

# Tunable $3 \times 3$ Nolen Matrix Network for Power-Saving Phased Array

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**Abstract**—In this article, a tunable Nolen matrix with small phase tuning range phase shifters and simple control is presented. The proposed circuit topology is constructed from  $3 \times 3$  Nolen matrix embedded with three tunable phase shifters with small tuning range of  $0^\circ$ – $120^\circ$ . For each input port excitation in this proposed network, a continuous  $120^\circ$  tunable progressive phase difference is obtained at the output ports, and the full  $360^\circ$  tunable phase progression is achieved by exciting all three input ports. Most importantly, all tuning range relaxed phase shifters are controlled simultaneously by only two-channel dc voltages to achieve full progressive phase tuning range, which can simplify the control method of beam steering. Compared to the conventional feeding matrix, the proposed design achieves a flexible progressive phase, compact size, easy control, and low power consumption. The theoretical analysis is carried out for the proposed circuit topology, and the closed-form equations are derived to minimize the phase shifter's tuning range and reduce the control complexity. To verify the proposed design concept, a prototype tunable Nolen matrix operating at 5.8 GHz is designed and tested, and the experimental results agree well with simulation and theoretical analysis.

**Index Terms**—Beamforming network, control-relaxed, phase shifter-relaxed, reconfigurable Nolen matrix.

## I. INTRODUCTION

WITH the emerging 5G/6G, Wi-Fi 6E, autonomous driving, and the Internet of Things (IoT), where the phased array and beamforming are one of the key techniques to enable high data rate and increase the network capacity and spectrum usage, currently, digital beamforming, local oscillator (LO) phase shifting, and radio frequency (RF) phase shifting are the primary techniques to steer the radiation beam continuously. For RF phase shifting, the progressive phase delay feeding to antenna elements is the key design aspect to control the direction of angle, so that the series feed [1], [2], parallel feed [3], and matrix feed [4], [5] have been investigated to radiate fixed directional beam. The tuning

capabilities for those feeding topologies are investigated in [6], [7], [8], and [9], where the large number of phase shifters with wide tuning range or vector modulators is applied as tuning elements in the networks, which results in high power loss, low efficiency, and complex control mechanism. In the previous works [10], a tunable  $4 \times 4$  Butler matrix is proposed by embedding the tunable phase shifters, where the two  $90^\circ$  tuning range phase shifters and two  $180^\circ$  tunable phase shifters are applied to configure the progressive phase delay. Comparing with Butler matrix, Nolen matrix features more advantages, i.e., flexible beams generation, smaller number of components, low loss, and compact size. In the work [11], [12], the Nolen matrix has been investigated in different scales to generate 3-D beams or symmetrical beams and reduce the size by using lumped elements. However, all of them concentrated solely on fixed radiation beams that were unable to be appropriately adjusted.

In this article, a tunable  $3 \times 3$  Nolen matrix is designed to continuously configure the progressive phase difference, so that the radiation beams can be steered to resolve the issue in fixed beam Nolen matrix. The design features are summarized as follows.

- 1) Smaller number components as three couplers, three tunable phase shifters, and one fixed phase shifter are applied to achieve lower power loss.
- 2) Only  $120^\circ$  tuning range is required for all the tunable phase shifters to enable  $360^\circ$  progressive phase difference among the outputs of the feeding network for full coverage.
- 3) All the tunable phase shifters are tuned concurrently by two-channel control voltage.
- 4) It can be expanded to accommodate a larger array with less power consumption. The substantial novelty in this article is to derive the topology of tunable Nolen matrix with minimum phase tuning range for tunable phase shifters and minimum number of tunable phase shifters in Nolen matrix network, and the design equations are derived to find the control relationships among tunable phase shifters and progressive phase differences of the Nolen matrix. To verify the design concept, a proposed prototype is designed at 5.8 GHz. The simulations and measurements align well to prove the proposed design theory.

