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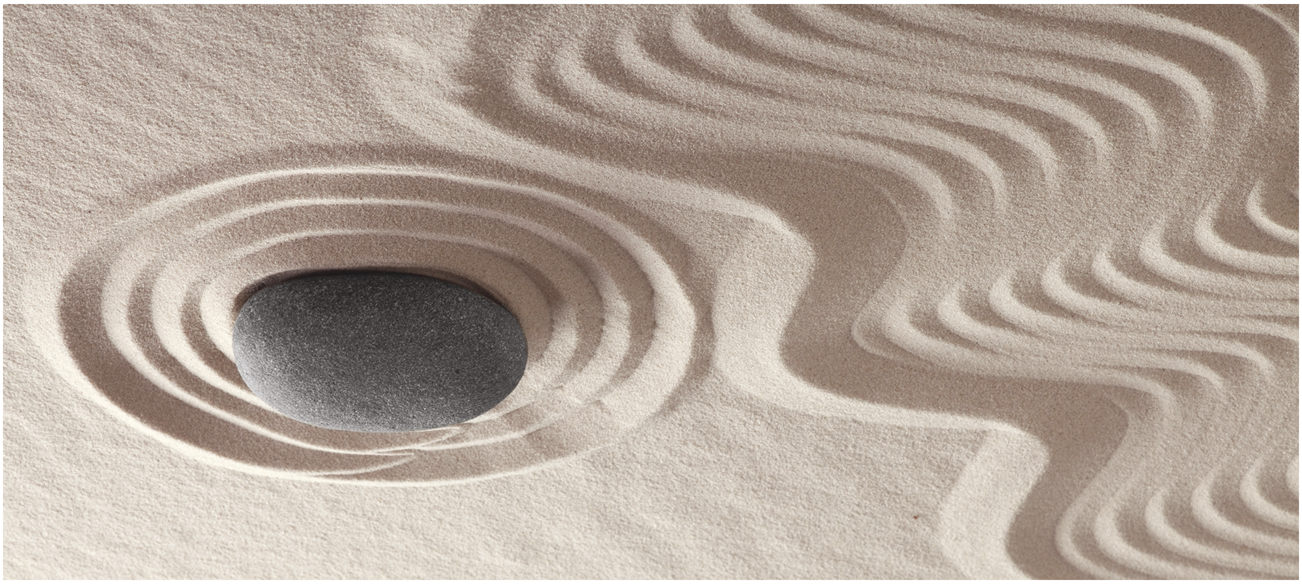

# Being Mindful in the Mathematical Moment

Consider dimensions of equity to enact more responsive mathematics teaching in the elementary classroom.

Jonathan N. Thomas, Cindy Jong, and Molly H. Fisher

**Teachers' actions matter greatly** in the mathematics classroom, but it is not always clear what guides our actions or from where they come. As we strive to create rich and engaging learning environments, how we capitalize on a particular moment is often unclear. Sometimes just identifying potential pathways we might take can be challenging. In this article, we will explore a certain kind of *mindfulness of the mathematical moment* that can help us examine our own assumptions, interpretations, and thought processes prior to making a particular instructional decision. While mindfulness is a word that has been used in many different contexts (e.g., mindful meditation), we use the term to describe ways we may intentionally direct our attention, interpretation, and decision-making in mathematical moments. We will focus on how to better notice the mathematical activity within our classrooms, including our own teaching, so that we may create broader and more equitable experiences for students. In doing so, we may consider the moments of teaching and learning in different ways, which, in turn, opens more equitable spaces for instructional decisions and student learning. Part of identifying a productive decision is an awareness of the possible pathways, including those that create more just and equitable spaces for students, and that is what we aim to explore in this article. However, before we dive into the how and why of mindfulness, let's look at a mathematical moment from a second-grade classroom. In this instance, a small group of students are engaged in a number talk around the task, $10 + 10 = \_\_ + 5$.

