A Broadcast Channel Framework for Joint Communications and Sensing-Part I: Feasible Region

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***Abstract*—In various cyber physical systems (CPSs), communications and sensing are conducted simultaneously. Therefore, the mechanism of joint communications and sensing (JCS) is envisioned to integrate both functions in the same waveform, frequency band and hardware. It is expected to be one of the major features of 6G wireless communication networks. A major challenge to the design and analysis of JCS is a unified framework that incorporates the distinct functions of communications and sensing. In the first par of this paper, the framework of broadcast channel that has been intensively studied in data communications and information theory is adopted for JCS, in which communication and sensing signals are broadcast to the concrete communication users and virtual sensing users. Such a broadcast channel framework benefits the applications of existing multiplexing schemes, such as dirty paper coding (DPC) or frequency division multiplexing (FDM). Based on the framework, the feasible performance region bound is derived, based on the broadcast-multiaccess duality. The design of dedicated sensing signal is studied for the scenarios of communication-first (or sensing-first) priority, based on the ambiguity function (AF) of radar sensing. The proposed scheme is numerically demonstrated using typical short-range communication and sensing setups. The scheme based on superposition coding will be discussed in the second part of this paper.**

I. Introduction

Joint communications and sensing (JCS) [1] is expected to be a featuring technology in 6G cellular networks. One of the motivations for JCS is its intensive applications in cyber physical systems (CPSs) such as vehicular ad hoc networks (VANETs) or urban air mobility (UAM), in which each mobile node needs to communicate with neighbors and sense the environments. The performance analysis and system design for JCS bring new challenges since it integrates the two closely related but significantly different functions in the same waveform, thus requiring a unified framework. However, in the history, communications and radar sensing are

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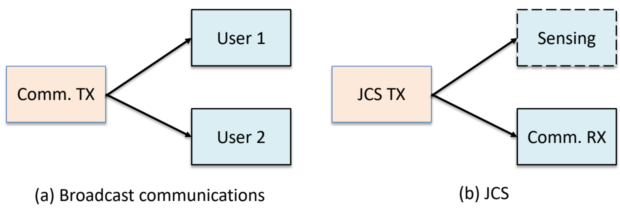

Fig. 1: Analogy between broadcast communications and JCS

based on information theory and detection/estimation theory, respectively. Although the two theories have many overlaps (e.g., using the information-theoretic metrics for analyzing the detection/estimation performance, such as in the Stein's Lemma [2]), they have different performance metrics (e.g., the channel capacity in communications and minimum mean square error (MMSE) in parameter estimation, respectively) and different arguments (e.g., random coding and Cramer-Rao bound, respectively).

In this paper, we leverage the broadcast channel (BC) [3], [4] framework for analyzing the performance bounds of JCS, as well as the waveform synthesis. We compare JCS to the downlink broadcast communications, as illustrated in Fig. 1, in which the downlink communications broadcast different messages to different communication receivers, while JCS broadcasts information to 'virtual sensing user' and concrete communication user. The benefits of using the BC framework in JCS include: (a) There have been substantial studies on BC, particular on the multiple-in-multiple-output (MIMO) case. When the conclusions are applied to the context of JCS, it helps us to understand how the information is superimposed in the layered structure and delivered to different destinations. (b) Concrete algorithms of data multiplexing in the study of BC can be applied to JCS; e.g., the powerful dirty paper coding (DPC) [5], or linear precoding [6], can be employed to mitigate the interference of signals dedicated to sensing on the signals for communications.

In the first part of this paper, we focus on the feasible region of JCS. The remainder of this paper is organized as follows. The studies related to this paper are introduced in Section II. Then, the system model is briefly introduced in

Section III. Based on the BC framework, the feasible region of JCS performance metrics is studied in Section IV, based on the information-theoretic argument. Then, the waveform synthesis algorithms are discussed in Section V. Numerical results are given in Section VI. Finally, conclusions are drawn in Section VII. The detailed coding scheme, motivated by the broadcast channel framework, will be discussed in the second part of this paper.

