

# Optical model potentials for deuteron scattering off $^{24}\text{Mg}$ , $^{28}\text{Si}$ , $^{58}\text{Ni}$ , $^{90}\text{Zr}$ , $^{116}\text{Sn}$ , and $^{208}\text{Pb}$ at $\approx 100$ MeV/nucleon

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Angular distributions of the elastic and inelastic deuteron-nucleus scattering off  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ ,  $^{58}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ , and  $^{208}\text{Pb}$  have been measured at a beam energy of 98 MeV/nucleon, with the goal of constraining the deuteron optical potential in this kinematical regime and to extract the reduced transition probabilities for the ground-state transitions to low-lying excited states of these nuclei. Two potential models were used in the analysis of the measured  $(d, d)$  and  $(d, d')$  data within the optical model and the distorted-wave Born approximation: the phenomenological optical model potential associated with the collective model of nuclear scattering, and the semimicroscopic double-folding model of the deuteron-nucleus potential based on a realistic density-dependent M3Y interaction. The deuteron optical potential and inelastic  $(d, d')$  scattering form factors were calculated using these two potential models, allowing for a direct comparison between the potential models as well as the validation of the deduced  $E\lambda$  transition rates.

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## I. INTRODUCTION

With the increasing availability of radioactive ion beams of reasonable intensities, there has been enhanced recent interest in the investigation of giant resonances (GRs), which are highly collective oscillations of atomic nuclei. In particular, the isoscalar giant monopole resonance (ISGMR) in nuclei far from the stability line has the potential to enable significant strides in our understanding of nuclear incompressibility,  $K_\infty$ , and especially the asymmetry term,  $K_\tau$  [1]. Such measurements have to be performed in inverse kinematics, with deuterium and helium gases being the best targets available so far. While inelastic scattering of  $\alpha$  particles has been the

mainstay of such studies for a long time now [1], there has not been much work with deuterons since the mid-1970s primarily because of various experimental constraints [2,3]. On the other hand, most of the measurements of GRs with radioactive ion beams so far have employed active-target time projection chambers (AT-TPCs) with deuterium as the component gas [4–7]. Because of paucity of GR data with deuterons, it was important to validate in known cases the results obtained in  $(d, d')$  measurements via direct comparison with results of inelastic  $\alpha$  scattering. Such a detailed investigation was carried out by measuring small-angle inelastic deuteron scattering off  $^{116}\text{Sn}$  and  $^{208}\text{Pb}$ , and it was established that the extracted ISGMR strength distributions, using a multipole decomposition analysis similar to that done for inelastic  $\alpha$  scattering, agree very well with those deduced from the  $(\alpha, \alpha')$  data [8].

Such analyses hinge upon obtaining good-quality optical model (OM) parameters from elastic scattering data. The OM potential is widely used to generate wave functions for elastic scattering. These wave functions serve, in turn, as the door waves, widely known as the distorted waves, for the analysis of other direct reaction channels within the distorted-wave Born approximation (DWBA) or coupled-channel formalism. The inelastic scattering leading to the excitation of collective states of the target nucleus is essential for the determination of the underlying nuclear structure properties from the measured

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TABLE I. Target specifications.

Target	Thickness (mg/cm <sup>2</sup> )	Target	Thickness (mg/cm <sup>2</sup> )
<sup>24</sup> Mg	50.0	<sup>90</sup> Zr	4.2
<sup>28</sup> Si	58.5	<sup>116</sup> Sn	10.0
<sup>58</sup> Ni	1.5	<sup>208</sup> Pb	10.0

angular distributions. The nuclear interaction potential is, however, inherently complicated. Therefore, the phenomenological optical potential (OP) of a simple functional form is often employed in the OM and DWBA calculations to describe direct nuclear reactions and to extract nuclear properties from the angular distribution data. It represents a simple “effective” interaction used in the collective model of nuclear scattering [9] to describe both the elastic and inelastic scattering channels.

For elastic deuteron-nucleus scattering, the existing OM studies of the deuteron OP are mainly based on the phenomenological potential model using the Woods-Saxon (WS) functional form as described, for example, in Ref. [10]. This simple OP was successfully used to describe the elastic as well as inelastic angular distributions of deuteron scattering off heavy nuclei ( $A \gtrsim 40$ ), based on the collective model of nuclear scattering [9]. However, the collective model approach seems to overestimate the (one-step) transition probabilities from the ground state to the low-lying excited states of some light nuclei. For example, as noted in Ref. [11], the OM parameter set that gives the best fit of the elastic angular distribution requires 40% lower value of the deformation parameter  $\beta_2$ , as compared to the adopted value, to reproduce the experimentally observed  $2_1^+$  angular distribution in the nucleus <sup>16</sup>O.

In this work, we report on the elastic and inelastic deuteron-nucleus scattering measurements at the beam energy of 98 MeV/nucleon for several targets, ranging from light- to medium- and heavy-mass nuclei. The measured  $(d, d)$  and  $(d, d')$  scattering data were analyzed within the OM and DWBA, respectively, using both the WS phenomenological potential model as well as a semimicroscopic deuteron-nucleus potential obtained in the double-folding model [12–14]. The extracted OM parameters are expected to be useful in analyzing the giant resonance data with radioactive ion beams.

The experiments were performed at the ring cyclotron facility of the Research Center for Nuclear Physics (RCNP), Osaka University, Japan. The  $E_d = 196$  MeV deuteron beam was scattered off six highly enriched (more than 90%) self-supporting targets listed in Table I. Elastic and inelastic deuteron-nucleus scattering measurements were made over an angular range of  $\theta_{\text{lab}} \approx 3.5^\circ$ – $32^\circ$ . Each experimental angular opening ( $\approx 2^\circ$  wide) was subdivided into three parts for the analysis, each corresponding to a solid angle of 0.42 msr.

The scattered particles were momentum analyzed by the magnetic spectrometer Grand Raiden and focused onto the focal-plane detector system [15] consisting of two multiwire drift chambers (MWDCs) and two plastic scintillators [16].

The time-of-flight and energy-loss techniques were used for the identification of the scattered particles. Grand Raiden was used in the double-focusing mode in order to identify and eliminate practically all instrumental background from the final spectra [8]. Particle tracks were reconstructed using the ray-tracing technique described in Refs. [15,17]. This, in turn, allowed for the reconstruction of the scattering angle. The experimental angular resolution was  $\approx 0.15^\circ$ , including the nominal broadening of the scattering angle due to the emittance of the beam and the multiple Coulomb scattering effects. Further details of the experimental and data analysis procedures are provided in Refs. [8,18].

