time. They responded to feedback from peers that the talking time might take too long by allowing a buffer time and assuming that tour guides might talk while walking. They also took up feedback from the instructor by responding to weather and accessibility considerations. When we consider Figure 14, we can see the mathematics they used in their model stayed the same, but they were able to make functional changes to support weather and accessibility. Instead of allocating a fixed amount of time to stop and discuss locations on campus, they reallocated this time as multipurpose.

In their final model, Group 3 also added consideration of the weather (Figure 15). All the identified problems came together to help inform their mathematical model. In considering how long the tour would take, PSTs added an assumption that people typically walk at a certain speed. They integrated this assumption with their plan for winter, allowing a slower walking rate and quantifying it to 3 ft/s, assuming that people walk in the snow at a similar speed as they walk backward.

After transitioning into the mathematical space, Group 3 moved among arranging POIs, measuring distances, and applying walking rates to assign an appropriate tour. They responded to peer feedback of the tour being too long by shifting their model from three color codes to two and focusing more on the walking rate as a determinant of time, rather than estimates that were based only on experience. They also identified spaces that were not part of their tour but that participants could visit at their leisure if time allowed. Once walking times were established, the group drew on their initial model to assign stopping times according to importance. The group responded to the instructor's feedback to consider bad weather and accessibility by slowing the travel speed. Overall, both peer and instructor feedback allowed for greater precision in their model.

