Note: Snapshot PDF is the proof copy of corrections marked in EditGenie, the layout would be different from typeset PDF and EditGenie editing view.

## Author Queries & Comments:

**Q1 :** Please note that the journal allows a maximum of journal upper limit keywords. Please edit keywords accordingly.

Response: I don't understand the query there. Do you mean that we already have the maximum number of keywords and you request us to remove one? If that's the case, we can remove team performance.

**Q2 :** The disclosure statement has been inserted. Please correct if this is inaccurate.

Response: This is correct.

**Q3 :** The CrossRef database (www.crossref.org/) has been used to validate the references. Mismatches between the original manuscript and CrossRef are tracked in red font. Please provide a revision if the change is incorrect. Do not comment on correct changes.

Response: Resolved

**Q4 :** Please provide missing city/state for the [37] references list entry.

Response: Resolved

**Q5 :** Please provide missing city/state/publisher for the [38] references list entry.

Response: Resolved

**CM1 :** Please replace NSF-CN in the funding information with **National Natural Science Foundation of China.** The grant number keeps the same.

---

# Communication dynamics of a two-agent interaction model with applications to human-autonomy teaming

**Recto running head :** JOURNAL OF DIFFERENCE EQUATIONS AND APPLICATIONS

**Verso running head :** L. RODRIGUEZ RODRIGUEZ ET AL.

Lucero Rodriguez Rodriguez[a], Carlos Bustamante Orellana[a], Nancy Cooke[b,c], Mustafa Demir[c], Yun Kang[d]

[a] Simon A. Levin Mathematical and Computational Modeling Sciences Center, Arizona State University, Tempe, AZ, USA

[b] Human Systems Engineering, Arizona State University, Mesa, AZ, USA

[c] Center for Human, Artificial Intelligence and Robot Teaming, Arizona State University, Tempe, AZ, USA

[d] Sciences and Mathematics Faculty, College of Integrative Sciences and Arts, Arizona State University, Mesa, AZ, USA

[1] **CONTACT** Yun Kang yun.kang@asu.edu Supplemental data for this article can be accessed online at http://dx.doi.org/10.1080/10236198.2024.2343834.

**History :** received : 2023-7-4 accepted : 2024-4-11



**ABSTRACT**

Communication is pivotal for the emergence of coordination and cooperation within teams. As a result, communication plays an important role in team dynamics as better communication between teammates can lead to more efficient and successful teams. To better understand communication dynamics and their impact on team performance, we develop a modelling framework of two-agent interaction dynamics in a discrete-time fashion. Our proposed model considers the communications between two agents based on each agent's personality and their individual impact on any received communication. Combined with data, we perform mathematical analysis and bifurcation diagrams to study how agents' personality and training may impact the quality of the team's communications and therefore their performance in task completion. The validations and parameter estimations of our model from data could potentially help us to select team members that could work together efficiently, and train members in the established team to collaborate better.

**Keywords**

- Communication dynamics
- team performance
- mathematical modelling
- discrete model
- model validation
- parameter estimation
- human behaviour**[Q1]**

**Mathematics Subject Classifications**

- 39-08
- 39A60

**FUNDING**

This research is partially supported by the NSF-MathBio [grant numbers 1716802 & 2052820], James S.McDonnell Foundation (DOI:10.37717/220020472), NSF-CN [grant number 1227116], and by the Army Research Office through Cooperative Agreement Number W911NF-20-2-0252.

## 1. Introduction

As a social species, human beings have been using language to communicate with each other. Communication plays a crucial role in facilitating cooperation and coordination among humans in the completion of various tasks [1,6,19,21,26,27]. For instance, in software engineering, effective communication among team members is imperative to ensure the delivery of high quality projects [31,35]. The sharing of information and effective communication among team members significantly contribute to the success of a project [22]. Studies conducted with college students have demonstrated a strong correlation between improved communication quality and their ability to achieve higher financial revenue through business projects [32]. Brewer and Holmes conducted a communication exercise with undergraduate students [4]. Their findings revealed that clear and precise communication among team members enhances overall team functioning and reduces instances of miscommunication. Moreover, a clear understanding of terminology among all team members helps mitigate communication problems [4].

With advancing technology, human interaction has expanded to include interactions between various agents, encompassing autonomous agents. Autonomous agents refer to ‘a computer-based entity that is individually recognized as occupying a distinct team member role’ [30]. O'Neill [30] defines Human-Autonomy teams as

> interdependence in activity and outcomes involving one or more humans and one or more autonomous agents, wherein each human and autonomous agent is recognized as a unique team member occupying a distinct role on the team, and in which the members strive to achieve a common goal as a collective.

Currently, on the day to day life, civilians have a human-automation communication, which primarily involves humans issuing commands to the automation and the automation then executes the commands or responds to the questions posed by the human. For example, individuals can use their phones to perform simple tasks, such as making calls or sending texts, and drivers can delegate the driving task to the car's built-in automation. However, in the near future, we will need to consider that autonomous agents could possess significantly improved communication capabilities with humans, and have the ability to take initiative, issue orders to both human and autonomous counterparts [14], and make their own decisions.

It is crucial to understand the team dynamics between humans and autonomous devices, as robots are not only utilized for daily tasks but also assigned increasingly important roles, such as aiding urban search and rescue (USAR) teams. Robots are used for tireless searching, positioning sensors, assessing damage, providing survivors with radio transmitters or supplies, guiding tool placement, and determining survivor position and location under rubble [28]. However, currently, USAR robots are manually controlled by human operators[29]. Ideally, a USAR robot should posses the ability to communicate effectively, efficiently, and autonomously with human team members, as well as adapt to difficult and stressful situations. What kind of communication interactions would occur if we introduced a completely autonomous agent capable of being an ideal team member? Bartlett and Cooke conducted an experiment that sheds light on the dynamics of human and automation interactions [2]. The study concluded that a fully autonomous robot will lead to a better team performance and lower workload since the operator could transfer some responsibilities to the robot.

Nonlinear dynamics and mathematical modelling have been powerful tools in studying interactions of species in ecological communities [8–11]. Those approaches have become important tools implemented in complex psychological systems to gain a better understanding of human behaviour [3]. Linear and/or nonlinear differential equations have been used to model interactions between two individuals since the work of Strogatz [37], who modelled the affection between Romeo and Juliet using a two-dimensional system of linear differential equations. Sprott [36] expanded upon these models by introducing a third person and modelling a love triangle. Sprott modified Strogatz's model by using a four-dimensional system of nonlinear differential equations. Killworth and Bernard derived a three-dimensional differential equation to model dynamic changes in human affective and effective interaction within a closed group [20]. These equations are capable of representing the intensity of person A's feelings towards person B, measuring the probability of A talking to B at any given moment, and quantifying A's sensitivity to external opinions over time. The equations incorporate functions that regulate person A's desire to engage in a conversation based on available time, measure the difficulty of finding time for a conversation, and assess the probability of person A discussing person B with person C, which may influence the interaction between A and B.

There have been several discrete-time models proposed for two-person interactions [18,24,38,39]. Jaffe and his collaborators derived a probabilistic model in which each individual makes statistically independent decisions, influenced by their immediate prior state and the state of the opposite individual [18]. The parameters in the model allow for measuring person A's bias towards a certain response, sensitivity to their own and the other's previous behaviour, and the interaction between person A's response tendencies and the previous behaviours [39]. Malone's model is similar to Jaffe's, but it incorporates the assumption that both person A and B have individual predispositions to make specific responses [24]. Thus, the probability of person A and person B having a certain response is a weighted average of their predisposition and the situational effect determined by the previous response [39]. These two weights are expressed as probabilities. Thomas and Martin's model introduces dynamics to the intra-individual and inter-individual variables [40]. Consequently, the current interactive behaviour is determined by the probability summation of self-regulatory and interactive effects. Suppes and Atkinson's model allows for Persons A and B to be reinforced independently during each iteration [38]. This reinforcement occurs after the response is given and depends on the responses of A and B, allowing for two scenarios. In the first scenario, and individual can be conditioned to a response if it is reinforced. In the second scenario, the individual can be conditioned to the other individual's response using a probability if the response is not reinforced [39].

Gottman [17] developed nonlinear discrete models for marriage dynamics, represented through their coding system, called the Specific Affect Coding System (SPAFF). SPAFF enables the coding of emotional interactions in marital and family settings [16]. Gottman's marital models are two-dimensional, representing the behaviour scores of husbands and wives. Each mathematical equation incorporates a linear function to describe the individual's own dynamics or uninfluenced behaviour, bilinear and object functions to describe the influence of one individual on the other, and repair and damping functions. Repair functions respond to excessive negativity from a partner, while damping functions respond to excessive positivity. These models can determine which couples are in a happy and stable marriage, in an unhappy but stable marriage, or have divorced. Additionally, Gottman's models have been used to analyse gay and lesbian marriages, revealing distinct differences compared to heterosexual marriages. Gottman's modelling approach provides a foundation for modelling communication dynamics between two agents in a nonlinear fashion.

Motivated by Bartlett and Cooke's work and experiment on cooperation between two agents in a USAR team [2], we derive a two-dimensional discrete-time model with delay to understand the communication dynamics in a team based on experimental data. Building upon Gottman's modelling approach [17], our model is tailored to describe the communication between team members during task execution, which involves a turn-taking dynamic. Additionally, our proposed nonlinear second-order discrete-time model can be validated using collected data obtained at discrete time intervals. We aim to address the following questions through the study and validation of our model: (1) How may different factors impact communication dynamics? What factors benefit or harm the team? (2) How do these factors differentially affect teams with varying levels of performance? (3) What characteristics are present in a high-performing team?

The remainder of this paper is structured as follows: Section 2 presents the derivation of a discrete-time mathematical model that describes speech dynamics for an individual as well as communication dynamics between two individuals. In Section 3, we analyse the proposed model and explore potential dynamics, including the number of equilibrium points and their stability. Section 4, focuses on bifurcation analysis and validates the model using experimental data to gain insights into how key parameters impact team performance, how communication can enhance performance in pre-established teams, and how to select team members with high expected performance. Finally, in the concluding section, we summarize our work and provide a brief discussion of our results.

## 2. Model derivation

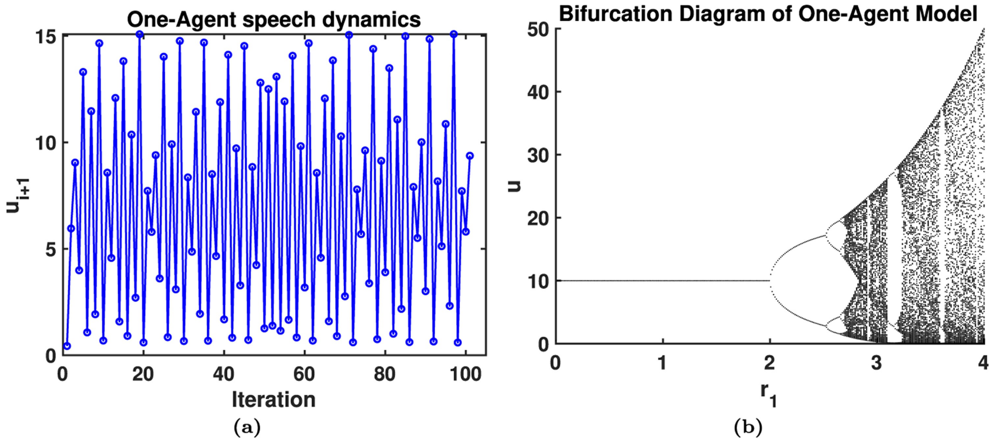
Communication is influenced by personality variables, including self-esteem, cognitive complexity, authoritarianism, optimism, empathy, interaction involvement, and conversational sensitivity [12]. A study conducted by Joanne Chung-Yan Chan and Po Yi Sy (2016) found a positive correlation between intercultural communication and personality traits such as agreeableness, openness, and conscientiousness in nursing students [7]. Similarly, another study showed a positive relationship between communication skills and empathy in hospice nurses, as well as a correlation between social and communication skills and effective performance in hospice nursing [34]. Thus, we begin by constructing a model for an individual that illustrates the intrinsic dynamics of their communication style. Building upon the modelling approaches by [5,17], we assume that in the absence of the other agent, the thought process and uninfluenced behaviour can be represented by the following discrete time map

$$u_{i+1} = u_i e^{r_1\left(1-\frac{u_i}{k_1}\right)},$$

where $r_1$ denotes the person's emotional inertia or resistance to change, while $k_1$ represents the average communicative level, reflecting their ability to verbally convey information. The discreet time model (1) is well-known as ‘Ricker's Map’ in ecology whose dynamics are well studied (see [8,11] Jim Cushing's work).

The intrinsic communication dynamics of (1) are determined by the emotional inertia $r_1$, as shown in the bifurcation diagram in Figure 1(b). A high emotional inertia indicates that a person's emotional state is less likely to change, making them more impervious to external or internal influences [23]. In this case, a lower value of $r_1$ (e.g. $r_1 < 2$) signifies higher emotional inertia, as the system reaches a steady state more quickly and is less prone to change (see Figure 1(b)). Conversely, low emotional inertia implies that a person's emotional state is more susceptible to change, suggesting that a greater susceptibility to environmental or psychological demands [23]. This behaviour is depicted in Figure 1(b) when $r_1 > 2$, where chaotic behaviour is observed in the system. Therefore, a higher emotional inertia, represented by a smaller value of $r_1$, leads to a higher level of predictability in the person's emotional state. Thus, the bifurcation diagram in Figure 1(b) proves to be useful in determining the person's emotional inertia. To illustrate this, the speech dynamics of a single agent are depicted in Figure 1(a). This time series provides insights into the dynamics of an individual's uninfluenced behaviour. In this example, we selected an emotional inertia value of $r_1 = 2.8$, which resulted in chaotic behaviour that captures the agent's frequent changes in state of mind.

