

Quantum Annealing for Distribution System Restoration via Resilient Microgrids Formation

Nima Nikmehr, *Senior Member, IEEE*, Peng Zhang, *Senior Member, IEEE*, Honghao Zheng, *Senior Member, IEEE* and Yacov A. Shamash, *Life Fellow, IEEE*

Abstract—Microgrids (MGs) formation enables distribution networks to enhance the resilience of the system after natural disasters and faults. In this paper, a quantum computing (QC) method is devised to resolve distribution grid restorations, which establishes a promising computational platform for grid resilience applications. The new method is based upon resilient MGs formation formulated as a combinatorial optimization problem to restore critical loads after natural disasters. Our main breakthrough consists of a quantum optimization model for resilient MGs formation and restoration, as well as a quantum annealing solution to combinatorially complex problems which are difficult for classical methods to tackle. To validate the efficacy of the quantum grid restoration/MGs formation, test results obtained by D-Waveare compared with those from classical solver, Gurobi.

Index Terms—Quantum computing, Quantum annealing, Microgrids formation, Load restoration, Microgrids

I. INTRODUCTION

Natural disasters can cause widespread blackouts on power grids with economic upheavals. Enhancing the resilience of power systems facing low-probability, high-impact events becomes an inevitable research [1]. To cope with the catastrophic impacts of natural disasters, microgrids (MGs) formation is a promising solution to restore critical loads [2] and the main grid. As an operational optimization problem modeled in the form of mixed-integer linear programming (MILP), however, MGs formation has been a daunting problem due to its combinatorial complexity [3], [4].

Most studies on critical load restoration and MGs formation take advantage of an MILP type of models, and then solve the problem using optimization solvers [5]–[7]. The mathematical model for MGs formation problem leads toward the combinatorial complexity, which cannot guarantee optimal solutions. Furthermore, the classical optimization solvers face difficulties in finding near-optimal or global optimal solution when the scale of the problem increases. To resolve these issues, quantum computing (QC) is found promising to solve a combinatorial problem including integer decision variables

This work was supported in part by the Advanced Grid Modeling Program under Department of Energy's Office of Electricity under Agreement No. 37533, and in part by the National Science Foundation under Grant Nos. OIA-2040599 and OIA-2134840. This research used resources of the Oak Ridge Leadership Computing Facility, which is a DOE Office of Science User Facility supported under Contract DE-AC05-00OR22725. This work relates to Department of Navy award N00014-20-1-2858 issued by the Office of Naval Research. The United States Government has a royalty-free license throughout the world in all copyrightable material contained herein.

N. Nikmehr, P. Zhang, and Y. A. Shamash are with the Department of Electrical and Computer Engineering, Stony Brook University, Stony Brook, NY, 11790 USA.

H. Zheng is with Smart Grid and Technology, Commonwealth Edison, Oakbrook, IL 60523 USA.

with many constraints [8], [9]. In [10], a thorough review of applications of quantum computing to power system problems is carried out, and the potential of QC in different areas, such as fundamental power analytics [11], [12], power system operation and optimization, power system stability and control, communication security, and AI/machine learning applications, is represented. As a first attempt to solve one of the important operational optimization problems in power systems, in [8], [9], distributed unit commitment problem is considered as an MILP optimization problem, and a quantum model of unit commitment is developed. Afterwards, a quantum version of alternating direction method of multipliers (ADMM) is developed to solve it in a distributed manner through decomposing the main problem into several subproblems, enabling current quantum machines to solve those combinatorial problems that are intractable for current classical computers. In [13], a combination of QC and Surrogate Lagrangian Relaxation (SLR) [14] allowed for solving large unit commitment problems.