Figure 13

Decision Map Capturing Group 2's Final Model

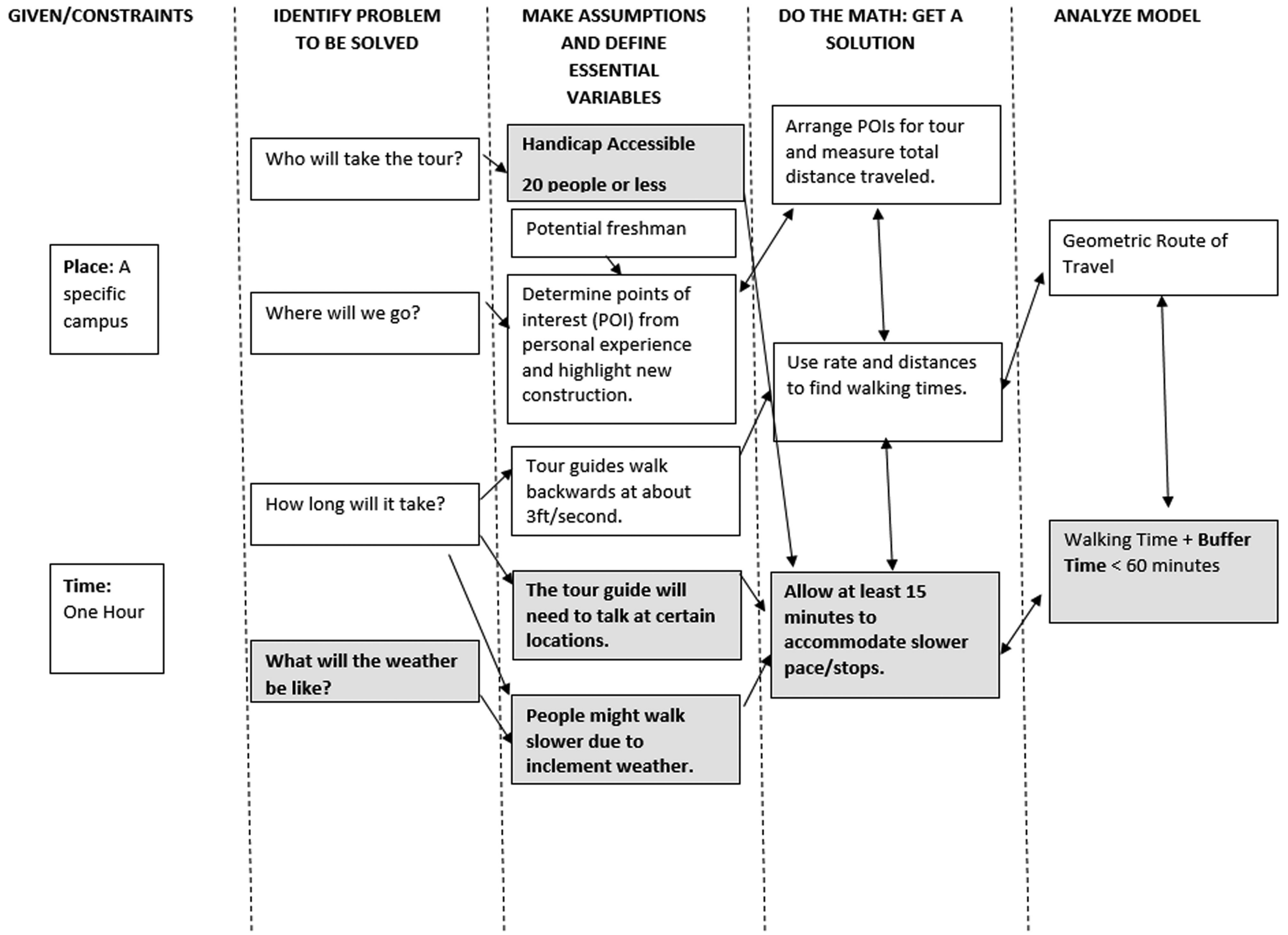


Figure 14

Example of Group 2's Model

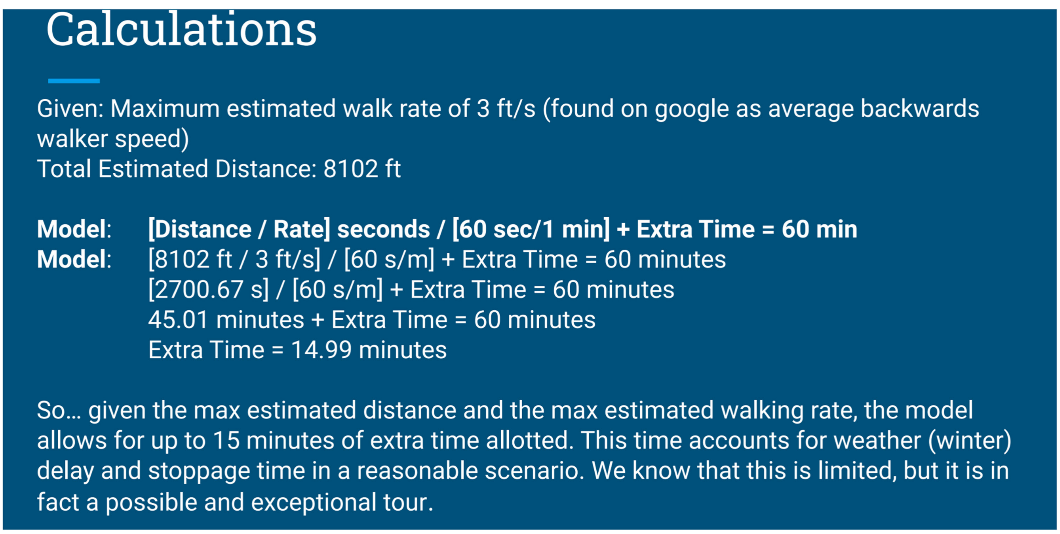
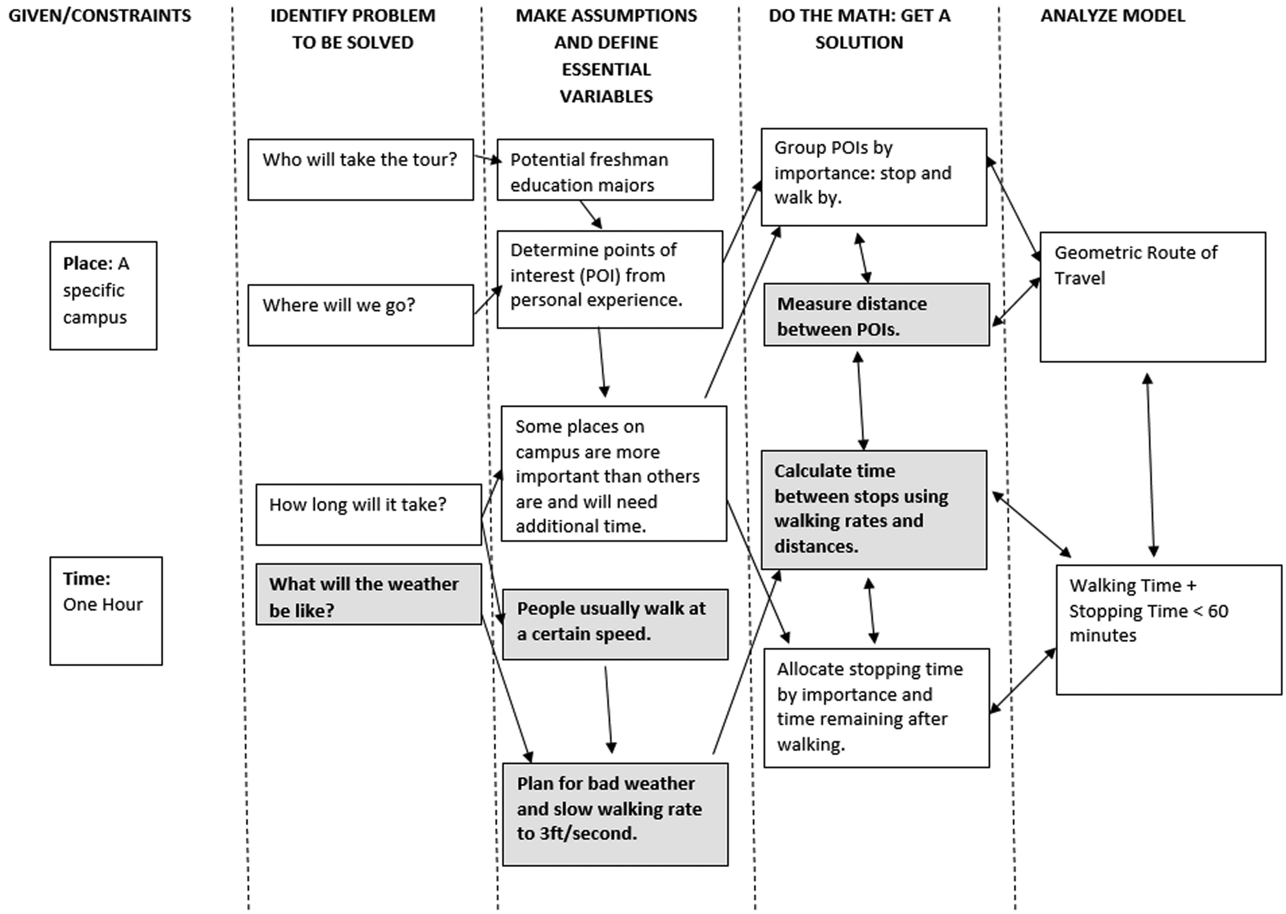


