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Nodal Operating Envelopes vs. Network-wide Constraints: What is better for network-safe coordination of DERs?

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ABSTRACT

For network-safe coordination of distributed energy resources (DERs), many approaches impose some form of constraint set to guarantee the safe operation of a distribution network. This paper presents a comparative analysis of two distinct approaches that leverage a constraint set for network-safe coordination: nodal operating envelopes versus network-wide constraints on the action of a DER aggregator. We investigate their respective strengths and limitations by considering information and communication requirements and trade-offs in terms of flexibility and fairness. The availability of potentially private information to the aggregator or to the distribution system operator determines which of the approaches is feasible. The results of the case studies suggest that if the goal is to maximize DER flexibility, a nodal constraint approach should be used. However, if the aggregator does not have information on how the constraints map to its DERs and/or does not have the ability to send out node-specific control inputs, the nodal constraint approach is not feasible and a network-wide constraint approach is needed. We show that a network-wide constraint approach constraining the aggregator's control input provides a good balance between flexibility and fairness.

1. Introduction

As distributed energy resources (DERs), like roof-top solar, energy storage, and thermostatically controlled loads (TCLs) become more prevalent, interest in actively managing DERs within the distribution network to provide grid services and participate in electricity markets increases. Active participation of DERs can improve reliability, decrease operating costs, and lower the environmental impacts of power system operations [1]. Compared to traditional transmission-level generation, most individual DERs are too small to provide significant benefit to the system. But coordinated together they can have a much bigger impact. However, if not properly managed, DERs and DER aggregations can lead to unsafe distribution network operation, for example, over- and under-voltages, over-current violations, and transformer overheating [2]. This has led to calls for increased coordination, for example, between transmission system operators (TSOs) and distribution system operators (DSOs) [3,4].

DER aggregations can be operated by either the DSO or by a third-party aggregator. In the case that the DSO is coordinating DERs to provide grid services, it could explicitly manage network constraints through a centralized algorithm, e.g., the framework proposed in [5,6]. However, in competitive electricity markets, DSOs usually cannot participate in wholesale electricity markets (and utility companies that have both distribution and generation businesses must follow strict rules ensuring key information that could affect market competitiveness, e.g., the DSO's network models, cannot flow from one business to the other). Therefore, in the United States, Australia, and parts of Europe it is likely that third-party aggregators will coordinate DERs to provide market-based services [7]. For security and privacy reasons, third-party aggregators cannot have access to detailed distribution network information needed to assess the network impacts of their control strategies. This means that some level of communication between the aggregator and DSO (and/or between the DSO and market operator) is required to ensure network safety. The structure that this communication should take is still an open question [8,9]. Distribution network and operational data could be leveraged by the aggregator to improve the performance of its DER coordination strategy. However, the DSO is reluctant to share, or in many cases prohibited from sharing, its network details. This implies a trade-off between the performance of grid services through network-safe DER coordination and the DSO's need for privacy. Therefore, it is crucial to understand the data requirements of different approaches for ensuring network-safe DER coordination.

In this work, we consider two approaches for network-safe DER coordination: calculating net power injection limits at every node in the network or leveraging a network-wide constraint for an aggregator's

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control action. Net power injection limits, which are specified for each node in the network and updated as system conditions change, have gained attention in recent years, particularly in Australia where they are known as dynamic operating envelopes, or just operating envelopes [10,11]. In other places, this is called dynamic hosting capacity [12]. These limits can be obtained by solving a modified version of the optimal power flow (OPF) problem, where the objective is to maximize a function of the net power injections at every node without violating any network constraints [10–13]. The form of the objective, and whether or not equity/fairness is considered can significantly impact the bounds at each node [11,14]. In [10], a linearized, three-phase optimal power flow (OPF) problem incorporating tap-changing transformers is used to calculate power export and import limits at every node, and then the effectiveness of those limits are assessed using a probabilistic approach. A convex inner approximation of the feasible operating region of the nodal power injections is presented in [12], where an approximation of power flow's nonlinearities is used to ensure feasibility for the worst-case current. A market-based mechanism to allocate portions of operating envelopes to each aggregator is proposed in [15]. These approaches require the DSO, who has necessary network information, to calculate the nodal limits and share them with individual resources or aggregators, such that the limits can be incorporated into control strategies.