Today's quantum computers are using either gate-based or annealing-based approaches. In the gate-based quantum machines, input qubits are initialized to quantum states, and after passing through unitary operators and quantum gates, the final state of qubit(s) are measured. This quantum path from input qubits to the measurement is called quantum circuit. According to IBM's QC roadmap, a Condor processor with 1,121 qubits is realizable by 2023, [15] and a 4,000+ qubit processor built with multiple clusters of modularly scaled processors is expected to be produced by 2025. However, current gate-based quantum devices face a major challenge in solving large-scale problems because of limitations on qubit numbers and connectivity. To resolve this issue, quantum annealers take advantage of quantum adiabatic evolution to solve combinatorial problems in the form of quadratic unconstrained binary optimization (QUBO). For instance, D-Wave's computers utilize quantum annealing (QA) to solve large QUBO problems using lattice of qubits [16].

The main contributions of this paper are listed as follows:

- A quantum-amenable model is developed for networked MGs formation-based load restoration problem in the form of QUBO, where Hamiltonian of the Ising model is solvable via quantum annealing.
- A quantum-amenable load restoration algorithm using MGs formation is successfully implemented and thoroughly verified with D-Wave quantum machines using D-Wave's hybrid quantum-classical solver.

II. PROBLEM FORMULATION

The main goal of networked microgrids formation is to alter the distribution system topology so that a maximum amount of

prioritized loads are picked up for fast restoration via controlling the line and load switches.

This paper focuses on the use of MGs formation to restore distribution networks, where the following assumptions are considered:

- Without loss of generality, only one aggregated distributed generation (DG) is assigned to each MG to supply the load.
- After a natural disaster, only DGs are responsible to energize the critical loads before restoring the main grids.

A. Networked MGs formation constraints

In this problem, the constraints are as follows:

1) *Belonging of nodes and lines to MGs*: Node $i \in B^*$ should only belong to one of the microgrids in the set M . To realize the node belonging constraint, a binary decision variable w_i should be defined to node $i \in B^*$. Node i belongs to MG m if $w_{im} = 1$, otherwise $w_{im} = 0$.

$$\sum_{m \in M} w_{im} = 1, i \in B^*. \quad (1)$$

Furthermore, if node $i \in B^*$ and node $\tilde{i} \in B^*$ are two nodes of MG m , the connected line $l_{i\tilde{i}}^m$ between nodes i and $\tilde{i}$ belongs to the same microgrid. Therefore,

$$l_{i\tilde{i}}^m = w_{im} \cdot w_{\tilde{i}m}, \quad \forall i, \tilde{i} \in B^*, m \in M \quad (2)$$

The equality constraint (2) can be converted to the following set of inequality constraints:

$$l_{i\tilde{i}}^m \leq w_{im}, \quad \forall i, \tilde{i} \in B^*, m \in M, \quad (3a)$$

$$l_{i\tilde{i}}^m \leq w_{\tilde{i}m}, \quad \forall i, \tilde{i} \in B^*, m \in M, \quad (3b)$$

$$l_{i\tilde{i}}^m \geq w_{im} + w_{\tilde{i}m} - 1, \quad \forall i, \tilde{i} \in B^*, m \in M. \quad (3c)$$

2) *Connectivity of nodes in each microgrid*: To satisfy the connectivity constraint in each microgrid, a parent-children node matrix should be established. A child node can belong to an MG if its parent belongs to the same MG. This constraint can be written as follows:

$$w_{im} \leq w_{\tilde{i}m}, \quad \forall i, \tilde{i} \in B^*, m \in M, \quad (4)$$

where, $w_{\tilde{i}m}$ is the belonging status of parent node $\tilde{i}$ to MG m .

3) *Energized loads constraint*: For an energized load in microgrid m , the load should be connected to node n of MG m . Therefore, the energized load status should be indicated as:

$$e_{im} = w_{im} \cdot s_i, \quad \forall i \in B^*, m \in M, \quad (5)$$

where, s_i is a binary decision variable, which indicates the connection status of i^{th} energized load to node $i \in B^*$ of microgrid m . If load i in microgrid m is connected to node i ($s_i = 1$) and node i belongs to MG m ($w_{im} = 1$), as a result the load is energized with the indication $e_{im} = 1$. Similar conversion as (3a)-(3c) should be applied to (5).

