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PROCEEDING

Noncommutative branch-cut quantum gravity with a self-coupling inflation scalar field: Dynamical equations

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Abstract

This article focuses on a recently developed formulation based on the noncommutative branch-cut cosmology, the Wheeler-DeWitt (WdW) equation, the Hořava–Lifshitz quantum gravity, chaotic and the coupling of the corresponding Lagrangian approach with the inflaton scalar field. Assuming a mini-superspace of variables obeying the noncommutative Poisson algebra, we examine the impact of the inflaton scalar field on the evolutionary dynamics of the branch-cut Universe scale factor, characterized by the dimensionless helix-like function $\ln^{-1}[\beta(t)]$. This scale factor characterizes a Riemannian foliated spacetime that topologically overcomes the primordial singularities. We take the Hořava–Lifshitz action modeled by branch-cut quantum gravity as our starting point, which depends on the scalar curvature of the branched Universe and its derivatives and which preserves the diffeomorphism property of General Relativity, maintaining compatibility with the Arnowitt–Deser–Misner formalism. We then investigate the sensitivity of the scale factor of the branch-cut Universe's dynamics.

KEYWORDS

branch cut cosmology, scalar field, inflation

1 | INTRODUCTION

In a recent publication included in the present volume (Hess et al. 2024), we examine the implications of a noncommutative formulation of the branch-cut quantum gravity on the wave function of the Universe. Based on a mini-superspace structure that obeys the noncommutative Poisson algebra, combined with the branch-cut gravity (Bodmann et al. 2022; Bodmann, et al. 2023a, 2023b; Hess et al. 2022; Weber et al. 2024a, 2024b; Zen Vasconcellos et al. 2019; Zen Vasconcellos et al. 2021a, 2021b; Zen Vasconcellos et al. 2022), the Wheeler-DeWitt equation DeWitt (1967) and Hořava–Lifshitz quantum gravity (Hořava 2009), we investigate the impact of a scalar field of the inflaton-type in the evolution of the Universe's wave function. The results indicate that the noncommutative algebraic structure captures small and large spacetime scales, driving the exponential acceleration of the Universe.

In this work, we continue this study by examining the impact of this formulation on the spacetime evolution of the scale factor of the branched Universe.

2 | HOŘAVA–LIFSHITZ BRANCH-CUT HAMILTONIAN

The starting point of our formulation is the noncommutative branch-cut Hořava–Lifshitz canonical super-Hamiltonian equation for the wave function of the Universe (for the details, see (Bodmann, Hadjimichef et al., 2023a; Hess et al. 2024)),

$$\begin{aligned}\mathcal{H}(\xi,\eta)\Psi(\xi,\eta) = \frac{1}{2}\frac{N}{\eta}\Big[& -p^2_{\eta,\gamma,\alpha} + g_r - g_m\eta - g_k\eta^2 - g_q\eta^3 \\ & + g_\Lambda\eta^4 + \frac{g_s}{\eta^2} + \frac{\alpha}{\eta^{3\alpha-2}} - \frac{\alpha\xi}{\eta^{3\alpha-1}} + \frac{1}{\eta^{3\alpha-1}}p_\xi\Big] \\ & \Psi(\xi,\eta) = 0,\end{aligned} \tag{1}$$

where $-p^2_{\eta,\gamma,\alpha}$ is defined as

$$\begin{aligned}-p^2_{\eta,\gamma,\alpha} &\equiv \frac{\partial^2}{\partial\eta^2} + \frac{\gamma}{\eta^{3\alpha-1}}\frac{\partial}{\partial\eta} \\ &= -\left(-i\frac{\partial}{\partial\eta}\right)\left(-i\frac{\partial}{\partial\eta}\right) + \frac{i|\gamma|}{\eta^{3\alpha-1}}\frac{\partial}{\partial\eta} \equiv -p^2_\eta - p_{\eta,\gamma,\alpha},\end{aligned} \tag{2}$$

with

$$p_{\eta,\gamma,\alpha} \equiv -\frac{i\mid\gamma\mid}{\eta^{3\alpha-1}}\frac{\partial}{\partial\eta} \mapsto \frac{\gamma}{\eta^{3\alpha-1}}\left(-i\frac{\partial}{\partial\eta}\right) = \frac{\gamma}{\eta^{3\alpha-1}}p_\eta. \tag{3}$$