***Teacher:*** So what about ten plus ten equals blank plus five? [Writes $10 + 10 = \_\_\_ + 5$ on the board] What do you think the answer would be? Ethan?
***Ethan:*** Twenty-five.
***Teacher:*** Why twenty-five?
***Ethan:*** Because if you add ten plus ten and then add the five, it's twenty-five.
***Teacher:*** [Records 25 on the board] OK, other thoughts? Mateo?
***Mateo:*** It has to be four, because if you add four five times, then that gives you twenty.
***Teacher:*** Interesting, four, huh? [Records 4 on the board]. How about you, Jaden?
***Jaden:*** I think it's fifteen. See, if you count like fifteen, sixteen, seventeen, eighteen, nineteen, twenty [raises five fingers sequentially], then it has to be fifteen.
***Teacher:*** So you counted up, Jaden? [Jaden nods to confirm his strategy and teacher records 15 on the board]. OK, let's represent this last strategy in another way.

Examining this moment from a mathematical standpoint, we might attend to Ethan's response and interpret that while his addition processes are correct, he seems to hold an operational (versus relational) understanding of equality that commonly manifests as adding all the numbers in a given task, regardless of the positioning of the equal sign (Carpenter et al., 2003; Leavy et al., 2013). Attending to Mateo's response, we might interpret some accurate arithmetic (and possibly multiplicative) reasoning hinged on, perhaps, some oversight of the operational sign at hand (i.e., multiplication versus addition). Lastly, attending to Jaden's response, we see a more relational understanding of equality and the task at hand, as well as an arithmetic approach of counting-on from a numerical composite (Steffe, 1992). Each response presents an opportunity to explore an important mathematical idea related to the task at hand. For example, the teacher might decide to put Ethan's thinking to the test using a balance scale or represent Jaden's strategy on the board using an empty-number-line to illustrate the possibility of "jumping by five" instead of counting by ones. However, what if we adjusted our point of view a bit and considered this moment through some different lenses related to equity? What possible decisions could we explore by attending to and interpreting this moment from a different perspective?

## PROFESSIONAL NOTICING AND DIMENSIONS OF EQUITY

In the example above, we applied some mindfulness to a particular mathematical moment in our noticing of key detail, what those details mean, and where we might go next with our teaching. We engaged in a responsive practice that mathematics education researchers refer to as professional noticing of children's mathematical thinking (Jacobs et al., 2010). This involves navigating three interrelated processes: *attending* to children's actions and words, *interpreting* the meaning of those actions and words, and *deciding* upon a productive course of action based on our interpretation. Often, this practice is focused on specific mathematical strategies and the children's thinking that

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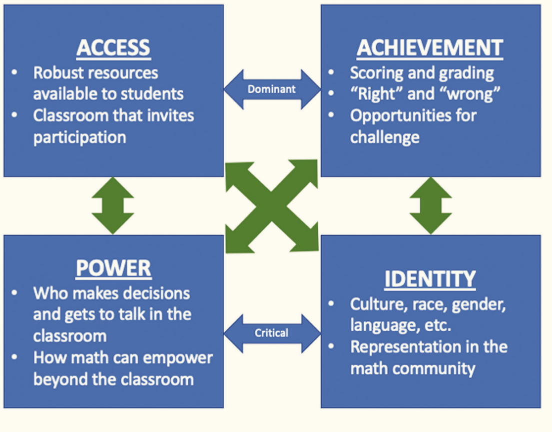
doi:10.5951/MTLT.2022.0224

underlies those strategies (Sherin et al., 2011); however, the framework of noticing is adaptable for many different purposes, including considerations of equity in the mathematics classroom. On this front, Gutiérrez (2009) provides useful lenses—or dimensions—to structure one's professional noticing (see Figure 1).