**Figure 1.** Time series simulation of one-agent speech dynamics is displayed on (a); and bifurcation diagram from the one-agent model on (b).



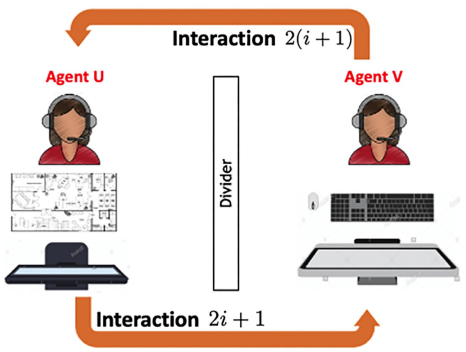

We extend the model from Equation (1) to include interaction and influencing behaviour from a second agent, denoted as $V$. Let $u_i$ represent the rate of speech in words per response time (s.), reflecting communication from Agent U to Agent V during interaction $i$. Similarly, $v_i$ denotes communication from Agent V to Agent U at interaction $i$. We assume that the communication follows a turn-taking dynamic, making discrete-time representation appropriate. In this scenario, Agent U initiates the interaction with Agent V. Figure 2(a) provides a diagram illustrating these assumptions. Additionally, each agent possesses their own uninfluenced behaviour, as described by Equation (1), and their own functional response (influenced behaviour) in response to the opposing agent, which will be explained below. Motivated by Gottman's modelling approach for married couples [17] and the available data (Figure 2(b)), we propose the following two-agent communication model:

$$u_{2i+1} = u_{2i-1}\exp\left(r_1\left(1-\frac{u_{2i-1}}{k_1}\right) - \underbrace{\frac{\beta_{uv}v_{2i}}{1+\gamma_{uv}u_{2i-1}v_{2i}}}_{\textbf{influences from V}}\right)$$

$$v_{2(i+1)} = v_{2i}\exp\left(r_2\left(1-\frac{v_{2i}}{k_2}\right) - \underbrace{\frac{\beta_{vu}u_{2i+1}}{1+\gamma_{vu}v_{2i}u_{2i+1}}}_{\textbf{influences from U}}\right) \qquad \text{for } i = 1, 2, 3, \ldots, n$$

Here, $i$ represents the turns taken by Agent U and Agent V to communicate with each other (i.e. odd turns correspond to $u$ and even turns correspond to $v$). The functional response of each individual, reflecting the influenced behaviour, follows Holling's Type II and is expressed as $\frac{\beta_{uv}v_{2i}}{1+\gamma_{uv}u_{2i-1}v_{2i}}$ for Agent U; and $\frac{\beta_{vu}u_{2i+1}}{1+\gamma_{vu}v_{2i}u_{2i+1}}$ for Agent V. In these expressions, the parameters $\beta_{uv}$ and $\beta_{vu}$ can be positive or negative, representing nonlinear impacts from previous communications. The parameters $\gamma_{uv}$ and $\gamma_{vu}$ are non-negative and represent the damping effects that Agent U and Agent V, respectively, use to regulate and respond to previous communications from both agents. Both $\beta$ and $\gamma$ are related to the impact of communication during conversations within a given task. Therefore, $\beta$ and $\gamma$ can be adjusted and adapted through proper training.

**Figure 2.** (a) Diagram of the testbed performed in article 'Human-robot teaming in urban search and rescue' [2] and describes the dynamics of our two-agent model. Additional details of the experimental testbed can be found in Section 4.1. (b) An example of the collected data in Bartlett and Cooke's study.




(a)

| | Agent | Time (s.) | Time Lag (s.) | # of Words | # of Words / Time Lag |
|---|---|---|---|---|---|
| $u_1$ | Operator | 0 | 1 | 4 | 4.00 |
| $v_2$ | Robot | 2 | 2 | 3 | 1.50 |
| $u_3$ | Operator | 6 | 4 | 9 | 2.25 |
| $v_4$ | Robot | 18 | 4 | 3 | 0.75 |
| $u_5$ | Operator | 20 | 2 | 25 | 12.50 |
| $v_6$ | Robot | 35 | 6 | 1 | 0.17 |
| $u_7$ | Operator | 37 | 2 | 16 | 8.00 |
| $v_8$ | Robot | 45 | 3 | 1 | 0.33 |
| $u_9$ | Operator | 49 | 4 | 42 | 10.50 |
| $v_{10}$ | Robot | 73 | 3 | 3 | 1.00 |

(b)

To establish a connection between our model and the experimental data presented in Figure 2(b), we assign initial conditions $u_1$ and $v_2$. This allows us to determine the values of $u_3$ and $v_4$ using the following equations:

$$u_3 = u_1 \exp\left(r_1\left(1 - \frac{u_1}{k_1}\right) - \frac{\beta_{uv} v_2}{1 + \gamma_{uv} u_1 y_2}\right)$$

$$v_4 = v_2 \exp\left(r_2\left(1 - \frac{v_2}{k_2}\right) - \frac{\beta_{vu} u_3}{1 + \gamma_{vu} v_2 u_3}\right)$$

This relationship can also be understood through the diagram shown in Figure 2(a). For the convenience of mathematical analysis and expressions, we introduce new variables by letting $x_i = u_{2i-1}$, $y_i = v_{2i}$, and $\{x_0, y_0\}$ be the initial conditions. Consequently, our model (2) can be equivalently represented by the following set of equations:

$$x_{i+1} = x_i \exp\left(r_1\left(1 - \frac{x_i}{k_1}\right) - \frac{\beta_{xy} y_i}{1 + \gamma_{xy} x_i y_i}\right)$$

$$y_{i+1} = y_i \exp\left(r_2\left(1 - \frac{y_i}{k_2}\right) - \frac{\beta_{yx} x_{i+1}}{1 + \gamma_{yx} y_i x_{i+1}}\right) \qquad \text{for } i = 1, 2, 3, \ldots, n$$

where $i$ denotes the turns taken by Agent U, now referred to as Agent X, and Agent V, now referred to as Agent Y, during their communication. Notably, there is a delay term $x_{i+1}$ in the expression for $y_{i+1}$ in (2b). Consequently, the model (2a)–(2b) can be expressed as follows:

$$x_{i+1} = x_i \exp\left(\mathrm{r}_1\left(1 - \frac{x_i}{k_1}\right) - \frac{\beta_{xy} y_i}{1 + \gamma_{xy} x_i y_i}\right)$$

$$y_{i+1} = y_i \exp\left(\mathrm{r}_2\left(1 - \frac{y_i}{k_2}\right) - \frac{\beta_{yx} x_i \exp\left(r_1\left(1 - \frac{x_i}{k_1}\right) - \frac{\beta_{xy} y_i}{1+\gamma_{xy} x_i y_i}\right)}{1 + \gamma_{yx} y_i x_i \exp\left(r_1\left(1 - \frac{x_i}{k_1}\right) - \frac{\beta_{xy} y_i}{1+\gamma_{xy} x_i y_i}\right)}\right)$$

We will use Equation (3) to study the model's local stability of steady states.

In summary, our two-dimensional communication discrete-time Model (2) incorporates delay impacts to describe the communication dynamics between two agents, based on the following five assumptions:

**1** The thought processes and communication style of each agent are modelled by the Ricker's map, Equation (1), representing their uninfluenced behaviour.

**2** Communication between the two agents occurs in a turn-taking manner and is conveyed discretely.

**3** Agent X initiates the communication.

**4** Agent X's response at turn $i+1$ is influenced by the communications that took place at turn $i$ from Agent X and Agent Y.

**5** Agent Y's response at turn $i+1$ is influenced by its own communication at turn $i$ and the response from Agent Y at turn $i+1$.

Our derived Model(2) provides a structured framework for understanding and analysing the complexity of human communication. This allows us to identify the key variables, relationships, and assumptions that underlie the verbal interactions. Our theoretical analysis, in the following section, will inform us of the potential dynamics of the system, as well as the effect of the parameters in the dynamics of communication, which will help us address our first question. We will use the available data to validate our proposed model (2), and then do parameter estimations under varied conditions to predict and examine the dynamics. This is explored in three data fittings in Section 4. After the initial model and data fitting in Section 4.2, we perform additional two scenarios of parameter estimations with the aim of gaining insights into different potential outcomes and their implications our second and third questions. Thus, through model fitting and parameter exploration, we can apply the model to the selection of two agents who can collaborate effectively to achieve optimal team performance.

## 3. Model dynamics

Model (2) is continuously differentiable on $\mathbb{R}^2_+ \to \mathbb{R}^2_+$ with nonnegative initial conditions $x_0, y_0 \in \mathbb{R}^2_+$. As a starting point, we establish the reasonableness of our model by proving it is positive invariant and bounded as follows:

**Theorem 3.1**

**Positive Invariance and Boundedness**

Model (2) is positive invariant and bounded in $\mathbb{R}^2_+$.

The detailed proof of Theorem *3.1* can be found in Appendix A.

### 3.1. Equilibria

Next, we examine the equilibria of Model(2) by solving Equations (2a) and (2b) when they are equal to zero. This yields the following equations:

$$x^{*} \exp\left(r_1\left(1 - \frac{x^{*}}{k_1}\right) - \frac{\beta_{xy} y^{*}}{1 + \gamma_{xy} x^{*} y^{*}}\right) - x^{*} = 0$$

$$y^{*} \exp\left(r_2\left(1 - \frac{y^{*}}{k_2}\right) - \frac{\beta_{yx} x^{*}}{1 + \gamma_{yx} y^{*} x^{*}}\right) - y^{*} = 0$$

#### 3.1.1. Boundary equilibria

From equations (4a) and (4b), Model (2) can have three boundary equilibria: (1) the *no-communication equilibrium* (where there is no communication between Agent X and Agent Y): $\mathbf{E_{0,0}} = (0,0)$; (2) the *no communication from Agent X equilibrium* $\mathbf{E_{0,y^{*}}} = (0, k_2)$; (3) the *no communication from Agent Y equilibrium* $\mathbf{E_{x^{*},0}} = (k_1, 0)$. The following theorem discusses the existence and stability of the three boundary equilibria. The dynamics of the interior equilibria (existence and stability) are explored in the subsequent subsection.

**Theorem 3.2**

**Boundary Equilibria Dynamics**

Model (2) always has the no-communication equilibrium $\mathbf{E_{0,0}} = (0,0)$ which is always unstable since $r_1 > 0$ and $r_2 > 0$. Moreover, Model (2) always has an additional boundary equilibrium $\mathbf{E}_{\mathbf{0},\mathbf{y}\square} = (0, k_2)$ which is locally asymptotically stable whenever $\beta_{xy} > \frac{r_1}{k_2}$ and $0 < r_2 < 2$, otherwise, it is unstable. Lastly, the boundary equilibrium $\mathbf{E}_{\mathbf{x}\square,\mathbf{0}} = (k_1, 0)$ always exists and is locally asymptotically stable whenever $\beta_{yx} > \frac{r_2}{k_1}$ and $0 < r_1 < 2$.

The detailed proof of Theorem 3.2 can be found in the Appendix A. The implications of the boundary equilibria stability conditions are as follows:

- The equilibrium point $\mathbf{E}_{\mathbf{x}\square,\mathbf{0}}$ is stable if and only if $\beta_{yx} \geq \frac{r_2}{k_1}$ and $0 \leq r_1 \leq 2$. This implies that Agent Y is more affected by Agent X's communication ($\beta_{yx}$) compared to Agent Y's personality ($r_2$) and Agent X's communication level ($k_1$). Due to Agent Y's personality, Agent Y becomes overwhelmed by Agent X's high level of communication ability. Finally, Agent X's personality value ($r_1$) should be between 0 and 2. Thus, Agent X has a high emotional inertia and dominates the conversation, overwhelming Agent Y and hindering their ability to communicate back.
- The equilibrium point $\mathbf{E}_{\mathbf{0},\mathbf{y}\square}$ is stable if and only if $\beta_{xy} \geq \frac{r_1}{k_2}$ and $0 \leq r_2 \leq 2$. Given the symmetry of the model, the implications for this boundary equilibrium are similar to $\mathbf{E}_{\mathbf{x}\square,\mathbf{0}}$. Hence, in this boundary equilibrium Agent Y's personality value ($r_2$) should be between 0 and 2. Agent Y has a high emotional inertia and dominates the conversation, overwhelming Agent X and inhibiting their ability to communicate back.