Figure 15

Decision Map Capturing Group 3's Final Model



Comparing Final Models

Looking across all three groups, we gain insight into how PSTs' models developed and into the role of targeted peer and instructor feedback. All the groups initially succeeded at defining mathematical problems and making assumptions. Similar to the study by Zeytun et al. (2017), two groups primarily drew on personal experiences without incorporating mathematics into the model. From the initial model to the final model, they grew in their ability to draw on mathematics to refine their models and make connections between the assumptions and variables they identified and the mathematics they engaged in.

Looking across groups, the process of analyzing and assessing one another's models was informative for PSTs as they revised their work. By reviewing other groups' models, PSTs were able to consider multiple approaches to the task and incorporate new ideas into the problems they identified and the assumptions and variables they considered. Looking at their models across time, we can see which mathematical ideas took hold and in what ways. In the initial models, several mathematical ideas were introduced to help make sense of creating the tour: determining a walking rate, assigning importance to locations, and breaking time into multiple variables. Some ideas, like walking rate, were picked up across the class as a usable mathematical tool, while others, like assigning importance, did not take hold. For many PSTs, discovering the walking rate allowed them to transition their experiences and assumptions about walking around campus to a mathematical space. One PST stated:

> A pivotal moment for me was during giving feedback to another student. One of my partners found an equation that helped her use math to figure out if the other group's tour was plausible rather than just using their best estimate to figure out if it was plausible or not. . . . An equation that everyone could use was just an equation that calculated the distance that everyone walked and at a certain rate. Therefore, I was able to apply this to my own tour and make efficient and effective changes to my own tour that improved it greatly.

This change was notable in Group 1, who created a second model that determined the total distance that could be traveled in an hour as a second way to determine if their tour was feasible. We hypothesize that PSTs did not draw on Group 3's approach (assigning importance to places visited) because they had not yet experienced models for rating and ranking.

Once groups determined whether the walking rate was feasible, other mathematical approaches became supplemental. For example, most groups determined that it would take a good portion of an hour to walk to most places they wanted to go on campus. They had to assign that time first before they could assign the remaining time to stopping and talking. For Group 2, instead of determining how the time should be spent, they assumed tour guides would talk as they walked, negating stopping time and allowing extra time as a buffer. Other mathematical ideas, like assigning importance, allowed PSTs to make choices about where to go and why.