Examining these dimensions in turn, Gutiérrez (2009) writes, "*Access* relates to the resources that students have available to them to participate in mathematics, including such things as: quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom environment that invites participation" (p. 5). *Achievement* is described as course-taking patterns, standardized test scores, and students' progress through the mathematical pipeline. Diving deeper into this dimension, achievement reflects some fundamental value of mathematical rightness and wrongness with respect to one's thinking and activity. The extent to which one's thinking or activity is deemed right or wrong in a mathematical moment influences one's broader achievement. *Identity* relates to the extent to which an individual feels valued within, represented, or connected to a particular learning community or discipline: "drawing on students' cultural backgrounds and encouraging learners to view themselves as mathematicians capable of having an impact on broader society" are examples of how identity might connect to a mathematical moment (Bucheister et al., 2019, p. 226). *Power* deals with the way control and authority are distributed across a community of learners within a classroom space (including the teacher). Questions about who gets to talk and when, and who decides which tasks and strategies are worthy of attention, are examples of how power intersects with a mathematical moment. Viewed from a more structural perspective, *power* may frame school structures (e.g., tracking) and policies that harm marginalized students (e.g., district funding). Further, Gutiérrez (2009) describes access and achievement as existing along a "dominant axis where access is a precursor to achievement" while identity and power exist along a critical axis, where "identity can be seen as a precursor to power" (p. 6). In this sense, the dominant axis relates to how well students can "play the game called mathematics" as it exists in a typical classroom, while the critical axis relates to the acknowledgment of students' frames of reference to "help build critical citizens so that they may change the game" (Gutiérrez, p. 6). Researchers have examined noticing in concert with these dimensions and found that teachers' attention, interpretations, and decisions—within a particular dimension—are shaped by their own biases and perceptions of students with respect to race, culture, and ethnicity (Jackson et al., 2018). As such, we find it worthwhile to commit ourselves to more intentional and mindful noticing of our own mathematics teaching with respect to equitable experiences for students.

*Figure 1* Dimensions of Equity




*Note.* Adapted from Gutiérrez, 2009.

Revisiting the discussion on the open number sentence ($10 + 10 = \_\_ + 5$) with these dimensions of equity as lenses, we can dive deeper by raising questions about the instructional decisions. Would the task have been more accessible if students were using or seeing a number balance? Did the teacher stop after a student shared the correct answer because they were focused on helping students understand the concept of equality? Why did the teacher not call on any girls to share their answers and strategies? How is the teacher affirming or elevating student voices in the learning process? Bucheister et al. (2019) provide a description of how these equity dimensions might influence instructional planning. We will build upon these ideas to discuss how teachers may mindfully consider these dimensions as they engage in their professional noticing of individual mathematical moments and connect those moments to broader patterns, structures, and policies that shape mathematics teaching and learning.

## PUTTING IDEAS INTO PRACTICE

Considering how one might synthesize professional noticing and dimensions of equity within a mathematical moment, let's return to our hypothetical classroom above. Jaden, Caleb, Aliyah, and Mateo are working together as a table group. Their teacher has tasked them with solving 400-198, and the teacher is asking how they got started on the problem.

***Teacher:*** So before we talk answers, let's talk strategies. There's a bunch of ways we could get started, but what way made most sense to you?
***Jaden:*** Write it on paper. [points to 400-198 written in standard algorithm on his white board]
***Caleb:*** [looking at Jaden's white board] Yeah, you stack it up.
***Teacher:*** Did everyone start out that way?
[***Aliyah:*** shakes her head.]
***Teacher:*** No Aliyah? How did you get started?
***Aliyah:*** Just go backward.
***Teacher:*** Backward? What do you mean?
***Aliyah:*** Like, four hundred, and go backward a hundred, and then another hundred . . . or almost a hundred.
***Caleb:*** But that's not going to work all the time. I mean, maybe it works, but this way is right. [points to Jaden's white board]
***Teacher:*** Let's hold off on right ways to do things for now. Let's just talk about how we got started. Mateo, what about you?
***Mateo:*** I don't get Aliyah's way. I think the right way to get started is like that. [motions to Caleb]
***Teacher:*** Is that how you got started? I thought I saw you doing something with blocks in the middle. [motions to pile of base-10 blocks in center of table and the smaller pile in front of Mateo]
***Mateo:*** Yeah.
***Teacher:*** So tell me about that.
***Mateo:*** I mean, it's basically the same way. I just wanted to make it with these. [touches the blocks]
***Teacher:*** Yeah, how is what you made the same?
***Mateo:*** Like, I got four of these [motions to 100's 'flats'] and then imagined taking off a bunch of these strips. Like, if you could break up these flat pieces into strips, but they don't come apart. But you need to take off some strips.
***Teacher:*** How many strips?
***Mateo:*** Nineteen, but it's not done. You still have to do the ones place.
***Aliyah:*** But that's more like my way.
***Caleb & Jaden:*** [somewhat in unison] No it's not . . .
***Mateo:*** I mean, I guess it's kinda both? I don't know.

Using professional noticing, we can consider different aspects of this moment through each dimension of equity.