Alternatively, the DSO can impose network-level constraints on either the behavior of the resource aggregation (i.e., aggregate power) or the action taken by the aggregator to ensure safe network operation. Ref. [16] introduces a method to quantify the network-safe aggregate power flexibility at a substation in an unbalanced distribution network. A methodology for calculating network-safe aggregate power flexibility that takes into account temporal coupling constraints is proposed in [17]. Ref. [18,19] introduced a control algorithm for aggregate TCLs that limit the number of TCLs turned ON or OFF, thereby constraining their collective power consumption and [20] proposed a method to compute network-safe bounds on the norm of power deviation across all nodes of the network. The approach in [21,22] employs Monte Carlo simulation and the bisection method to compute a constraint set on the aggregator's broadcast control input to DERs. Under any input in this constraint set, a chance constraint on network safety is guaranteed.

Given the existing literature on nodal and network-wide constraints, it is unclear whether one method or the other is generally better or if certain scenarios are more suited for each method. A deeper understanding of the strengths, weaknesses, and trade-offs of these two approaches is needed. Motivated by this, our paper conducts a comprehensive comparison of the performance of nodal versus network-wide constraints for the safe operation of DERs in distribution networks.

We note that there are a variety of alternative approaches to DSO-aggregator coordination as well. For example, [23] proposes an iterative approach based on distribution locational marginal prices, wherein aggregator power schedules are updated until they converge. Ref. [24] proposes a market-based approach for a DSO to procure flexibility from aggregators. Both of these approaches assume the existence of DSO markets, which are unlikely to emerge in the United States and therefore not considered in this paper. Another type of non-market based approach is DSO-centric as opposed to aggregator-centric [25] in that, instead of the DSO sending constraints, the aggregator sends its desired actions/constraints and the DSO must determine if they are feasible. For example, [26] proposed a blocking strategy used by the DSO to block aggregator control inputs that would cause network constraint violations. Ref. [27] proposes an iterative approach wherein the aggregator shares its desired operating envelope with the DSO, who determines whether network violations would occur. Then, the DSO calculates a penalty for operating envelopes that would violate network constraints and the aggregator updates its operating envelopes to avoid the penalty. While these approaches are valid and interesting, we limit our analysis in this work to aggregator-centric approaches.

The contributions of our paper are as follows. We provide a qualitative discussion of nodal versus network-wide constraints for maintaining safe operations in active distribution networks. We then provide an analytical analysis using case studies to compare the two approaches. Specifically, we begin by discussing the differences between these approaches in terms of the information required by each entity and further assumptions made in each approach. We then detail each approach mathematically. Subsequently, we present numerical simulations to evaluate the level of flexibility provided to the network by the resources under each approach. For the case study, we utilize methods based on [21,28], for nodal and network-wide constraints, respectively.

The structure of this paper is as follows. Section 2 provides a qualitative discussion of the two approaches. In Sections 3 and 4, we will briefly describe the details of the methods used to determine the nodal and network-wide constraints used in the case studies. In Section 5, we will present three case studies to highlight how the two approaches compare in practice. The network details will be given in 5.1. The results are presented in 5.2. Lastly, Section 6 will present our conclusions.

2. Discussion of approaches

In this section, we provide a general discussion on nodal constraints and network-wide constraints for maintaining safe operations within active distribution networks. We then describe the specific frameworks for DER coordination, the nodal constraint approach, and the network-wide constraint approach that are used to draw a more detailed comparison in the remainder of this paper. We discuss the differences in the type of constraints computed, communication requirements, and assumptions for each of the approaches.

2.1. General discussion

Generally, safe operation in power systems means that bus voltages and line currents are within some bounds defined by equipment limitations. Network safety could be ensured by directly constraining bus voltages and line currents or by indirectly constraining these quantities by constraining net power injections. Based on the information available to the DSO and aggregator, power injections can be managed either at a nodal or network-wide level. Bounding power injections at a nodal level means defining individual limits at every node in the network in such a way that if every node is operating within its limits, then the voltage at every bus and the current on every line in the network should be within safe limits. This approach can be used when DERs at every node are acting independently or DERs across the network are being coordinated.

Alternatively, constraints on network-wide power injections, or by proxy an aggregator's control input, could be used for network-safe DER coordination. This type of constraint is appropriate when the aggregator cannot directly control nodal power injections, i.e., the aggregator does not know which DERs are connected to which nodes. When the DSO computes this type of constraint, it must consider the worst-case permitted by the network-wide constraint, e.g., assuming the aggregator leverages the full capacity of the constraint using only the DERs connected at the farthest node from the substation. This makes the network-wide constraint approach more conservative than the nodal power constraint in general.

However, additional information can be used to reduce the conservativism of the network-wide approach, and equity considerations can make the nodal approach more conservative [24,29]. It is unclear whether the nodal approach would always outperform the network-wide approach, or what trade-offs may exist. In order to shed some light on this, we consider the network-wide approach proposed in [21], which relies on an understanding of how an aggregator's control policy will directly impact DER behavior, and we consider a nodal approach

Table 1
Comparing the assumptions for nodal vs. Network-wide constraint approaches.