Even though the three groups all assimilated walking rates into their models, they did so in different ways. Some groups changed the walking speed, whereas others allowed for a buffer time in their model. This was also true when the instructor asked them to account for weather and accessibility. We can see that the PSTs relied on both their peers' and the instructor's feedback to make informed changes. All the groups modified their models to respond to the instructor's comments about accessibility and weather and each group used at least one of their peers' recommended changes to improve their model. All the PSTs' models benefited from the discussion of different approaches.

Table 3

PSTs' Perceptions of Mathematical Modeling With Respect to Four Dimensions

Dimension	Unique PSTs	Theme	Code	PSTs ($N = 11$)
Theoretical	11	Aims and perspectives of modeling	Multiple approaches can happen for the same problem.	A, C, K
			There is not "one" right solution in a modeling task.	B, G, I
			Modeling shows that mathematics is intertwined in our daily lives.	H
			Modeling is a process. It is OK if it takes time to formulate a solution.	A, G, K
		Modeling cycles	Modeling problems have multiple steps.	D
			Modeling involves making assumptions.	F
			Modeling involves revision. Revision can bolster student confidence and make solutions stronger.	D, E, F, H, K
Task	9	Development of modeling tasks	Modeling tasks give purpose to mathematics.	A, B, C, K
			Modeling tasks can connect to students' lived experiences.	B, C, D, G, I, K
			Modeling tasks can connect many different mathematical ideas.	E
		Multiple solutions to modeling task	Modeling tasks allowed me to consider different approaches and perspectives to the problem.	E, G, H, I
Instructional	5	Planning lessons with modeling tasks	The task put me in my students' shoes to understand modeling from the learner's perspective.	D, J
			Students bring their own experiences and background knowledge when solving modeling problems.	B
		Carrying out modeling lessons	Students can take different approaches when solving a problem; I need to be aware of that.	B
		Interventions, support, and feedback	Allowing students to see different perspectives and collaboration strengthens problem solving.	J, K
			It is important for the teacher to monitor students to help them grow their model.	B, J
			Open-ended tasks help students learn how to make decisions when problem solving.	H
Diagnostic	1	Recognizing challenges in the modeling cycle	The teacher can modify a modeling task to leverage mathematical exploration.	E

PSTs' Perspectives on Modeling

This section explores 11 PSTs' perceptions of mathematical modeling as a pedagogical practice with respect to Borromeo Ferri's (2018) four dimensions for teaching mathematical modeling (see Table 3). Our third research question asked about modeling from the PSTs' perspective. This section helps us understand what the PSTs took from the modeling task as learners and what insights they drew from the process as future teachers. The table shows the number of PSTs who received a particular code, with letters denoting individual PSTs.

When we implemented the Campus Tour Task, we hypothesized that by doing modeling tasks, PSTs would experience each of the four dimensions of teaching mathematical modeling, primarily the theoretical and task dimensions. Results from their journals highlight that they were also observing attributes of modeling across the instructional dimension, and one reflection referred to the diagnostic dimension.

In relation to the theoretical dimension, PSTs discussed that by working through the modeling task, they gained insight into the aims and perspectives of the modeling process and features of the modeling cycle. In particular, six PSTs realized that multiple approaches and multiple solutions can exist to a problem and no one correct solution to modeling tasks exists. Three PSTs discussed that modeling is a process that takes time and careful thought. If a solution does not present itself immediately, that is OK. One PST expressed this when she stated, "I learned from working through this modeling task that it is a lot of work and steps to complete. It is okay if not everything makes sense right away. It taught me it is okay to step back and reflect." One PST also described that she learned that modeling helps us see that mathematics is intertwined within our daily lives, often without realization.

When we consider the task dimension, PSTs reflected on the development of modeling tasks and multiple solutions to modeling, in particular, the role of modeling tasks in giving mathematics a purpose, connecting to students' lived experiences, connecting different content areas, and illuminating different perspectives. Four PSTs reflected on the idea that the modeling task allowed for consideration of different approaches and perspectives to the problem. The following quote highlights a PST discussing the task dimension with respect to giving purpose, connecting to lived experiences, and considering different approaches to the problem:

> Within our three groups in class, we all came up with slightly different ideas and strategies. We found different buildings and talking points to be significant and it shows that there isn't one right way to go about this problem. I think that encouraging kids to use calculations, whether that's creating an equation or determining distance in feet/second, that will be the most useful to them. There are numerous opportunities for students to use what they know about mathematics and apply that into a meaningful exploration.