In order to reach this super-Hamiltonian, the noncommuting variables and momenta are mapped to new commuting variables and momenta. The expressions of the noncommuting variables and momenta in terms of the commuting ones are substituted into the Hořava–Lifshitz Hamiltonian, giving rise to (1). (In Bodmann, Hadjimichef, et al., 2023a; Hess et al. 2024, the new commuting variables were assigned a tilde above, which is now skipped.) As a consequence of these steps, new interaction terms appear, which were not present before in the commuting case.

In expression (1), g_i represents the following running coupling constants: g_k, g_Λ, g_r, and g_s which denote respectively the curvature, cosmological constant, radiation, and stiff matter running coupling constants (Bertolami & Zarro 2011; Maeda et al. 2010), $g_m u$ in turn describes the contribution of baryon matter combined with dark matter, and $g_q u^3$ corresponds to the quintessence contribution. The g_r, and g_s running coupling constants can be positive or negative, without affecting the stability of the solutions. Stiff matter contribution is determined by the $p = \omega\rho$ equation of state condition. A stiff matter era was introduced in the Zel'dovich's cosmological model (Zel'dovich 1972) where the primordial universe was assumed to be made of a cold gas of baryons. It may also characterize dark matter made of a relativistic self-gravitating Bose-Einstein condensates (BECs) (Chavanis 2015). The parametrization of curvature, cosmological constant, radiation, stiff matter, baryon matter combined with dark matter, and quintessence running coupling constants are in tune with the Wilkinson Microwave Anisotropy Probe (WMAP) observations (Hinshaw et al. 2013). The solutions of the WdW equation, represented by a geometric function of compact manifolds and matter fields, describe the evolution of the quantum wave function of the Universe (Hartle & Hawking 1983; Hawking 1982), $\Psi(\eta,\xi)$. This expression is defined here in terms of the variables $\eta(t)$ that characterizes the scale factor of the branch-cut cosmology and its dual complementary counterpart, represented by $\xi(t)$.

3 | MINI-SUPERSPACE OF VARIABLES

We consider in this section an extended mini-superspace of variables $(\eta(t), \xi(t), \phi(t))$, obeying the noncommutative Poisson algebra, where $\phi(t)$ represents the scalar inflaton quantum field.

We adopt for the action of the scalar field the following expression (Kiritsis & Kofinas 2009; Tavakoli et al. 2021)

$$S_\phi = \int_{\mathcal{M}} d^3x\,dt\,N\sqrt{g}\Big[\frac{1}{N^2}\big(\dot{\phi} - N^i\partial_i\phi\big)^2\Big]. \tag{4}$$

Assuming homogeneous and isotropic cosmological settings we have $N_i = 0$ (Kiritsis & Kofinas 2009; Tavakoli et al. 2021), and the action of the scalar field $\phi(t)$, given by expression (4), may be written as

$$S_\phi = \int_{\mathcal{M}} d^3x\,dt\,N\sqrt{g}\left(\frac{1}{2}\frac{1}{N^2}F(\phi)\dot{\phi}^2\right), \tag{5}$$

where $F(\phi)$ represents a coupling function.

The conjugate momenta to the dynamical variables $(\eta(t), \xi(t), \phi(t))$ can be obtained by definition as $p_q = \partial L/\partial \dot{q}$, where L defines the total Lagrangian of the system, resulting in

$$p_\eta = -\frac{1}{N}\eta\eta'; p_\xi = -\frac{1}{N}\xi\xi'; \quad \text{and} \quad p_\phi = \frac{1}{N}F(\phi)\eta^3\phi'. \tag{6}$$