## ACCESS

Beginning with access, we attend to the availability of physical resources (i.e., base-10 blocks) and the extent to which these resources supported some students' mathematical thinking. Further, the well-positioned task along with the small group setting allowed for each student to participate in the activity and engage in mathematical reasoning. Similarly, we might interpret Mateo's remarks about not being able to take the base-10 blocks apart as signaling a desire for groupable models that are more adaptable (e.g., unifix cubes). Working toward a decision, we could introduce a new model or open representational space for Mateo to enact his strategy (see Figure 2). In one possible response, the teacher suggests bundles and sticks, which is a common tool for engaging in place-value activities (Wright et al., 2015). Typically involving popsicle/craft-sticks and rubber bands, this tool allows for bundling (and unbundling) of groups of 10, 100, and even 1,000.

## ACHIEVEMENT

Returning to the idea that achievement is grounded in the presence or absence of value judgments when considered in the context of a specific mathematical moment, attending to this exchange reveals very little (if any) signaling from the teacher regarding correct answers or preferred strategies. Indeed, we see some tacit and explicit signals from Caleb, Jaden, and Mateo regarding their notion of a correct way to approach this task (i.e., "I think the right way to get started is like that"). Having interpreted students' remarks that signal correctness, one possible decision could be to preempt this discussion to better balance the value of

***Figure 2*** Access-Oriented Decision

**CREATING ACCESS VIA RESOURCE INTRODUCTION**
**TEACHER:** "*So, Mateo, if you had something different, it might help you think on this one a bit. What if we used the bundles and sticks from that tub over there. Would that get you further with this one because you can break those apart?*

**CREATING ACCESS VIA REPRESENTATION**
**TEACHER:** "*I get that you want to take these blocks apart, Mateo. Could you grab your white board there and draw me a picture of what you are thinking in your mind?*

competing strategies (see Figure 3). More critically, the teacher could challenge notions of correctness altogether. In this instance, the teacher—perhaps, after Mateo's remark about the "right way to get started"—confronts the notion of correctness with respect to a particular strategy and refocuses attention on the nature of correctness with respect to mathematical viability. This language has the effect of mitigating prevailing notions of correctness (within the achievement dimension) and creating more freedom for mathematical reasoning.

## IDENTITY

Diving into the critical axis, there are myriad decisions we could make that would connect to the dimension of identity. In Figure 4, we present two possible pathways that a teacher might take in this moment.

In the first decision, the teacher explicitly lifts students' thinking as worthwhile and interesting. Further, the teacher positions the students' themselves as teachers of mathematics which, in turn, fosters their identity as mathematically capable individuals. The second example tasks students with connecting this mathematical activity to some aspect of their daily lives or lived experience. This sort of decision links students' lives beyond the classroom and their activity as mathematical thinkers. In both cases, the decisions focus on using mathematical reasoning as a catalyst for some deeper connection to self.

## POWER

Continuing along the critical axis, power provides considerable terrain for productive instructional decision-making. In Figure 5, we present some potential decisions that align with this dimension.

In these examples, students are empowered to take control of some aspect of the problem-solving space, whether it be the examination of a critiqued strategy or the creation of a new and more applicable task to showcase one's strategy. At its core, within a mathematical moment, power is about the apportionment of control and authority. Being mindful of power dynamics in the moment allows for the more deliberate apportionment of agency and authority among students and teacher—that teachers may be aware of how much power they wield in a mathematical moment and identifying opportune times to cede such power to students by extending them space to share their own ideas. It is important to note, in this dimension, that power differentials across students are also worthy of consideration. In addition to thinking of how we as teachers may cede power (at times) to our students, we are also managers of power distribution across our students as they interact. We may ask that students who have shared much of their thinking hold back on subsequent tasks and invite those who have not yet shared to enter the reasoning space. Such deliberate shifting of

***Figure 3*** Achievement-Oriented Decision

**REFRAMING ACHIEVEMENT BY PRE-EMPTING DISCUSSION OF CORRECTNESS**
**TEACHER:** "*Mateo and Caleb, I noticed you all talked about a right way of doing this problem. Let's hold back on calling a particular approach right or wrong and just focus on how the thinking works*"

**REFRAMING ACHIEVEMENT BY CHALLENGING NOTIONS OF CORRECTNESS**
**TEACHER:** "*Mateo and Caleb, you both mentioned a right way of doing this problem. If other folks at the table have strategies that are mathematically sound, why wouldn't those by right, too? Why are only some ways right?*