Borromeo Ferri's (2018) idea of instruction with teachers being two-fold was evident in PSTs' reflections that related to the instructional dimension. Specifically, their comments reveal that the ways we teach modeling tasks matter. PSTs reflected on their experiences as learners, instructional strategies that they experienced in the modeling process, and ways those experiences might be incorporated into their future classrooms. Five PSTs' reflections referred to the instructional dimension. They discussed that engaging in the modeling process allowed them to experience modeling first hand—from the learner's perspective—and helped them to realize that students bring their own experiences to bear on the problem. One PST reflected that when carrying out modeling lessons, teachers must be aware of different approaches students might take. With respect to intervention, support, and feedback, PSTs highlighted that feedback and discussion allow students to strengthen their models and that open-ended activities help to foster students' decision-making processes. When discussing feedback and describing how collaboration strengthens problem solving, one PST stated:

> Taking the time to analyze other tours showed me some of the different thought processes my peers have and then gave me a chance to work on giving positive and constructive feedback that didn't change the thought processes they had but instead helped them to expand and strengthen their ideas.

Last, with regard to the diagnostic dimension, one PST discussed that when the instructor added the considerations of winter weather and accessibility to the Campus Tour Task, it made her realize that the teacher can modify a task to foster deeper mathematical exploration. She stated:

> One part of the task that I found useful as a teacher was, having to configure winter weather into this task. I think this made me look even deeper into the math, which is exactly what I hope my students are going to be doing.

Looking across perceptions of modeling, we can see several reasons why experiencing modeling as a learner, and the collaborative learning experiences involved, has an impact on PSTs as learners and may influence them to integrate modeling tasks into their future classrooms. First, it helped to bolster the idea that problems can be approached in different ways and that different solutions can exist in response to a problem (theoretical dimension). It gave them ideas of what kinds of tasks are meaningful to students and first-hand knowledge of what students will experience when they engage in a modeling task (task dimension). Specifically, they saw the value in connecting mathematics to students' lives and the value of collaboration. PSTs paid attention to how the task is enacted and reflected on teaching strategies that they found meaningful in the classroom, such as feedback (instructional dimension) and leveraging the demands of the task (diagnostic dimension).

Discussion and Implications

This section shows how answers to our three research questions contribute to the field, and we raise implications for future research. In our first question, we sought to understand how PSTs construct, refine, and grow their mathematical models. The way we illustrated PSTs' initial and final models through a Modeling Decision Map is novel and a contribution that other researchers may use as they look into capturing how modelers move through the modeling process. For us, comparing Modeling Decision Maps at different points in time helped highlight the changes PSTs made from the initial to the final models. We know from prior researchers that PSTs demonstrate improvement across modeling competencies (Çiltaş & Işık, 2013; Karacı Yaşa & Karataş, 2018; Tidwell et al., 2023). In our work, differences between the first and final models show evidence of PSTs' growth, and the maps allowed us to point to specific parts of their models that were modified. Further research is needed to document whether Modeling Decision Maps help document how other modelers (e.g., K–12 students, undergraduate students) develop and refine mathematical models.

Our second question sought to understand how collaborative learning practices like discussion and targeted feedback help PSTs refine their models. Researchers have documented that PSTs find it challenging to use multiple procedures to solve a modeling problem and revise their models (Durandt & Lautenbach, 2020). In our work, we addressed known challenges by incorporating revision as a pedagogical practice. Giving feedback in purposeful ways was powerful. By reviewing one another's work, PSTs gained mathematical insight, glimpsed the problem from multiple perspectives, and understood that multiple approaches could be equally valid. Revision helped PSTs develop the mathematics they used in the model and the ways in which they justified ideas across the modeling cycle. Implications for future research include examining other pedagogical practices in the modeling cycle and exploring how they could prompt growth within a particular modeling competency.

Last, our third question sought to understand how PSTs perceive the modeling process as a pedagogical experience. We situated our work in the realistic perspective of modeling (Pollak, 2007, 2016) because we wanted to help PSTs become more familiar with modeling and see it as a tool that they could use to make sense of realistic, messy situations (Biccard & Wessels, 2011; Blum, 2011; Ludwig & Reit, 2013). We sought to create modeling situations in which they could develop agency and feel comfortable drawing on their lived experiences as a foothold to enter the task. With respect to their identities as teachers, our study helps support the claim that PSTs need to experience the modeling process as learners before they begin teaching it. Our work contributes to the field in new ways in that we documented that PSTs not only learned about the modeling cycle and task design but also made observations about teaching modeling and specific practices they want to incorporate into their future classrooms. Even though we did not draw attention to the pedagogical practices we used, PSTs still made observations about the structure of task enactment. Future research could examine how drawing attention and encouraging reflection on teaching moves affects PSTs' understanding of modeling with respect to Borromeo Ferri's (2018) four dimensions.