## II. DESIGN THEORY OF $3 \times 3$ TUNABLE NOLEN MATRIX

As depicted in Fig. 1(a), the proposed tunable Nolen matrix feeding network has three input ports 1–3 and three output ports 4–6. It is composed of three couplers with two coupling ratios, three tunable phase shifters, and one fixed phase shifter,

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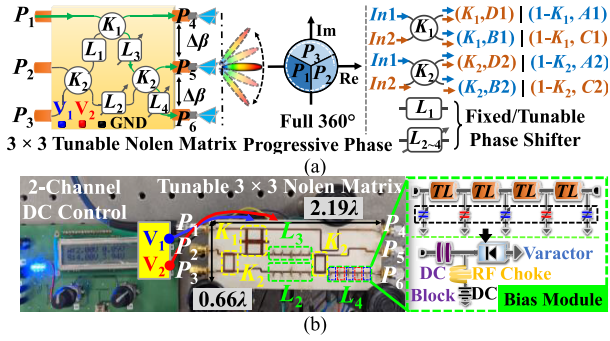


Fig. 1. Proposed  $3 \times 3$  tunable Nolen matrix feeding network with two channel dc control. (a) Schematic. (b) Fabricated circuit.

and they are formed in a three-row-three-column lattice layout. At the top of the lattice layout, one coupler has coupling ratio  $K_1$ , while the other two have the coupling ratio  $K_2$ . Three tunable phase shifters denoted as  $L_2$ – $L_4$  are placed around the coupler with coupling ratio  $K_2$  at the right bottom. A fixed delay line marked as  $L_1$  connects two couplers with ratios  $K_1$  and  $K_2$  in the left. Specifically, when port In1 is excited, the coupling port has a magnitude with  $K_1$ , and the through port has the rest of the magnitude  $1 - K_1$ . Meanwhile, the phase shifts are  $A1$  and  $B1$  in the through and coupling paths. When port In2 is excited, the same magnitude responses are realized at the coupling and through ports, while the phase shifts  $C1$  and  $D1$  are introduced in two paths, respectively. For other two couplers, the coupling ratio is  $K_2$ , while the magnitudes are  $K_2$  at the coupling port and  $1 - K_2$  at the through port. Its corresponding phase shifts in the two paths under In1 and In2 port excitations are also shown in Fig. 1(a). The design detail of two couplers is presented in our previous work [11]. In this design, the phase differences of couplers under different port excitations follow the rules:  $(B1 - A1) + (D1 - C1) = (B2 - A2) + (D2 - C2) = \pi$ . Considering equal magnitude distributions and three unique progressive phases across the matrix output ports under the input port excitations, the coupling ratios  $K_1$  and  $K_2$  are derived to  $2/3$  and  $1/2$ , respectively. Then, the phase responses of tunable phase shifters and fixed phase shifters are derived as

$$L2 - L4 - L1 = D2 - C1 \mp 90^\circ \quad (1)$$

$$L3 - L4 = \Delta K1 - \Delta K2 + A2 \quad (2)$$

where  $\Delta K_1 = B1 - A1$  and  $\Delta K_2 = B2 - A2$  are the phase difference of the coupler with coupling ratio  $K_1$  and  $K_2$ , respectively. Then, the progressive phase difference of  $3 \times 3$  tunable Nolen matrix by exciting each port can be further derived as

$$\Delta 1 = \Delta K_2 - L_4 \quad (3)$$

$$\Delta 2 = \Delta K_2 - L_4 \pm 120^\circ \quad (4)$$

$$\Delta 3 = \Delta K_2 - L_4 \mp 120^\circ \quad (5)$$

where  $\Delta 1$ ,  $\Delta 2$ , and  $\Delta 3$  present the phase difference among the output ports of  $3 \times 3$  Nolen matrix when the incident wave is applied at input ports  $P_1$ – $P_3$ , respectively. Phase differences ( $\Delta 1$ – $\Delta 3$ ) are determined by  $\Delta K_2$  (the phase difference of  $K_2$  coupler) and tunable phase shifter  $L_4$ . Here, for the coupler with coupling ratio  $K_2$ , the phase difference  $\Delta K_2$  is a predetermined constant value. Thus, tunable phase shifter  $L_4$  enables controllable progressive phase delay in the proposed matrix. From (3)–(5), it is found that the progressive phase difference under each input port excitation features  $120^\circ$  offset

