MATHEMATICS TEACHER
Learning & Teaching PK-12

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

Bridging Modeling and Environmental Issues

AUTHOR NAMES:

Jung, Hyunyi; Wickstrom, Megan H.; Piasecki, Chris

DIGITAL OBJECT IDENTIFIER: 10.5951/MTLT.2020.0369

VOLUME: 114

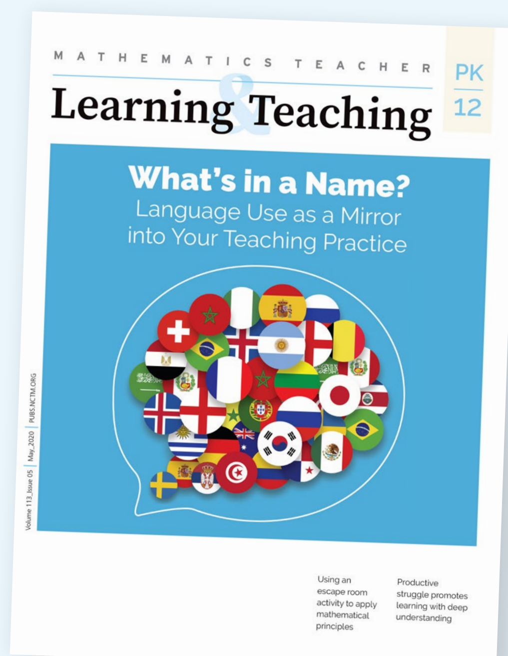
ISSUE NUMBER: 11

Mission Statement

The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

Approved by the NCTM Board of Directors on July 15, 2017.

CONTACT: mtlt@nctm.org



NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Access digital content at nctm.org/mtlt11411g6.

Bridging Modeling and Environmental Issues

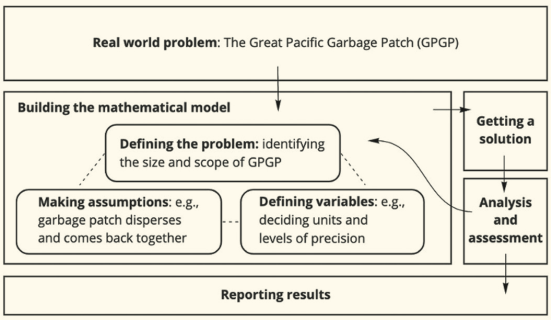
The Great Pacific Garbage Patch activity involves an urgent environmental issue that students can discuss. It engages students in the interpretation of visual data, measurements, units, and the area of regular and irregular figures.

Hyunyi Jung, Megan H. Wickstrom, and Chris Piasecki

Have you ever noticed trash collecting in streams or waterways? How does trash influence our environment, and what can be done to enact change? These are questions middle school students explored as we launched the Great Pacific Garbage Patch (GPGP) task. Environmental and climate changes are important issues facing our world currently and ones that our students will grapple with in the decades to come. Exploring environmental issues requires the skills for visualization, understanding, and relating these findings back to our impact on the environment. Mathematical modeling is one way we can understand environmental issues through a mathematical lens. For our students, this involved using mathematics to translate a real-world problem that is difficult to conceptualize into something tangible to gain a deeper understanding of the problem. It also involved important measurement competencies such as relating scale to actual measurements (CCSSM 7.G.A.1) and solving real-world problems involving the surface area of different objects (CCSSM 7.G.B.6) (NGA Center and CCSSO 2010). Mathematical modeling involves defining a mathematical problem in relation to an authentic situation (see figure 1).