**Table 1.** Description of parameters from Model (2).

| Parameter | Description | Value |
|---|---|---|
| $r_1$ | Emotional inertia of Agent X | $(0, 4]$ |
| $r_2$ | Emotional inertia of Agent Y | $(0, 4]$ |
| $k_1$ | Average communicative level of Agent X | $\mathrm{R}_+$ |
| $k_2$ | Average communicative level of Agent Y | $\mathrm{R}_+$ |
| $\beta_{xy}$ | Nonlinear impacts that Agent Y has on Agent X | R |
| $\beta_{yx}$ | Nonlinear impacts that Agent X has on Agent Y | R |
| $\gamma_{xy}$ | Damping effects that Agent X can regulate and respond to Agent Y's communication | $\mathrm{R}_+$ |
| $\gamma_{yx}$ | Damping effects that Agent Y can regulate and respond to Agent X's communication | $\mathrm{R}_+$ |

**Table 2.** Summary of boundary equilibrium dynamics. LAS: locally asymptotically stable.

| Equilibrium | Existence | Stability |
|---|---|---|
| $\mathbf{E}_{\mathbf{0},\mathbf{0}}$ | Always | Since $r_1 > 0$ and $r_2 > 0$, $\mathbf{E}_{\mathbf{0},\mathbf{0}}$ is always unstable. |
| $\mathbf{E}_{\mathbf{x}\square,\mathbf{0}} = (k_1, 0)$ | Always | LAS iff $\beta_{yx} > \frac{r_2}{k_1}$ and $0 < r_1 < 2$, otherwise it is unstable. |
| $\mathbf{E}_{\mathbf{0},\mathbf{y}\square} = (0, k_2)$ | Always | LAS iff $\beta_{xy} > \frac{r_1}{k_2}$ and $0 < r_2 < 2$, otherwise it is unstable. |

#### 3.1.2. Interior equilibria of the general model

We can explore the potential number of interior equilibria of the general Model (2) by analysing the nullclines. Solving Equations ((2a)) and (2b) for $y$ and $x$, respectively, we obtain the following equations:

$$y = \frac{r_1(k_1 - x)}{r_1\gamma_{xy}x^2 - r_1k_1\gamma_{xy}x + k_1\beta_{xy}}$$

$$x = \frac{r_2(k_2 - y)}{r_2\gamma_{yx}y^2 - r_2k_2\gamma_{yx}y + k_2\beta_{yx}}$$

From the numerators of Equations ( (5a)) and ( (5b)), we define $h_1(x) = r_1\gamma_{xy}x^2 - r_1k_1\gamma_{xy}x + k_1\beta_{xy}$ and $h_1(y) = r_2\gamma_{yx}y^2 - r_2k_2\gamma_{yx}y + k_2\beta_{yx}$. Since both equations are quadratic, we consider three distinct cases based on the expression $h_1(x) = 0$.

$$\text{\it{Case 1.} The equation } h_1(x) = 0 \text{ has no roots when } \beta_{xy} > \frac{r_1k_1\gamma_{xy}}{4}.$$

$$\text{\it{Case 2.} The equation } h_1(x) = 0 \text{ has one root when } \beta_{xy} = \frac{r_1k_1\gamma_{xy}}{4}.$$

$$\text{\it{Case 3.} The equation } h_1(x) = 0 \text{ has two roots when } \beta_{xy} < \frac{r_1k_1\gamma_{xy}}{4}.$$

Similarly, the expression $h_1(y) = 0$ follows the same three cases with their corresponding parameters. To determine the possible number of interior equilibria numerically, we utilize Equation ( (5a)) and find solutions for the following quadratic equation

$$y^2(-r_2\gamma_{yx}x) + y(k_2r_2\gamma_{yx}x - r_2) + k_2r_2 - k_2\beta_{yx}x = 0$$

**Theorem 3.3**

**Interior equilibria existence conditions for General Model**

Model(2) can have either five, three, two, or one interior equilibria depending on the values of its strictly positive parameters $r_1, r_2, k_1, k_2, \gamma_{xy}, \gamma_{yx}$ and $\beta_{xy}, \beta_{yx}$ □ R defined in Table *1* and on conditions in (6).

- Model (2) has exactly **five interior equilibrium** points if
- *Case (5a):*

$$\frac{\beta_{xy}}{\gamma_{xy}} > \frac{r_1k_1}{4}, \quad \frac{\beta_{yx}}{\gamma_{yx}} > \frac{r_2k_2}{4}, \text{ and } \frac{\beta_{xy}}{\gamma_{xy}} > r_1\left(1 - \frac{1}{k_1\left(2\gamma_{yx}\sqrt{\frac{k_2\beta_{yx}}{r_2\gamma_{yx}}} - k_2\right)}\right)^2$$

OR *Case (5b):*

$$\frac{\beta_{xy}}{\gamma_{xy}} \le \frac{r_1 k_1}{4}, \quad \frac{\beta_{yx}}{\gamma_{yx}} > \frac{r_2 k_2}{4}, \quad 0 < k_1 < \frac{1}{\gamma_{xy}}, \quad \frac{1}{2} \ge \frac{r_1}{k_2\beta_{xy}} + \sqrt{1 - \frac{4\beta_{yx}}{r_2 k_2 \gamma_{yx}}},$$

$$\text{and} \quad \frac{\beta_{yx}}{\gamma_{yx}} > \frac{r_2}{\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}} - k_1\right)}\left(1 - \frac{1}{k_2\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}} - k_1\right)}\right)^2$$

OR *Case (5c):*

$$\frac{\beta_{xy}}{\gamma_{xy}} \le \frac{r_1 k_1}{4}, \quad \frac{\beta_{yx}}{\gamma_{yx}} \le \frac{r_2 k_2}{4}, \quad \frac{1}{2} \ge \frac{r_1}{k_2\beta_{xy}} + \sqrt{1 - \frac{4\beta_{yx}}{r_2 k_2 \gamma_{yx}}}, \text{ and } \quad 0 < k_1 < \frac{1}{\gamma_{xy}}$$

- Model (2) has exactly **three interior equilibrium** points if
- *Case (3a):*

$$\frac{\beta_{yx}}{\gamma_{yx}} > \frac{r_2}{\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}} - k_1\right)}\left(1 - \frac{1}{k_2\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}} - k_1\right)}\right)^2,$$

$$\frac{1}{2} \ge \frac{r_1}{k_2\beta_{xy}} + \sqrt{1 - \frac{4\beta_{yx}}{r_2 k_2 \gamma_{yx}}}, \quad \frac{\gamma_{xy}}{k_2^2\gamma_{yx}^2} < k_1 < \frac{r_2\gamma_{xy}}{k_2\gamma_{yx}\left(\sqrt{r_2 k_2 \gamma_{yx}} - 2\sqrt{\beta_{yx}}\right)^2},$$

$$\text{and} \quad k_2 > \frac{\beta_{yx}}{r_2\gamma_{yx}}$$

OR *Case (3b):*

$$\frac{\beta_{xy}}{\gamma_{xy}} \le \frac{r_1k_1}{4}, \quad \frac{\beta_{yx}}{\gamma_{yx}} \le \frac{r_2k_2}{4}, \quad k_2 > \frac{\beta_{yx}}{r_2\gamma_{yx}}, \quad \frac{1}{2} \ge \frac{r_1}{k_2\beta_{xy}} + \sqrt{1-\frac{4\beta_{yx}}{r_2k_2\gamma_{yx}}},$$

$$\text{and} \quad \frac{\gamma_{xy}}{k_2^2\gamma_{yx}^2} < k_1 < \frac{r_2\gamma_{xy}}{k_2\gamma_{yx}\left(\sqrt{r_2k_2\gamma_{yx}}-2\sqrt{\beta_{yx}}\right)^2}$$

- Model (2) has exactly **two interior equilibrium** points if
- *Case (2):*

$$\frac{\beta_{xy}}{\gamma_{xy}} < \frac{r_1k_1}{4}, \quad \frac{\beta_{yx}}{\gamma_{yx}} > \frac{r_2k_2}{4}, \text{ and}$$

$$\frac{\beta_{yx}}{\gamma_{yx}} > \frac{r_2}{\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}}-k_1\right)}\left(1-\frac{1}{k_2\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}}-k_1\right)}\right)^2$$

OR *Case (2b):*

$$\frac{\beta_{xy}}{\gamma_{xy}} \le \frac{r_1k_1}{4}, \text{ and } \frac{\beta_{yx}}{\gamma_{yx}} \le \frac{r_2k_2}{4}$$

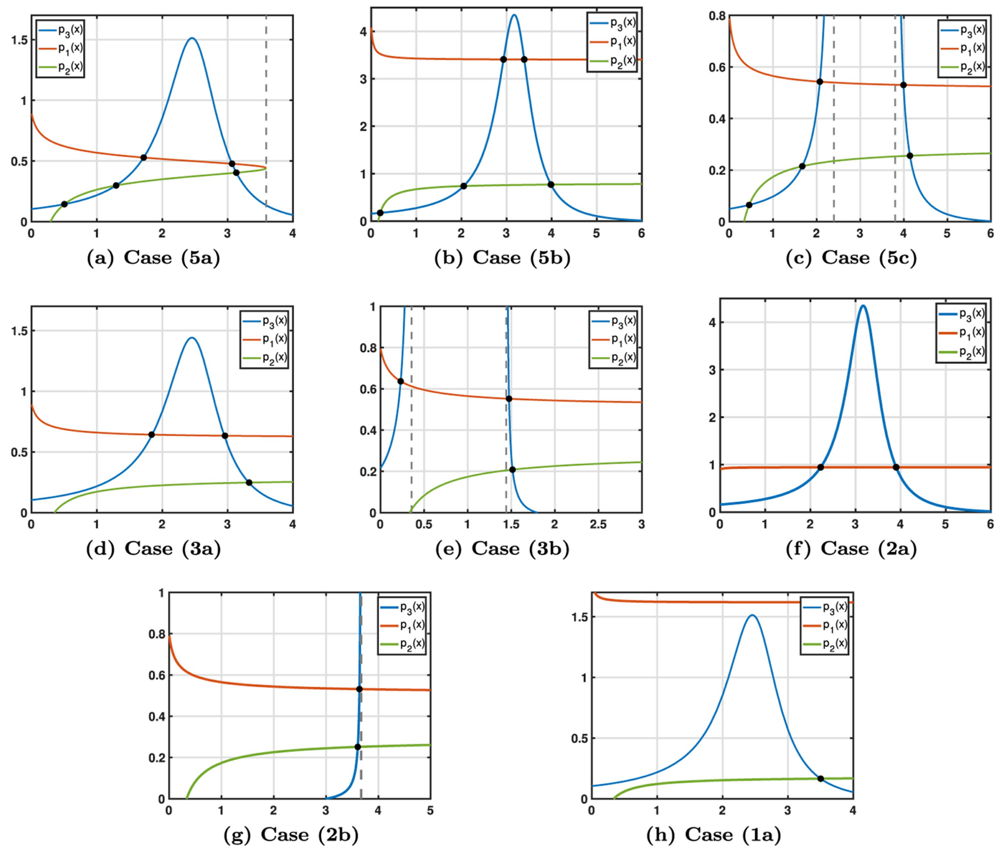
- Model (2) has at least **one interior equilibria** point if
- *Case (1a):*

$$\frac{\beta_{xy}}{\gamma_{xy}} \ge \frac{r_1k_1}{4}, \quad \frac{\gamma_{xy}}{k_2^2\gamma_{yx}^2} < k_1 < \frac{r_2\gamma_{xy}}{k_2\gamma_{yx}\left(\sqrt{r_2k_2\gamma_{yx}}-2\sqrt{\beta_{yx}}\right)^2}, \quad k_2 > \frac{\beta_{yx}}{r_2\gamma_{yx}}, \text{ and}$$

$$\frac{\beta_{yx}}{\gamma_{yx}} < \frac{r_2}{\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}}-k_1\right)}\left(1-\frac{1}{k_2\gamma_{xy}\left(2\sqrt{\frac{k_1\beta_{xy}}{r_1\gamma_{xy}}}-k_1\right)}\right)^2$$

The detailed proof of Theorem 3.3 can be found in Appendix A. In Figure3, we observe the number of possible interior equilibria of Model (2) through the behaviour of the system's nullclines. Each subfigure represents an example of each case in Theorem 3.3.

**Figure 3.** Plots of Equations (5a) and (7) reflecting the examples of cases described in Theorem 3.3. The figures reflect the possible number of interior equilibria points (**black points**). The grey dashed lines represent the asymptotes of nullclines. Lines coloured in blue, orange and green correspond to each nullcline of Model (2). (a) **Case (5a)**, (b) **Case (5b)**, (c) **Case (5c)**, (d) **Case (3a)**, (e) **Case (3b)**, (f) **Case (2a)**, (g) **Case (2b)**, (h) **Case (1a)**.




The implication of the interior equilibria existence conditions is as follows:

- *Case 5a and 5b* (see Figure 3(a ,b))**:** Larger positive values of the ratios $\frac{\beta_{xy}}{\gamma_{xy}}$ and $\frac{\beta_{yx}}{\gamma_{yx}}$ might contribute to satisfying all conditions. This emphasizes that the nonlinear impacts ($\beta_{xy,yx}$) are relatively strong compared to the damping effects ($\gamma_{xy,yx}$). Thus, an increase in the impacts of one agent on another is beneficial for the existence of five equilibrium points.
- *Case 5c* (see Figure 3(c))**:** Smaller values of the ratios $\frac{\beta_{xy}}{\gamma_{xy}}$, possibly negative, and $\frac{\beta_{yx}}{\gamma_{yx}}$ will help satisfy conditions 1, 2, and 3. This indicates that the nonlinear impacts are relatively weaker compared to damping effects. Additionally, a smaller ratio of how emotionally inert Agent X is to the combined influence of the average communicative level and nonlinear impacts from Agent Y ($\frac{r_1}{k_2\beta_{xy}}$) and a smaller value of $\frac{\beta_{yx}}{\gamma_{yx}}$ can satisfy condition 3. A larger value of Agent X's damping capability ($\gamma_{xy}$) could satisfy condition 4.
- *Case 3a and 3b* (see Figure 3(d ,e))**:** It is necessary to have a larger value of communication level ($k_2$) compared to the value of the ratio of how strongly Agent X's actions impact Agent Y to the combined influence of emotional inertia and damping effects that Agent Y can regulate ($\frac{\beta_{yx}}{r_2\gamma_{yx}}$). Additionally, a larger value of the ratio $\frac{\beta_{yx}}{\gamma_{yx}}$ is needed, indicating the nonlinear impacts are relatively strong compared to damping effects. Conversely, we need a smaller value of the ratio $\frac{\beta_{xy}}{\gamma_{xy}}$, indicating the nonlinear impacts are

relatively weaker compared to damping effects. Lastly, for Case 3b, it is possible for the ratio $\frac{\beta_{xy}}{\gamma_{xy}}$ to be negative, whereas $\frac{\beta_{yx}}{\gamma_{yx}}$ will be positive. Conversely, in Case 3a, both ratios can only be positive.