	Nodal constraints on power	Network-wide constraints on control input
DSO has detailed network info like topology, line impedances, and DER locations	Yes	Yes
DSO has accurate uncontrollable load forecasts	Yes[a]	Yes[a]
DSO knows DER capacities	Yes[a]	Yes
DSO knows how control input affects DER power consumption	No	Yes
Aggregator knows which DERs correspond to each constraint	Yes	No
All DERs in the network are operated by one aggregator	No	Yes[a]
All DERs at a single node are operated by the aggregator	Yes[a]	Yes[a]
The relationship between voltage and power consumption of the DERs is monotonic	Yes	Yes

[a] Assumption can be relaxed but could result in more conservative operational constraints.

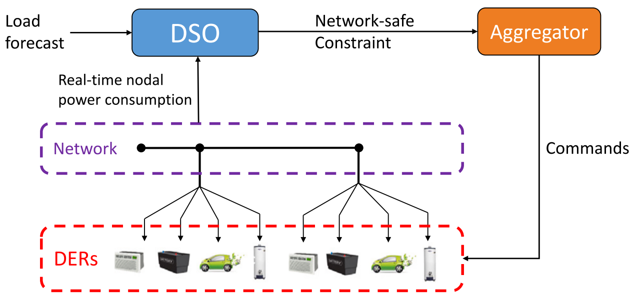

Fig. 1. Communications between the DSO, aggregator, network, and DERs.

with and without equity considerations [28,30]. While the nodal constraint approach assumes that the aggregator utilizes DERs' private information (e.g., each DER's node in the network), the network-wide approach assumes that the DSO leverages the aggregator's private information (e.g., aggregator's DER control policy). Thus, comparing these two approaches is expected to reveal a trade-off between DER flexibility and the privacy of each entity.

2.2. Considered framework

We consider a framework in which there is a single aggregator coordinating all of the DERs in a distribution network. The DSO and the aggregator communicate with each other for network-safe DER coordination, as illustrated in Fig. 1. Here, the DSO sends some form of network-safety constraints to the aggregator and then the aggregator coordinates the DERs to provide grid-services while satisfying those constraints. This architecture is consistent with aggregator-centric network-safe DER coordination as defined in [25].

We assume that the DSO has detailed information needed for power flow calculations, such as the network topology, line parameters, and DER locations. Using this information, the DSO computes network-safe constraints; the type of the constraints depends on the approach used. Then, the DSO sends the obtained network-safe constraints to the aggregator. While the aggregator coordinates the participating DERs, it must satisfy the received constraints.

2.3. Discussion of specific approaches

Under the nodal constraint approach, $2|\mathcal{N}|$ constraints are constructed and updated as the network state changes, where $\mathcal{N}$ is the set of nodes in the network and $|\cdot|$ denotes cardinality. At each node i, the DSO computes the DER power consumption lower bound $\underline{p}_i$ and upper bound $\overline{p}_i$. Then, the nodal constraints ensuring network safety are given by

$$\underline{p}_i \le \sum_{k\in\mathcal{I}_i} p_k^{\mathrm{D}} \le \overline{p}_i, \quad \forall i \in \mathcal{N}. \tag{1}$$

where p_k^{D} is the power consumption of DER k, and $\mathcal{I}_i$ is the set of the indices of the DERs at node i. This approach assumes that the aggregator knows $\mathcal{I}_i$ for every node i and the corresponding $(\underline{p}_i, \overline{p}_i)$ in order to satisfy (1). Assuming the DSO knows $\mathcal{I}_i$ for every node i, it may be unwilling or unable to provide the aggregator with that information due to privacy concerns. Without that information, the aggregator cannot directly control the nodal DER power consumption $\sum_{k\in\mathcal{I}_i} p_k^{\mathrm{D}}$, leaving the nodal power bounds $(\underline{p}_i, \overline{p}_i)$ useless to the aggregator.

The network-wide constraint approach considers a scenario in which the aggregator cannot directly control the nodal DER power consumption. It is assumed that the aggregator sends a scalar input, denoted by u, to the DERs throughout the network. Upon receipt of the input u, the DERs adjust their power consumption in response to u. We assume that the power consumption of the DERs is monotonic with respect to u, enabling the aggregator to steer the direction of changes to the aggregate power consumption. Such a framework is considered in [21] wherein the input u signals the probability that TCLs should switch ON or OFF. As a result, the number of TCLs turned ON or OFF is likely proportional to the aggregator's input. The DSO leverages its knowledge of the impact of u on DERs' power consumption to compute network-safe input bounds $(\underline{u}, \overline{u})$ and sends them to the aggregator. After receiving these bounds, the aggregator must choose an input u such that $\underline{u} \le u \le \overline{u}$ holds, ensuring safe operation of the network.

