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PREFACE FOR THE FIRST SPECIAL ISSUE IN HONOR OF BOB PEGO

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This is the first of two issues of the Quarterly of Applied Mathematics dedicated to Bob Pego on the occasion of his retirement from the Department of Mathematics at Carnegie Mellon University. The papers presented here were edited in collaboration with Gautam Iyer, Jian-Guo Liu, and Dejan Slepčev. I express my gratitude to them for their assistance in bringing this collection to completion.

No one who has discussed science with Bob can fail to be impressed by his broad curiosity and love of learning. He has an unerring ability to ask just the right question and to tug patiently at just the right strands, unraveling one mystery after another. He has written more than a hundred papers, mainly in mathematical analysis and modeling. All his work reflects a careful balance between a model with a sound scientific basis and the use of precise and often unexpected tools of analysis to obtain sharp results. This sense of taste seems to have emerged almost fully formed in his seminal Ph.D. thesis in 1982, supervised by Andy Majda, on stable viscosity matrices for systems of conservation laws on the line [20]. This work, which continues to be of interest to the conservation laws community, initiated a lifelong interest in stability problems for traveling waves.

Bob's interests expanded in the years immediately following his thesis to include phase transitions in nonlinear viscoelasticity [28] and the stability of traveling waves, broadly construed. The best-known works during this period are perhaps his articles with Jack Carr on metastability in phase transitions [4, 6] and his papers with Michael Weinstein on solitary waves [33, 34]. These works remain the foundation for rigorous analysis of two distinct classes of fundamental phenomena in nonlinear waves. They also represent beautiful forays into infinite-dimensional dynamical systems. Critical ideas in these papers are the use of invariant manifold techniques (which have a different character for dissipative and Hamiltonian flows), subtle analyses of spectral problems, and a particular eye for unexpected phenomena such as the ultra slow motion of fronts.

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His work on solitary waves broadened in the 1990s and early 2000s to include several other models for nonlinear waves, such as the Korteweg-de Vries (KdV) [32], Kadomtsev-Petviashvili (KP) [12], and Benney-Luke equations [31], spectral stability in the Boussinesq system [35], and a series of results on traveling waves in lattices, especially his comprehensive work with Gero Friesecke on the Fermi-Pasta-Ulam lattices [8–11]. Other results on traveling waves include several papers on reaction-diffusion systems and shearing motions in non-Newtonian flows [26, 27].

His work with Carr [4–6], and the related influence of John Ball and Oliver Penrose, initiated a continuing interest in the kinetics of phase transitions in materials science [1]. Bob's versatility with techniques and his interests in the dynamics of phase transitions is represented in his celebrated 1989 paper on front migration in the Cahn-Hilliard equation [29]. This paper opened the door to many studies on the Mullins-Sekerka instability in crystal growth. These papers reflect both a growing interest in materials science, as well as a refined understanding of the subtleties of gradient flows in infinite dimensions. The methods in his work include both formal asymptotic analysis and rigorous analysis based on the infinite-dimensional Riemannian geometry of fundamental models such as the Allen-Cahn and Cahn-Hilliard equations. By balancing these aspects, Bob obtained sharp results on domain coarsening for several important models in epitaxial growth and the kinetics of phase transitions [30]. They also left him with an inexhaustible supply of gradient flows that stubbornly refuse to converge!

By the end of the 1990s, Bob's interests in materials science moved towards kinetic theories and their relation to PDE models. Along with Barbara Niethammer, he began a careful analysis of the relation between the Becker-Döring model and the Lifshitz-Slyozov-Wagner (LSW) theory of Ostwald ripening [25]. A striking insight from this work was that the subtle phenomena of "fat tails" and non-self-similar behavior could be understood precisely in terms of Karamata's theory of regular variation [24]. Subtle Tauberian theorems now suddenly made their presence felt in several kinetic theories. I was fortunate to catch this wave; Bob and I worked out a classification theory for self-similar behavior in the solvable cases of Smoluchowski's coagulation equations [21, 22]. This work led to unexpected links with shock clustering in Burgers turbulence [23] and, years later, a similar classification in the theory of continuous state branching processes (with Gautam Iyer and Nicolas Leger) [13, 14].