From the total action determined by adding the Hořava–Lifshitz and the scalar field actions, the Hamiltonian associated with the mini-superspace of variables may be obtained:

$$\begin{aligned}\mathcal{H} = &\frac{N}{2\eta}\left[-p_\eta^2 - \frac{\gamma}{\eta^{3\alpha-1}}p_\eta + g_r - g_m\eta - g_k\eta^2 - g_q\eta^3 + g_\Lambda\eta^4 + \frac{g_s}{\eta^2}\right] \\ &+ \frac{N}{2\eta}\left[\frac{\alpha}{\eta^{3\alpha-2}} - \frac{\alpha\xi}{\eta^{3\alpha-1}} + \frac{1}{\eta^{3\alpha-1}}p_\xi\right], \\ &+ \frac{N}{2\eta}\left[\frac{1}{\eta^{3\omega-1}F(\phi)}p_\phi^2\right].\end{aligned} \tag{7}$$

Adopting usual canonical quantization procedures, we then promote the canonical conjugate momenta p_η, p_ξ, and p_ϕ into operators

$$p_\eta \to -i\frac{\partial}{\partial\eta}; \quad p_\xi \to -i\frac{\partial}{\partial\xi}; \quad \text{and} \quad p_\phi \to -i\frac{\partial}{\partial\phi}. \tag{8}$$

3.1 | Hamilton equations

Hamilton equations may be synthesized in the form

$$\eta' = \frac{\partial\mathcal{H}}{\partial p_\eta} \quad \text{and} \quad p_\eta' = -\frac{\partial\mathcal{H}}{\partial\eta}, \tag{9}$$

$$\xi' = \frac{\partial\mathcal{H}}{\partial p_\xi} \quad \text{and} \quad p_\xi' = -\frac{\partial\mathcal{H}}{\partial\xi}, \tag{10}$$

and

$$\phi' = \frac{\partial\mathcal{H}}{\partial p_\phi} \quad \text{and} \quad p_\phi' = -\frac{\partial\mathcal{H}}{\partial\phi}. \tag{11}$$

Combining these equations with (7), we obtain the following Hamilton equations:

$$\begin{aligned}&\eta' = -\frac{p_\eta}{\eta} - \frac{\gamma}{2\eta^{3\alpha}}; \quad \xi' = \frac{1}{2\eta^{3\alpha}}; \quad p_\xi' = \frac{\alpha}{2\eta^{3\alpha}}; \\ &\phi' = N\frac{p_\phi}{\eta^{3\omega}F(\phi)}; \quad p_\phi' = \frac{N}{2}\left[\frac{p_\phi^2}{u^{3\omega}}\frac{F'(\phi)}{F^2(\phi)}\right],\end{aligned} \tag{12}$$

and

$$\begin{aligned}p_\eta' = &-\frac{N}{2\eta}\left[\frac{1}{\eta}p_\eta^2 + \frac{3\alpha\gamma}{\eta^{3\alpha}}p_\eta - \frac{g_r}{\eta} - g_k\eta - 2g_q\eta^2 + 3g_\Lambda\eta^3 - 3\frac{g_s}{\eta^3}\right] \\ &+ \frac{N}{2\eta}\left[\frac{\alpha(3\alpha-1)}{\eta^{3\alpha-1}} - \frac{3\alpha^2 t}{2\eta^{6\alpha-1}} + \frac{3\omega}{u^{3\omega}}\frac{p_\phi^2}{F(\phi)}\right].\end{aligned} \tag{13}$$

We have here taken an explicit time-dependence on the variables η and ξ, so

$$\xi = \int dt\dot{\xi} = \frac{t}{2\eta^{3\alpha}}, \quad p_\xi = \int dt\dot{p}_\xi = -\frac{\alpha t}{2\eta^{3\alpha}}. \tag{14}$$

Assuming $\omega = 1$ and the time gauge $N = \eta^n(t)$, by eliminating p_ϕ on the ϕ' and p_ϕ' equations above, we obtain

$$\frac{\phi''(t)}{\phi'(t)} - \frac{1}{2}\frac{F'(\phi)}{F(\phi)}\phi'(t) + \frac{F'(\phi)}{F(\phi)} + (3-n)\frac{\eta'(t)}{\eta(t)} = 0. \tag{15}$$

Integrating this equation we obtain

$$\log\left(\frac{\phi'}{\sqrt{F(\phi)}}F(\phi)^{\phi'(t)}\right) = \log\left(\eta(t)^{n-3}\right). \tag{16}$$