The remaining assumptions made by the two approaches used in the case studies are summarized in Table 1. One of the common assumptions is that the DSO has accurate forecasts for uncontrollable loads, i.e., the loads in the network that are not controlled by the aggregator. We note that this condition can be relaxed to account for uncertainty as long as the DSO can quantify the load uncertainty. Under the nodal constraint approach, it is not strictly necessary that the DSO knows the DER capacities. However, if the DSO has this information it allows them to more effectively define the constraints at each node. The assumption that only a single aggregator is controlling every DER in the network or every DER at a node could also be relaxed. However, relaxing it would require the DSO to assign bounds to each aggregator either at a network-wide or nodal level. The impacts of this require investigation in future work. One common assumption between the approaches is that the relationship between voltage and power consumption is monotonic. Specifically, the nodal constraint method assumes that if $\underline{p}_i$ and $\overline{p}_i$ define the safe power bounds according to the modified OPF, any p_i such that $\underline{p}_i \le p_i \le \overline{p}_i$ will result in safe voltages. The network-wide constraint method assumes that as power consumption increases at every node, network voltages will decrease. We note that this may not always hold in practice due to the nonlinearities in the power flow equations.

3. Constructing nodal constraints

As noted in Section 1, nodal constraints on power consumption can be obtained by solving a modified version of the OPF problem. In this section, we will provide a brief overview of the optimization problem, based on the formulation in [28], that we used to determine the nodal constraints, or operating envelopes, in the case studies. There are two similar optimization problems, one to find the upper bound and one to find the lower bound. Both problems must be re-solved at each time step to determine the appropriate power bounds as the system loading changes.

Our modified OPF problem uses the branch flow formulation [31], where voltages and current angles are omitted by writing the power flow and voltage difference equations in terms of the squared magnitudes. We denote the set of lines in the network by $\mathcal{L}$. Let $z_{ij} = r_{ij} + jx_{ij}$ represent the impedance on the line connecting nodes i and j. The uncontrollable loads' active and reactive power consumption at node i are p_i^{l} and q_i^{l}, respectively. The per unit squared voltage limits at each bus are $\underline{v}, \overline{v}$.

First, we will present the formulation for finding the upper bounds on nodal power consumption. The decision variables are the upper bound on DER power consumption $\overline{p}_i$ at each node i, p_{ij} representing the active power flowing from node i to node j; q_{ij} representing the reactive power flowing from node i to node j; the squared voltage magnitude $v_i = |V_i|^2$ at each node i; and $l_{ij} = |I_{ij}|^2$ representing the squared current magnitude on the line connecting node i and node j. The constraints of the problem are

$$\sum_{k\in\mathcal{I}_i} p_k^{\mathrm{Dmin}} \le \overline{p}_i \le \sum_{k\in\mathcal{I}_i} p_k^{\mathrm{Dmax}}, \quad \forall i \in \mathcal{N} \tag{2a}$$

$$\sum_{i:i\to j} (p_{ij} - r_{ij}l_{ij}) - \overline{p}_j - p_j^{\mathrm{l}} = \sum_{k:j\to k} p_{jk}, \forall j \in \mathcal{N} \tag{2b}$$

$$\sum_{i:i\to j} (q_{ij} - x_{ij}l_{ij}) - q_j^{\mathrm{l}} = \sum_{k:j\to k} q_{jk}, \forall j \in \mathcal{N} \tag{2c}$$

$$v_i = v_j + 2(r_{ij}p_{ij} + x_{ij}q_{ij}) - (r_{ij}^2 + x_{ij}^2)l_{ij}, \forall i \in \mathcal{N} \tag{2d}$$

$$p_{ij}^2 + q_{ij}^2 = l_{ij}v_i, \quad \forall ij \in \mathcal{L} \tag{2e}$$

$$\underline{v} \le v_i \le \overline{v}, \quad \forall i \in \mathcal{N} \tag{2f}$$

Constraint (2a) enforces that the upper limit on total power consumption at each node is not lower than the minimum total power consumption of the DERs at that node and not greater than the maximum total power consumption of the DERs at that node, where p_k^{Dmin} and p_k^{Dmax} are the minimum and maximum power consumption of DER k, respectively. Constraints (2b) and (2c) enforce active and reactive power balance, where notation $i : i \to j$ specifies that we should sum over all lines ij injecting power into j, and $k : j \to k$ specifies that we should sum over all lines jk consuming power from j. Constraint (2d) defines the voltage drop between bus i and the downstream bus j. Lastly, (2f) enforces the voltage limits at each bus.