TABLE I  
COMPARISON BETWEEN THE PROPOSED TUNABLE  $3 \times 3$   
NOLN MATRIX AND OTHER FEEDING NETWORK

	[6]	[7]	[9]	[10]	This Work
<b>Feeding Type</b>	Series	Parallel	Butler	Butler	Nolen
$f_0$ (GHz)	2.4	8	2.4	5.8	5.8
<b>Reflection (dB)</b>	$\leq -10$	$\leq -10$	$\leq -11$	$\leq -11$	$\leq -11$
<b>Progressive Phase</b>	$197^\circ$	$160^\circ$	$360^\circ$	$360^\circ$	$360^\circ$
<b>Phase Shifter Range</b>	$197^\circ$	$260^\circ$	$360^\circ$	$180^\circ$	$120^\circ$
<b>Circuit Size (<math>\lambda</math>)</b>	$1.8 \times 0.6$	N.A.	$1.1 \times 0.7$	$2.7 \times 1.7$	$2.19 \times 0.66$
<b>DC Control Number</b>	7	6	6	2	2
<b>Max DC Control</b>	15V	50V	20V	19V	19V

from each other (i.e.,  $\Delta 2 - \Delta 1 = \Delta 3 - \Delta 2 = \Delta 1 - \Delta 3 = \pm 120^\circ$ ), minimizing the tuning range of the tunable phase shifter  $L_4$  to  $120^\circ$  achieving a full-progressive phase-tuning range. For each input port, the progressive phase delay is tuned in a range of  $120^\circ$  by sweeping the tunable phase shifters within  $120^\circ$ ; then, full  $360^\circ$  tuning range is covered by switching three input ports. Meanwhile, from (1) and (2), tunable phase shifters  $L_2$  and  $L_3$  must follow  $L_4$  with the same value or fixed offset and be tuned simultaneously, which further reduces the complexity of control for beam steering.

Based on the analysis above, the couplers in the proposed  $3 \times 3$  tunable Nolen matrix have coupling ratios  $K_1 = 2/3$  and  $K_2 = 1/2$ . Our previous design in [13] is referenced to realize the coupler with  $2/3$  coupling ratio, where two hybrid couplers are connected in parallel with two open-end stubs inserted between them. The conventional quadrature hybrid coupler [14] is applied for realizing the coupler with coupling ratio  $1/2$ . After deciding the couplers, the phase differences of each coupler  $\Delta K_1$  and  $\Delta K_2$  are obtained, and the phase shifters  $L_1$ – $L_4$  are derived by applying (1) and (2). To achieve phase tuning and reduce the complexity of control, a tunable transmission line in our previous design [15] is applied for tunable phase shifter design. It is composed of two identical microstrip lines ( $TL$ ) in series and three short-ended varactors at two ends and the center, as shown in Fig. 1(b). Using ABCD-matrix analysis method, the capacitances of varactors are derived to realize tunable electrical length with impedance matching so as functioning as a tunable phase shifter. Here, MA46H120 varactor is adopted as the tunable capacitor, and the phase tuning is controlled by two bias voltages. Two proposed phase shifters are cascaded to implement the tunable phase shifters ( $L_2$ – $L_4$  in Fig. 1), where the phase is tuned in a range of  $120^\circ$  with low insertion loss variation. The design flow of the proposed tunable  $3 \times 3$  Nolen matrix is summarized: 1) determine the coupling ratios  $K_1$  and  $K_2$  of two couplers based on the matrix magnitude distributions; 2) design the tunable phase shifter  $L_4$  according to the progressive phase ( $\Delta 1$ – $\Delta 3$ ) and output phase differences of the coupler ( $\Delta K_2$ ) in (3)–(5); and 3) design the phase shifters  $L_1$ – $L_3$  based on (1) and (2).