## II. DWBA ANALYSIS BASED ON THE PHENOMENOLOGICAL OM POTENTIAL

The two available global deuteron-nucleus optical potentials were developed by Daehnick *et al.* [10] and Bojowald *et al.* [19] in the 1980s using the phenomenological WS form for both the real and imaginary OPs, covering the mass range of  $27 \leq A \leq 238$  and energy range of  $E_d \approx 10$ –90 MeV. The deuteron carries one unit of spin ( $s = 1$ ) in its ground state, and this requires the inclusion of a real spin-orbit term  $V_{\ell s}$  in the OP. For the imaginary OP, in addition to the volume absorption  $W$ , an imaginary surface term  $W_D$  is also included to account for the surface absorption which is significant due to the deuteron breakup [20,21]. Thus, the total OP is determined explicitly as

$$U(r) = V(r) + iW(r) + iW_D(r) + V_{\ell s}(r)(\ell \cdot s) + V_C(r), \quad (1)$$

where

$$V(r) = -Vf(r, r_V, a_V), \quad (2)$$

$$W(r) = -Wf(r, r_W, a_W), \quad (3)$$

$$W_D(r) = 4a_D W_D \frac{d}{dr} f(r, r_D, a_D), \quad (4)$$

$$V_{\ell s}(r) = V_{\ell s} \left[ \frac{\hbar}{m\pi c} \right]^2 \frac{1}{r} \frac{d}{dr} f(r, r_{\ell s}, a_{\ell s}), \quad (5)$$

and  $V_C(r)$  is the Coulomb potential of a uniformly charged sphere of radius  $R_C = 1.3A^{1/3}$  fm. The functional form  $f$  is chosen in the WS form for all the terms:

$$f(r, r_i, a_i) = \left[ 1 + \exp \left( \frac{r - r_i A^{1/3}}{a_i} \right) \right]^{-1}. \quad (6)$$

The phenomenological OM potential given by Eqs. (1)–(6) is used in the OM analysis of the measured elastic  $(d, d)$  scattering data, and to generate the distorted waves for the DWBA description of the inelastic  $(d, d')$  scattering data. The associated OM parameters were obtained from a  $\chi^2$ -minimization fit to the elastic  $(d, d)$  data using the code ECIS97 [22], and they are given explicitly in Table II. The global deuteron OPs [10,19] were used for the parameter initialization in the  $\chi^2$  search. The OM results obtained using the best-fit OP parameter set are shown as dashed lines in Figs. 1 and 2, in comparison with elastic  $(d, d)$  scattering data measured at  $E_d = 196$  MeV for the targets under study.

TABLE II. Best-fit parameters of the phenomenological OP (1)–(6). Because of spin convention, the  $V_{\ell s}$  value must be divided by 2 when used in the numerical input of the code ECIS97 [22]. The errors were deduced from the weight of each parameter in the covariant multiparameter  $\chi^2$  search, with  $r_V$  and  $V_{\ell s}$  kept fixed during the search.

Target	$^{208}\text{Pb}$	$^{116}\text{Sn}$	$^{90}\text{Zr}$	$^{58}\text{Ni}$	$^{28}\text{Si}$	$^{24}\text{Mg}$
$V$ (MeV)	$46.54 \pm 0.01$	$44.33 \pm 0.01$	$42.95 \pm 0.13$	$39.07 \pm 0.14$	$35.58 \pm 0.13$	$31.92 \pm 0.13$
$r_V$ (fm)	1.18	1.18	1.18	1.18	1.18	1.18
$a_V$ (fm)	$0.938 \pm 0.001$	$0.911 \pm 0.001$	$0.997 \pm 0.013$	$0.914 \pm 0.004$	$0.911 \pm 0.013$	$0.977 \pm 0.004$
$W$ (MeV)	$20.59 \pm 0.01$	$20.87 \pm 0.03$	$20.20 \pm 0.20$	$21.41 \pm 0.20$	$22.67 \pm 0.13$	$24.39 \pm 0.14$
$r_W$ (fm)	$1.160 \pm 0.001$	$1.070 \pm 0.001$	$1.060 \pm 0.013$	$1.100 \pm 0.019$	$0.850 \pm 0.001$	$1.000 \pm 0.012$
$a_W$ (fm)	$0.361 \pm 0.002$	$0.670 \pm 0.002$	$0.538 \pm 0.012$	$0.456 \pm 0.004$	$0.420 \pm 0.005$	$0.501 \pm 0.003$
$W_D$ (MeV)	$7.00 \pm 0.01$	$7.00 \pm 0.01$	$7.90 \pm 0.19$	$7.60 \pm 0.19$	$7.95 \pm 0.04$	$7.50 \pm 0.14$
$r_D$ (fm)	$1.230 \pm 0.001$	$1.110 \pm 0.001$	$1.100 \pm 0.019$	$1.050 \pm 0.020$	$1.000 \pm 0.004$	$1.022 \pm 0.005$
$a_D$ (fm)	$0.790 \pm 0.001$	$1.080 \pm 0.001$	$0.997 \pm 0.006$	$1.030 \pm 0.015$	$0.980 \pm 0.001$	$0.920 \pm 0.003$
$V_{\ell s}$ (MeV)	4.22	4.22	4.22	4.22	4.22	4.22
$r_{\ell s}$ (fm)	$1.150 \pm 0.001$	$1.130 \pm 0.001$	$1.200 \pm 0.001$	$1.190 \pm 0.017$	$1.130 \pm 0.015$	$1.163 \pm 0.003$
$a_{\ell s}$ (fm)	$1.230 \pm 0.001$	$1.110 \pm 0.013$	$0.985 \pm 0.002$	$1.110 \pm 0.012$	$1.110 \pm 0.020$	$1.164 \pm 0.013$

The DWBA analysis of inelastic deuteron-nucleus scattering data is restricted to the direct one-step excitation of a collective state of the target, neglecting the contributions of indirect two- or three-step transitions and the channel coupling effects. The  $(d, d')$  scattering cross section is calculated using the following DWBA inelastic scattering amplitude [23]:

$$T_{\text{DWBA}} = \int [\chi^-(\mathbf{k}', \mathbf{r}')]^* \langle dA' | V_{d-A} | dA \rangle \chi^+(\mathbf{k}, \mathbf{r}) d^3r d^3r', \quad (7)$$