***Figure 4*** Identity-Oriented Decisions

**BUILDING IDENTITY AROUND MATHEMATICAL CAPABILITY**
**TEACHER:** "*So, Caleb and Jaden, you all think Aliyah's way isn't the best way to get started. Mateo, you did something with the blocks. Each of you had an interesting way to get started, though, and let's make some time for each of your ideas. Caleb, pair up with Aliyah and really listen closely to her strategy. Jaden, pair up with Mateo. I want you all to think on these things. How are your ways similar and how are they different? I want you all to teach one another because there is some really interesting and powerful thinking going on here with all of you.*"

**CONNECTING MATHEMATICS TO IDENTITY VIA LIVED EXPERIENCE**
**TEACHER:** "*I think we have some interesting ideas out here and, in a minute, I want to hear more about your process. I am wondering though, if you were to find this problem out in the world, would you still approach it the same way. Take a minute and think about where you might face a problem like 400-198 in your life and then tell me where that might be.*"

***Figure 5*** Power-Oriented Decisions

**CEDING POWER TO STUDENTS FOR MATHEMATICAL EXAMINATION**
**TEACHER:** "*We definitely want to talk about these different strategies, but sounds like we have some disagreement around Aliyah's thinking – that it might not work all the time. Mateo and Caleb, Aliyah and Jaden, pair up for 1 minute and see if you all can find a problem where Aliyah's method doesn't work.*

**CEDING POWER TO STUDENTS FOR STRATEGY ELABORATION**
**TEACHER:** "*So, you all have shared how you got started, but I really want to know more about the nuts and bolts of your thinking. How about we quickly go around the table and each of you walk me through your process. If you aren't sharing, I want you to think about ways that what's being shared is similar to your own thinking, and how it's different.*"

**CEDING POWER TO STUDENTS FOR TASK CREATION**
**TEACHER:** "*I think you all have gotten some interesting thinking out on the table, and I definitely want to know more about your strategies. Take a minute and come up with a good problem other than 400-198 that you can walk us through that will really highlight how your strategy works. At the end, I want to know why the problem you thought up was good for your strategy.*"

power among students is also an important aspect of power consideration in the mathematics classroom.

## TIPS FOR ENHANCING MORE EQUITABLE PRACTICE

It is important to note that these ideas are quite complex, and there is considerable overlap and interplay among the dimensions of equity. For example, lack of resource availability (access) may impact one's view of themselves as mathematically capable (identity). Perhaps the first step in acclimating to these ideas is to give ourselves space to experiment and explore a bit. Toward that end, below are a few suggestions to guide such exploration.

### A New Set of Goggles

One of the most powerful aspects of professional noticing is that it provides teachers with the lens and language to dive deeply into the individual moments of mathematics teaching and learning. For example, attending to Mateo's lament regarding the inability to separate base-10 blocks, we might interpret a propensity for counting-by-ones. Thus, one productive deciding avenue might focus on introducing Mateo to models and representations that involve working with composite quantities and exploring non-count-by-ones strategies (Steffe, 1992; Thomas & Tabor, 2012). Throughout each of the exchanges in this article and within our own teaching, there are many opportunities to glean information about a child's mathematical thinking from a subtle finger movement, or a gesture, or a remark to another student. While teachers may not be able to analyze every moment or occurrence, for those instances we find interesting or perplexing, we might ask ourselves:

(1) What is noteworthy about this moment? (*Attend*)
(2) What do I think it means for the students' mathematical experience or learning? (*Interpret*)
(3) Based on my interpretation, what is a productive pathway forward? (*Decide*)

Being intentional about thinking through these questions as we teach allows for an enhanced mindfulness of our own perceptions and how we respond to what we are perceiving in the classroom. It allows us to look deeply into a seemingly small moment and shape a more responsive experience.