The GPGP, one of many floating islands of debris in the Pacific Ocean, is an environmental problem for which we can use mathematics to help understand (more about the GPGP task can be found in this video [link online] [Watson 2019] or at the National Geographic website [link online] [Sue 2019]). Many of us are unaware of the scope of the garbage patch because we do not experience it directly, but mathematical modeling can help us understand the size and density of the garbage patch by making assumptions and defining variables with regard to the situation. For example, modelers may assume that the garbage patch disperses and comes back together. They may select small areas of the garbage patch and decide what to keep or ignore about the areas. This would allow them to identify a simplified version of the messy question and approach it mathematically. Once modelers propose a solution, they must then consider whether it is useful in understanding the situation or if it needs to be modified. When modeling, students have the freedom to bring together separate strands of mathematical knowledge and analyze a situation in creative ways (Greer, Verschaffel, and Mukhopadhyay 2007). Students are encouraged to share ideas and consider multiple approaches as they refine their model to best meet the needs of the situation.

Fig. 1



The GPGP problem provides an opportunity to engage in the mathematical modeling process. Adapted from Bliss, Fowler, and Galluzzo 2014.

THE GPGP LESSON AND IMPLEMENTATION

Our decades of collective experience as teachers and researchers of mathematical modeling led us to design and refine this GPGP task (see figure 2) multiple times. This task was originally designed for 24 students in grades 6–8 who attended the first author's after-school mathematical modeling program in person. It was revised and taught for 19 students in grade 7 who participated in synchronous online mathematical modeling summer camp sessions led by the first two authors. In this article, we focus on the lesson structure, teacher questions, and student work from the summer camp because we believe they reflect the improvement of the lesson after revision. Table 1 shows an overview of the lesson that includes the amount of time we spent on the different sections of the lesson, the teacher actions, and the important discussion questions for students.

Hyunyi Jung, hyunyi.jung@coe.ufl.edu, is an assistant professor of mathematics education at the University of Florida in Gainesville She is interested in creating and implementing culturally sustaining mathematical modeling tasks with teachers and identifying the theoretical perspectives of instructional design and discourse that become apparent from the practice.

Megan H. Wickstrom, megan.wickstrom@montana.edu, is an associate professor of mathematics education at Montana State University, Bozeman. She is interested in the teaching and learning of mathematical modeling and finding ways to bolster students' mathematical identities through relevant and meaningful problems.

Chris Piasecki, christopher.piasecki@bellinghamschools.org, teaches eighth-grade mathematics at Fairhaven Middle School in Bellingham, Washington. He enjoys fostering creativity, collaboration, and sense making through open-ended problems and mathematical discourse.

doi:10.5951/MTLT.2020.0369

This lesson may take one or two class periods depending on students' prior experiences and knowledge; the "Developing the Model" and "Sharing Out and Revising the Model" phases may take less or more time than what is presented in table 1.

We opened the lesson by asking, "What questions do you have about the garbage patch?" Students shared these questions:

1. "Where does the trash come from?"
2. "How long is garbage sitting there?"
3. "Are people actually aware of how much plastic and debris are getting into the bodies of water?"
4. "What is the rate? How much garbage is added to the ocean each year?"
5. "How many animals die due to the garbage patch?"
6. "What is being done to help with the garbage patch?"

The first question is asking about the cause of the problem. The second, third, and fourth questions involve the current status and changes. The fifth question concerns the effect of the problem; and the last one asks for the actions that have been taken to help solve the problem.

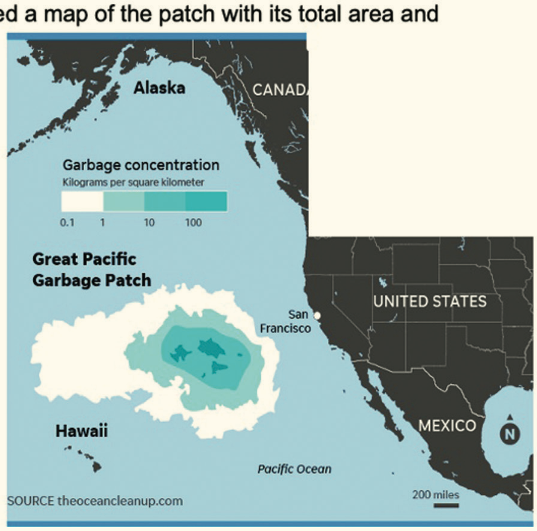
We acknowledged all these great questions and wonderings students shared. As a starting point, we decided to focus on the development of a descriptive model. Often before we can understand how something is growing or affecting other phenomena (e.g., the life of animals), we have to be able to create a descriptive model that captures the current state of the situation. Therefore, we invited students to develop a model that could be used to convey the size and density of the garbage patch to the general public.