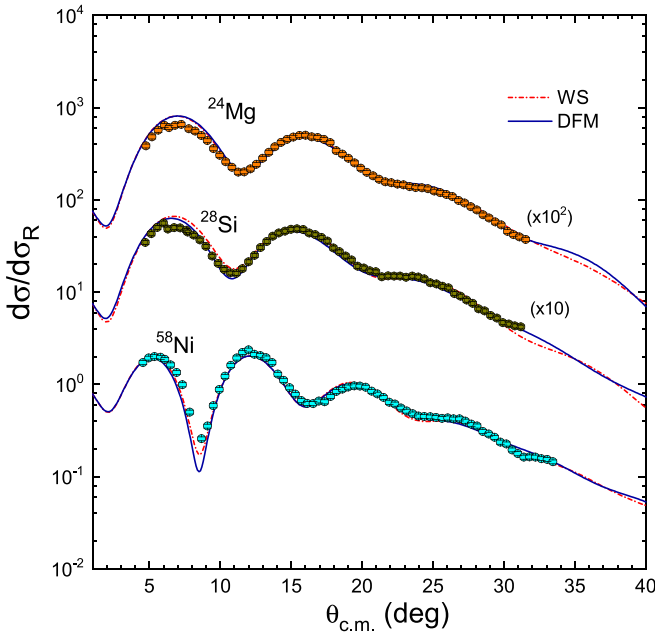


FIG. 1. Elastic  $(d, d)$  scattering data measured at  $E_d = 196$  MeV (in ratio to the corresponding Rutherford cross sections) for  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ , and  $^{58}\text{Ni}$  targets (solid points). The dashed and solid lines are the OM results given by the phenomenological OP (1)–(6) and hybrid folded OP (13)–(18), respectively.

where the distorted waves  $\chi^\pm$  are generated by the OP (1)–(6). The transition matrix element of the deuteron-nucleus interaction  $\langle dA' | V_{d-A} | dA \rangle$  is dubbed the inelastic scattering form factor (FF). A widely adopted method is to obtain the nuclear inelastic scattering FF by radially deforming nuclear part of the OP, based on the collective vibrational (or rotational) model of nuclear scattering [9]. For the  $(d, d')$  scattering to a  $2^+$ -pole collective excited state of target, the inelastic scattering FF is determined as

$$F_\lambda(r) = F_N^{(\lambda)}(r) + F_{\ell s}^{(\lambda)}(r)(\ell \cdot s) + F_C^{(\lambda)}(r), \quad (8)$$

$$F_N^{(\lambda)}(r) = \delta_\lambda^{(N)} \frac{d}{dr} [V(r) + iW(r) + iW_D(r)], \quad (9)$$

$$F_{\ell s}^{(\lambda)}(r) = \delta_\lambda^{(N)} \frac{d}{dr} V_{\ell s}(r). \quad (10)$$

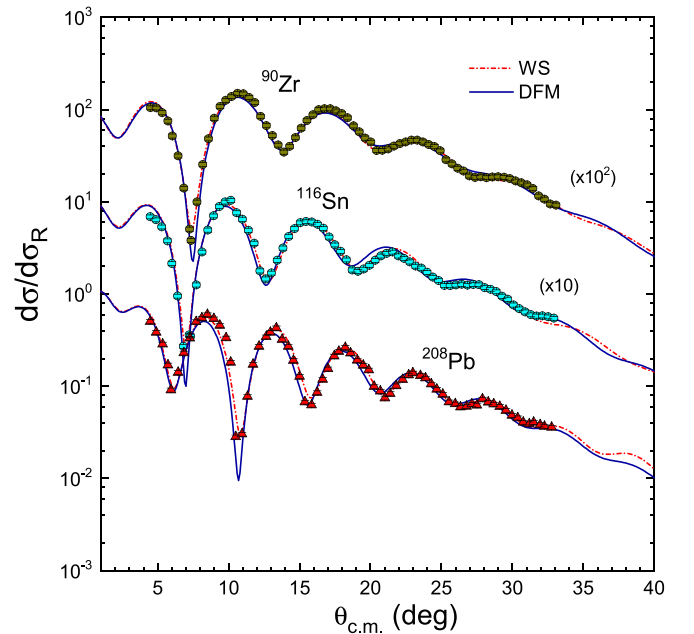


FIG. 2. The same as Fig. 1 but for  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ , and  $^{208}\text{Pb}$  targets.

TABLE III. Experimentally adopted values for the reduced electric transition rates  $B(E2 \uparrow)$  [24,25],  $B(E3 \uparrow)$  [26], and the best-fit transition rates  $B(E\lambda)_{\text{DOP}}$  and  $B(E\lambda)_{\text{DFM}}$  deduced from the DWBA analysis of the  $(d, d')$  data using the inelastic FF based on the phenomenological OP and that based on the hybrid folded deuteron-nucleus potential, respectively. The errors (italicized numbers in parentheses) were estimated from those of the measured angular distributions, which refer to the uncertainties in the last digits of the quoted  $B(E\lambda)$  values.

Target	$E_x$ (MeV)	$\lambda^\pi$	$B(E\lambda)_{\text{adopted}}$ ( $e^2 \text{ b}^\lambda$ )	$B(E\lambda)_{\text{DOP}}$ ( $e^2 \text{ b}^\lambda$ )	$B(E\lambda)_{\text{DFM}}$ ( $e^2 \text{ b}^\lambda$ )
$^{24}\text{Mg}$	1.368	$2^+$	0.0432 (11)	0.0284 (14)	0.0410 (21)
$^{28}\text{Si}$	1.779	$2^+$	0.0326 (12)	0.0196 (10)	0.0326 (16)
$^{58}\text{Ni}$	1.454	$2^+$	0.0695 (20)	0.0695 (35)	0.0695 (35)
$^{90}\text{Zr}$	2.186	$2^+$	0.061 (4)	0.0641 (32)	0.0641 (32)
	2.748	$3^-$	0.098 (5) <sup>a</sup> 0.037–0.079 <sup>b</sup>	0.0510 (37)	0.0640 (32)
$^{116}\text{Sn}$	1.293	$2^+$	0.209 (6)	0.241 (12)	0.240 (12)
	2.266	$3^-$	0.132 (18) <sup>c</sup> 0.112–0.202 <sup>b</sup>	0.142 (7)	0.160 (8)
$^{208}\text{Pb}$	2.610	$3^-$	0.611 (9) <sup>c</sup> 0.419–0.836 <sup>b</sup>	0.611 (31)	0.642 (32)

<sup>a</sup>Adopted from  $(e, e')$  data.

<sup>b</sup>From inelastic nucleon- and light-ion scattering data.

<sup>c</sup>Adopted from Coulomb excitation data.