4) *MG operational constraint*: The linearized DistFlow model is used for power flow constraints [17]. The linearized DistFlow model can be written as follows [18]:

$$P_{i,m}^{flow} = \sum_{\hat{i}} P_{i-\hat{i},m}^{flow} + e_{i,m} \cdot P_{i,m}^L, \quad \forall i, \hat{i} \in B^*, m \in M, \quad (6a)$$

$$Q_{i,m}^{flow} = \sum_{\hat{i}} Q_{i-\hat{i},m}^{flow} + e_{i,m} \cdot Q_{i,m}^L, \quad \forall i, \hat{i} \in B^*, m \in M, \quad (6b)$$

$$V_{i,m} = V_{\tilde{i},m} - \frac{R_{i-\tilde{i}} \cdot P_{i,m}^{flow} + X_{i-\tilde{i}} \cdot Q_{i,m}^{flow}}{V_{0,m}}, \quad \forall i, \tilde{i} \in B^*, \quad (6c)$$

where, $P_{i,m}^{flow}$ and $Q_{i,m}^{flow}$ are real and reactive power in-flow at node i of MG m , respectively. Real and reactive powers $P_{i-\hat{i},m}^{flow}$ and $Q_{i-\hat{i},m}^{flow}$ represent the injected real and reactive power to children nodes $\hat{i}$ from node i in microgrid m , respectively. Moreover, $P_{i,m}^L$ and $Q_{i,m}^L$ are real and reactive loads, respectively. While the voltage at node i is denoted by V_i , $V_{0,m}$ describes the reference voltage at DG node in MG m . The resistance and reactance of distribution lines $i-\hat{i}$ are described by $R_{i-\hat{i}}$ and $X_{i-\hat{i}}$, respectively.

Additionally, other constraints are considered such that the in-flow real and reactive powers and nodes voltage are within a range between minimum and maximum values.

B. Objective function for networked MGs formation

The objective function is to maximize the load restoration after a disaster occurs. Different weights are assigned to loads, and loads with higher priority to restore should have larger weights (c). The weighted load restoration objective function is written as follows:

$$\max_{w,l,s,e,P_{flow},Q_{flow},V} \sum_{i \in B^*} \sum_{m \in M} c_i \cdot e_{im} \cdot P_{i,m}^L. \quad (7)$$

III. QUANTUM-AMENABLE RESILIENT MICROGRIDS FORMATION

In this section, a quantum-amenable model of resilient MG formation is developed.

A. Quantum Optimization Procedure

Quantum annealing is an optimization algorithm that is used by adiabatic quantum computers such as D-Wave devices. The QA algorithm is mainly used to find the near-optimal or even global optima of combinatorial optimization problems with non-continuous decision variables [16], [19]. The quantum processors units (QPUs) of D-Wave computers in quantum annealing optimization procedure is adaptable to solve quadratic unconstrained binary optimization (QUBO) problems. A classical QUBO problem can be defined as follows:

$$x = \underset{x}{\operatorname{argmin}} (x^T Q x + C x), \quad (8)$$

where, x is an $N \times 1$ vector of binary decision variables. Real-valued matrices Q and C are $N \times N$ and $1 \times N$ vectors, respectively.

In an optimization problem, finding the ground state of an Ising Hamiltonian is equivalent to the minimum value of the corresponding objective function [20]. To achieve the Ising model of a QUBO problem, a graph $G = (V, E)$ is used. In [8], the steps of building the Hamiltonian Ising model using vertices and edges of an graph has been explained in detail.

According to the graph and explained steps in [8], we can formulate the Hamiltonian Ising model $\mathcal{H}$ as follows:

$$\mathcal{H} = \sum_{k=1}^N \sum_{j=1, j \neq k}^N L_{kj} \cdot \xi_k \cdot \xi_j + \sum_{k \in V} q_k \cdot \xi_k. \quad (9)$$

where, the first term of Ising model in (9) indicates the interaction/coupling between two qubits k and j through the edge between them (L_{kj}) while the second term has to do with the local field caused by external magnetic q with a spin ξ . To map the classical QUBO problem to the Hamiltonian Ising model, the binary decision variables $x \in \{0, +1\}$ in (8) should be replaced by spin variables $\xi \in \{-1, +1\}$ in (9). To fulfill this conversion, the transformation $x = \frac{\xi+1}{2}$ is employed. This conversion results in:

$$L_{kj} = \frac{1}{4} Q_{kj}, \quad \forall (k, j) \in E, \quad (10a)$$

$$q_k = \frac{1}{2} \left(C_k + \sum_j Q_{kj} \right), \quad \forall k \in V. \quad (10b)$$