STUDENTS' APPROACHES TO THE TASK

As students considered the area of the garbage patch, they explored ways to represent the size using other objects, such as familiar places. Teachers asked questions that led to discussion about the comparisons between objects shown on the map. Because the seventh graders lived in Wisconsin, comparing the garbage patch to the size of their state came up naturally.

Fig. 2

In the world's oceans, there are five distinct garbage patches, and the Great Pacific Garbage Patch is the largest. Most of the garbage is plastic and sits near the surface of the ocean. The plastic can affect the amount of sunlight that enters the ocean, which in turn can stop important ocean plants from growing. As an initial attempt to solve this problem, scientists have constructed a map of the patch with its total area and concentration. Because people cannot see the garbage patch, it is difficult for them to imagine the size and scope of the problem and how their waste might be contributing. They need your help getting the word out about the size and density of the garbage patch, as shown in this map.



The GPGP task invites students to determine the size and density of the garbage patch. Adapted from The Ocean Cleanup 2019.

Table 1 An Overview of the GPGP Lesson

Lesson Part (Time)	Teacher Action	Discussion Questions for Students
Introduction (10 minutes)	Introduce the term *garbage patch* and ask students to share their experiences. Give background information (link online) about GPGP.	• Has anyone heard of a garbage patch? • What is the GPGP? • What questions do you have about the garbage patch?
Envisioning the mathematical model (10 minutes)	Introduce the main task including the GPGP sketch (see figure 2).	• Using the sketch, we would like you to determine the size and density of the patch to help others become aware of this problem. What information is given in the sketch? • What further information would help people understand more about this problem? • How should we present the information?
Developing the model (30 minutes)	Encourage students to work in teams of three or four to begin drafting their model that would help them determine the size and density of the garbage patch. Observe students and ask questions as needed.	• What is a square kilometer, and how big is it? • How would you use the map scale to approximate the area? • What unit is suitable to convey your approximation? Is this unit understandable and relatable to a broader audience? • How do you make sense of areas with different densities of trash?
Sharing out and revising the model (25 minutes)	Have each group of students explain their model and discuss and reflect on what they learned about mathematical modeling.	• What do you like about your classmate's approaches? • How does the area of each concentration type relate to the total area of the patch? • Are there any components you would change about your approach after seeing other examples? • What did you learn about mathematical modeling?

Wickstrom: Do you think the state of Wisconsin is bigger or smaller than this garbage patch?
Student: Much bigger.
Wickstrom: Wisconsin is bigger?
Student: No, the garbage patch.
Jung: How do you know?
Student: If you look at the map—here, I can share the screen [*the student shares her screen*], the garbage patch is that [*pointing to the patch*]. Compared to Nevada or California, it [the patch]'s much, much, much bigger than that. Wisconsin is about that size or even smaller than that. So, you can kind of compare it to that.
Other students: [*showing agreement by nodding*]
Jung: That's interesting.
Wickstrom: So, if you are going to convince someone how big it is, it would be interesting to see how many Wisconsins big it is.

As shown in this episode, the student was comparing the size of the garbage patch with a familiar place (i.e., Wisconsin), and she further made the comparison between the size of Wisconsin and the size of other states to efficiently make the comparisons. Such conversations may have helped students come up with a related strategy of identifying the size of the garbage patch using other objects. When we invited students to work in small groups in Zoom breakout rooms, each group of students came up with different strategies to find the area of the patch and how that translated to estimating the amount of trash in the GPGP as described in the following sections.