Prescription (9)–(10) is known as the deformed optical potential (DOP) method to generate the inelastic nuclear scattering FF. The spin-orbit contribution (10) to the total  $(d, d')$  cross section is minor, but it helps to better reproduce the observed oscillation pattern of inelastic  $(d, d')$  cross section for medium mass targets. For an isoscalar excitation, the nuclear deformation length  $\delta_\lambda^{(N)}$  is usually assumed to be the same as the proton deformation length  $\delta_\lambda^{(p)}$ , determined from the reduced transition rate  $B(E\lambda \uparrow)$  [23] as

$$\delta_\lambda^{(p)} = \frac{4\pi\sqrt{B(E\lambda \uparrow)/e^2}}{3ZR_A^{\lambda-1}}, \quad R_A = 1.3A^{1/3} \text{ fm}. \quad (11)$$

The Coulomb inelastic FF in Eq. (8) can be approximately obtained in a “model independent” form [23]:

$$F_C^{(\lambda)}(r) = \frac{4\pi\sqrt{B(E\lambda \uparrow)}e}{(2\lambda+1)r^{\lambda+1}}. \quad (12)$$

Thus, the OP parameters (1)–(6) and the reduced transition rate  $B(E\lambda \uparrow)$  of the target excitation are the main inputs for the DWBA analysis of inelastic  $(d, d')$  scattering using the collective model of nuclear scattering [9] associated with the phenomenological OP. The DWBA calculation of the  $(d, d')$  scattering cross section was done using two choices of the  $B(E\lambda)$ :

- $B(E\lambda \uparrow)$  is fixed to the experimentally adopted transition rates [24–26], denoted as  $B(E\lambda)_{\text{adopted}}$  in Table III.
- $B(E\lambda \uparrow)$  is deduced from the best DWBA fit of the calculated  $(d, d')$  cross section to the  $(d, d')$  scattering data and denoted as  $B(E\lambda)_{\text{DOP}}$  or  $B(E\lambda)_{\text{DFM}}$  in Table III.

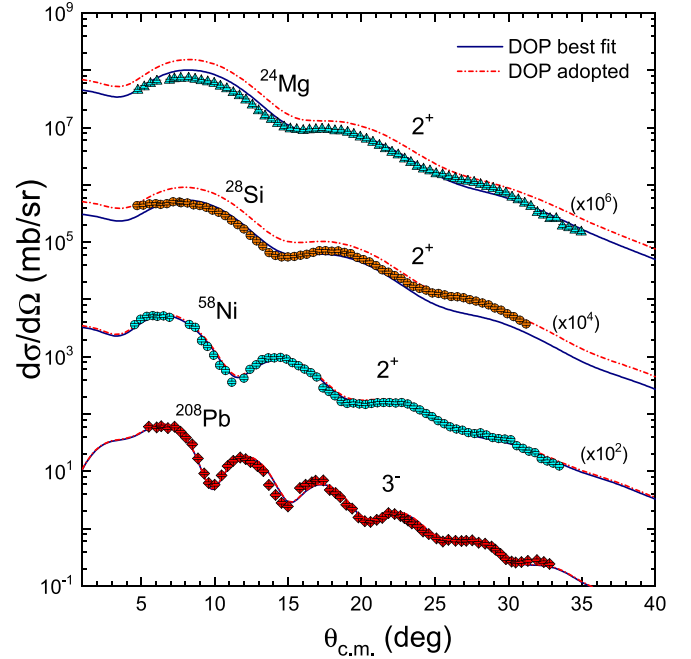


FIG. 3. The inelastic  $(d, d')$  scattering data measured at  $E_d = 196$  MeV (solid points) for the excitation of  $2_1^+$  states of  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ ,  $^{58}\text{Ni}$  targets, and the  $3_1^-$  state of the  $^{208}\text{Pb}$  target. The DWBA results given by the collective model FF (8)–(12) based on the adopted  $B(E\lambda)_{\text{adopted}}$  and best-fit  $B(E\lambda)_{\text{DOP}}$  values (see Table III) are shown as dashed and solid lines, respectively.

The DWBA description of inelastic  $(d, d')$  scattering data measured for the excitation of  $2_1^+$  states of  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ ,  $^{58}\text{Ni}$  targets, and  $3_1^-$  state of  $^{208}\text{Pb}$  target given by the collective-model inelastic FF (8)–(12) is shown in Fig. 3. As can be seen, the  $3_1^-$  angular distribution measured for  $^{208}\text{Pb}$  target is reproduced well with the adopted  $B(E3)_{\text{adopted}}$  value [26]. However, the DWBA description of the measured  $2_1^+$  cross section using the adopted  $B(E2)$  value,  $B(E2)_{\text{adopted}}$ , seems to get worse for light  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  nuclei, where the calculated  $(d, d')$  cross section overestimates data at small angles, and then becomes non-oscillatory for angles above  $\approx 25^\circ$ . The best DWBA fit to inelastic  $(d, d')$  data given by the collective-model FF requires  $B(E2)_{\text{DOP}}$  value of around 40% lower than  $B(E2)_{\text{adopted}}$  value. This likely indicates a deficiency of the collective model of nuclear scattering [9] based on the phenomenological OP (1)–(6) for light-mass nuclei, as also found earlier for  $^{16}\text{O}$  in Ref. [11].

The DWBA has been proven to be a reliable approximation for the direct reaction studies at energies around or above 100 MeV/nucleon. Therefore, a good agreement of the  $(d, d')$  cross section calculated in the DWBA with  $(d, d')$  data measured for  $2_1^+$  and  $3_1^-$  states of target must be a prerequisite for the validation of the phenomenological OP (1)–(6). In the present work, the DWBA analysis of inelastic deuteron scattering off  $^{90}\text{Zr}$  and  $^{116}\text{Sn}$  targets was chosen as test ground for the phenomenological deuteron-nucleus OP. The DWBA results obtained for  $^{116}\text{Sn}$  and  $^{90}\text{Zr}$  targets are shown in Figs. 4 and 5, respectively. At variance with light targets, the DWBA cross sections given by the collective model inelastic FF based