### III. SIMULATION AND MEASUREMENT RESULTS

In the experiment, a 5.8-GHz tunable  $3 \times 3$  Nolen matrix is designed and fabricated on Rogers RT/Duroid 6002 PCB (thickness = 0.508 mm,  $\tan \delta = 0.0012$ , and  $\epsilon_r = 2.94$ ), as shown in Fig. 1(b). The overall size of the circuit is about  $2.19\lambda \times 0.66\lambda$  at 5.8 GHz. Specifically, the control circuit designed in our previous work [10] is used to provide two-channel dc control. The proposed tunable Nolen matrix is designed using Keysight ADS and evaluated

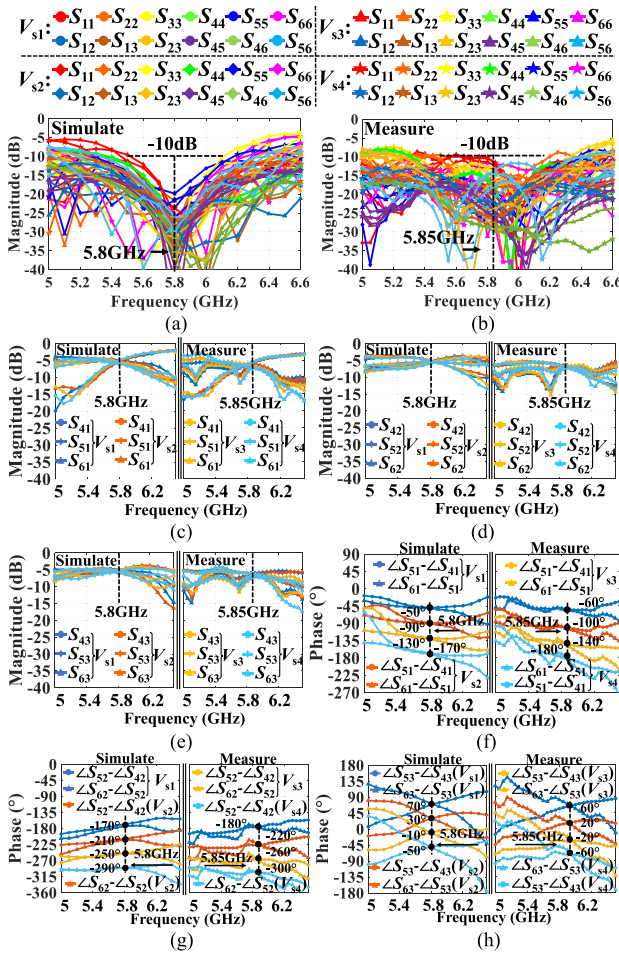


Fig. 2. Experimental results of the proposed tunable  $3 \times 3$  Nolen matrix. (a) and (b) Return losses and isolations. (c)–(e) Insertion losses with exciting  $P_1$ – $P_3$ . (f)–(h) Output phase differences with excitation on  $P_1$ – $P_3$ .

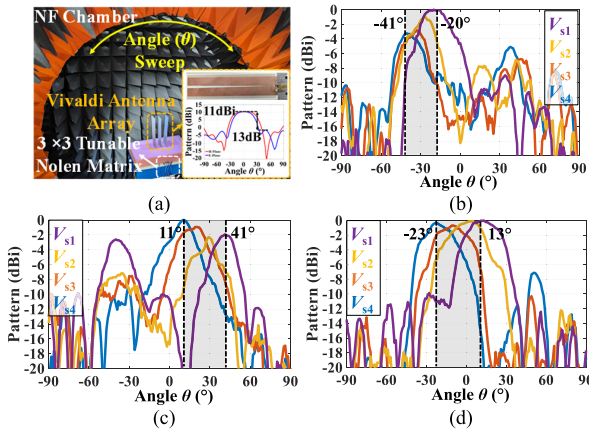


Fig. 3. Normalized radiation patterns of the proposed tunable  $3 \times 3$  Nolen matrix with  $1 \times 3$  Vivaldi antenna array under port excitations. (a) Measurement setup. (b)  $P_1$  excitation. (c)  $P_2$  excitation. (d)  $P_3$  excitation.