- *Case 2a* (see Figure 3(f))**:** A smaller and positive value of the ratio $\frac{\beta_{xy}}{\gamma_{xy}}$ can satisfy condition 1, while a larger value of the ratio $\frac{\beta_{yx}}{\gamma_{yx}}$ can satisfy conditions 2 and 3. This means that the nonlinear impacts on Agent X are relatively weaker compared to their damping capabilities. Conversely, Agent Y's nonlinear impacts are relatively strong compared to their damping capabilities.
- *Case 2b* (see Figure 3(g))**:** Smaller, possibly negative, values of the ratios $\frac{\beta_{xy}}{\gamma_{xy}}$ and $\frac{\beta_{yx}}{\gamma_{yx}}$ can satisfy both conditions. This suggests that both agents' nonlinear impacts are relatively weaker compared to their damping capabilities.
- *Case 1a* (see Figure 3(h))**:** A larger value of $k_2$ compared to $\frac{\beta_{yx}}{r_2\gamma_{yx}}$ could satisfy condition 3. In this case, a larger value of the ratio $\frac{\beta_{xy}}{\gamma_{xy}}$ can satisfy conditions 1 and 4, while a smaller value of $\frac{\beta_{yx}}{\gamma_{yx}}$, possibly negative, can satisfy conditions 2, 3, and 4. This indicates that the nonlinear impacts on Agent X are relatively strong compared to their damping capabilities. Conversely, Agent Y's nonlinear impacts are relatively weaker compared to their damping capabilities.

For all conditions in (6), Model (2) can only have either five, three, two, or one interior equilibria. The number of interior equilibria is determined by the positive intercepts of the nullclines,

$$p_3(x) = \frac{r_1(k_1 - x)}{r_1\gamma_{xy}x^2 - r_1k_1\gamma_{xy}x + k_1\beta_{xy}} \quad \text{and}$$

$$p_{1,2}(x) = \frac{-1}{2\gamma_{yx}x}\left(1 - k_2\gamma_{yx}x \pm \sqrt{r_2(r_2(1 + k_2\gamma_{yx}x)^2 - 4k_2\beta_{yx}\gamma_{yx}x^2)}\right)$$

Figure C1 shows the possible number of equilibria by graphing all three nullclines and using the three cases for $h_1(x) = 0$ and $h_1(y) = 0$ as conditions for $\beta_{xy}$ and $\beta_{yx}$. In Figure C1, we use blue, orange, and green lines to represent Case 1, 2, or 3, respectively, from the conditions in (6). The circle markers are coloured purple, magenta, black, and dark green to identify five, three, two, or one intersection(s), respectively, between the functions. Solid, dashed, and dotted lines are used to represent the change of parameters with respect to one function and therefore indicate in the number of positive intersections. Appendix D contains the details of the parameter values used in each plot.

We observe that all conditions yield either five or three possible interior equilibria. Nullcline $p_2(x)$ will always intersect the positive y-axis and remain in the first quadrant, resulting in two intersections with $p_3(x)$ in most cases. Thus, Figure 4 exemplifies the conditions under which nullcline $p_1(x)$ will intersect $p_3(x)$ once or thrice, leading to one out of three interior equilibria or three out of five interior equilibria. These two nullclines will intersect thrice if $x^c = \frac{r_2}{\beta_{yx}} < \varepsilon = \sqrt{\frac{k_1}{\gamma_{xy}}}$. Otherwise, they will intersect only once. Therefore, if the impact of previous communication on Agent Y's emotional inertia (small $x^c$) is greater than Agent X's damping effect on their communicative level ($x^c < \varepsilon$), we can have up to five interior equilibria. A distinct finding is that if there are two possible interior equilibria under a given case, we could not find conditions for the existence of only one interior equilibrium. Similarly, if we found one possible interior equilibrium, we could not find conditions for the existence of two interior equilibria under the same conditions. Refer to Figures C1a and C1c to observe these dynamics. Moreover, we observe a minimum of two interior equilibria whenever $\beta_{xy} \leq \frac{r_1k_1\gamma_{xy}}{4}$ or $\beta_{yx} \leq \frac{r_2k_2\gamma_{yx}}{4}$. Thus, the impact of Agent X's or Agent Y's previous communication should be less than or equal to one-fourth of their emotional inertia, communication ability, and damping effect capability. Otherwise, the agents can be overwhelmed by the interaction. We study the stability of the interior equilibria in Section 5 through simulated bifurcation diagrams.

#### 3.1.3. Interior equilibrium of the symmetric model

In this section, we explore the existence and stability of the interior equilibria in the symmetric model, where $k = k_1 = k_2$, $r = r_1 = r_2$, $\gamma = \gamma_{xy} = \gamma_{yx}$ and $\beta = \beta_{xy} = \beta_{yx}$. This means that both agents in the team have the same personality and training, and Model (2) becomes:

$$x = x\exp\left(r\left(1 - \frac{x}{k}\right) - \frac{\beta y}{1 + \gamma xy}\right)$$

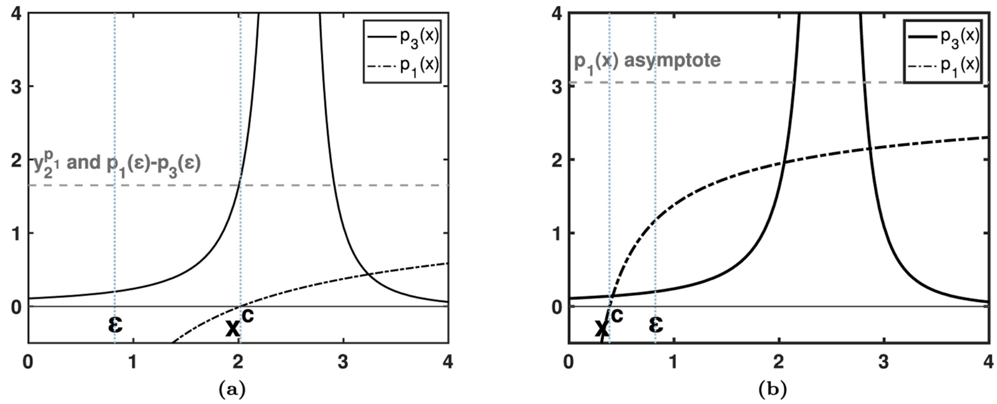
$$y = y\exp\left(r\left(1 - \frac{y}{k}\right) - \frac{\beta x}{1 + \gamma yx}\right)$$

From the exponential function in system (8), we can derive the following equations to plot the system's nullclines:

$$y = \frac{r(k-x)}{r\gamma x^2 - rk\gamma x + k\beta}$$

$$x = \frac{r(k-y)}{r\gamma y^2 - rk\gamma y + k\beta}$$

**Figure 4.** Nullclines $p_1(x)$ and $p_3(x)$ showing the existence of one interior equilibria when $\varepsilon = \frac{\sqrt{k1}}{\sqrt{\gamma_{xy}}} < x^c$ and the existence of three interior equilibria when $\varepsilon > x^c$.




Let $x^{\square}$ and $y^{\square}$ be equilibrium of Model (8). Then by substitution of Equation (9a) in Equation (9b), we obtain the fifth degree polynomial

$$f(x) = -k\beta(-r\gamma x^2 + kr\gamma x + r - k\beta)(-r\gamma x^3 + kr\gamma x^2 - rx - k\beta x + kr)$$

The quadratic function $g_1(x) = -r\gamma x^2 + kr\gamma x + r - k\beta$ can be solved explicitly from which we derive two equilibria $\mathbf{E}_{\mathbf{x}^{\square}_{1,2},\mathbf{y}^{\square}_{1,2}}$ in Equation (10). Equilibria $\mathbf{E}_{\mathbf{x}^{\square}_{j},\mathbf{y}^{\square}_{j}}, j = 3, 4, 5$ is obtained from the cubic function $g_2(x) = -r\gamma x^3 + kr\gamma x^2 - rx - k\beta x + kr$. The system has five possible interior equilibrium points. However, only two of them can be expressed explicitly as follows:

$$\mathbf{E}_{\mathbf{x}^{\square}_1,\mathbf{y}^{\square}_1} = \left(\frac{k}{2}\left[1 - \sqrt{1 + \frac{4(r-k\beta)}{k^2 r\gamma}}\right], \frac{k}{2}\left[1 + \sqrt{1 + \frac{4(r-k\beta)}{k^2 r\gamma}}\right]\right)$$

$$\mathbf{E}_{\mathbf{x}^{\square}_2,\mathbf{y}^{\square}_2} = \left(\frac{k}{2}\left[1 + \sqrt{1 + \frac{4(r-k\beta)}{k^2 r\gamma}}\right], \frac{k}{2}\left[1 - \sqrt{1 + \frac{4(r-k\beta)}{k^2 r\gamma}}\right]\right)$$

Thus, the interior equilibrium points $\mathbf{E}_{\mathbf{x}_j^*,\mathbf{y}_j^*}$, where $j = 1, 2$ exist if $\frac{r}{k} < \beta < \frac{r(k^2\gamma+4)}{4k}$.

**Theorem 3.4**

**Interior equilibria existence conditions for Symmetric model**

Model (8) can have the following dynamics depending on the values of its strictly positive parameters $r, k, \gamma$ and $\beta \in \mathbb{R}$ defined in Table 1.

**(1) Equilibria Existence:**

- Model (8) has **five interior equilibrium** points if

$$\gamma > \frac{27}{k^2} \quad \text{and} \quad \frac{2r}{k} + \frac{2kr\gamma}{9} < \beta < \frac{r(k^2\gamma - 3)}{3k}.$$

- Model (8) has **three interior equilibrium** points ($\mathbf{E}_{\mathbf{x}_j^*,\mathbf{y}_j^*}, j = 1, 2, 3$) if

$$\gamma \leq \frac{27}{k^2} \quad \text{and} \quad \frac{r}{k} < \beta < \frac{r(k^2\gamma + 4)}{4k},$$

OR

$$\gamma > \frac{27}{k^2} \quad \text{and} \quad \frac{r}{k} < \beta < \frac{2r}{k} + \frac{2kr\gamma}{9}$$

- Model (8) will always have **one interior equilibria** ($\mathbf{E}_{\mathbf{x}_3^*,\mathbf{y}_3^*}$).

**(2) Equilibria Stability:**

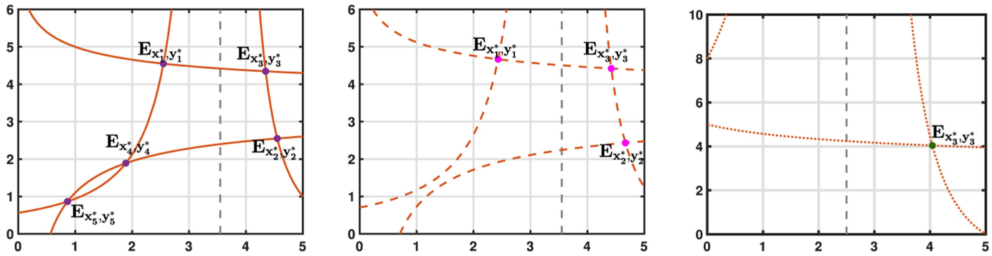
- **Equilibria $\mathbf{E}_{\mathbf{x}_j^*,\mathbf{y}_j^*}$**, $j = 1, 2, 4, 5$ are always <u>unstable</u>.
- **Equilibria $\mathbf{E}_{\mathbf{x}_3^*,\mathbf{y}_3^*}$** is locally asymptotically stable if

$$\beta > 0, \qquad r < 1, \qquad \frac{k}{2} < x_3^*, \quad \text{and} \quad k\beta - r\gamma(k - x_3^*)^2 > 1.$$

The proof of Theorem 3.4 can be found in Appendix A. Figure 5 illustrates the number of possible interior equilibria for the symmetric Model (8). The corresponding parameter values used for each plot are provided in the figure. The symmetric Model(8) can have only five, three, or one interior equilibrium point. The existence of only two equilibrium points were not found in the system. This can be attributed to the team being formed by two individuals with the same personality and receiving identical training to complete their task (strictly symmetric model). Thus, when forming a team with similar personalities and training, the possibility of finding bistability decreases. The number of interior equilibria in Model (8) is determined by the positive intercepts of the functions $p_{1,2}(x) = \frac{\sqrt{r(k\gamma x-1)} \pm \sqrt{r(1+k\gamma x)^2 - 4k\beta\gamma x^2}}{2\sqrt{r\gamma x}}$ and $p_3(x) = \frac{r(k-x)}{r\gamma x^2 - rk\gamma x + k\beta}$. In Figure 5, the number of intersections is represented by purple, magenta, and green dots for five, three, and one interior equilibria, respectively. Figure 6 depicts the bifurcation diagrams for both agents with respect to parameter $\beta$. The figure shows the three boundary and five interior equilibria that can exists in Model 8. The blue points represent source equilibria, the red indicate sink equilibria, and green indicate saddle points. Thus, only one interior equilibria can be stable. The proof of Theorem 3.4 can be found in the

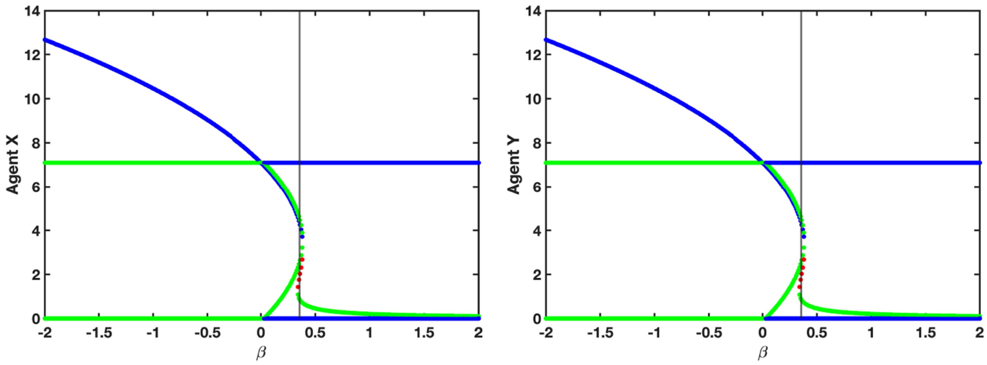
Appendix, and a summary of the conditions for each case can be found in Table 3. The stability conditions for $\mathbf{E}_{\mathbf{x}_1^*,\mathbf{y}_1^*}$ and $\mathbf{E}_{\mathbf{x}_2^*,\mathbf{y}_2^*}$ are provided in Theorem 3.4.