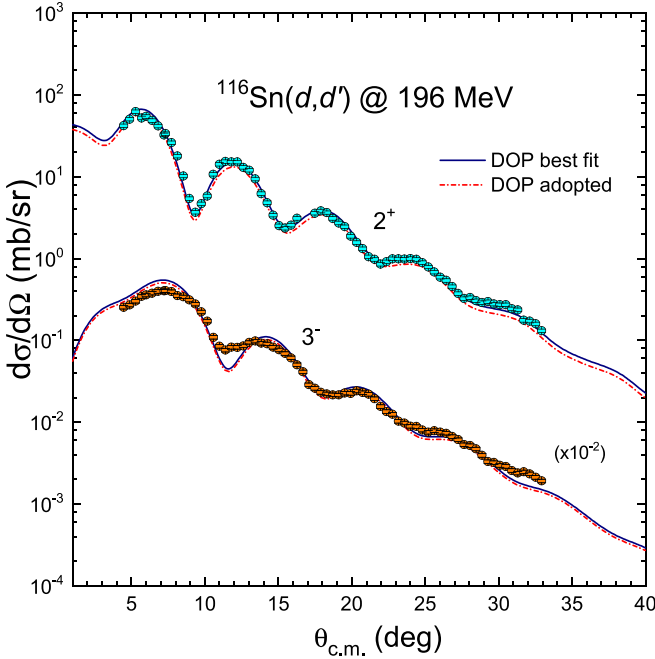


FIG. 4. Inelastic  $^{116}\text{Sn}(d, d')$  scattering data (solid points) measured at  $E_d = 196$  MeV for the excitation of the  $2_1^+$  and  $3_1^-$  states of  $^{116}\text{Sn}$  target. The DWBA results given by the collective model FF (8)-(12) based on the adopted  $B(E\lambda)_{\text{adopted}}$  and best-fit  $B(E\lambda)_{\text{DOP}}$  values (see Table III) are shown as dashed and solid lines, respectively.

on the best-fit OP and adopted  $B(E2)_{\text{adopted}}$  values [24,25] agree well with the measured  $2_1^+$  angular distribution for both  $^{90}\text{Zr}$  and  $^{116}\text{Sn}$  targets. The best DWBA fit yields  $B(E2)_{\text{DOP}}$

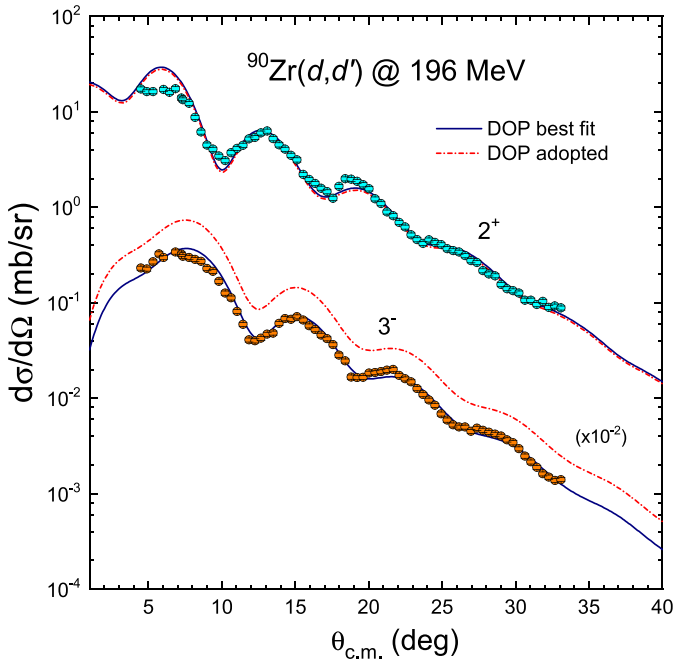


FIG. 5. The same as Fig. 4 but for inelastic  $^{90}\text{Zr}(d, d')$  scattering data.

values around 5% and 15% larger than  $B(E2)_{\text{adopted}}$  values for  $^{90}\text{Zr}$  and  $^{116}\text{Sn}$ , respectively (see Table III).

The situation is different, on the other hand, for the  $3_1^-$  angular distributions measured for these targets. While the DWBA calculation using the collective model inelastic FF based on the adopted  $B(E3)_{\text{adopted}}$  value [26] describes well the  $(d, d')$  data measured for the  $3_1^-$  state of  $^{116}\text{Sn}$ , the same prescription overestimates the  $3_1^-$  angular distribution measured for the  $^{90}\text{Zr}$  target over the whole angular range. Given  $B(E3)_{\text{adopted}} \approx 0.098 e^2 b^3$  adopted from  $(e, e')$  data, and the empirical  $B(E3)$  values of 0.037 to 0.079  $e^2 b^3$  deduced from inelastic nucleon- and light-ion scattering data (see Table V in Ref. [26]), such a disagreement with the  $3_1^-$  angular distribution for  $^{90}\text{Zr}$  would not be unexpected. Indeed, the best DWBA fit to the measured  $3_1^-$  cross sections yields a lower value of  $B(E3)_{\text{DOP}} \approx 0.051 e^2 b^3$ .

### III. DWBA ANALYSIS BASED ON THE FOLDED DEUTERON-NUCLEUS POTENTIAL

The microscopic description of the nucleus-nucleus interaction is usually based on a chosen effective pairwise nucleon-nucleon ( $NN$ ) interaction  $v$  between a projectile nucleon and a bound nucleon of the target. The density dependence of  $v$  presumably takes into account the three-nucleon interaction and higher-order  $NN$  correlations. Among such models, the double-folding model (DFM) has been used widely to calculate the  $\alpha$ -nucleus and heavy-ion OP [12–14]. The established success of the DFM in describing the observed elastic scattering for many nucleus-nucleus systems indicates that it produces the dominant part of the nucleus-nucleus OP. Within the DFM, the deuteron-nucleus scattering potential can be formally determined [14] as

$$U_F = \langle dA' | V_{d-A} | dA \rangle = \mathcal{A} \sum_{i \in d} \sum_{j \in A} \sum_{j' \in A'} \langle ij' | v_{NN} | ij \rangle, \quad (13)$$

where the diagonal ( $A' = A$ ) and nondiagonal ( $A' \neq A$ ) matrix elements describe elastic and inelastic deuteron-nucleus scattering, respectively. The antisymmetrization  $\mathcal{A}$  accounts for all single-nucleon exchanges between deuteron and target, giving rise to a nonlocal exchange term of the potential (13),

$$U_F = \sum_{i \in d} \sum_{j \in A} \sum_{j' \in A'} [\langle ij' | v_D | ij \rangle + \langle ij' | v_{EX} | ji \rangle], \quad (14)$$

where  $v_D(\text{EX})$  is the direct (exchange) part of  $v$ . Given the nonlocal exchange potential, one has to solve an integrodifferential OM equation involving a nonlocal kernel of the deuteron-nucleus OP, which has not been done so far due the complexity of the nonlocal DFM computation. In fact, the exact solution of the OM equation with the nonlocal folded OP was obtained only for elastic nucleon scattering (see, e.g., Ref. [27] and references therein). Like previous DFM calculations [14,28], we have used in the present work the well-proven local approximation for the exchange potential, based on the Wentzel-Kramers-Brillouin (WKB) approximation for the change in the deuteron-nucleus relative motion induced by the exchange of spatial coordinates of each interacting nucleon pair (see more details in

TABLE IV. Parameters of the Fermi distribution (15).