Combining professional noticing and the dimensions of equity (access, achievement, identity, power) expands the terrain for how we might attend to and

***Figure 6*** Reflection Questions on Equitable Teaching Within and Across Dimensions

**QUESTIONS FOR REFLECTION**

| ACCESS | ACHIEVEMENT |
| --- | --- |
| • In what ways do my students have access to high quality mathematics instruction? *Ex: How would you rate the mathematical tasks and availability of resources in the problem scenarios*<br>• How do school funding structures impact the resources available to students in particular schools? | • How do standardized tests influence instructional decisions and mathematics experiences for students? *Ex: How well does the students' mathematical activity relate to the assessments they will likely face?*<br>• In what ways do inflexible grouping practices and tracking place labels on students that limit their mathematical growth? |
| **POWER** | **IDENTITY** |
| • How can I invite more student voices and encourage more ownership of the mathematics? *Ex: How is power apportioned amongst the students and teachers in the problem scenario and does this apportionment lend itself to ownership of mathematics?*<br>• In what ways can students explore how mathematics influences their community to further empower them? | • How am I using students' cultural and linguistic backgrounds as assets to build upon and highlight in mathematics? *Ex: To what extent does the activity in the problem scenario create space for students to form an identity of themselves as capable mathematical thinkers?*<br>• What can I do to broaden participation in mathematics and STEM opportunities for students and communities that are marginalized? |

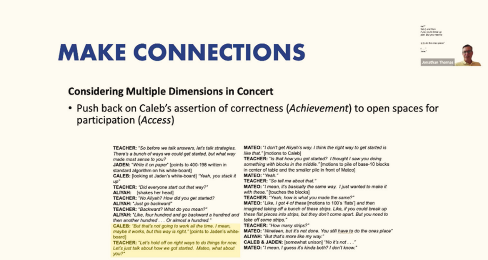
  

interpret a moment as well as the productive decisions available to us. One way to capitalize on this combination of noticing and equity is to think of these dimensions as goggles. At the onset, we might foreground a particular set of goggles, say *achievement* for example, and channel our professional noticing through that lens. This might lead us to detect moments where we, as teachers, signal and ascribe (perhaps unwittingly) correctness and incorrectness to students' thinking and activity. Certainly, there are times when ascribing value to a particular strategy or approach is appropriate; however, using professional noticing for equity allows us to be more mindful and intentional about when and where to assign correctness. For example, in the exchange around the task 400-198, we may attend to Mateo's remarks on the standard algorithm (i.e., "the right way to do it") and interpret those remarks as a held belief regarding some fundamental correctness of that traditional algorithm. From there, one productive decision—as seen in the exchange above—might be to push back on this belief to maintain space for other students to reason productively and achieve mathematically with their own strategies. Indeed, using professional noticing on classroom activities—including our own—through the goggles of equity dimensions creates new opportunities and terrain to consider a mathematical moment and how we might respond productively and intentionally. Moreover, we may also focus our noticing skills on broader structures and dynamics within our classroom and beyond. Such decisions regarding which googles to use may be driven by observances of structural inequities, personal growth goals, or mathematical intent. Whatever the motivation, being mindful of why we do what we do allows for a more thoughtful and reflective mathematical environment. Figure 6 provides some reflection questions aimed at noticing equity within a bigger picture.

## CONCLUSION

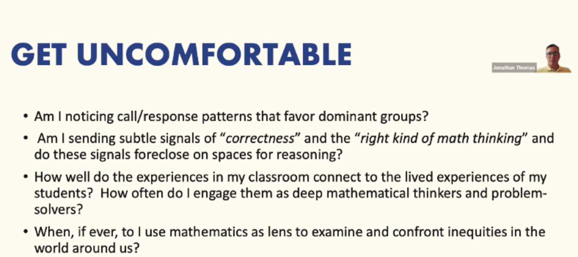
We reiterate the potential of professional noticing cojoined with key dimensions of equity. Striving to attend to a moment, interpret that moment, and formulate a productive decision allows for a mindful and intentional approach to mathematics teaching and learning. Connecting this mindfulness and intentionality to specific aspects of equity propels us into deeper and more critical instructional space. It allows us to consider not only a students' reasoning, but also the intersection of such reasoning on the broader mathematical experience of the student with respect to access, achievement, identity, and power. In Videos 1 and 2, we examine ways to make such connections and perhaps get a bit uncomfortable in our teaching practice regarding our mindful noticing. Engaging in this critical practice may shine a light on areas for growth, and it will also likely illuminate things that may have otherwise gone unnoticed and unexamined. It is this mindfulness and intentionality with our teaching that allows us to intertwine mathematics and equity.

***Video 1*** Make Connections




Watch the full video online

***Video 2*** Get Uncomfortable

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