Expanding $F(\phi)^{\phi'(t)}$, around $t = 0$, it results

$$\begin{aligned}F(\phi)^{\phi'(t)} = &F(\phi)^{\phi'(0)}\{1 + \log(F(\phi))\phi''(0)F(\phi)) \\ &+ \tfrac{1}{2}\log(F(\phi))\left(\log(F(\phi))\phi''(0)^2 + \phi''(0)^{(3)}\right) \\ &F^2(\phi)\ \dots\}\end{aligned} \tag{17}$$

Adopting the boundary condition $\phi'(0) = 1$, we obtain from the expression above, in the first order of $\phi(t)$ time derivative (for convergence reasons) (see for instance Tavakoli et al. (2021)),

$$\phi'^2(t)F(\phi) = \mathcal{K}\eta(t)^{2(n-3)}, \tag{18}$$

where $\mathcal{K}$ represents an integration constant.

In what follows, we use Cauchy's implicit function theorem, assuming that the variable η has an explicit time dependence, so the variables η and t can be separated, thus ensuring that these variables are differentiable. We also assume, for simplicity, in view of the high difficulty to solve the above equations, that η' and η act as independent variables.

Combining Equations (13) and (18), the following expression follows:

$$2\eta(t)\eta''(t) + \eta'^2(t) + \frac{3\alpha\gamma\eta'(t)}{\eta^{3\alpha}(t)} + V(\eta, t) = 0, \tag{19}$$

where the potential $V(\eta, t)$ has an explicit time dependence:

$$V(\eta,t)=g_k+2g_q\eta(t)-3g_\Lambda\eta^2(t)+\frac{g_r}{\eta^2(t)}+3\frac{g_s}{\eta^4(t)}+\frac{\alpha(3\alpha-1)}{\eta^{3\alpha}(t)}-\frac{3\alpha^2 t}{\eta^{6\alpha-1}(t)}+\frac{\mathcal{K}}{\eta^4(t)}. \tag{20}$$

Comparing Equation (19) with the corresponding equation obtained through the commutative formulation (Weber et al. 2024a, 2024b) brings to light the formal structural differences between the two approaches:

$$2u(t)u''(t)+u'(t)^2+g_k+2g_q u(t)-3g_\Lambda u^2(t)+\frac{g_r}{u^2(t)}+3\frac{g_s+\mathcal{K}}{u^4(t)}=0, \tag{21}$$

in which $\alpha=\gamma=\chi=0$.

4 | RESULTS

Figure 1 shows a graphical representation of Equation (20). In the contour graphs, color variations indicate the behavior of the potential $V(\eta, t)$, with regions of greater amplitudes (lighter colors) or, conversely, of smaller amplitudes (darker colors) indicating the different intensities of the potential in function of the variables $\eta(t)$ and t. A word of caution regarding the meaning of intensity: greater intensity means in our conception effects that may increase the dynamical behavior of the branch-cut cosmology scale factor, driving the acceleration of the Universe, while lower intensity means the opposite. Therefore, a region where a singularity predominates is, in this sense, a region of lower intensity from a dynamical point of view and therefore with a lower probability of driving the acceleration of the Universe. In this sense, a region of this type may drive the opposite effect and can lead to a big-crunch effect. The blue color signifies the region with lower values of potential intensity, normally around a singularity or a region where decelerating effects of the branch-cut Universe predominate from a gravitational point of view. The cream color in turn designates regions with higher values, where the region of greatest impulse of the primordial Universe predominates. The white regions, although not always clearly visible in some figures, correspond to the exact region where the singularity that increases the probability to generate the big-crunch effects predominates.

It is important to remember that although branch-cut cosmology may overcome the main problem of the primordial singularity, the divergence of the solutions, the model does not eliminate it. What the model does is transform the primordial singularity into a branch-point and describe a foliated region around the branch-point, a branch-cut foliated family of Riemann surfaces in the form $\ln^{-1}[\beta(t)]$, which comprehends the BCG scale factor, and which in this contribution is (partially) described by the function $\eta(t)$. These Riemann surfaces correspond to stacks of cut complex planes, joined along the cut. The term used, "partially", is due to the fact that, in the noncommutative formulation, the transformation of variables that leads to a canonical equation that describes the system's Hamiltonian makes the variables $\eta(t)$ and $\xi(t)$ into dual, complementary variables. Thus, the dual complementary character of these variables makes the structurally descriptive characteristics of the original form factor, $\ln[\beta(t)]$, are shared between the two new variables, although it is still an open problem to determine the extent and meaning of this sharing.