Strategy 1: Spanning the Patch

Across both implementations, several groups found the area of the garbage patch by measuring the length and width of the patch or finding its span. They copied the scale bar (200 miles) to use it as a unit and found the number of bars that covered the horizontal and vertical distances across the patch, as shown in figure 3.

Students found that the horizontal distance of the garbage patch was 11 bars across, and they multiplied it by the scale (200 miles) to get a distance of 2,200 miles. Similarly, they determined the vertical distance to be 6 bars, and multiplied it by 200 miles to get a distance of 1,200 miles. Then, the team multiplied these two distances to get a rectangular area of 2,640,000 square miles. In continuing to describe her team's strategy, a student reported her estimation rationale.

> So, then we would round down with both of them, because you have all this extra space here. Because it was just a rectangle is what the area we found, so we would round down because there's a bunch of this extra space. So we round down to 2,500,000 square miles.

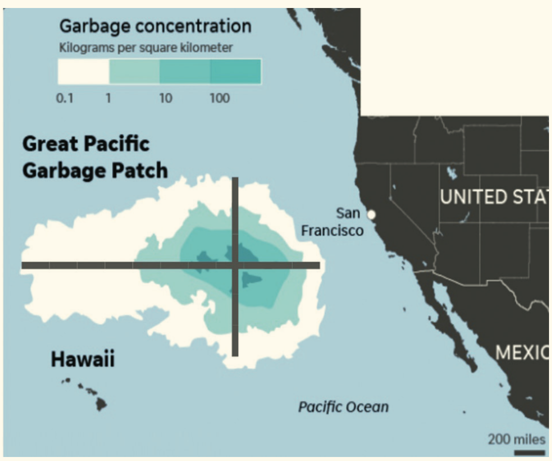
Because the team knew that the rectangular area is greater than the actual area of the garbage patch, they rounded the number down to 2,500,000 square miles. Although the team members rounded the number down without a precise estimation approach, they recognized that their strategy would result in a larger size of the garbage patch than the actual size. The team decided to take the more efficient method that would result in a quick way to estimate the garbage patch size.

Strategy 2: Units Approach

In this strategy, students used units, sometimes of multiple sizes, to determine the size of the garbage patch. In this example, students devised a square unit (200 miles × 200 miles) using the scale bar (200 miles) (see figure 4).

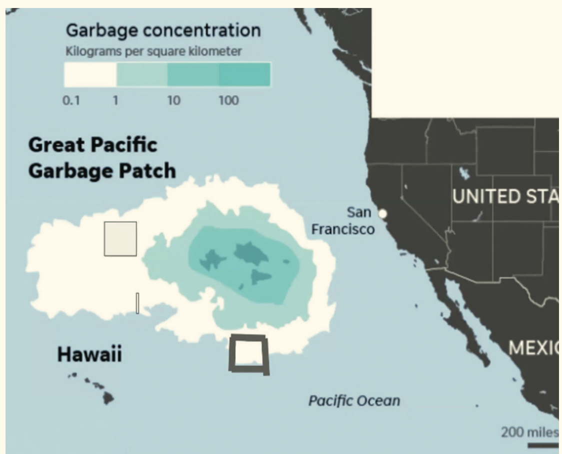
The students asked themselves how many squares would fit over the entire garbage patch. They dragged the square around and found that 24 squares would cover the patch. Because the area of one square is 40,000 square miles (about 103,600 km^2), they multiplied 24 squares by 103,600 km^2 to find the total area of the garbage patch, which was about 2,486,400 km^2. Notice that this approach may require a longer time than the first group's approach to reach a conclusion because they are dragging a square and keeping track of the number of squares they have already counted. This approach

Fig. 3



A student group shared a rectangular area approximation approach. Source: The Ocean Cleanup (link online).

Fig. 4



A student group shared a square unit approach. Source: The Ocean Cleanup (link online).

results in a more precise way to estimate the garbage patch size than the previous approach by using a smaller unit to measure the irregular shape of the patch.