by Keysight PNA-X. Both simulated and measured results are shown in Fig. 2, where the tunable phase shifters are tuned within  $120^\circ$  phase delay by two-channel dc control voltages to achieve a full  $360^\circ$  progressive phases with the same transmission across three output ports. Specifically, the simulated and measured reflections and isolations of tunable Nolen matrix are shown in Fig. 2(a) and (b), where the simulated results are greater than 19 dB at 5.8 GHz, and measured results are better than 11 dB at 5.85 GHz

with 10-dB bandwidth of 540 MHz (9.2%). The simulated and measured insertion losses are shown in Fig. 2(c)–(e). For three different input port excitations, the simulated transmissions are between 4.85 and 6 dB, and the tolerance of measured insertion losses are 2.5 dB at 5.85 GHz. The 3-dB bandwidth of all transmission varies from 50 to 570 MHz. The output progressive phase differences under different input port excitations are plotted in Fig. 2(f)–(h). The two channel control voltages [ $V_{s1} = (19, 19 \text{ V})$ ,  $V_{s2} = (7.5, 4 \text{ V})$ ,  $V_{s3} = (4.3, 3.9 \text{ V})$ , and  $V_{s4} = (2.9, 0.9 \text{ V})$ ] in simulations and [ $V_{s1} = (19, 19 \text{ V})$ ,  $V_{s2} = (7.5, 6.5 \text{ V})$ ,  $V_{s3} = (6, 3.5 \text{ V})$ , and  $V_{s4} = (4.5, 3 \text{ V})$ ] in measurement are applied to tune four progressive phased difference at each input port excitation. For  $P_1$  excitation, the simulated phase difference at 5.8 GHz is tuned from  $-50^\circ$  to  $-170^\circ$  within  $120^\circ$  range, and the measured is tuned from  $-60^\circ$  to  $-180^\circ$ . For  $P_2$  excitation, the simulated phase difference is tuned from  $-170^\circ$  to  $-290^\circ$  within  $120^\circ$  range and the measured result is tuned from  $-180^\circ$  to  $-300^\circ$ . For  $P_3$  excitation, the simulated phase difference is tuned from  $70^\circ$  ( $-290^\circ$ ) to  $-50^\circ$ , while the measured result is tuned from  $60^\circ$  ( $-300^\circ$ ) to  $-60^\circ$  that reaches to  $120^\circ$  range. The tolerances of the simulated and measured phase difference are about  $5^\circ$  and  $13^\circ$  at 5.8 GHz, respectively. The measured bandwidth varies between 136 and 898 MHz within  $13^\circ$  tolerance. Overall, the measurement results agree well with the simulation results, and the slight frequency shift and control voltages difference are attribute to fabrication and variations of component values on tunable phase shifters. Also, the variation of insertion loss is mainly caused by the parasitic and diode model applied for tunable phase shifters design. To tackle the losses, a new phase shifter with active loss compensation can be employed to improve the overall system loss [16]. The comparison Table I highlights our proposed tunable  $3 \times 3$  Nolen matrix, which can achieve full  $360^\circ$  progressive phase tuning using small-tuning range phase shifters. The radiation patterns are evaluated in a near-field chamber by connecting three Vivaldi antenna to the proposed tunable  $3 \times 3$  Nolen matrix, as shown in Fig. 3(a). For input port  $P_1$  excitation, as in Fig. 3(b), the radiation beam is steered from  $-41^\circ$  to  $-20^\circ$  when two dc controls are swept from 4.5 to 19 V and 3 to 19 V, respectively. In Fig. 3(c), the beam is steered between  $+11^\circ$  and  $+41^\circ$  for port  $P_2$  excitation. For port  $P_3$  excitation, the radiation beam is steered from  $-23^\circ$  to  $-13^\circ$ , as shown in Fig. 3(d). In total, the radiation beams can be steered from  $-41^\circ$  to  $+41^\circ$  by controlling two control voltages. Some grating lobes are observed in radiation pattern measurement, which is mainly due to the  $\pm 180^\circ$  progressive phase differences at the output ports of Nolen matrix.

#### IV. CONCLUSION

A novel beamforming phased array using tunable  $3 \times 3$  Nolen matrix with relaxed phase tuning and simple control is proposed in this article. The matrix is composed of three couplers with two coupling ratios and three tunable phase shifters. From theoretical analysis, it is found that the proposed beamforming network requires tunable phase shifters with minimal phase tuning range and only two control voltages to steer the beam covering whole scanning angle. The detailed design for each component is provided, and experimental measurements are shown to verify the proposed design concept. This proposed tunable  $3 \times 3$  Nolen matrix makes an alternative method for beamforming and beam steering designs in the 5G-FR2 phased array system.

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