In short, the scale factor $\ln^{-1}[\beta(t)]$ corresponds to a single-valued and holomorphic function, except at branch point helix-like superposition of cut planes, with upper edge of cut in n-th plane joined with lower edge of cut in $(n+1)$-th plane. And BCG describes a map of solutions corresponding to equipotential regions that make possible overcoming the primordial singularity that would otherwise be inescapable in the standard model. The model implies that, as one of these solutions approaches the branch-point, there is a jump of 2π from a Riemann sheet, in a continuous and coordinated manner, to a new family of solutions but contained in another Riemann sheet. Evidently, the physical and cosmological meaning of this jump requires additional studies so that its meaning can be understood in more depth. Two possibilities in the first approach stand out. In the first, the succession of Riemann leaves could represent the evolutionary process of the Universe in the branch-cut composition. On the other hand, each Riemann sheet could represent a Universe from a composition of multiverses, tightly connected.

Back to the potential $V(\eta, t)$, the plotting results indicate that the noncommutative algebraic structure contributes in an intense way to the configuration and distribution of matter in the early Universe, driving the acceleration of the primordial Universe.

Figure 2 shows typical sample family solutions of the evolution in time of the scalar factor $\eta(t)$ given by Equation (19). On the left figure, the solutions correspond to sampling $\eta'(-1)$ and boundary condition $\eta(-1)=-1$. The solutions on the right figure correspond to sampling $\eta'(1)$ and boundary condition $\eta(1)=1$. Figure shows the dramatic and intense acceleration of the Universe through the highest exponential growth of the scale factor $\eta(t)$. In our conception, it is exactly the potential that describes the presence of matter and energy in the primordial Universe combined with the noncommutative algebraic structure, which captures the small and large scales of the space-time structure that are the sources of this dramatic