A system Hamiltonian with the know ground state is initialized and prepared as follows:

$$\mathcal{H}_0 = \sum_{k=1}^N \sigma_k^x, \quad (11)$$

where, Pauli-x operator $\sigma_k^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is applied to qubit k .

The system Hamiltonian is gradually move toward problem Hamiltonian $\mathcal{H}_p$, which is written as follows:

$$\mathcal{H}_p = \sum_{k=1}^N \sum_{j=1, j \neq k}^N L_{kj} \cdot \sigma_k^z \cdot \sigma_j^z + \sum_{k \in V} q_k \cdot \sigma_k^z, \quad (12)$$

where, Pauli-z operator $\sigma_k^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is applied to qubit k .

The problem Hamiltonian $\mathcal{H}_p$ is the classical Ising model in (9), and therefore, the solution of the Ising model is achieved by finding the eigenvectors of corresponding problem Hamiltonian.

According to the adiabatic theorem of quantum mechanics, if the required transition (annealing) time from $\mathcal{H}_0$ to $\mathcal{H}_p$ is sufficiently slow, the system guarantees the ground state, meaning that the optimal solution is achieved [21], [22].

B. Quantum Model for Resilient Microgrids Formation

In this subsection, the quantum-amenable resilient MGs formation model is developed and the optimization solution using quantum annealing (QA) in a D-Wave machine is discussed.

Objective Function. The objective function (7) should be mapped to the Ising model by converting the binary variables into spins using the transformation $x = \frac{\xi+1}{2}$:

$$\max_{\xi} \mathcal{H}_{obj} : \max_{\xi} \sum_{i \in B^*} \sum_{m \in M} \sum_{k \in V} c_i \cdot \frac{1 + \xi_{im,k}^e}{2} \cdot P_{i,m}^L, \quad (13)$$

where, $\xi_{im,k}^e$ is the spin of vertex (qubit) k associated with energized load i in MG m .

Node Belonging Constraint. The constraint Hamiltonian associated with classical constraint (1) is written as follows:

$$\mathcal{H}_{c1} = \lambda_1 \left(\sum_{k \in V} \sum_{m \in M} \frac{1 + \xi_{im,k}^w}{2} - 1 \right)^2, \quad i \in B^*, \quad (14)$$

where, $\xi_{im,k}^w$ is the spin of vertex (qubit) k for node i in MG m . Furthermore, since the problem should be in unconstrained form, the existing constraints should be added to the objective function in a quadratic form by a penalty coefficient. Hence, λ_1 is the penalty coefficient of node belonging constraint.

Line Belonging Constraint. The line belonging constraint (3a-3c) is transformed to the Hamiltonian model as follows:

$$\mathcal{H}_{c2} = \lambda_2 \left(\sum_{(k,j) \in E} \frac{1 + \xi_{im,(k,j)}^l}{2} - \sum_{k \in V} \frac{1 + \xi_{im,k}^w}{2} \right)^2, \quad (15a)$$

$$\mathcal{H}_{c3} = \lambda_3 \left(\sum_{(k,j) \in E} \frac{1 + \xi_{im,(k,j)}^l}{2} - \sum_{k \in V} \frac{1 + \xi_{im,k}^{\bar{w}}}{2} \right)^2, \quad (15b)$$

$$\mathcal{H}_{c4} = \lambda_4 \left(\sum_{k \in V} \frac{1 + \xi_{im,k}^w}{2} + \sum_{k \in V} \frac{1 + \xi_{im,k}^{\bar{w}}}{2} - \sum_{(k,j) \in E} \frac{1 + \xi_{im,(k,j)}^l}{2} - 1 \right)^2, \quad \forall i, \tilde{i} \in B^*, \quad m \in M, \quad (15c)$$

where, $\xi_{im,(k,j)}^l$ is the spin of edge or distribution line l between nodes/vertices k and j in MG m . $\xi_{im,k}^w$ is the spin of vertex (qubit) k for node i in MG m . Moreover, the penalty terms are indicated by λ_2 , λ_3 and λ_4 .