Strategy 3: Units and Regions Approach

Another student group made a 200-mile × 200-mile square to use as a unit and showed a sophisticated strategy—using the square to consider the garbage concentration. Students asked themselves how many squares would fit into the different areas of the garbage patch that contain varying concentrations of garbage. These students counted the number of squares that fit into each of the white, lightest teal, darker teal, and darkest teal areas of the garbage patch. Then they multiplied each number of squares by 0.1, 1, 10, and 100 kg/km^2, respectively, per the "Garbage concentration" scale shown in the top left corner of the map. The students added the multiplied numbers to find the total size of the patch. Most groups that used a unit approach eventually decided to organize their units by region to help calculate the total amount of trash.

After each group presented how students found the area, we discussed the benefits and challenges of each across the whole class. Students described how they would convey the size of the garbage patch to the general public. Using units like football fields, states, or king-size beds would help to emphasize the scope of the problem, but it was cumbersome accounting for different concentrations of different areas to determine the total concentration.

Class Discussion of Strategies

To deepen students' understanding of the garbage concentration, we had students discuss how we could count trash and what would be a helpful unit to use. Students discussed that using one item may help people envision the total amount of trash in a tangible way, and the instructors suggested plastic water bottles (see figure 5) as a common unit that would be easy for the general population to understand.

We also discussed why measuring the patch by weight mattered. Students discussed that sometimes trash can become compacted and take up less space but that its weight should stay the same. After finding the area of the garbage patch, students determined the concentration in varying ways. Some groups considered how many water bottles make up 1 kg because they found the amount of trash in kilograms. In describing her team's strategy, one student explained,

> Well, we just found out that around 73 water bottles would be one kilometer . . . I mean, one kilogram. And then since they found out . . . we already knew the amount of trash that was in the patch, we kind of just swapped out each kilogram with 73 bottles.

They divided 1 kg by the weight of 1 water bottle, as found on the internet (0.0139 kg), and determined that approximately 73 water bottles would be equal to 1 kg in weight. Then they multiplied 73 water bottles by the size of the total amount of the garbage in the garbage patch in kilograms that they had previously found.

Other groups thought of the weight of the water bottles in grams so that they did not have to deal with decimals (e.g., 0.019 kg). They converted the size of the garbage patch in each area (i.e., white, lightest teal, darker teal, and darkest teal) from kilograms to grams. A student said,

> We divided them by 19. So, when we converted them, we converted them to grams because the average water bottle is 19 grams. So, then we added them up and divided them by 19 to see how many water bottles could fit in the total amount of space.

After adding all the values to find the total amount of garbage in the patch, they divided it by the weight of a water bottle in grams. Because each group found its own weight of the water bottles from the internet, students came up with different weights for a water bottle and different final numerical solutions.

Fig. 5

Teachers suggested using plastic water bottles as a common unit of measurement.

Prior experience engaging in open-ended problems in this camp helped students embrace these diverse approaches and numerical solutions shared in this activity.

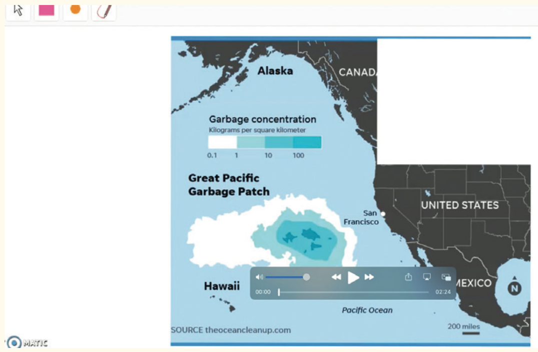
FUTURE WORK AND EXTENSION

Our initial intent was for students to build a description of the garbage patch and use the description to launch into exploration to predict how the garbage patch may grow over time, given current societal waste production. Our time ran short, and we were unable to ask these questions that we had planned: “How do you expect the garbage patch to grow each year?” “How long do you think it would take to double in size?” and “What are the environmental consequences of the garbage patch doubling?” If students had been asked to make predictions, they may have considered the proportion of each concentration type, where to add the additional plastics, and why.