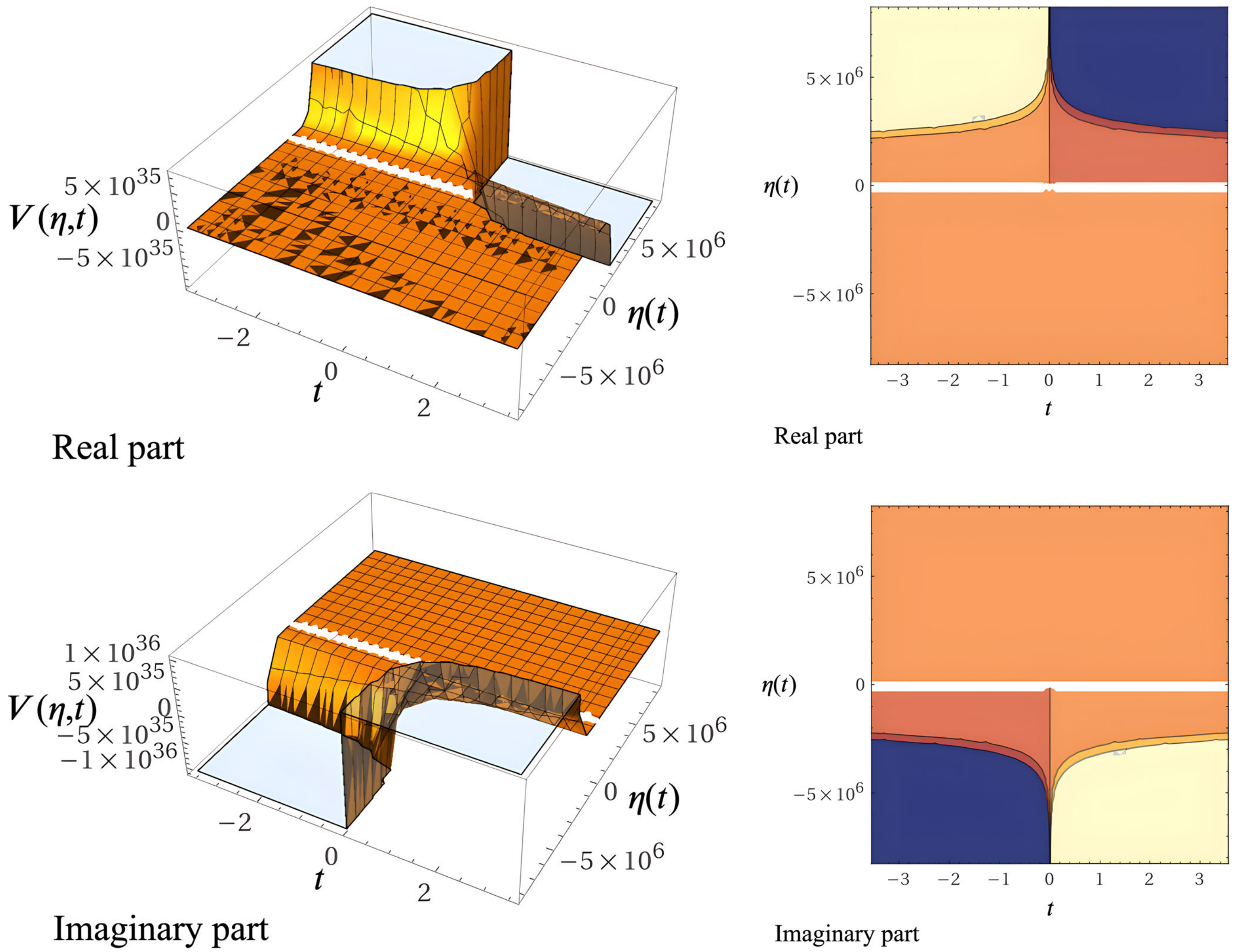

FIGURE 1 On the left figures, plotting of the potential $V(\eta, t)$ given in Equation (20). On the right figures, the corresponding contour plots. The parameters are as follows: $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_r = 0.4$; $g_s = 0.03$; $\alpha = -3/4$; $\gamma = -1$; $\mathscr{K} = 1$. The parameter χ in turn is implicitly inserted in the structure of the dual variables η and ξ.

and accelerated expansion of the Universe. In short, these are the ingredients that, in our conception, drive the acceleration of the Universe.

The comparison of typical results for the dynamic equations corresponding to the commutative and noncommutative formulations is enlightening. Figure 3 shows typical results corresponding to the dynamic equation (21) showing that depending on the choices of parameters and initial conditions, the commutative formulation predicts the possibility of a big crunch of the early Universe or a moderate acceleration in its evolution.

5 | SUMMARY AND FINAL REMARKS

The paper investigates the implications of a noncommutative formulation of branch-cut quantum gravity on the wave function of the Universe. Utilizing a mini-superspace structure obeying a noncommutative Poisson algebra and combining it with branch-cut gravity, the study explores the impact of an inflaton-type scalar field on the evolution of the Universe's wave function. The results indicate that the noncommutative algebraic structure captures both small and large spacetime scales, leading to exponential acceleration of the Universe.

The study opens avenues for further exploration of the noncommutative formulation of branch-cut quantum gravity and its implications on the early Universe. Future research could delve into the physical and cosmological interpretations of the jump between Riemann sheets and the detailed consequences of the noncommutative algebraic structure on different scales of spacetime. Additionally, the model's predictions and compatibility with observational data could be investigated to validate its cosmological relevance.