**Figure 5.** Plots of nullclines (9a) and equation $g_1(x)$. The figures reflect the possible number of interior equilibria points for symmetric Model (8). The model has five (purple points), or three (magenta points), or one (dark green points) interior equilibria. The grey dashed line represents the asymptote of nullcline 9a. The solid, dashed and dotted lines correspond to the change in parameters for both nullclines. (a) $r=0.2,\ k=7.1,\ \beta=0.335\ \gamma=1$ (b) $r=0.2,\ k=7.1,\ \beta=0.284,\ \gamma=0.8$ (c) $r=1.4,\ k=5,\ \beta=0.175,\ \gamma=0.1$




(a) $r=0.2,\ k=7.1,\ \beta=0.335,\ \gamma=1$ (b) $r=0.2,\ k=7.1,\ \beta=0.284,\ \gamma=0.8$ (c) $r=1.4,\ k=5,\ \beta=0.175,\ \gamma=0.1$

**Figure 6.** Bifurcation diagram with respect to $\beta$ for the symmetric model (8). The diagram shows boundary and interior equilibria of the system, where blue represents stable equilibria, red indicates sink points, and green indicates saddle points. The following parameter values were used $r=0.2,\ k=7.1,\ \gamma=1$.




(a) (b)

**Table 3.** Summary for the existence of interior equilibrium for symmetric Model (8).

| Equilibrium | Existence | Stability |
|---|---|---|
| $\mathbf{E}_3$ | Always | LAS if $\beta>0$, $r<1$, $\frac{k}{2}<x^*$, and $k\beta-r\gamma(k-x^*)^2>1$, otherwise they are unstable. |
| $\mathbf{E}_j, j=1,2$ | $\frac{r}{k}<\beta<\frac{r(k^2\gamma+4)}{4k}$ | Unstable |
| $\mathbf{E}_j, j=4,5$ | $\gamma>\frac{27}{k^2}$ and $\frac{2r}{k}+\frac{2kr\gamma}{9}<\beta<\frac{r(k^2\gamma-3)}{3k}$ | Unstable |

Note: LAS: locally asymptotically stable.

In Figure 5, we can observe that the equilibria $\mathbf{E}_{\mathbf{x}_1^*,\mathbf{y}_1^*}$ and $\mathbf{E}_{\mathbf{x}_2^*,\mathbf{y}_2^*}$ exist only when there are a total of three or five interior equilibria. As indicated in Table 3, due to the symmetry of $\mathbf{E}_{\mathbf{x}_1^*,\mathbf{y}_1^*}$ and $\mathbf{E}_{\mathbf{x}_2^*,\mathbf{y}_2^*}$, both equilibrium points are unstable since the condition for their existence ($\beta<\frac{r(k^2\gamma+4)}{4k}$) contradicts stability condition. On the other hand, equilibrium $\mathbf{E}_{\mathbf{x}_3^*,\mathbf{y}_3^*}$ always exists and can be stable if the psychological and environmental effects of both agents are positive ($\beta>0$), both agents exhibit a high level of emotional inertia ($r<1$), the agents' communication level is greater than half of their communicative level ($\frac{k}{2}<x$), and the combined impact of the agents'

communication, emotional inertia ($r$), damping capability ($\gamma$), and communicative level ($k$) is smaller than the impact of the psychological and environmental effects ($\beta$) on the communication level.

## 4. Application in team dynamics

Similar to the application of Gottman's model to married couples [17], Model (2) can be applied to the communication between any two individuals. However, our focus lies on studying communication within a two-agent team. Communication serves as the foundation for coordination and cooperation, making it crucial in teamwork. Various agencies, such as the military, aeronautics and space researchers, and first responders, regularly undertake high-risk and critical operations that require collaboration among team members. Whether these operations involve protecting national security or saving civilian lives, the teams must effectively achieve their primary goals while minimizing risks and avoiding casualties. Consequently, agencies, such as the military and NASA, invest in and foster partnerships with research institutions to further studies in team dynamics with the aim to study the effects on team performance, situation awareness, and team coordination [13,15,25].

### 4.1. Dataset

To gain insights into the required autonomy level for an effective two-agent human-robot team, we refer to Bartlett and Cooke's 2015 study [2]. Their experiment involved a two-agent interaction, similar to what we can expect in USAR situations. Bartlett and Cooke devised a search and rescue scenario using the videogame Minecraft. The virtual environment they created represented a collapsed office building, in which the two-agent team has to navigate. The internal agent, a human, acting as an independent and intelligent robot, was responsible for navigating the environment and 'rescuing' victims. The external agent, a human, had the task of guiding the internal agent. Both agents were seated side-by-side with a divider between them (see Figure 2(a)). The external agent could only observe the events inside the videogame and was provided with a map of the building to direct the internal agent's movements. However, the environment deliberately contained inconsistencies such as missing or additional walls and misplaced doorways, which were not reflected in the map provided to the external agent. Victims were represented by green and blue blocks, while hazards were depicted by pink blocks. The internal agent had to locate and click on all the green and blue blocks within eight minutes, while avoiding the pink blocks. Clicking on pink or blue blocks after eight minutes would negatively impact the team's overall performance score. Successful completion of the task required constant communication, coordination, and shared knowledge among team members. The recorded data included information such as the speaker (operator or robot), communication time (in seconds), lag between communications (in seconds), and word count per message (see Figure 2(b)). As participants were recruited to form the two-agent teams, we treat the recorded data from the robot as if it were generated by a second human agent.

The data collected by Bartlett and Cooke consists of a set of 40 teams with team performance scores ranging from 13 to 41 [2]. We divided the teams into three categories: low-performing, medium-performing, and high-performing teams. Among the teams, there were nine low-performing teams with scores between 13 and 19, twenty medium-performing teams with scores between 20 and 29, and eleven high-performing teams with scores between 30 and 41. It is worth noting that only one team achieved a score of 41, making it an outlier.

Throughout the remainder of this section, we will refer to the external agent in the data as Agent X, and the internal agent as Agent Y. It is important to note that in the figures presented, the $x$-axis represents interactions. For example, *Interaction 1* corresponds to one verbal communication from Agent X and the subsequent response from Agent Y. The y-axis depicts the value of $\mathrm{words}/\mathrm{lag}(\mathrm{sec.})$, which indicates the ratio of the speaker's communication density to the time taken to respond to the previous communication. Additionally, the figures illustrate the data and the corresponding model fitting for the three performance levels. The data is segregated based on Agent X' communication (Figures (a), (c) and (e)) and Agent Y's communication (Figures (b), (d) and (f)). For each performance group, we calculated the average $\mathrm{words}/\mathrm{lag}(\mathrm{s.})$ per interaction, represented by black dots in the figures. For example, we determined the average for Agent X's communication at *Interaction 1* across all low-performing teams, which corresponds to the first black dot in Figures (a). Finally, the standard error of the calculated mean is visualized using a grey shaded area.

The data, as shown in Figure C2, highlights several notable differences among the performance groups and agents, and Table 4 presents the mean data for each agent at various performance levels. In the following data observations we define *density of communication* as the value of $words/lag$ on each interaction, and *communication variability* as the change of $words/lag$ value between interaction $i$ and interaction $i+1$.

- **Agent X demonstrates greater variability and density of communication across all performance levels compared to Agent Y.** This means that, on average, Agent X communicates more frequently than Agent Y in the given scenarios.
- **The team dynamics established by the experimental task involve Agent X as the leader and decision-maker.** In this team dynamic, it is expected that Agent X will communicate more by requesting information and conveying strategy changes, while Agent Y navigates the scenario.

- **Communication density increases for both agents as performance improves.**
- **Some low-performing teams exhibit inadequate or sporadic communication from either agent.**
- **In low-performing teams**, Agent Y exhibits a decrease in communication density and variability after approximately one hundred interactions.
- **In high-performing teams,**
  - Agent Y's communication variability and density show an upward trend, with values ranging from 1 to 10 words per lag.
  - Agent X maintains a more consistent variability (4–9 words per lag) during the intermediate section of the task.

- **Agents in medium-performing teams display less variability between interactions.** That is, the difference between the value of words per lag at interaction $i$ and interaction $i+1$ is small.
- **A higher difference in communication between the agents leads to lower team performance.** The difference in mean data between Agent X and Agent Y ($\bar{Y}-\bar{X}$) decreases for the medium-performing and high-performing groups. In the low-performing group, the difference is 4.5, while in the medium-performing and high-performing groups, the differences are 2.9 and 3.6, respectively.
  - Having a smaller difference may not be optimal for the team (e.g. in a medium-performing team).

**Table 4.** Summary of experimental data collected by Bartlett and Cooke [2].

| | | Agents | | |
|---|---|---|---|---|
| Data Mean | Performance | $\bar{X}$ | $\bar{Y}$ | $\bar{Y}-\bar{X}$ |
| | *Low* | 6.56 | 2.04 | −4.52 |
| | *Medium* | 5.59 | 2.61 | −2.98 |
| | *High* | 6.78 | 3.13 | −3.65 |

### 4.2. Model fitting

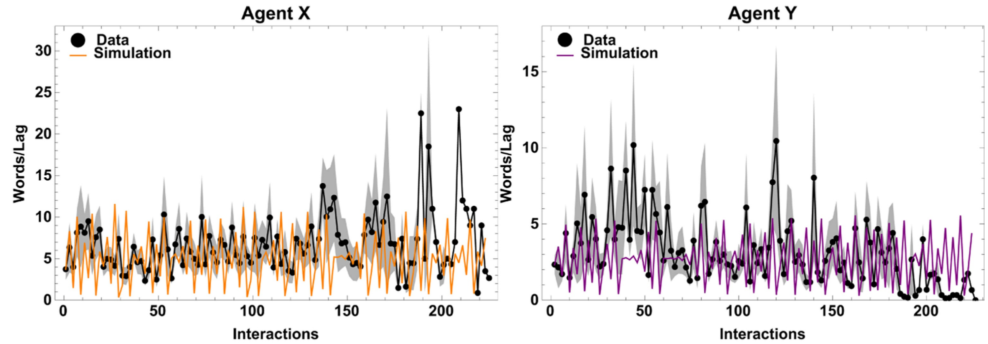
Model (2) assumes that communication is influenced by personality, such as emotional inertia and communicative level; and training, such as environmental impact and damping capabilities. The available data provides us an opportunity to validate our proposed model and provide us a baseline value of those key parameters after validation. Thus, in this section we will first fit the eight parameters of Model(2) to estimate the agents' emotional inertia, their psychological and environmental impacts during the task completion, and their damping capability. Our initial fitting (see Figure C2 and Table 5) suggests that our model is plausible and thus can obtain a baseline for our parameter values. With this baseline, we can now study how does the agents' training change if the team is composed of two agents with similar personality. Thus, we perform a second fitting were fix the parameters of emotional inertia and mean communicative level ($r_{1,2}, k_{1,2}$). Additionally, we want to explore how we can compose the best team possible if the same training is given to both agents. Then, our third fitting only is performed for parameters $r_{1,2}$ and $k_{1,2}$, while $\beta_{xy,yx}$ and $\gamma_{xy,yx}$ are fixed. The methodology for the parameter estimations in this section was completed with the aid of MATLAB R2022a software and the coded algorithm for the fitting can be found in Appendix B.