In the past decade, this body of work has expanded into kinetic models of animal size distributions [7, 18] (with Pierre Degond, Jian-Guo Liu, and Barbara Niethammer). The study of the Bose–Einstein condensate has also been a continuing theme of interest in Bob's work. His work in the area began with the spectral stability of vortices (with Kollár [15]). More recently, Bob began studying the long-time dynamics of the photon energy spectrum in high temperature plasmas. The governing equation (introduced by Kompaneets in 1957) allows for an "outflux" of photons at zero energy analogous to Bose–Einstein condensation. Using a mixture of parabolic and hyperbolic techniques, Bob and coauthors completely describe the long-time dynamics of the photon energy spectrum and identify a new mechanism that leads to the zero-energy photon outflux [2, 3, 16]. Bob's continuing interest in fluids has also expanded to include least action principles and optimal transport [19] (with Jian-Guo Liu and Dejan Slepčev).

In addition to these selected papers, which reflect our biased understanding of his work, Bob's contributions include several papers on numerical methods. One particularly noteworthy method was a stable and efficient discretization of the Navier–Stokes equations (with Jian-Guo Liu and Jie Liu [17]), using a novel commutator estimate between the Laplacian and Leray projections. His work also includes several small notes with sharp observations on surprising mathematical and physical phenomena and many other papers that expand on the themes above. These influences are reflected in the papers collected here.

(1) Alama, Bronsard, Lu, and Wang study a geometric scaling limit of a free energy functional for diblock copolymers.
(2) Caballero and Wayne study metastable breathers for discrete nonlinear Schrödinger equations with damping and driving (thus combining two of Bob's favorite phenomena!).
(3) Degond, Frouvelle, Merino-Aceituno, and Trescases study hydrodynamic descriptions of swarming motivated by studies of collective animal behavior.
(4) Eichenberg, Niethammer, and Velazquez present the analysis of a singular LSW model.
(5) Han, Slepčev, and Yang study a Riemannian geometry on the space of signals, termed the HV geometry.
(6) Iyer, Lu, and Nolen study mixing times of a Bernoulli map on the torus.
(7) Jabin and Zhou study subtle numerical oscillations for apparently simple advection phenomena on non-Cartesian grids.
(8) Liu and Wang provide another description of the sharp constant problem for Gagliardo-Nirenberg-Sobolev inequalities.
(9) Mizumachi analyzes the linear stability of elastic 2-line solitons for the KP-II equation.
(10) Murray and Wilcox study vortex sheets in the Birkhoff-Rott model focusing on singularity formation.
(11) Quintero studies the stability of standing waves for the Benney-Roskes system.

These papers reflect the scientific impact of a career that has spanned several areas of applied analysis. They reflect the esteem in which we hold Bob's work, as well as his mathematical depth, modesty, and extraordinary capacity for detail.

I have often been asked if I was Bob's student. Bob was not my thesis advisor, but we met when I was a fresh postdoc in clear need of mentoring. Felix Otto had suggested a problem on domain coarsening in fluids that had me completely stuck. While casting around for simpler models, I stumbled on Bob's work with Jack Carr [5]. Serendipitously, a talk by Peter March led me to Smoluchowski's coagulation equations, which had intriguing similarities with [5]. Shortly thereafter, on a visit to the University of Maryland, I arrived at Bob's office with a vague idea and an equation in hand. Things didn't seem to go very well during that meeting, but a few days later, when I was back in Wisconsin, I received a detailed email with a surprising calculation. Soon I was back in Maryland and the ideas began to flow. The first paper led into another and then another, and before I knew it, I had built my career on our joint work. Never before or after has a collaboration enriched my life as deeply. Things came easily, like magic, as we shared insights and looked over one another's shoulders at calculations. Bob taught me the craft—how to write a paper, how to balance rigor and meaning, how to follow a suggestive calculation to its essence, how to hold on without letting go until a problem is well and truly solved. He is, in every sense, my teacher.

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