In closing, we highlight the importance of incorporating modeling into teacher preparation programs as a tool to both deepen mathematical understanding and encourage the teaching and learning of modeling. We think the Modeling Decision Map can serve as a tool to examine and compare the preliminary and final models created by PSTs. We encourage other researchers to continue to examine the role of modeling tasks in influencing PSTs as learners and teachers.

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APPENDIX A

Modeling Task Rubric

For each modeling task we tackle, your write-up will consist of 3 parts: conceptualization of the problem, communication of results, and analysis of results. These portions of the write-up are described below.

Conceptualization: Conceptualization refers to the way in which you frame the real-world problem in the mathematical world. It involves acknowledging your positionality, or perspective on the problem, in relation to other potential perspectives. It also involves discussing why the mathematics you used to explore the problem makes sense to use. In particular, I would like you to address the following questions:

- Defining the Modeling Problem:
  - What is the mathematical problem that you are solving?
  - Who cares about this problem? Are there different perspectives that could be taken? Why does this perspective make sense?
  - Are there issues of equity or social justice in this problem? If so, what were they and how did you take them into account?
  - What mathematics did you use? Why did it make sense in terms of the problem?
- Building the Model:
  - What assumptions did you make when building your model? (This could also tie back into who cares about the problem)
  - What variables or parameters did you set (if any) and why?

Communication of Results: Communication of results is both explaining the mathematics you used to solve the task as well as communicating your findings to a broader audience. When communicating mathematical ideas, it is important to discuss the mathematics you used and why. To make it accessible to a broader audience, it is important to provide an overall summary that could include charts and figures.

- Mathematical Calculation/Meaning of Mathematics:
  - What mathematics did you use to solve the problem?
  - What were constraints that affect how you solved the problem? Describe how you dealt with constraints and in what ways.
  - What were aspects of the problem that could vary? What were aspects of the problem that were fixed? Describe how you dealt with variables and constants in the problem and in what ways.
  - What do your mathematical findings mean in terms of the problem?
  - If you used different approaches in conversation with each other, why? How do they come together to inform your solution?
- Accessibility:
  - How does your answer fit with the original problem statement?
  - Are your results communicated clearly? Can a friend or colleague, unfamiliar with your process, understand your results?

Analysis: Analysis is reflecting back on the entire process to consider your model across several perspectives. It involves looking back at your own perspective to consider the choices and revisions you made and why. It also involves considering the generalizability of your model and if and how it can be applied to other situations. Finally, it is also important to consider your model in comparison to your colleagues' models to discuss how their ideas inform your own.

- Reflection:
  - How, if at all, did you change your approach while working through the task?
  - What revisions did you make to your initial model? Why?

- Generalizability:
  - How applicable is your model to other situations? What other types of problems could you use this process to understand?
- Comparison:
  - How did you model compare to other solutions? (classmates and other sources)
  - How, if at all, would you take this information into account if you were to continue working on your model?

APPENDIX B

Modeling Journal

Throughout the course, you will keep a journal detailing your thinking on each of the modeling tasks. This is a private document that will only be shared with you and your instructor. Please create a google doc to share with the instructor. After each modeling task, write a short reflection for each of the following prompts:

Context of the Problem

- Tell us what task this journal entry refers to.
- What aspects of solving the task went well and what aspects were more challenging? Why?

Utility of the Task

- How useful have you found this task to you personally? Describe specific parts of the task that you found useful and why.
- How useful have you found this task to you as a teacher? Describe specific parts of the task that you found useful and why.

Emotions/Self-Efficacy

- Describe any pivotal moments (both negative and positive) you had while working on the task. What experiences prompted these feelings?
- How confident do you feel in mathematical modeling after working through this task? What, if any, aspects of the task helped build your confidence?

Social/Group Work

- In what ways did you contribute to the development of the solution?
- In what ways did your group members help or inhibit your progress in the modeling task? Provide specific examples.

Reflection

- What have you learned so far from working through this task (about mathematical modeling, your mathematical self, mathematical modeling in relation to teaching)?
- What, if anything, have you learned that will impact how you go about a future modeling task?
- What, if anything, have you learned about your mathematical self?