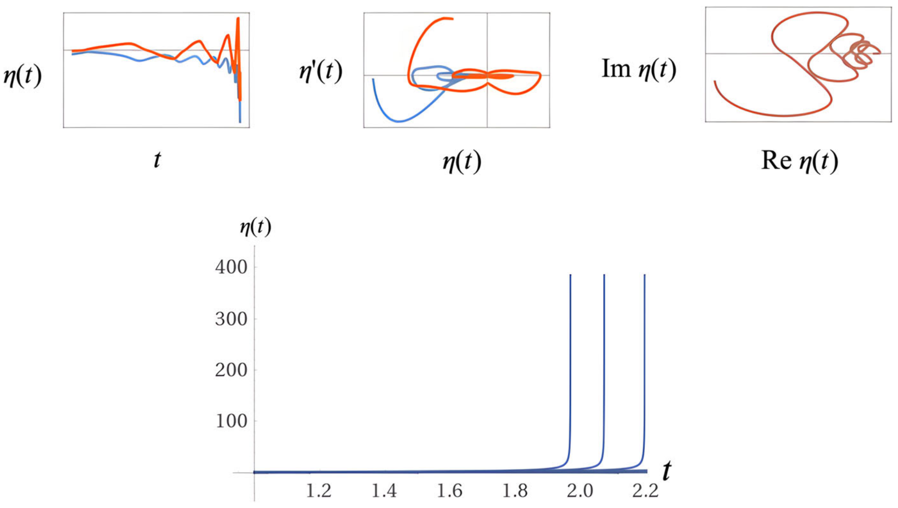

FIGURE 2 Typical sample individual solutions of the evolution in time of the scalar factor $\eta(t)$ given by Equation (19). On the up figures, the solutions correspond the negative sector of the solutions, and to sampling $\eta'(-1)$ and boundary condition $\eta(-1) = -1$. The solutions on the below figure correspond to sampling $\eta'(1)$ and boundary condition $\eta(1) = 1$. The parameter adopted in the calculations of the solutions are as follows: $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_r = 0.4$; $g_s = 0.03$; $\alpha = -1/4$; $\gamma = -1$; $\mathcal{K} = 1$. The parameter χ in turn is implicitly inserted in the structure of the dual variables η and ξ.

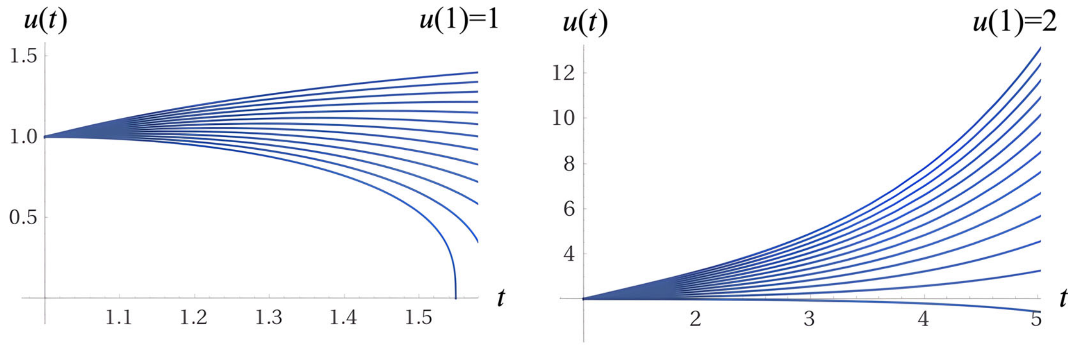

FIGURE 3 Typical sample family solutions of the evolution in time of the scalar factor $u(t)$ of the commutative formulation for different initial conditions. The figures correspond to sampling $u'(1)$. The parameter adopted in the calculations of the solutions are as follows: $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_r = 0.4$; $g_s = 0.03$; $\mathcal{K} = 1$.

AUTHOR CONTRIBUTIONS

Conceptualization: C.A.Z.V.; *Methodology*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., D.H., F.W., and M.M.; *Software*: C.A.Z.V., B.A.L.B., M.R., M.M.; *Validation*: C.A.Z.V., B.A.L.B., D.H., P.O.H., J.A.deF.P., and F.W.; *Formal analysis*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., D.H., and F.W.; *Investigation*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., M.R., G.A.D., M.M., and F.W.; *Resources*: C.A.Z.V.; *Data curation*: C.A.Z.V. and B.A.L.B.; *Writing-original draft preparation*, C.A.Z.V.; *Writing-review and editing*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., D.H., G.A.D., M.R., M.M., and F.W.; *Visualization*: C.A.Z.V. and B.A.L.B.; *Supervision*: C.A.Z.V.; *Project administration*: C.A.Z.V. All authors have read and agreed to the published version of the manuscript.

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