QUARTERLY OF APPLIED MATHEMATICS
VOLUME LXXXI, NUMBER 3
SEPTEMBER 2023, PAGES 429–432
https://doi.org/10.1090/qam/1671
Article electronically published on May 15, 2023

PREFACE FOR THE SECOND SPECIAL ISSUE IN HONOR OF C. M. DAFERMOS

By

GOVIND MENON

Division of Applied Mathematics, Brown University, 182 George Street, Providence, RI 02912

This is the second issue in a collection of papers dedicated to Constantine Dafermos in honor of his 80th birthday. The first issue was prefaced by an outline of his work written jointly with Marshall Slemrod and Athanasios Tzavaras, my coeditors for this collection. This preface places these papers in a broader context.

Dafermos's oeuvre builds, as he so eloquently put it in his magisterial treatise [1], on the rich stream of nourishment carried by the umbilical cord that connects the laws of continuum physics with mathematical analysis. This nourishment takes many forms and unites many intellectual strands. On one hand, his work reflects the influence of the great masters of the 19th century — Boltzmann, Clausius, Gibbs, and Maxwell — on the character of physical law. It also reflects the classical beauty of the mathematical tradition of Cauchy, Euler, and Lagrange, uniting geometry, material properties, and Newton's laws to create continuum mechanics.

His work is also a direct outgrowth of the rich tradition of mathematical analysis and natural philosophy that blossomed after the Second World War. During this period, several problems arising in the rapidly expanding needs of science and technology were given firm mathematical foundations. These include a systematization of continuum physics led by Truesdell; Hopf, Leray, and Serrin's work on the Navier-Stokes equations; mathematical formulations of material behavior once thought exotic, such as Ericksen's work on liquid crystals and Rivlin and Treloar's work on rubber; and an appreciation of the subtleties of irreversible material behavior, as for example in Prager's work on plasticity and Reiner's development of rheology. Several developments in continuum dynamics that were closely tied to military needs began to live a mathematical life of their own during this period under the influence of von Neumann. Of special note was the revitalization of the study of shock waves that began with Riemann. This discipline is rich enough to encompass particular investigations of great ingenuity, as well as the broad principles outlined above. It is territory perfectly suited to Dafermos.

Received March 24, 2023, and, in revised form, April 7, 2023.
2020 *Mathematics Subject Classification.* Primary 35L65, 76-02.
This work was supported by the National Science Foundation (DMS 2107205).
Email address: govind_menon@brown.edu

A persistent strand within this tapestry of ideas is the fundamental importance of thermodynamics. No mathematical scientist in the past fifty years has thought as carefully as Dafermos about the role of the second law within the context of partial differential equations. The role of the Clausius-Duhem inequality on thermoelasticity had been studied by others, especially Coleman and Noll, Ericksen, and Gurtin; however, their results were restricted to smooth processes, a far too narrow class to include shock waves, the phenomena of greatest interest. It was only Dafermos who understood how to bridge the mathematical theory of shock waves, especially the work of Di Perna, Glimm, and T. P. Liu, with the classical framework of continuum physics. In a fertile period in the 1970s, he developed this idea in several lecture notes and papers providing the foundational framework for the analysis of entropy conditions in the theory of conservation laws.

In addition to his own work, thanks to his broad understanding of science, as well as his personal warmth and generosity, Dafermos helped catalyze several important new ideas that transformed PDE theory. He helped revitalize the interaction between the calculus of variations and nonlinear elasticity through his early support of Ball's work. He also immediately understood the importance of the weak convergence methods being pioneered by Di Perna, Murat, and Tartar. These influences are reflected in several articles in this volume.

(1) Ancona, Bianchini, Bressan, Colombo, and Nguyen study regularity of solutions to balance laws, a subtle problem even for a scalar conservation law on the line with a source, and conjecture extensions to 2×2 hyperbolic systems in accordance with Di Perna's work.
(2) Cao and Chen provide new insights into minimal entropy conditions for uniqueness of solutions to conservation laws, a classic problem studied by several leading figures, including Kruzkov.
(3) Christoforou presents a surprising extension of the relative entropy method to the isometric embedding problem of negatively curved surfaces in $\mathbb{R}^3$.
(4) The subtle effect of distinct regularization mechanisms is studied in the papers by Bolbot, Mitsotakis, and Tazavaras (dissipation and dispersion) and Liu (dissipation and relaxation).
(5) Serre presents a new application of his compensated integrability theory to gas dynamics.
(6) Vasseur reviews Dafermos's relative entropy method and discusses the extension of the method to include wild solutions constructed by convex integration.