This approach only ensures network safety when the nodal power is precisely $\overline{p}_i$, not necessarily for all nodal powers below this bound. However, the assumption of a monotonic relationship between the nodal powers and the network voltages lends that any $\sum_{k\in\mathcal{I}_i} p_k^{\mathrm{D}} \le \overline{p}_i$ is safe. When using a linear approximation of the power flow equations, as is commonly done when constructing operating envelopes [10,30], this monotonic relationship can be proven. In general, monotonicity does not hold for the nonlinear power flow equations, but it often holds empirically.

As noted in Section 1, the form of the objective function depends on the goals of the DSO. For example, the DSO may want to maximize the total allowable power consumption by DERs across the network, which could lead to significant discrepancies in limit sizes depending on the location of each node relative to the substation. This is because the power consumption of customers farther down the feeder will generally have a larger impact on voltages due to the radial structure of distribution networks and the nonlinearities of power flow [30]. Considering fairness would lead to different nodal constraints. To provide a more thorough comparison between nodal constraints and network-wide constraints, we will solve the modified OPF problem using two different objective functions.

The first objective function used maximizes the sum of the real power limit across every node, given by

$$\max \sum_{i\in\mathcal{N}} \overline{p}_i. \tag{3}$$

For the remainder of this paper, the nodal constraint approach using this objective function will be referred to as the MaxSum operating envelope approach or *MaxSum OE approach*. This will lead to the greatest total network flexibility, but will favor nodes closer to the substation. To obtain more evenly distributed limits, the second objective maximizes the smallest nodal limit of the network. This "fair" objective is written as

$$\max \ \gamma \tag{4}$$

and requires the additional constraint

$$\gamma \le \overline{p}_i, \quad \forall i \in \mathcal{N}, \tag{5}$$

where (5) transforms a $\max\min$ objective into a linear objective by defining the smallest operating envelope to be maximized. For the remainder of this paper, the nodal constraint approach using this objective function will be referred to as the Fair operating envelope approach or *Fair OE approach*.

The formulation for finding the lower bounds on nodal power consumption is the same except that the objective is minimized and $\overline{p}_i$ is replaced by $\underline{p}_i$.

4. Constructing network-wide constraints

In this section, we provide an overview of a coordination framework between an aggregator and DSO utilizing a network-wide constraint, developed in [21]. We also explain how the approach can be compared to the nodal constraint approach.

These earlier studies [21] consider a stochastic setting, wherein the uncontrollable loads are uncertain and the aggregator's input u signals the probability of TCL mode switching. They impose the chance constraint to manage under-voltages

$$\Pr\left(\min_{i\in\mathcal{N}} v_i(u) \ge \underline{v}\right) \ge 1 - \epsilon, \tag{6}$$

where $v_i(u)$ represents the voltage at node i under the aggregator's input u, and $1 - \epsilon$ denotes the desired probability of network safety. A similar constraint could be used to manage over-voltages. Ref. [21] shows that (6) holds at a specified confidence level provided that the empirical safe probability, derived from a sufficiently large number of uncertainty realizations, is adequately high.

It is assumed that the DSO has knowledge of how the input u influences the power consumption of the DERs, and hence can conduct simulations of the nodal voltages within the network under any input u. Specifically, the DSO computes the nodal voltages for a large number of realizations of the uncertainties, evaluates if the number of realizations and the resulting empirical safe probability are sufficiently large, and then verifies whether or not (6) is satisfied for an input u at the desired confidence level. Employing the bisection method [32], the DSO iteratively updates u and computes the nodal voltages to determine the maximum upper bound $\overline{u}$ such that (6) is satisfied for any input at or below $\overline{u}$. Subsequently, the DSO sends the upper bound $\overline{u}$ to the aggregator, who must select an input $u \le \overline{u}$.

For comparison with the nodal constraint approach in Section 3, we adapt the network-wide constraint approach to suit a deterministic scenario. In this scenario, both the uncontrollable load p_j^{l} and the power consumption of the DERs under an input u are deterministic. Thus, the voltage at each node $v_i(u)$ is a deterministic function of the input u and the DSO can compute the voltages by solving the power flow equations.