**Table 5.** Parameter estimations of all eight parameters of Model (2) for teams with different levels of performance.

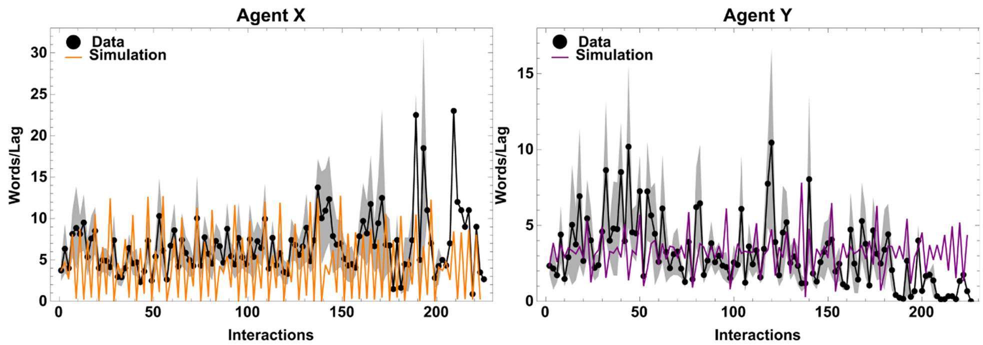
| | Parameters | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Performance | $r_1$ (Ngreen↓) | $r_2$ (Ngreen↓) | $k_1$ (Ngreen↓) | $k_2$ (red↑) | $\beta_{xy}$ (Ngreen↓) | $\beta_{yx}$ (red↑) | $\gamma_{xy}$ (Ngreen↓) | $\gamma_{yx}$ (red↑) | Agent X *MSE* | Agent Y *MSE* |
| *Low* | 3.1993 | 3.3383 | 5.8158 | 2.0598 | 6.3591 | 0.3511 | 3.9948 | 0.1202 | 12.2030 | 2.0230 |
| *Medium* | 3.2793 | 3.2931 | 5.2387 | 2.4998 | 2.9034 | 2.5929 | 2.0798 | 4.0432 | 10.1178 | 1.5699 |
| *High* | 3.0519 | 2.9977 | 5.3669 | 2.8423 | 1.1647 | 1.207 | 2.2948 | 4.1082 | 13.0897 | 2.3115 |

Figure 7( a,b) shows the fitted model for high-performing teams where all eight parameters were estimated. The remaining plots for low- and medium-performing team can be found in Figure C2. Table 5 contains the fitted values of all parameters of different team performance levels with the mean squared error (MSE) corresponding to each agents' data. Table 6 contains the rate of change of the parameters corresponding to each agent on a certain level of performance. Based on the fitted parameters, we draw the following conclusions:

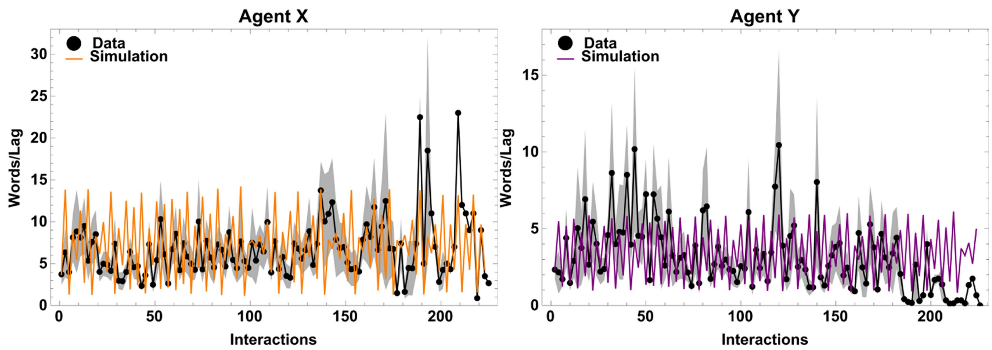
**Figure 7.** All figures correspond to data from high-performing teams. **Black** dots represent the data mean, the *grey area* is the standard error of the mean at each interaction, and the **purple** and **orange** lines represent the model predicted values for Agent X and Agent Y communication, respectively. Model fitting of all eight parameters of Model (2) are depicted on figures (a) and (b), fitting of parameters corresponding to agents' training when agents have consistent personality are depicted on figures (c) and (d), and fitting of parameters corresponding to agents' personality when agents have consistent training are depicted on figures (c) and (d). The fitted parameter values can be found in Tables 5, 7, and 9. (a) Fitting of all model parameters (b) Fitting of all model parameters (c) Fitting with consistent agent personality. $r_1 = r_2 = 3.2$ (d) Fitting with consistent agent personality. $k_1 = k_2 = 3.97$ (e) Fitting with consistent agent training. $\beta_{xy} = \beta_{yx} = 2.42$ (f) Fitting with consistent agent training. $\gamma_{xy} = \gamma_{yx} = 2.77$.




**(a)** Fitting of all model parameters **(b)** Fitting of all model parameters




**(c)** Fitting with consistent agent personality. $r_1 = r_2 = 3.2$ **(d)** Fitting with consistent agent personality. $k_1 = k_2 = 3.97$




**(e)** Fitting with consistent agent training. $\beta_{xy} = \beta_{yx} = 2.42$ **(f)** Fitting with consistent agent training. $\gamma_{xy} = \gamma_{yx} = 2.77$

**Table 6.** Rate of change calculation between all the parameters corresponding to Agent X and Agent Y for different levels of performance.

| | Rate of Change | | | |
|---|---|---|---|---|
| Performance | $\Delta r = r_2 - r_1$ | $\Delta k = k_2 - k_1$ | $\Delta\beta = \beta_{yx} - \beta_{xy}$ | $\Delta\gamma = \gamma_{yx} - \gamma_{xy}$ |

| | | | | |
|---|---|---|---|---|
| *Low* | 0.139 | −3.7560 | −6.008 | −3.8746 |
| *Medium* | 0.0138 | −2.7389 | −0.3105 | 1.9634 |
| *High* | −0.0542 | −2.5246 | 0.0423 | 1.8134 |

**Table 7.** Parameter estimations for three performance levels.

| | Parameters | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Performance | $r_1$ | $r_2$ | $k_1$ | $k_2$ | $\beta_{xy}$ (Ngreen↓) | $\beta_{yx}$ (red↑) | $\gamma_{xy}$ (Ngreen↓) | $\gamma_{yx}$ (red↑) | Agent X *MSE* | Agent Y *MSE* |
| Low | 3.2 | 3.2 | 3.97 | 3.97 | 4.4047 | 2.2785 | 5.8693 | 0.6649 | 12.9475 | 1.4254 |
| Medium | 3.2 | 3.2 | 3.97 | 3.97 | 0.2291 | 2.8316 | 0.1902 | 1.0917 | 13.5473 | 1.6651 |
| High | 3.2 | 3.2 | 3.97 | 3.97 | −0.692 | 8.9071 | 4.3172 | 4.5713 | 13.2696 | 2.2004 |

Note: We fixed parameters $r_1 = r_2 = 3.2$ and $k_1 = k_2 = 3.97$ for all performance levels.

**Table 8.** Rate of change calculation between all the parameters related to training corresponding to Agent X and Agent Y for different levels of performance.

| | Rate of Change | |
|---|---|---|
| Performance | $\Delta\beta = \beta_{yx} - \beta_{xy}$ | $\Delta\gamma = \gamma_{yx} - \gamma_{xy}$ |
| *Low* | −2.1262 | −5.2044 |
| *Medium* | 2.6025 | 0.9015 |
| *High* | 9.5991 | 0.2541 |

**Table 9.** Parameter estimations for three performance levels.

| | Parameters | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Performance | $r_1$ (Ngreen↓) | $r_2$ (Ngreen↓) | $k_1$ (red↑) | $k_2$ (red↑) | $\beta_{xy}$ | $\beta_{yx}$ | $\gamma_{xy}$ | $\gamma_{yx}$ | Agent X *MSE* | Agent Y *MSE* |
| Low | 3.0943 | 3.1864 | 5.5501 | 3.1799 | 2.42 | 2.42 | 2.77 | 2.77 | 9.17523 | 2.0804 |
| Medium | 3.0372 | 3.1054 | 5.5441 | 3.4890 | 2.42 | 2.42 | 2.77 | 2.77 | 10.3117 | 2.5278 |
| High | 2.9519 | 3.1227 | 7.7731 | 3.8441 | 2.42 | 2.42 | 2.77 | 2.77 | 13.2334 | 3.2802 |

Note: Parameters $\beta_{xy} = \beta_{yx} = 2.42$ and $\gamma_{xy} = \gamma_{yx} = 2.77$ are fixed for all performance levels.

- **Both $r_1$ and $r_2$ decrease as team performance increases.** This suggests that high-performing teams have agents with higher and similar emotional inertia, indicating that their emotional stability is not easily affected by their surroundings and experiences.
- **The value of $\Delta r$ also decreases as performance increases.** This indicates that high-performing teams have agents with more similar emotional inertia, contributing to their overall emotional stability.
- **The parameter $k_1$ decreases, while $k_2$ increases as performance increases, resulting in a decrease in the distance between the parameters ($|\Delta k|$).** This implies that in higher-performing teams, Agent Y participates more in the conversation by transmitting more information.
    - *Agent X must decrease their mean communicative level to allow for a more dense interaction from Agent Y.* This adjustment in communicative levels ensures that Agent Y's increased participation in the conversation is accommodated.

- *The fitting reflects the data's behaviour.* A larger value of $k$ is fitted for Agent X ($\Delta k < 0$).

- **The parameter $\beta_{xy}$ decreases, while an overall increase in $\beta_{yx}$ is observed.** This indicates that the psychological and environmental impacts are higher on Agent Y in medium-performing teams compared to low and high-performing teams.
- **Agent X has a higher psychological and environmental impact than damping capabilities ($\beta_{xy} > \gamma_{xy}$).** This imbalance affects Agent Y since Agent X, as the leader responsible for strategy and planning, directly impacts and influences Agent Y.
- **Agent X's damping capabilities ($\gamma_{xy}$) generally decrease, while Agent Y's damping capabilities ($\gamma_{yx}$) increase.** This dynamic is favourable for the team, as Agent Y's ability to better assess situations and the environment allows Agent X to have a lower negative psychological impact and focus on strategy and planning.
- **The rate of change of $\beta$ and $\gamma$ becomes positive, and the distance between the parameters ($|\Delta\beta|$, $|\Delta\gamma|$) significantly decreases as performance increases.** This highlights the importance of both agents having the ability and training to assess different obstacles and be minimally affected by unique events based on their past experiences and training.

#### 4.2.1. Consistent agent personality

We conducted a second fitting in which the parameters related to the personality of Agent X and Agent Y ($r$, $k$) were fixed across team performance levels and among agents, i.e. $r_1 = r_2$, $k_1 = k_2$. The objective was to examine how training for the agents should be modified if both agents in the team have the same or similar personality. For this fitting, we calculated the averages of $r_1$, $r_2$, $k_1$, and $k_2$ obtained in the previous fitting (Table 5), resulting in $r_1 = r_2 = 3.2$ and $k_1 = k_2 = 3.97$.

Figure 7( c,d) illustrates the fitted model for high-performing teams, where only the $\beta$s and $\gamma$s were estimated. The remaining plots for low- and medium-performing team can be found in Figure C3. Table 7 presents the fitted values of all parameters for different team performance levels, along with the mean squared error corresponding to each agent's data. Table 8 displays the rate of change of the fitted parameters for each agent at specific performance levels. Based on the fitted parameters, the following conclusions can be drawn:

- **The estimated value of $\beta_{xy}$ shows a significant decrease, while $\beta_{yx}$ increases as performance increases.** Thus, resulting in an increased $\Delta\beta$ of 9.5991 for the high-performing team. This indicates that Agent Y receives and manages a majority of the negative psychological and environmental impacts, and a larger $\Delta\beta$ value contributes to improved team performance.
- **The parameter $\gamma_{xy}$ exhibits an overall decrease, whereas $\gamma_{yx}$ steadily increases with team performance.** This suggests that Agent X's damping capability decreases, while Agent Y's damping capability increases as team performance improves.
  - *In the medium-performing team, the lower value of $\gamma_{xy}$ may be attributed to the decreased value of $\beta_{xy}$.* This implies that the subpar performance of the medium-performing team is due to both agents having low damping capability and experiencing minimal psychological and environmental effects.
  - *The rate of change of $\gamma$ becomes positive, and the difference between both parameters ($|\Delta\gamma|$) decreases as the team's performance improves.* This indicates that as team performance improves, both agents' damping capabilities undergo positive changes, and the difference between their damping capabilities decreases.

- **A higher $\Delta\beta$ and lower $\Delta\gamma$ are beneficial to the team's performance.** This suggests that a larger difference in $\beta$ values and a smaller difference in $\gamma$ values contribute to improved team performance.
- **A team with better performance includes an Agent Y who, as the first agent to encounter challenging situations, can assess and control obstacles and situations more effectively.** This implies that in high-performing teams, Agent Y's ability to handle challenging situations leads to better overall team performance.
- **Agent X does not experience the negative effects of these events and can focus on strategy and decision-making.** This highlights that when Agent Y effectively handles challenging situations, Agent X is not negatively impacted, allowing them to concentrate on strategic planning and decision-making.

#### 4.2.2. Consistent agent training

The third data fitting starts with the assumption that $\beta_{xy} = \beta_{yx}$, $\gamma_{xy} = \gamma_{yx}$, and then estimates the values of other parameters to investigate how the personalities of both agents would change across different levels of performance when they receive identical training. Thus, we aimed to determine how we can form a team based solely on the agents' personality to achieve higher performance. For this fitting, we calculated the averages of $\beta_{xy}$, $\beta_{yx}$, $\gamma_{xy}$, and $\gamma_{yx}$ obtained in the first fitting (Table 5), resulting in $\beta_{xy} = \beta_{yx} = 2.42$ and $\gamma_{xy} = \gamma_{yx} = 2.77$.