After teaching this lesson, we also noticed a limitation in working with students virtually and with tools like Google Docs™ and Slides™, which can be challenging to use when measuring spaces. Because of this, we reached out to the third author to help us design a GeoGebra application. The third author also created a tutorial that would allow other educators to more easily use this task in a virtual environment with students (see video 1 [link online]).

As described in the video clip, this tool allows students to construct circles and rectangles and free draw to help make sense of the area and concentration of the patch and should be beneficial for readers looking to try the task.

Video 1 A GeoGebra Tutorial Allows Students to Use This Task in a Virtual Environment



Watch the full video online.

FINAL THOUGHTS

From the outside looking in, mathematical modeling can look messy and difficult. This messiness, however, resembles problems that people often face in the real world like the GPGP that our students explored. Many of our students reported that they learned mathematics in a way that procedures are taught first, and applications followed, often only after procedures were mastered. This way of learning mathematics implies that the learned procedures are the preferred method to solve the application problems. What is missing in this approach is the opportunity for students to draw on multiple knowledge bases and practice ways to *create* their mathematical models in response to the problem at hand. This is a vital life skill because most real-life problems we face, such as this environmental problem, are mostly new to us and do not offer an idea of what procedures to use.

Our goals for engaging students in the GPGP task were for students to experience an urgent problem occurring in the world and create their models to approach the problem. To solve the problem, they had to understand the problem situation, develop their mathematical model, and apply the model to the situation. When engaged in the GPGP task as shown in this article, students devised diverse approaches and mathematical models to determine the size and density of the garbage patch.

Students also showed their awareness of an environmental problem through this task. They wondered about the cause of the problem, length of time the garbage patch remains, the negative effect of the garbage patch on animals, and actions that have been taken to remove the garbage patch. After the lesson, when students were asked to share the next step that scientists could take, a middle school student mentioned that cleaning up all the garbage and creating a safer and healthier environment are important next steps. The opportunity to use mathematics to analyze and understand this environmental problem builds vital awareness as described earlier. This awareness allows students to see that each and every person on our planet matters and can make a difference through informed and responsible decisions. Mathematical modeling is a tool, among many, that allows us to explore and share information on environmental issues, including the climate crisis, food waste, and deforestation. It can serve as a connecting link between environmental problems and mathematical understanding.

REFERENCES

Bliss, Karen, Kathleen Fowler, and Benjamin Galluzzo. 2014. "Math Modeling: Getting Started and Getting Solutions." https://m3challenge.siam.org/sites/default/files/uploads/siam-guidebook-final-press.pdf.

Greer, Brian, Lieven Verschaffel, and Swapna Mukhopadhyay. 2007. "Modelling for Life: Mathematics and Children's Experience." In *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, edited by Werner Blum, Peter L. Galbraith, Hans-Wolfgang Henn, and Mogens Niss, pp. 89–98. New York: Springer.

National Geographic Resource Library. 2019. "Great Pacific Garbage Patch." *National Geographic*. Encyclopedic Entry. https://www.nationalgeographic.org/encyclopedia/great-pacific-garbage-patch/.

National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). 2010. *Common Core State Standards for Mathematics*. Washington, DC: NGA Center and CCSSO. http://www.corestandards.org.

The Ocean Cleanup. 2021. "The Great Pacific Garbage Patch." https://theoceancleanup.com/great-pacific-garbage-patch/.

Watson, TJ. 2019. "The Great Pacific Garbage Patch." YouTube video, 4:02. https://www.youtube.com/watch?v=MnCbTTTi7ic.

ACKNOWLEDGMENTS

This article is based on work supported by the Wisconsin Department of Public Instruction and by the National Science Foundation under Grant No. 2053155 and Grant No. 1924678. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. The authors would like to thank Eryn Maher for feedback on an earlier draft and Emily Zawoy and Bibi Altmix on the final draft of this article.