The chance constraint (6) is replaced by the deterministic constraint

$$\underline{v} \le v_i(u) \le \overline{v} \qquad \forall i \in \mathcal{N}. \tag{7}$$

The aggregator's input u, which must be in the range $[0,1]$, adjusts the power consumption of DER k according to

$$p_k^{\mathrm{D}}(u) = p_k^{\mathrm{Dmin}} + u\left(p_k^{\mathrm{Dmax}} - p_k^{\mathrm{Dmin}}\right), \tag{8}$$

which is a linear and increasing function of the aggregator's input u. Then, DER power consumption at node i under the input u is

$$p_i(u) = \sum_{k\in\mathcal{I}_i} p_k^{\mathrm{D}}(u) = \sum_{k\in\mathcal{I}_i} p_k^{\mathrm{Dmin}} + u \sum_{k\in\mathcal{I}_i}\left(p_k^{\mathrm{Dmax}} - p_k^{\mathrm{Dmin}}\right). \tag{9}$$

Thus, DER power consumption at each node $p_i(u)$ is linear and monotonic with respect to the input u.

While the approach proposed in [21] leveraged a large number of uncertainty realizations in the stochastic setting, we leverage an adaptation wherein we solve an optimization problem to obtain $\overline{u}$ that satisfies (7) for the deterministic setting. Since the DSO knows that the power consumption of the DERs at node i changes according to (9), to find $\overline{u}$, the DSO solves the following optimization problem,

$$\max_{\overline{u}} \ \overline{u} \tag{10a}$$

$$\text{s.t. (2b)–(2f) with } \overline{p}_j \text{ replaced by } p_j(\overline{u}). \tag{10b}$$

Here, instead of solving this problem using an optimization solver, we use the bisection method as in [21].

While this approach provides less degrees of freedom in DER control, it allows the DSO to understand the type of control the aggregator uses and also the correlation between the DERs' nodal power consumptions. Consequently, this knowledge could lead to less conservative operating envelopes at certain nodes when compared to the Fair OE approach, where no correlation is known/assumed. This consequence is illustrated in the case studies in Section 5.

For the remainder of this paper, the network-wide constraint approach outlined in this section will be referred to as the Input Constraint approach or the *Input Const approach*.

5. Case studies

5.1. Setup

The network used in the case studies presented below is the 56-bus balanced distribution network introduced in [33]. It is a modified version of the IEEE 123-bus network [34]. The network is illustrated in Fig. 2. For simplicity, we assume there are no capacitor banks in the network and no voltage regulators except for one at the substation. The voltage at the substation is set to 1.02 pu and the bounds on the voltages are $\underline{v} = 0.95$ pu and $\overline{v} = 1.05$ pu. Any operation leading to values below or above these limits is considered unsafe. Lastly, we assume that all DERs in the network are controllable TCLs, i.e., they are only capable of consuming power. We also assume that every node has at least one TCL connected to it and that all TCLs are coordinated by a single aggregator.

We consider three scenarios to illustrate the differences in operational outcomes when using nodal versus network-wide constraints for maintaining safe operations in active distribution networks. In Scenario 1, there is an even distribution of DERs across the nodes participating in the network and the uncontrollable load at every node is the same. Specifically, we assume that the same number of DERs, each with the same p_k^{Dmax}, are at each node in the network. While this scenario is unlikely to occur in the real world, its simplicity will provide a clear picture of how each of the methods works in principle. In the second, more realistic scenario, the number of DERs at each node, the power capacity of each DER, and the uncontrollable loads at each node are varied. Scenario 3 is identical to Scenario 2 except that all DERs have been removed from nodes 20–32, the nodes farthest away from the substation. This scenario is used to highlight how the relative performance of the approaches changes with changes in the DER distribution.

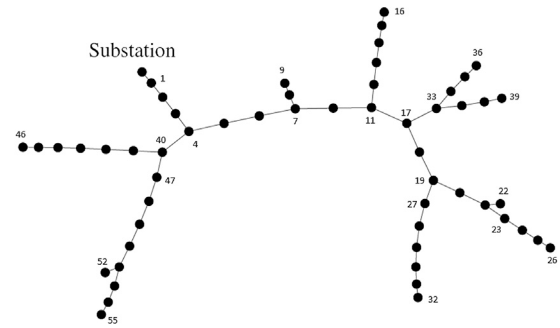

Fig. 2. Single-line diagram of 56-bus network presented in [33].

For all three scenarios, nodal and network-wide constraints were constructed using the methods outlined in Sections 3 and 4 under various loading conditions. We define the nodal nominal real and reactive load $\tilde{p}_j$ and $\tilde{q}_j$ at each node j, and set the load p_j^{l} and q_j^{l} to be proportional to the nominal power as follows

$$p_j^{\mathrm{l}} = \alpha\tilde{p}_j, \quad q_j^{\mathrm{l}} = \alpha\tilde{q}_j. \tag{11}$$

where α is a coefficient used to adjust the level of the uncontrollable load. In the case studies, we vary the value of α to see how loading conditions impact the bounds constructed by each approach.