Figure 7( e,f) illustrates the fitted model for high-performing teams, where only the $r$s and $k$s were estimated. The remaining plots for low- and medium-performing team can be found in Figure C4. Table 9 presents the fitted values of the parameters for different team performance levels, along with the mean squared error corresponding to each agent's data. Table 10 displays the rate of change of the fitted parameters for each agent at specific performance levels. Based on the fitted parameters, the following conclusions can be drawn:

**Table 10.** Rate of change calculation between all the parameters related to personality corresponding to Agent X and Agent Y for different levels of performance.

| | Rate of Change | |
|---|---|---|
| Performance | $\Delta r = r_2 - r_1$ | $\Delta k = k_2 - k_1$ |
| Low | 0.0921 | −2.3702 |
| Medium | 0.0682 | −2.0551 |
| High | 0.1708 | −3.9290 |

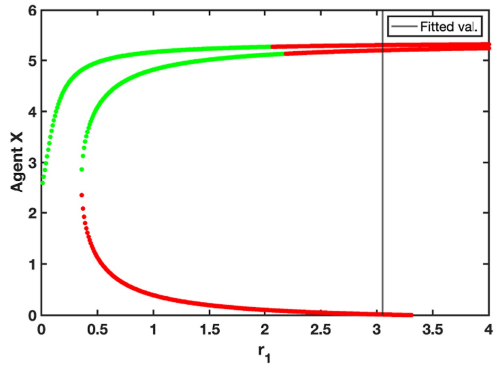
- **The estimated value of $r_1$ exhibits a decrease, while $r_2$ shows an overall decrease as the team's performance improves.** This suggests that both agents' emotional inertia decreases with improved team performance.
  - *Agent X demonstrates higher emotional inertia across all three performance levels ($r_1 > r_2$).*
  - *In the medium-performing team,* Agent Y has slightly higher emotional inertia (smaller $r_2$) compared to the high-performing team.
  - *High-performing teams have a slight increase of 0.2 for $\Delta r$.*
  - *For a high-performing team,* we require Agent X to have a higher emotional inertia $r_1 < 3$ and a distinctly higher communicative level $k_1 > k_2$.

- **The parameter estimation for $k_1$ is very similar for the low and medium-performing teams.** The estimated value for the high-performing team increases by 2.2 units.
- **On the other hand, $k_2$ steadily increases with performance improvement.** This indicates that as team performance improves, the value of $k_2$ also increases.
- **The model captures the behaviour of the data, where the mean communicative level is higher for Agent X ($k_1 > k_2$).** This finding suggests that the model interprets the behaviour of a higher communicative level exhibited by Agent X compared to Agent Y.
  - *There is a decrease of 1.6 units in $\Delta k$.* This implies that the difference between Agent X's and Agent Y's communicative level decreases by 1.6 units as team performance improves.

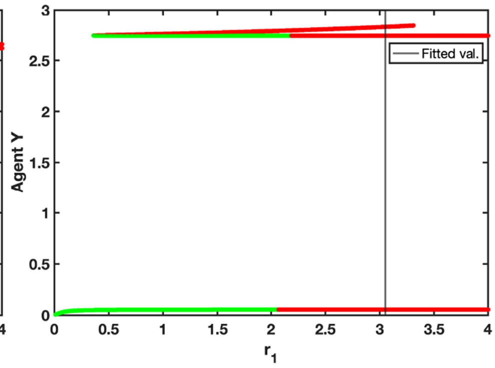
### 4.3. Bifurcation diagrams and parameter exploration

There are instances when it is not possible to change the members of an established team or create the ideal team. Therefore, studying the effect of each component(s) of communication on the dynamics can help us understand how communication is influenced and how it can be improved by exploring the parameters values of the high-performing team. Thus, we generated bifurcation diagrams for each of the eight parameters in the system and used the fitted parameters for the high-performing team from Table 5. Figure 8 specifically focuses on the parameters corresponding to Agent X, while the remaining diagrams can be found in Figure C5 in C. In Figure8, each row represents one parameter, each column represents an agent, and the vertical grey line represents the fitted value of the parameter for which we are plotting the bifurcation diagram. The blue points in the figure represent stable points, red represents unstable points, and green points represent saddle points. Stable points indicate unchanging communication between the agents, where either both agents or one agent maintains a consistent communication style (words/lag value).

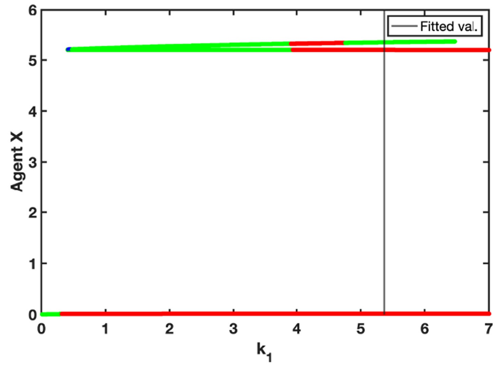
**Figure 8.** Bifurcation diagrams with respect to parameters $r_1$, $k_1$, $\beta_{xy}$, and $\gamma_{yx}$ in Model (2). The baseline parameters used correspond to high-performing teams and can be found in Table 5. The diagram shows interior equilibria of the system, where blue represents stable equilibria, red indicates sink points, and green indicates saddle points.



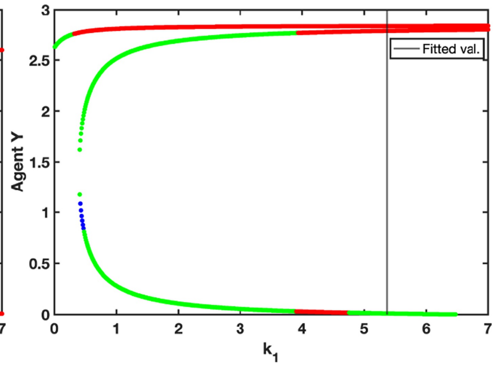

(a)



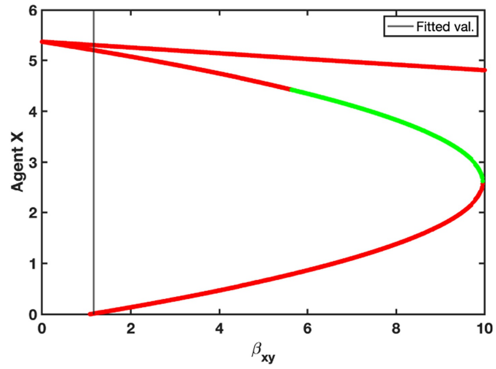

(b)



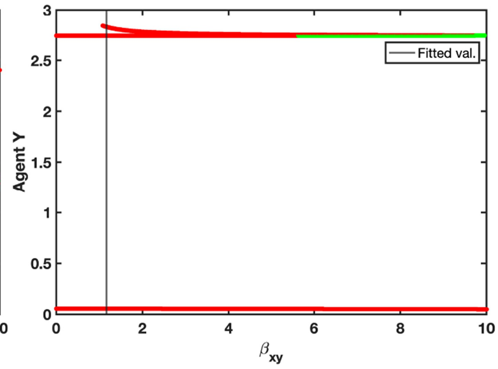

(c)



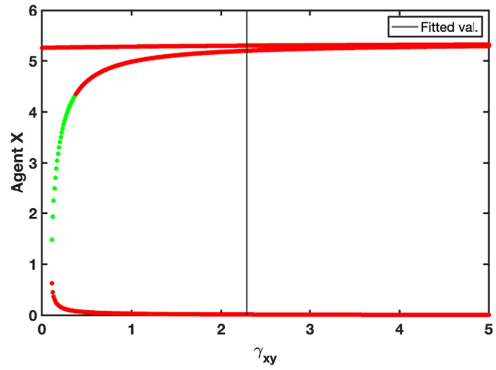

(d)



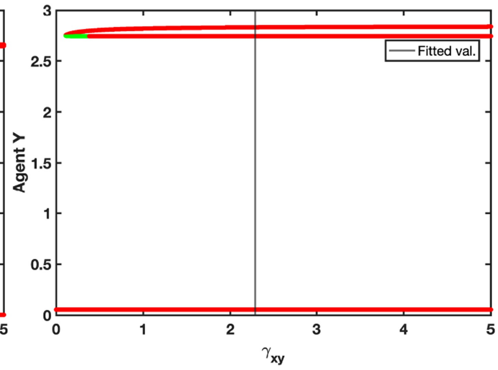

(e)




(f)




(g)

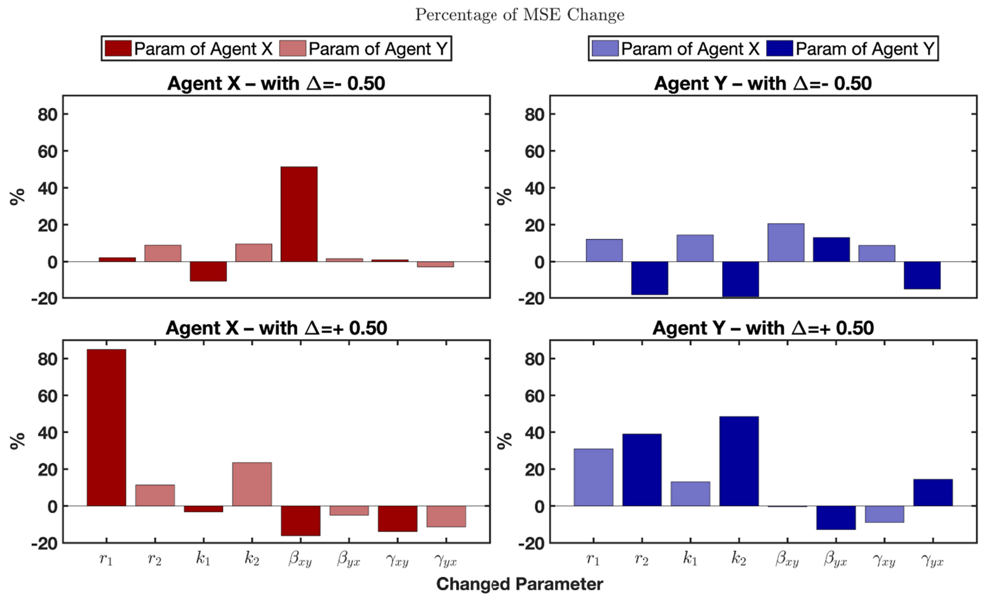



(h)

By analysing the estimated parameters from the experimental data and observing the variation in each parameter's value, we find that the model can exhibit one, two, or three equilibria. However, most of these equilibria tend to be saddles or sources (unstable), except for a small interval $[0.42, 0.47]$ for parameter $k_1$, where stability is observed. The fitted values and the range of values around parameters $r_1$, $\beta_{xy}$, and $\gamma_{xy}$ result in three sources. On the other hand, the fitted value and the range of values around parameter $k_1$ yield one saddle and two sources. Considering that the experimental data displays complicated behaviour, consequently, our bifurcation diagrams primarily depict unstable equilibria, further study of the parameters' effect on communication is warranted. Therefore, we conduct numerical simulations to explore how may each parameter impact communication dynamics as follows.

By using the parameters for high-performing teams from Table 5 as the baseline, we individually perform a $\Delta = \pm 0.5$ change in each parameter to explore the impact of changes in each of the eight parameters on the communication dynamics of the two-agent team. To measure the change in communication, we observe the percentage change in the calculated MSE from Table 5 and the new MSE of a new simulation (see Figure 9). Now, let's delve into the specific findings, highlighting the parameters that have the most significant influence on communication dynamics when their values are increased or decreased.

**Figure 9.** Percentage of MSE change in relation to the MSE values of each agent listed in Table 5. The calculation is performed by varying each parameter individually by a change of $\Delta = \pm 0.5$.




- **Parameters $r_1$, $r_2$, $k_2$, $\beta_{xy}$, and $\beta_{yx}$ are the most sensitive to change.** The change in these parameters has a more pronounced effect on communication dynamics when their values are decreased or increased by 0.5.
- **The $\beta$ parameters are the only ones sensitive to both increase and decrease.** Parameters $\beta_{xy}$ and $\beta_{yx}$ have different effects on communication depending on the direction of change.
    - *Decreasing $\beta_{xy}$ has a greater effect on communication than increasing it.* A very low value of psychological and environmental impact for the lead team member may lead to reduced focus and situational awareness of potentially high-risk events.
    - *Increasing $\beta_{yx}$ has a more significant impact on communication than decreasing it.* Agent Y, responsible for navigating and interacting with the high-risk environment, may inadequately communicate crucial details, leading to erroneous assessments and decisions by Agent X.

- **The emotional inertia is the least resilient parameter.** The parameter $r$ affects communication whether it increases or decreases.
    - *A low value of $r$ (high emotional inertia)* makes it challenging for an agent to transition between emotional states, potentially prolonging negative emotions.
    - *A high value of $r$ (low emotional inertia)* allows emotions to change more rapidly.

- **Agent Y's communicative level ($k_2$) is another parameter with low resilience.**
    - *Decreasing Agent Y's communicative level* reduces the information received by Agent X, limiting their understanding of the situation and decision-making abilities.
    - *Increasing Agent Y's communicative level* may overwhelm Agent X with excessive communication, hindering effective communication of the next plan of action.

It is important to note that the increase or decrease of these parameters does not inherently imply better or worse outcomes. The ideal level of emotional inertia, for example, may vary depending on the context and circumstances. Some situations may require individuals who posses stability and continuity in their emotional experiences, while others may necessitate individuals who can quickly adapt and shift their emotional states. Therefore, our parameter exploration provides insights into which elements of human communication are more likely to affect communication dynamics in an established team when perturbed.

## 5. Conclusion

In this study, we proposed a discrete-time mathematical model that considers communication between two agents based on their individual personalities and the impact of received communication. We adopted a population dynamics approach to establish our modelling framework, applying it to examine the communication dynamics between Agent X ($x_{i+1}$) and Agent Y ($y_{i+1}$) working collaboratively on a task through a two-dimensional system of difference equations with delay. Our model, coupled with data analysis, aims to address the following questions: (1) How may different factors impact communication dynamics? What factors benefit or harm the team? (2) How do these factors differentially affect teams with varying levels of performance? (3) What characteristics are present in a high-performing team?

Based on the available experimental data, we observed that high-performing teams exhibit continuous and active communication, with Agent X assuming a leadership role. Teams with poor or sporadic communication from either agent performed inadequately, resulting in low team scores. The experimental data displayed chaotic behaviour, indicating that communication between agents never stabilizes. This observation addresses question (3) from the data analysis perspective.