Node Connectivity Constraint. To ensure the connectivity of nodes in a microgrid, classical constraint (4) should be converted to the following constraint Hamiltonian:

$$\mathcal{H}_{c5} = \lambda_5 \left(\sum_{k \in V} \frac{1 + \xi_{im,k}^w}{2} - \sum_{k \in V} \frac{1 + \xi_{im,k}^{\bar{w}}}{2} \right)^2, \quad \forall i, \hat{i} \in B^*, \quad (16)$$

where, λ_5 is the penalty term associated with node connectivity constraint.

Energized Loads Constraint. Similar to the Hamiltonian of (15a)-(17c), the Hamiltonian model of energized loads constraint can be rewritten by assigning penalty coefficients λ_6 , λ_7 and λ_8 .

DistFlow Constraint. The Hamiltonian of DistFlow model is written as follows:

$$\mathcal{H}_{c9} = \lambda_9 \left(P_{i,m}^{flow} - \sum_{\hat{i}} P_{i-\hat{i},m}^{flow} - \sum_{k \in V} \frac{1 + \xi_{im,k}^e}{2} \cdot P_{i,m}^L \right)^2, \quad (17a)$$

$$\mathcal{H}_{c10} = \lambda_{10} \left(Q_{i,m}^{flow} - \sum_{\hat{i}} Q_{i-\hat{i},m}^{flow} - \sum_{k \in V} \frac{1 + \xi_{im,k}^e}{2} \cdot Q_{i,m}^L \right)^2, \quad (17b)$$

where, λ_9 and λ_{10} are penalty terms used for DistFlow constraint.

Line Flow Constraint. In-flow real and reactive powers have the similar Hamiltonian model as Distflow and line belonging constraints with the penalty coefficients of line flow constraint represented by λ_{11} , λ_{12} and λ_{13} .

After mapping the QUBO objective function and constraints to a Hamiltonian model, the following problem Hamiltonian or total energy function is constructed to find the minimum energy of the system using QA:

$$\mathcal{H}_p = -\mathcal{H}_{obj} + \sum_{d \in D} \mathcal{H}_{cd}, \quad (18)$$

where, D is the total number of constraints Hamiltonian. Since the classical optimization problem in (7) is a maximization problem, the sign of $\mathcal{H}_{obj}$ is changed to negative to be considered as a minimization problem solvable by QA.

IV. NUMERICAL RESULTS

In this section, a modified IEEE 37-bus distribution network [23] is selected as a test system to show the efficacy of quantum annealing solving the MILP problem of the grid restoration and MGs formation. The single-line diagram of modified IEEE 37-bus distribution system is depicted in Fig. 1.

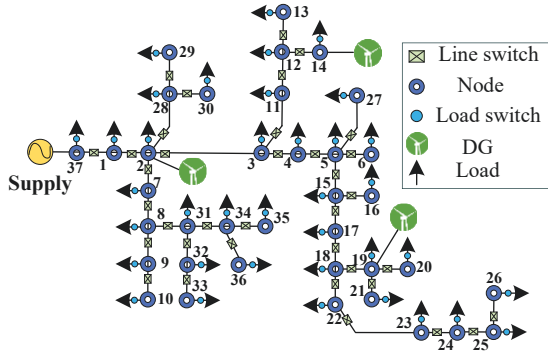


Fig. 1. The single-line diagram of modified IEEE 37-bus distribution system.

In this case study, there exist three DGs at nodes 2, 14 and 19 with the maximum real power generation capacities of 252.53 kW, 120.42 kW, and 202.99 kW, as well as maximum VAR capacities of 46.31 kVar, 171.72 kVar, and 197.48 kVar, respectively. Furthermore, the line switches and load switches can be in either *on* (1) or *off* (0) mode. According to the objective function, each load has a priority weight (c) to be restored once a disaster happens. The load weights along with real and reactive loads amount are adopted from [24].