5.2. Results

In this section, we present the results of the case studies on the 56-bus network. The figures illustrate the bounds $\overline{p}_i$ for the nodal constraint approaches and $p_i(\overline{u})$ for the network-wide constraint approach using (9). Specifically, we interpret the maximum power consumption $p_i(\overline{u})$ as an effective nodal bound, though the aggregator does not use these bounds.

5.2.1. Scenario 1

First, we present the results of the evenly distributed DERs scenario, as it provides the simplest comparison. Fig. 3 shows that under this network setup, the allowable power consumption by the DERs at each node was the same between the Fair OE approach and the Input Constraint approach. The MaxSum OE approach led to most of the nodes having larger allowable power consumption levels but prevented some nodes farthest from the substation from having any allowable power consumption. In terms of the total allowable aggregate power consumption by DERs in the network, to be referred to as *flexibility* for the remainder of the paper, the Fair OE and the Input Constraint approaches gave the same level of flexibility regardless of the uncontrollable loading conditions. The MaxSum OE approach provided about 20% more flexibility than the other two approaches under each of the tested loading conditions. However, this greater total flexibility came at the cost of no flexibility at certain nodes in the network.

5.2.2. Scenario 2

When the distribution of DERs and loads within the network is varied, a more realistic scenario, the comparison between nodal constraints and network-wide constraints is less cut and dried. Fig. 4 shows the real power limits at each node found by all three approaches under a single loading condition. As in the evenly distributed DER scenario, the MaxSum OE approach led to the largest power consumption limit at most nodes, but left the DERs at nodes 20–32 without the ability to consume power. The Fair OE approach led to the least variation in limit magnitudes out of the three approaches, but as shown in Fig. 5, it led to the lowest total power consumption across the network.

The Input Constraint approach did not achieve the same level of total network flexibility as the MaxSum OE approach (Fig. 5). However, it did result in similar limits at nodes away from the substation

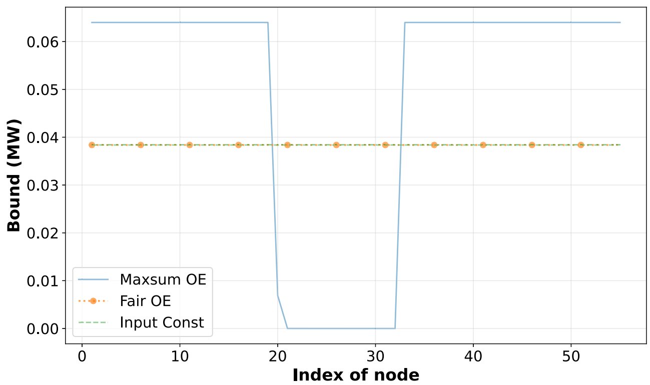

Fig. 3. Comparison of the real power limits based on nodal constraints found using the Maxsum OE approach, the Fair OE approach, and the Input Const approach at each node for Scenario 1.

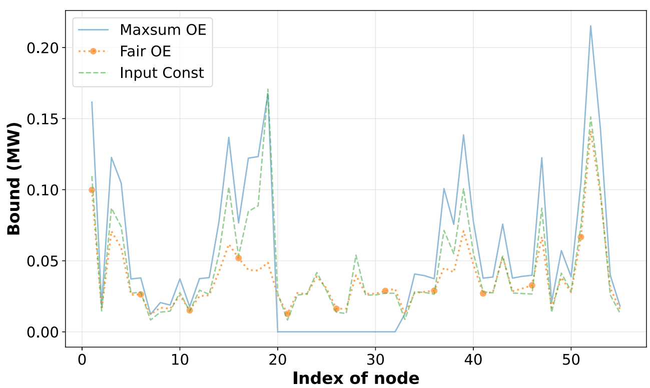

Fig. 4. Comparison of the real power limits found using the Maxsum OE approach, the Fair OE approach, and the Input Const approach at each node.

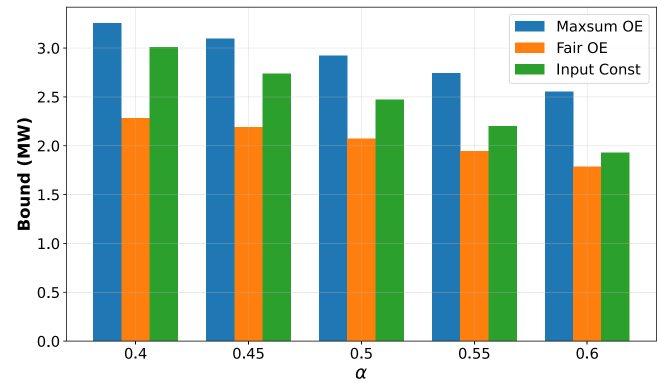

Fig. 5. Comparison of the total network flexibility found using the Maxsum OE approach, the Fair OE approach, and the Input Const approach with respect to network loading increases for Scenario 2.