To further investigate communication dynamics, we conducted a theoretical analysis of our proposed model (2). Our analysis revealed that the model can have multiple types of interior equilibria: five, three, two, or one. Considering the complexity of the general model, we studied the dynamics of the symmetrical model by assuming that two agents are identical. The symmetric Model(8) can possess the same types of interior equilibria as the general model with at least one equilibrium ($\mathbf{E}_{\mathbf{x}_3,\mathbf{y}_3}$) always existing. Our analysis implied that the symmetrical model can have only one stable interior equilibrium $\mathbf{E}_{\mathbf{x}_3,\mathbf{y}_3}$ when certain conditions are met: both agents exhibit positive psychological and environmental effects ($\beta > 0$), high emotional inertia ($r<1$), communication levels greater than half of their respective communicative levels ($\frac{k}{2} < x$), and the combined impact of communication, emotional inertia ($r$), damping capability ($\gamma$), and communicative level ($k$) is smaller than the impact of psychological and environmental effects ($\beta$) on the communication level.

Stable equilibria in the system indicate a consistent communication pattern between agents, where either both agents or one agent maintains a consistent communication style (words/lag value). Given that unchanging communication between agents is not reflective of the experimental data, then the mentioned conditions for stability of the equilibrium $\mathbf{E}_{\mathbf{x}_3,\mathbf{y}_3}$ can harm the team's communication and performance. These theoretical findings provide insights into the factors that can either benefit or harm the team, addressing the first question.

We further gained more insights into our research questions by exploring the effect of parameter variations on different scenarios through parameter estimation. By fitting all eight parameters of the model, we found that high-performing agents maintain emotional stability regardless of their surroundings and experiences. In high-performing teams, Agent Y contributes more to the conversation by transmitting more information (see Figure 7(a,b) and Table 5). Medium-performing teams exhibit higher psychological and environmental impacts on Agent Y compared to low and high-performing teams. As performance improves, Agent X's damping capabilities generally decrease, while Agent Y's damping capabilities increase. This allows Agent X to focus on strategy and planning while Agent Y better assesses situations and the environment. When both agents possess the same emotional inertia and communicative level, Agent Y receives and manages a majority of the negative psychological and environmental impacts, leading to improved team performance. This can be observed by the increased values of parameters $\beta_{yx}$ and $\gamma_{yx}$ in Table 7. Additionally, as the first agent to encounter challenging situations, Agent Y's effective assessment and control of obstacles and situations contribute to enhanced team performance. Agent Y's adept handling of obstacles and situations also reduces the negative impact on Agent X, enabling them to concentrate on strategic tasks.

We explored a scenario in which both agents undergo the same training to ensure equal responsiveness to their teammate's communication, with both agents being equally affected by their teammate's communication. As team performance improves, both Agent X and Agent Y exhibit increasing emotional inertia, with Agent Y demonstrating slightly higher emotional inertia in medium-performing teams (see Table 9). Additionally, Agent Y's communicative level consistently increases as team performance improves, while Agent X consistently maintains a higher mean communicative level than Agent Y across all performance levels. For optimal performance, Agent X should possess both higher emotional inertia and a notably higher communicative level compared to Agent Y.

Our bifurcation diagrams and parameter exploration demonstrated that the agents' emotional inertia, Agent Y's communicative level, and the agents' psychological and environmental impact are the most sensitive communication elements affecting the agents' communication dynamics as observed from the percentage MSE change in Figure9. High emotional inertia poses challenges for agents in transitioning between emotional states, while low emotional inertia allows emotions to change rapidly. A low value of psychological and environmental impact for the lead team member may lead to reduced focus and situational awareness, affecting their ability to identify high-risk events. Decreasing Agent Y's psychological and environmental impact may result in inadequate communication of crucial details, leading to incorrect assessments and decisions by Agent X. Reducing Agent Y's communicative level limits the information received by Agent X, hindering their understanding of the situation and decision-making abilities. Conversely, increasing Agent Y's communicative level may overwhelm Agent X with excessive communication.

The current model incorporates important features of two agents' interactions, and yet it could be improved in several aspects due to limitations. Firstly, it excludes the effect of different levels of workload, which can impact communication quality between agents and final team performance. Secondly, it does not consider the level of trust between team members, an important factor influencing decision-making and mutual reliance. Additionally, due to data limitations, we were unable to incorporate team performance as a function of time. Another major limitation of the model is the omission of information content and quality provided by each agent. Nevertheless, despite these limitations, our presented model lays a foundation and yields promising results regarding the implications of teams with members possessing similar personalities or receiving adequate training. Furthermore, it sheds light on how these dynamics change when one or both agents have different personalities or experience varied impacts from their teammate.

## Acknowledgments

We extend our sincere gratitude to Dr Jim Cushing on the occasion of his 80th birthday. His invaluable guidance and support have greatly influenced our research, and this paper is a tribute to his remarkable legacy and profound impact on our academic and professional lives.

## Disclosure statement

No potential conflict of interest was reported by the author(s). **[Q2]**

## ORCID

*Lucero Rodriguez Rodriguez* http://orcid.org/0000-0003-1842-1252

*Carlos Bustamante Orellana* http://orcid.org/0000-0002-7074-1811

*Nancy Cooke* http://orcid.org/0000-0003-0408-5796

*Mustafa Demir* http://orcid.org/0000-0002-5667-3701

*Yun Kang* http://orcid.org/0000-0001-5448-7609

## References

1 **D. Balliet**, *Communication and cooperation in social dilemmas: A meta-analytic review*. J. Conflict Resol. 54 (1) (2010), pp. 39–57. **[Q3]**

2 **C.E. Bartlett and N.J. Cooke**, *Human-robot teaming in urban search and rescue*. Proc. Hum. Factors Ergon. Soc. Annu. Meet. 59 (1) (2015), pp. 250–254.

3 **S. Barton**, *Chaos, self-organization, and psychology*. Am. Psychol. 49 (1) (1994), pp. 5–14.

4 **E.C. Brewer and T.L. Holmes**, *Better communication = better teams: A communication exercise to improve team performance*. IEEE. Trans. Prof. Commun. 59 (3) (2016), pp. 288–298.

5 **E.H. Buber**, *A nonlinear dynamic model of social interaction*. Communic. Res. 18 (2) (1991), pp. 174–198. https://doi.org/10.1177/009365091018002003

6 **E. Cha, Y. Kim, T. Fong, and M.J. Mataric**, *A survey of nonverbal signaling methods for non-humanoid robots*. Found. Trends Robotics 6 (4) (2018), pp. 211–323.

7 **J. Chan and P. Sy**, *The relationships among personality, intercultural communication, and cultural self-efficacy in nursing students*. J. Nurs. Res. 24 (4) (2016), pp. 286–290.

8 **J. Cushing**, *A Darwinian Ricker Equation*, in *Progress on Difference Equations and Discrete Dynamical Systems – 25th ICDEA, 2019*, S. Baigent, M. Bohner, and S. Elaydi, eds. Springer, Springer Proceedings in Mathematics and Statistics, 2020, pp. 231–243. Publisher Copyright: ^2020, Springer Nature Switzerland AG.; 25th International Conference on Difference Equations and Applications, ICDEA 2019 ; Conference date: 24-06-2019 Through 28-06-2019.

9 **J.M. Cushing**, *An Introduction to Structured Population Dynamics*, Society for Industrial Mathematics, 1998.

10 **J.M. Cushing, R.F. Costantino, B. Dennis, R. Desharnais, and S.M. Henson**, *Chaos in ecology: experimental nonlinear dynamics*, Vol. 1, Elsevier, 2003.

11 **J. Cushing, S. Levarge, N. Chitnis, and S.M. Henson**, *Some discrete competition models and the competitive exclusion principle*$^{\dagger}$. J. Differ. Equ. 10 (13-15) (2004), pp. 1139–1151.

12 **J.A. Daly and C.A. Diesel**, *Measures of communication-related personality variables*. Commun. Educ.41 (4) (1992), pp. 405–414.

13 **M. Demir, M. Cohen, C.J. Johnson, E.K. Chiou, and N.J. Cooke**, *Exploration of the impact of interpersonal communication and coordination dynamics on team effectiveness in human-machine teams*. Int. J. Hum. Comput. Interaction 39 (9) (2023), pp. 1841–1855. https://doi.org/10.1080/10447318.2022.2143004

14 **M. Demir, N.J. Cooke, and P.G. Amazeen**, *A conceptual model of team dynamical behaviors and performance in human-autonomy teaming*. Cogn. Syst. Res. 52 (2018), pp. 497–507.

15 **J.C. Gorman, N.J. Cooke, and J.L. Winner**, *Measuring team situation awareness in decentralized command and control environments*. Ergonomics 49 (12-13) (2006), pp. 1312–1325. https://doi.org/10.1080/00140130600612788 PMID: 17008258.

16 **J. Gottman, K. McCoy, J. Coan, and H. Collier**, *The Specific Affect Coding System (The SPAFF) for Observing Emotional Communication in Marital and Family Interaction*, 1995, 09.

17 **J.M. Gottman, J.D. Murray, C.C. Swanson, R. Tyson, and K.R. Swanson**, *The Mathematics of Marriage: Dynamic Nonlinear Models*, The MIT Press, 2002 12. doi:https://doi.org/10.7551/mitpress/4499.001.0001

18 **J. Jaffe and D.A. Norman**, *A simulation of the time patterns of dialogue*, Harvard Univ., Center for Cognitive Studies, 1964.

19 **N.L. Kerr and C.M. Kaufman-Gilliland**, *Communication, commitment, and cooperation in social dilemma*. J. Pers. Soc. Psychol. 66 (3) (1994), pp. 513–529.

20 **P. Killworth and H. Bernard**, *A model of human group dynamics*. Soc. Sci. Res. 5 (2) (1976), pp. 173–224.

21 **P. Kollock**, *Social dilemmas: The anatomy of cooperation*. Annu. Rev. Sociol. 24 (1) (1998), pp. 183–214.

22 **R.E. Kraut and L.A. Streeter**, *Coordination in software development*. Commun. ACM. 38 (3) (1995), pp. 69–81.

23 **P. Kuppens, N.B. Allen, and L.B. Sheeber**, *Emotional inertia and psychological maladjustment*. Psychol. Sci. 21 (7) (2010), pp. 984–991. doi:https://doi.org/10.1177/0956797610372634 PMID: 20501521.

24 **T.W. Malone**, *Computer simulation of two-person interactions*. Behav. Sci. 20 (4) (1975), pp. 260–267.

25 **A.R. Marathe, K.E. Schaefer, A.W. Evans, and J.S. Metcalfe**, *Bidirectional Communication for Effective Human-Agent Teaming*, in *Virtual, Augmented and Mixed Reality: Interaction, Navigation, Visualization, Embodiment, and Simulation*, J.Y. Chen and G. Fragomeni, eds., Cham. Springer International Publishing, 2018.

26 **L. Mingyue Ma, T. Fong, M.J. Micire, Y.K. Kim, and K. Feigh**, *Human-robot teaming: Concepts and components for design*, in *Field and service robotics*. Springer, 2018, pp. 649–663.

27 **D. Mishra and A. Mishra**, *Effective communication, collaboration, and coordination in extreme programming: Human-centric perspective in a small organization*. Hum. Factors Ergon. Manufact. Service Ind. 19 (5) (2009), pp. 438–456.

28 **R. Murphy**, *Marsupial and shape-shifting robots for urban search and rescue*. IEEE Intell. Syst. Appl.15 (2000), pp. 14–19.

29 **R. Murphy**, *Human-robot interaction in rescue robotics*. IEEE Trans. Syst., Man, Cyber., Part C (Appl. Rev.) 34 (2) (2004), pp. 138–153.

30 **T. O'Neill, N. McNeese, A. Barron, and B. Schelble**, *Human–autonomy teaming: A review and analysis of the empirical literature*. Hum. Factors. 64 (5) (2022), pp. 904–938.

31 **M. Pikkarainen, J. Haikara, O. Salo, P. Abrahamsson, and J. Still**, *The impact of agile practices on communication in software development*. Empir. Softw. Eng. 13 (3) (2008), pp. 303–337.

32 **J. Pöysä-Tarhonen, J. Elen, and P. Tarhonen**, *Student teams' development over time: tracing the relationship between the quality of communication and teams' performance*. High. Educ. Res. Dev.35 (4) (2016), pp. 787–799. doi:https://doi.org/10.1080/07294360.2015.1137887

33 **M. Radin**, *Difference Equations For Scientists And Engineering: Interdisciplinary Difference Equations*, World Scientific Publishing Company, 2019.

34 **R.E. Riggio and S.J. Taylor**, *Personality and communication skills as predictors of hospice nurse performance*. J. Bus. Psychol. 15 (2) (2000), pp. 351–359.

35 **K. Schneider, O. Liskin, H. Paulsen, and S. Kauffeld**, *Media, mood, and meetings: Related to project success?*. ACM Trans. Comput. Educ. 15 (4) (2015), pp. 1–33. doi:https://doi.org/10.1145/2771440

36 **J.C. Sprott**, *Dynamical models of love*. Nonlinear Dyn. Psychol. Life Sc. 8 (3) (2004), pp. 303–14.

37 **S.H. Strogatz**, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*, Perseus

Publishing, Cambridge, Massachusetts, 1994.**[Q4]**

38 **P. Suppes and R.C. Atkinson**, *Markov Learning Models for Multiperson Interactions*, Stanford University Press, Stanford, California, 1960.**[Q5]**

39 **E.A. Thomas and T.W. Malone**, *On the dynamics of two-person interactions*. Psychol. Rev. 86 (4) (1979), pp. 331–360.

40 **E.A. Thomas and J.A. Martin**, *Analyses of parent-infant interaction*. Psychol. Rev. 83 (2) (1976), pp. 141.156