There is another umbilical cord that matters here. The deep friendship between Constantine Dafermos and Walter Strauss set a tone for a harmonious coexistence between the Division of Applied Mathematics and the Department of Mathematics at Brown University. Rigorous analysis in continuum mechanics, kinetic theory, and partial differential equations expanded at a rapid pace thanks to their efforts to nurture a generation of students who felt few scientific boundaries. This broader influence is also reflected in several papers. It gives me particular pleasure to recognize, within the list of authors below, some who are in the third generation of this tradition.

(1) Abbatiello and Feireisl study the well-posedness of the Oberbeck-Boussinesq system with nonlocal boundary conditions.
(2) Acharya and Slemrod study dislocation dynamics in crystalline solids, a central problem in plasticity.
(3) Buckmaster, Cao-Labora, and Gómez-Serrano present a construction of imploding self-similar solutions to the isentropic, compressible Euler and Navier-Stokes equations.
(4) The papers by Freistühler, Hadžić, and Strauss concern astrophysical flows. Freistühler studies the well-posedness of relativistic fluid dynamics from the perspective of hyperbolic conservation laws; Hadžić studies the self-similar collapse and expansion of stars in both the Newtonian and relativistic regime; Strauss reviews the existence theory for rotating stars.
(5) Guo and Iyer apply a new compactness technique to the study of Prandtl layers with forcing.
(6) Haziot and Pausader provide a new proof of an interesting inequality that arises in the study of the Muskat problem.
(7) Lu and Tadmor present the study of hydrodynamic limits for multiagent models of flocking phenomena. This paper is a testament to the modern vitality of kinetic theory to capture 21st-century applications in social media, just as much as the laws of nature studied by Boltzmann and Maxwell.
(8) Wang and Ye study the well-posedness of the incompressible, inhomogeneous Navier-Stokes equations with a generalized dissipative mechanism.

I express my gratitude to the authors of these works, as well as several others who expressed interest in contributing, but whose articles were not received in time.

Continuum mechanics and thermodynamics represent some of the most successful scientific theories and a strand of persistent importance in applied mathematics. Each generation inherits a tradition that must be nurtured, and challenged, to ensure its continued vitality. I look forward to a future where studies of the second law in the context of shock waves provide a template for an understanding of the propagation of information, uniting two distinct notions of entropy.

Scholarship of such depth provides bonds of personal, as well as intellectual, kinship. Today these must include not just an understanding of continental intellectual traditions, but also the vast expansion of science made possible by the development of national universities in the second- and third-world, and the generosity of American universities, in the post-war period. Continuum mechanics was also a pathway to mathematics for many students who began in engineering. It is a path that both Costas and I understand with all the intimacy of lived experience. Like Costas, I too began as an engineer and gravitated towards mathematics when my eyes were opened to the beauty of continuum theories in Fung's delightful introduction. Like Costas, my path led to the Division of Applied Mathematics at Brown University through the (now defunct) Department of Theoretical and Applied Mechanics at Cornell.

It has been my great good fortune to know Costas in several avatars, as a student, colleague, and mentor. This collection also marks the moment of transition between us as Managing Editors of the Quarterly of Applied Mathematics. It brings me particular pride to begin with a celebration of Dafermos's work and, by extension, that of his teachers Ericksen and Truesdell, and the great scholars who have preceded us. As Maxwell reflects in his famous paper [2], few domains of knowledge capture the spirit of the eternal questions quite as well as thermodynamics. Perhaps good ideas are never really created or destroyed, just transformed from generation to generation. Tradition is not the transmission of ashes, but the passage of Promethean fire.

References

[1] C. M. Dafermos, *Hyperbolic conservation laws in continuum physics*, 4th ed., Grundlehren der mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 325, Springer-Verlag, Berlin, 2016, DOI 10.1007/978-3-662-49451-6. MR3468916

[2] J. C. Maxwell, *Molecules*, Nature **4** (1873), 437–441.