The new topology of the distribution system Fig. 1 after MGs formation upon the occurrence of a disaster is shown in Fig. 2.

In Fig. 2, after the blackout occurs, three lines become unavailable that is the lines switch are turned into the off mode. These lines are located between nodes 1-37, 2-28, and 8-31. The rest of the grid is considered for the MGs formation to restore the loads. As a result of MGs formation, three MGs are established. Each MG is empowered with a DG to feed its loads. In Table I, the node belonging status of each node to an MG is described. According to the results of resilient MGs formation shown in Fig. 2 and Table I, the switching mode of loads and lines leads toward belonging of nodes and lines for

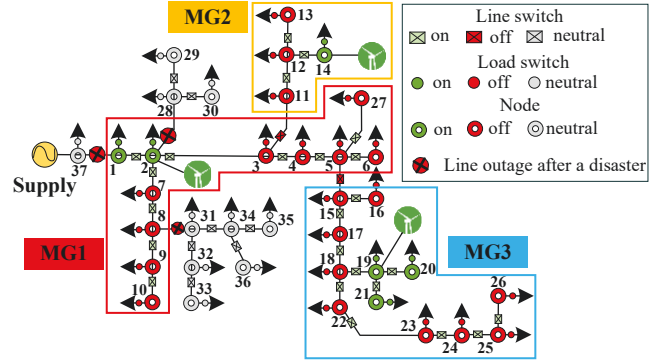


Fig. 2. Microgrids formation after a natural disaster.

TABLE I
BELONGING OF SYSTEM NODES TO MGs AND ENERGIZED STATUS OF NODES AFTER RESILIENT MGs FORMATION

MG	Nodes	Energized nodes
MG1	1,2,3,4,5,6,7,8,9,10,27	1,2
MG2	11,12,13,14	14
MG3	15,16,17,18,19,20,21,22,23,24,25,26	19,20,21

each MG. Furthermore, the load energized following the disaster is described for MGs.

The minimum energy level of the system is -700.4632 using D-Wave's hybrid quantum-classical solver, which is the same as minimum amount of objective function obtained by the classical solver Gurobi.

In MG1, the real power injections from nodes 1 and 2 are 30.4 kW and 49.01 kW, respectively, while the reactive power injections are 5.09 kVar and 28.69 kVar, respectively. In MG2, the real and reactive power outputs are 41.18 kW and 7.45 kVar, respectively. Similarly, in MG3, nodes 19, 20 and 21 send 60.14 kW, 35.12 kW and 12.45 kW, respectively. Moreover, the VAR injections from aforementioned nodes are 41.49 kVar, 19.28 kVar and 13.76 kVar, respectively.

In Fig. 3, the energy level of the problem's objective function is depicted. In a D-Wave quantum computer, low-energy states of a problem's objective function are sampled by samplers. The hybrid constrained quadratic model (CQM) sampler is utilized to sample the low-energy states of the critical load restoration problem. According to Fig. 3(a), D-Wave's quantum-classical hybrid CQM solver returns the number of samples for each achieved energy level or solution. The hybrid CQM sampler sampled from 47 solution. Because of the probabilistic nature of solutions obtained by D-Wave's hybrid solver, the distribution of solutions are depicted as a box plot in Fig. 3(b). In this distribution, minimum and maximum values of objective function are -4017.5212 and 0, respectively. The median value of load restoration-oriented objective function is -700.4632 with a probability of occurrence at 42.5%. Furthermore, the first quartile value (Q1 or 25th percentile) is -1531.2285.

The superposition and tunneling features of D-Wave quantum machines enable the hybrid solver to return a combination of feasible and infeasible samples. In the feasible solutions of load restoration/MGs formation problem, all the constraints (14)-(17) along with line flow constraints are satisfied. The feasibility of samples from annealing-based quantum computing is shown in Fig. 4.