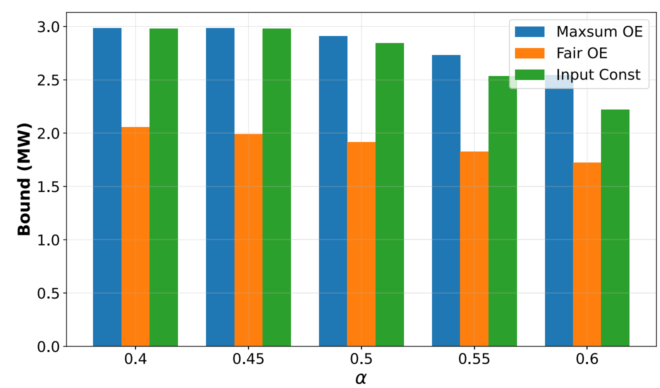

Fig. 6. Comparison of the total network flexibility from nodal constraints (Fair OE and MaxSum OE) and network-wide constraints (Input Const) with respect to network loading increases for Scenario 3.

compared to the Fair OE approach while still enabling larger limits at other nodes. In that sense, the Input Constraint approach was able to achieve a good compromise between total flexibility and fairness. This ability stems from a single characteristic of the approach, which is that the power consumption of each DER is directly proportional to the aggregator's control input. Unlike in the nodal constraint approach, where no correlation is assumed between the nodal power consumptions, this additional knowledge of the behavior of power consumption across the nodes can be leveraged to increase the total flexibility. Fig. 5, which shows the total power consumption by the DERs across the network as the loads increase, suggests that the advantage of having that additional information decreases as the load in the network increases.

5.2.3. Scenario 3

Removing DERs from the nodes farthest from the substation (nodes 20–32), which are typically the most constrained, gives the Input Constraint method an advantage similar to the one that the MaxSum OE approach gains by disregarding fairness. However, as network loading increases, shown in Fig. 6, the MaxSum OE approach is able to maintain a higher level of flexibility by removing flexibility at nodes 18 and 19.

5.2.4. Key takeaways

The above results suggest that if the single goal of the DSO or the aggregator is to maximize the total flexibility of the DERs, the MaxSum OE approach is best. However, the MaxSum OE approach is only feasible if the aggregator has information on which operating envelopes apply to which DERs and if it has the ability to send out node-specific control inputs. If fairness is a concern, the MaxSum OE approach is less appealing. In that case, the Fair OE approach can be used if, again, the aggregator has information on which operating envelopes apply to which DERs and the ability to send out node-specific control inputs. Alternatively, if this information is not available to the aggregator, and if the aggregator provides the DSO with information needed to assess the impacts of the aggregator's control input on the DERs, the Input Constraint approach can provide a good trade-off between fairness and flexibility. This balance arises from knowing that the DERs will respond linearly to the control input. In the nodal constraint approach, no correlation is assumed between the nodal power consumptions.

6. Conclusions

In this paper, we presented a qualitative and quantitative comparison of nodal and network-wide constraints for ensuring the safe operation of active distribution networks. Specifically, we discussed the assumptions and shared information requirements of each approach. Subsequently, we used three case studies to illustrate how the approaches compare under different conditions.

The discussion highlights that a key difference between the two approaches is who shares potentially private information with whom. The nodal constraint approach used in this paper requires that the DSO shares the set of DERs at each node with the aggregator, and the network-wide approach requires the aggregator to share information about its control policy with the DSO. The case studies highlight how considering equity increases the conservativism of the nodal constraint method, and how having additional information on how the DERs' actions are correlated can lead to the network-wide constraint method being less conservative when equity is considered. The key takeaway from this comparison is that each approach is only feasible under specific rules or market structures. One approach cannot simply be swapped for the other. Flexibility objectives should be considered when designing the market structures and regulations, as it is the structures and regulations that determine the possible approaches.

CRediT authorship contribution statement

Hannah Moring: Methodology, Writing – original draft, Writing – review & editing, Conceptualization, Data curation, Formal analysis, Investigation. **Sunho Jang:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Visualization, Writing – original draft, Writing – review & editing. **Necmiye Ozay:** Conceptualization, Funding acquisition, Methodology, Supervision, Writing – review & editing. **Johanna L. Mathieu:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Supervision, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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