Nuclear density distribution						
Target	<sup>24</sup> Mg	<sup>28</sup> Si	<sup>58</sup> Ni	<sup>90</sup> Zr	<sup>116</sup> Sn	<sup>208</sup> Pb
$\rho_0$ (fm <sup>-3</sup> )	0.170	0.167	0.176	0.165	0.154	0.157
$c$ (fm)	2.995	3.160	4.080	4.900	5.490	6.670
$a$ (fm)	0.478	0.523	0.515	0.515	0.515	0.545
Reference	[35]	[35]	[36]	[37]	[37]	[37]
Charge density distribution						
Target	<sup>24</sup> Mg	<sup>28</sup> Si	<sup>58</sup> Ni	<sup>90</sup> Zr	<sup>116</sup> Sn	<sup>208</sup> Pb
$\rho_0$ (fm <sup>-3</sup> )	0.0785	0.0841	0.0826	0.0726	0.0688	0.0628
$c$ (fm)	3.045	3.154	4.177	4.908	5.417	6.647
$a$ (fm)	0.523	0.523	0.523	0.523	0.523	0.523
Reference	[38]	[38]	[38]	[38]	[38]	[38]

Ref. [29]). In this case, both the direct and exchange terms of the deuteron-nucleus potential (14) are evaluated [14] by folding the deuteron and target densities with the chosen effective  $NN$  interaction  $v$ . The accuracy of such a WKB approximation was shown again in a recent OM study of elastic nucleon-nucleus scattering [27]. With a localized exchange potential, not only the OM calculation is much simpler, but also the comparison with the (local) phenomenological OP (1)–(6) is more direct, which is essential for the present study.

#### A. Effective density-dependent $NN$ interaction and nuclear densities

The CDM3Y6 density-dependent version of the M3Y interaction [28] is used in the present DFM calculation of the deuteron-nucleus potential (13)–(14). The real density dependence of the CDM3Y6 interaction was adjusted by a realistic HF description of nuclear matter, with nuclear incompressibility  $K \approx 252$  MeV [28]. The imaginary density dependence of the CDM3Y6 interaction was introduced in Ref. [30] to reproduce the Brueckner-Hartree-Fock results for nucleon OP in nuclear matter by Jeukenne, Lejeune, and Mahaux (the JLM potential) [31]. This (complex) density-dependent CDM3Y6 interaction was successfully used in the DFM calculation of the  $\alpha + ^{208}\text{Pb}$  OP and inelastic scattering FF [30] for the multipole decomposition analysis of  $(\alpha, \alpha')$  data measured for isoscalar giant resonances of  $^{208}\text{Pb}$  at 97 MeV/nucleon [32]. We note that, prior to the present work, the only DFM calculation of the deuteron-nucleus OP was done 42 years ago by Cook [33] using the density independent M3Y interaction.

Besides the effective  $NN$  interaction, the nuclear densities of projectile and target are essential inputs for the DFM calculation. In the present work, we have chosen the deuteron density given by the exact solution of the two-body problem using the Argonne V18 interaction as the free  $NN$  interaction [34]. For the DFM calculation of the diagonal ( $A' = A$ ) elastic deuteron-nucleus potential, the Fermi distribution has been adopted for the ground-state (g.s.) densities of target nuclei:

$$\rho_0(r) = \rho_0 / \{1 + \exp[(r - c)/a]\}, \quad (15)$$

with the parameters  $\rho_0$ ,  $c$ , and  $a$  listed in Table IV.

The main input for the DFM calculation of the nondiagonal ( $A' \neq A$ ) deuteron-nucleus potential (the folded inelastic scattering FF) is the nuclear transition density of the target excitation. The DFM method (13)–(14) is also used to calculate the Coulomb inelastic FF microscopically by folding the deuteron charge density and charge transition density of the target excitation with the Coulomb force acting between two protons [14]. For the  $2^\lambda$ -pole excitations under study, we adopt the so-called Bohr-Mottelson prescription [39] for the nuclear- and charge transition densities:

$$\rho_\lambda(r) = -\delta_\lambda \frac{d\rho_0(r)}{dr} \quad \text{and} \quad \rho_{\text{charge}}^{(\lambda)}(r) = -\delta_\lambda \frac{d\rho_{\text{charge}}^{(0)}(r)}{dr}, \quad (16)$$

where the g.s. charge density  $\rho_{\text{charge}}^{(0)}(r)$  is determined using the same Fermi distribution (15) with parameters listed in Table IV. The deformation length  $\delta_\lambda$  is determined from the transition rate  $B(E\lambda \uparrow)$  of the target excitation using the following relation:

$$B(E\lambda) = e^2 |M_\lambda|^2, \quad \text{where} \quad M_\lambda = \int_0^\infty r^{\lambda+2} \rho_{\text{charge}}^{(\lambda)}(r) dr. \quad (17)$$

At variance with the DOP prescription (11), the deformation length  $\delta_\lambda$  in Eq. (17) is the scaling factor of the nuclear transition density (16) used in the folding calculation (14) of inelastic scattering FF, which is constrained by the considered  $B(E\lambda)$  transition rate.

#### B. Folded deuteron OP and inelastic scattering FF

The deuteron breakup has been shown to affect significantly the deuteron OP at the surface [20,21], and a WS surface term  $W_D(r)$  has been added to the imaginary folded OP because the DFM does not take into account the dynamic polarization of the OP by the breakup effect. A slight renormalization of the strength of both the real and imaginary folded OP is also allowed in the OM analysis of elastic ( $d, d$ ) data, and the total OP is determined in the hybrid manner as

$$U(r) = U_F^{(0)}(r) + iW_D(r) + V_{\ell s}(r)(\ell \cdot s) + V_C(r), \quad (18)$$

where  $U_F^{(0)}(r) = N_V V_F^{(0)}(r) + iN_W W_F^{(0)}(r)$  is the diagonal folded deuteron-nucleus potential (14). The surface and spin-orbit terms of the OP (18) are determined in the same way as in Eqs. (4)–(5). The OM results obtained with the hybrid folded deuteron OP (18) are shown as solid lines in Figs. 1 and 2, with the best-fit  $N_{V(W)}$  coefficients, parameters of the surface, and spin-orbit terms of the OP listed in Table V. With the surface absorption taken into account by  $W_D(r)$  term of the imaginary deuteron OP (18), the impact of the deuteron breakup on the complex folded deuteron-nucleus OP seems to be small, with the obtained  $N_{V(W)}$  coefficients being quite close to unity (see Table V).