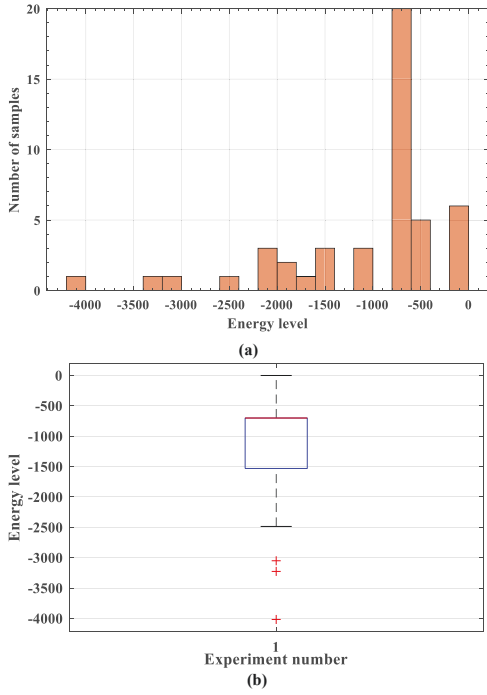


Fig. 3. Energy level of the system. (a) Energy level of the problem for different samples of the problem's objective function. (b) Box plot diagram of the energy level indicating the distribution of energy level of the objective function.

The MGs formation-based load restoration problem takes 0.032s to be solved by the quantum processing unit (QPU) while the classical solver Gurobi needs 0.055s.

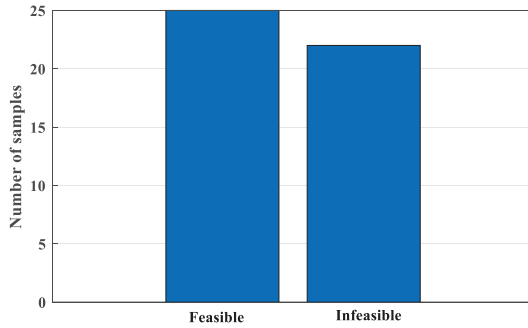


Fig. 4. The feasibility of samples from the sampled low-energy states.

Large number of qubits along with high connectivity between qubits are the key factors in solving large scale load restoration/MGs formation optimization problem using today's D-Wave hybrid solvers incorporating one million decision variables and 100,000 constraints.

V. CONCLUSION

Combinatorially complex operational optimization problems including non-continuous decision variables are good candidates for annealing-based quantum computing to overcome the barriers of current classical methods. In this paper, by leveraging quantum optimization methods, the quantum resilient MGs formation and load restoration problem is developed by deriving the Ising Hamiltonian model. To restore the critical loads in the distribution system, MGs formation is considered as an effective strategy where the switching status (0/1) of distribution lines and loads were deciding factors in keeping the critical

loads energized after a natural disaster occurs. D-Wave's hybrid quantum-classical solver has been used to solve a case study with 37-bus and 3 MGs considering versatile constraints. The results obtained by quantum annealing illustrated the accuracy of this computing platform compared to the classical solver.