The total inelastic ( $d, d'$ ) scattering FF is also determined in the hybrid manner as

$$F_\lambda(r) = U_F^{(\lambda)}(r) + F_{W_D}^{(\lambda)}(r) + F_{\ell s}^{(\lambda)}(r)(\ell \cdot s) + F_C^{(\lambda)}(r), \quad (19)$$

TABLE V. Best-fit  $N_{V(W)}$  coefficients, parameters of the surface, and spin-orbit terms of the hybrid folded deuteron OP (18). Because of spin convention, the  $V_{\ell s}$  value must be divided by 2 when used in the numerical input of the code ECIS97 [22]. The errors were deduced from the weight of each parameter in the covariant multiparameter  $\chi^2$  search, with  $W_D$ ,  $a_D$ ,  $V_{\ell s}$ , and  $a_{\ell s}$  kept fixed during the search.

Target	$^{208}\text{Pb}$	$^{116}\text{Sn}$	$^{90}\text{Zr}$	$^{58}\text{Ni}$	$^{28}\text{Si}$	$^{24}\text{Mg}$
$N_V$	$0.98 \pm 0.01$	$0.94 \pm 0.01$	$0.98 \pm 0.01$	$0.98 \pm 0.01$	$0.95 \pm 0.01$	$0.95 \pm 0.01$
$N_W$	$1.01 \pm 0.02$	$1.04 \pm 0.02$	$1.00 \pm 0.01$	$1.03 \pm 0.03$	$1.09 \pm 0.03$	$1.00 \pm 0.03$
$W_D$ (MeV)	7.60	7.60	7.60	7.60	7.60	7.60
$r_D$ (fm)	$0.70 \pm 0.09$	$0.70 \pm 0.04$	$0.55 \pm 0.03$	$0.92 \pm 0.04$	$0.54 \pm 0.07$	$0.75 \pm 0.08$
$a_D$ (fm)	0.65	0.65	0.65	0.65	0.65	0.65
$V_{\ell s}$ (MeV)	4.22	4.22	4.22	4.22	4.22	4.22
$r_{\ell s}$ (fm)	$1.03 \pm 0.01$	$1.00 \pm 0.02$	$1.03 \pm 0.02$	$1.05 \pm 0.01$	$1.03 \pm 0.02$	$0.82 \pm 0.02$
$a_{\ell s}$ (fm)	0.85	0.85	0.85	0.85	0.85	0.85

where the folded nuclear  $U_F^{(\lambda)}(r) = V_F^{(\lambda)}(r) + iW_F^{(\lambda)}(r)$  and Coulomb  $F_C^{(\lambda)}(r)$  terms of the inelastic scattering FF are kept unchanged (as given by the DFM calculation) in the DWBA analysis of  $(d, d')$  data. The surface and spin-orbit terms of the FF (19) are determined by the DOP method (9)–(10), using the same deformation length  $\delta_\lambda$  as that used for the nuclear- and charge transition densities (16). The DWBA results obtained with the hybrid folded inelastic FF (19) for  $(d, d')$  scattering on  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ ,  $^{58}\text{Ni}$ , and  $^{208}\text{Pb}$  targets are shown in Fig. 6. Unlike the DWBA description of the  $2_1^+$  cross sections measured for light  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  targets given by the collective model FF (9)–(12) shown in Fig. 3, the hybrid folded FF (19) reproduces these same  $(d, d')$  data very well using the  $B(E2)_{\text{adopted}}$  values. The oscillation pattern of the  $2_1^+$  cross section observed for these targets is also better reproduced by the hybrid folded FF

compared to the collective model FF, and the best-fit  $B(E2)_{\text{DFM}}$  value agrees nicely with the  $B(E2)_{\text{adopted}}$  value as shown in Table III.

It should be noted here that the deformation parameter  $\beta_2$  is obtained in one case from the OP geometry using the DOP method (9)–(10), while in the other case it is obtained (16) from the geometry of the g.s. density. A more accurate comparison could be made based on the deformation length  $\beta_2 R$  obtained from both geometries or even better to compare the full integral of both the folded inelastic FF and collective model FF determined with the respective deformation parameters.

The DWBA descriptions of the  $2_1^+$  and  $3_1^-$  angular distributions given by the hybrid folded inelastic FF (19) for  $^{116}\text{Sn}$  and  $^{90}\text{Zr}$  targets are shown in Figs. 7 and 8, respectively. One can see that the collective-model and hybrid folded inelastic

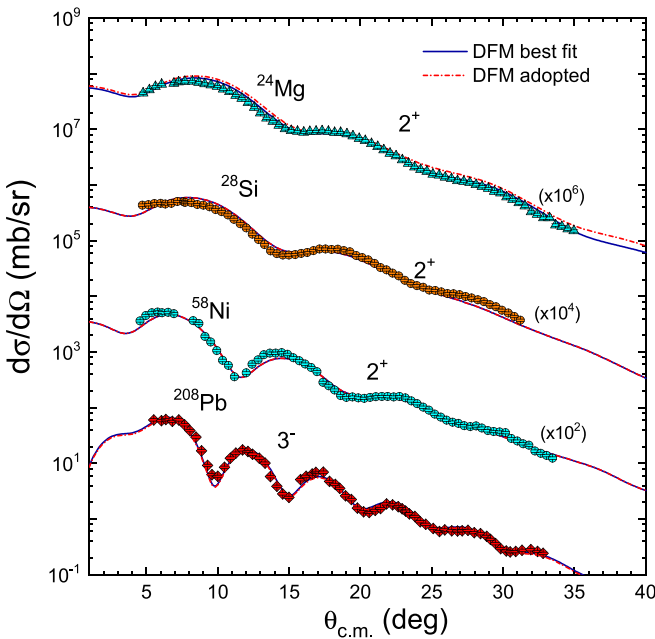


FIG. 6. The same as in Fig. 3 but obtained with the hybrid folded inelastic FF (19). The DWBA results based on the adopted  $B(E\lambda)_{\text{adopted}}$  and best-fit  $B(E\lambda)_{\text{DFM}}$  values (see Table III) are shown as dashed and solid lines, respectively.

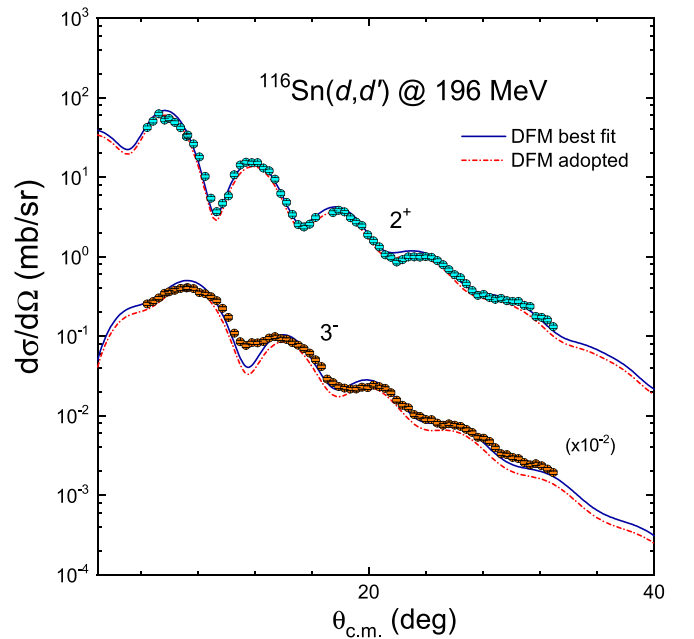


FIG. 7. The same as in Fig. 4, but obtained with the hybrid folded inelastic FF (19). The DWBA results based on the adopted  $B(E\lambda)_{\text{adopted}}$  and best-fit  $B(E\lambda)_{\text{DFM}}$  values (see Table III) are shown as dashed and solid lines, respectively.

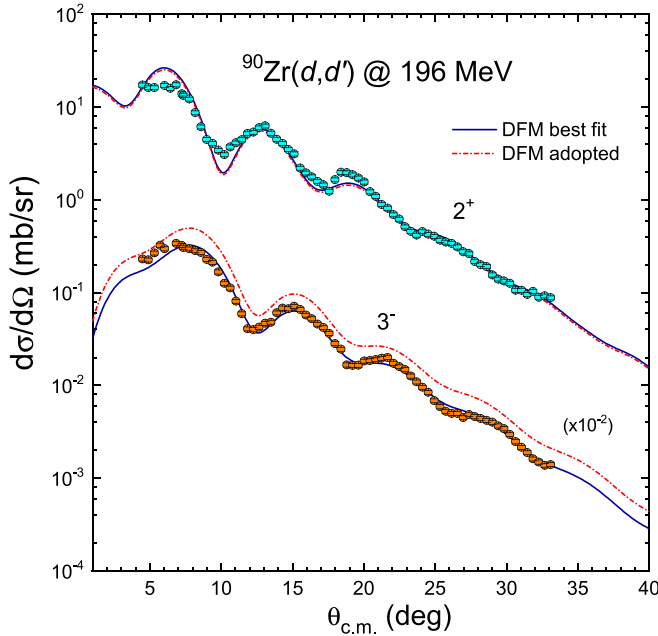


FIG. 8. The same as in Fig. 5, but obtained with the hybrid folded inelastic FF (19). The DWBA results based on the  $B(E\lambda)_{\text{adopted}}$  and best-fit  $B(E\lambda)_{\text{DFM}}$  values (see Table III) are shown as dashed and solid lines, respectively.

scattering FFs give nearly equivalent DWBA descriptions of  $(d, d')$  data measured for the  $2_1^+$  excitation of these nuclei. The best-fit  $B(E2)_{\text{DOP}}$  and  $B(E2)_{\text{DFM}}$  values obtained for  $^{90}\text{Zr}$  and  $^{116}\text{Sn}$  targets are larger than the adopted values by around 5% and 15%, respectively (see Table III).

Like the DWBA results given by the collective-model FF shown in Figs. 4 and 5, the DWBA results given by the hybrid folded FF (19) based on the  $B(E3)_{\text{adopted}}$  value [26] reasonably reproduce the  $(d, d')$  data measured for the  $3_1^-$  state of  $^{116}\text{Sn}$ , but overestimate the data measured for the  $3_1^-$  state of  $^{90}\text{Zr}$  over the entire angular range. The best-fit  $B(E3)_{\text{DFM}}$  value is around 35% smaller than  $B(E3)_{\text{adopted}}$  value from  $(e, e')$  data, but in a good agreement with the empirical  $B(E3)$  values deduced from inelastic nucleon- and light-ion scattering data [26].

We note further that the best-fit  $B(E3)_{\text{DOP}}$  value obtained for the  $3_1^-$  state of  $^{90}\text{Zr}$  using the collective-model FF is about 20% lower than the best-fit  $B(E3)_{\text{DFM}}$  value. A similar trend was also found for  $3_1^-$  states of  $^{116}\text{Sn}$  and  $^{208}\text{Pb}$  (see Table III) which shows that the DOP method consistently gives a lower  $B(E3)$  value compared to that given by the DFM approach, as discussed previously in Ref. [40]. However, the DOP method seems to better reproduce  $B(E3)_{\text{adopted}}$  value from the Coulomb excitation data [26].

#### IV. SUMMARY

Elastic and inelastic deuteron scattering have been measured off  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ ,  $^{58}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ , and  $^{208}\text{Pb}$  at an energy of 98 MeV/nucleon. The measured  $(d, d)$  and  $(d, d')$  angular distributions were analyzed within the OM and DWBA using the phenomenological deuteron OP associated with the collective model of nuclear scattering, and the hybrid potential model for the deuteron OP and inelastic FF based on the microscopic DFM calculation. The  $E2$  and  $E3$  transition rates of the  $2_1^+$  and  $3_1^-$  excitations of these target nuclei were deduced from the best DWBA fits to the  $(d, d')$  scattering data, which agree reasonably with the adopted  $B(E\lambda)$  values [24,26].

The results of the OM analysis of elastic  $(d, d)$  data using the hybrid folded OP show that the deuteron breakup does not affect significantly the volume part of the deuteron OP given by the DFM calculation. However, the breakup effect seems to imply an enhanced absorption at the surface, which can be taken into account effectively by a surface WS potential added to the imaginary deuteron OP.

While both potential models describe equally well the  $(d, d)$  and  $(d, d')$  angular distributions measured for medium- and heavy-mass target nuclei, the DWBA calculation using the collective-model inelastic FF (DOP) gives a poorer description of the  $2_1^+$  angular distribution measured for the light-mass  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  nuclei, in comparison with the DWBA description of the same data using the semimicroscopic folded inelastic FF (DFM).

The DWBA analysis of the  $3_1^-$  angular distribution measured for  $^{90}\text{Zr}$  target indicates that  $B(E3)_{\text{adopted}}$  value from  $(e, e')$  data [26] is too high, and the DWBA results given by the two potential models agree well with  $(d, d')$  scattering data, with the best-fit  $B(E3)$  values close to those deduced from inelastic nucleon- and light-ion scattering data.

A hindrance of the  $E3$  transition rate determined by the collective-model inelastic FF compared to that determined by the inelastic folded FF was also found, which illustrates the inconsistency between the DWBA description of nuclear excitation with  $\lambda \gtrsim 3$  based on the DOP approach and that based on the DFM approach discussed earlier by Beene *et al.* [40].

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