REFERENCES

- [1] X. Liu, M. Shahidehpour, Z. Li, X. Liu, Y. Cao, and Z. Bie, "Microgrids for enhancing the power grid resilience in extreme conditions," *IEEE Transactions on Smart Grid*, vol. 8, no. 2, pp. 589–597, 2016.
- [2] L. Che and M. Shahidehpour, "Adaptive formation of microgrids with mobile emergency resources for critical service restoration in extreme conditions," *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 742–753, 2018.
- [3] J. Li, X.-Y. Ma, C.-C. Liu, and K. P. Schneider, "Distribution system restoration with microgrids using spanning tree search," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 3021–3029, 2014.
- [4] Y. Xu, C.-C. Liu, K. P. Schneider, F. K. Tuffner, and D. T. Ton, "Microgrids for service restoration to critical load in a resilient distribution system," *IEEE Transactions on Smart Grid*, vol. 9, no. 1, pp. 426–437, 2016.
- [5] Y. Xu and W. Liu, "Novel multiagent based load restoration algorithm for microgrids," *IEEE Transactions on Smart Grid*, vol. 2, no. 1, pp. 152–161, 2011.
- [6] Y. Li, J. Xiao, C. Chen, Y. Tan, and Y. Cao, "Service restoration model with mixed-integer second-order cone programming for distribution network with distributed generations," *IEEE Transactions on Smart Grid*, vol. 10, no. 4, pp. 4138–4150, 2018.
- [7] Y. Huang, G. Li, C. Chen, Y. Bian, T. Qian, and Z. Bie, "Resilient distribution networks by microgrid formation using deep reinforcement learning," *IEEE Transactions on Smart Grid*, 2022.
- [8] N. Nikmehr, P. Zhang, and M. A. Bragin, "Quantum distributed unit commitment: An application in microgrids," *IEEE Transactions on Power Systems*, vol. 37, no. 5, pp. 3592–3603, 2022.
- [9] N. Nikmehr, P. Zhang, and M. Bragin, "Quantum-enabled distributed unit commitment," in *2022 IEEE Power Energy Society General Meeting*, 2022, pp. 1–5.
- [10] Y. Zhou, Z. Tang, N. Nikmehr, P. Babahajiani, F. Feng, T.-C. Wei, H. Zheng, and P. Zhang, "Quantum computing in power systems," *iEnergy*, 2022.
- [11] N. Nikmehr and P. Zhang, "Quantum-inspired power system reliability assessment," *IEEE Transactions on Power Systems*, pp. 1–14, 2022.
- [12] N. Nikmehr and P. Zhang, "Quantum distribution system reliability assessment," in *2022 IEEE Power Energy Society General Meeting*, 2022, pp. 1–5.
- [13] F. Feng, P. Zhang, M. A. Bragin, and Y. Zhou, "Novel resolution of unit commitment problems through quantum surrogate lagrangian relaxation," *IEEE Transactions on Power Systems*, 2022.
- [14] M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, and G. A. Stern, "Convergence of the surrogate lagrangian relaxation method," *Journal of Optimization Theory and applications*, vol. 164, no. 1, pp. 173–201, 2015.
- [15] J. Gambetta, "IBM's roadmap for scaling quantum technology," Available at: <https://research.ibm.com/blog/ibm-quantum-roadmap>, 2020.
- [16] T. Kadowaki and H. Nishimori, "Quantum annealing in the transverse ising model," *Physical Review E*, vol. 58, no. 5, p. 5355, 1998.
- [17] S. Tan, J.-X. Xu, and S. K. Panda, "Optimization of distribution network incorporating distributed generators: An integrated approach," *IEEE Transactions on power systems*, vol. 28, no. 3, pp. 2421–2432, 2013.
- [18] Z. Wang, H. Chen, J. Wang, and M. Begovic, "Inverter-less hybrid voltage/var control for distribution circuits with photovoltaic generators," *IEEE Transactions on Smart Grid*, vol. 5, no. 6, pp. 2718–2728, 2014.
- [19] E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, "Quantum computation by adiabatic evolution," *arXiv preprint quant-ph/0001106*, 2000.
- [20] T. Albash and D. A. Lidar, "Adiabatic quantum computation," *Reviews of Modern Physics*, vol. 90, no. 1, p. 015002, 2018.
- [21] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, "A quantum adiabatic evolution algorithm applied to random instances of an np-complete problem," *Science*, vol. 292, no. 5516, pp. 472–475, 2001.
- [22] S. Boixo, T. Albash, F. M. Spedalieri, N. Chancellor, and D. A. Lidar, "Experimental signature of programmable quantum annealing," *Nature communications*, vol. 4, no. 1, pp. 1–8, 2013.
- [23] W. H. Kersting, "Radial distribution test feeders," *IEEE Transactions on Power Systems*, vol. 6, no. 3, pp. 975–985, 1991.
- [24] C. Chen, J. Wang, F. Qiu, and D. Zhao, "Resilient distribution system by microgrids formation after natural disasters," *IEEE Transactions on smart grid*, vol. 7, no. 2, pp. 958–966, 2015.