II. Related Works

Surveys on JCS can be found in [1], [7]–[10]. In fact, JCS is not a novel technology; it has been proposed decades ago, while it receives intensive studies recently, particularly on spatial separation of functions in MIMO cases [1], [8], [10]. The separation of communication and sensing signals can also be in the time or frequency domain [11], [12]. In this paper, the signals for communications and sensing are superimposed and mutually adaptive, in a contrast.

Meanwhile, BC has been intensively studied in early 2000s. While the BC channel capacity region has been identified for degraded channels [2], it is still an open problem for the MIMO case. A major breakthrough is the introduction of DPC [5], disclosed by [3], [4]. Based on DPC, the duality between multiple-access (MAC) and BC is identified in [13], which will be leveraged in this paper. A comprehensive survey on the important results of MIMO broadcast communications can be found in [14].

Note that, in the pioneering study by D. Bliss [15], the multiaccess of communications and sensing are studied in which sensing is also considered as a user characterized by its information rate. Similar ideas are shared by Y. Liu in [16], where successive interference cancellation is employed for mitigating the interference between sensing and communication signals. Different from the multiple access channels considered by these studies, our paper focuses on the broadcast of both communications and sensing from a single transmitter, thus requiring substantially different coding methodologies.

III. System Model

In this section, we introduce the system model for JCS, including the signal model and the performance metric of radar sensing.

A. Transmit Signals

We consider a JCS transceiver (with transmit power P_t and bandwidth W), a communication receiver and a radar target (which could be identical to or different from the communication receiver). The analysis will be extended to the case of multiple radar targets in our future research. We denote by N_t, N_r and N_c the numbers of antennas at the JCS transmitter, JCS receiver, and communication receiver, respectively. Orthogonal frequency division multiplexing (OFDM) signaling is used with M subcarriers with frequency spacing δf. For simplicity, we consider analog beamforming for the JCS transmitter, where the scalar baseband transmit signal is given by

$$x(t) = \sum_{m=1}^{M} X_m e^{-j2\pi(m-1)\delta f t}, \tag{1}$$

where X_m is the symbol over the m-th subcarrier. The modulated signal for radio frequency (RF) radiation is then given by $x(t)e^{-j2\pi f_c t}$, where f_c is the carrier frequency. Here, the details of cyclic prefix of OFDM signal are omitted (which in fact may also be used for the radar sensing). Then, the transmitted signal at the JCS transmitter is given by $x(t)\mathbf{u}$, where $\mathbf{u}$ is the N_t-dimensional steering vector for analog beamforming.

In this paper, we consider the linear superposition of signals for communications and sensing[1], namely

$$x(t) = x_s(t) + x_c(t), \tag{2}$$

where x_s and x_c are signals dedicated to sensing and communications, respectively. Therefore, the symbols over different sub-carriers are decomposed as

$$X_m = X_m^s + X_m^c, \tag{3}$$

where X_m^s and X_m^c are the complex signals for sensing and communications, respectively, over the m-th subcarrier. For simplicity, we assume that X_m^c is a quadrature amplitude modulation (QAM) symbol, as in standard data communications.

B. Received Signals

For simplicity, we assume far field for the reflected signals at the radar target, such that the JCS receiver receives a planar EM wave. The received baseband signals at the different antennas are given by an N_r-vector $\mathbf{y}_r$, which is given by

$$\mathbf{y}_r(t) = \mathbf{a}_t^H \mathbf{u} x(t-\tau)\mathbf{a}_r + \mathbf{w}_r(t), \tag{4}$$

where $\mathbf{a}_t$ and $\mathbf{a}_r$ are the N_t-dimensional and N_r-dimensional signature vectors for the forward and backward propagations of EM wave for the JCS transceiver, τ is the time delay, where the differences of traveling time among the antennas are omitted due to the far field assumption[2], and $\mathbf{w}_r$ is the noise. Then, the JCS uses the maximal ratio combing for the received signal, and thus obtains the scalar signal

$$\begin{aligned} y_r(t) &= \mathbf{a}_r^H \mathbf{y}_r(t) \\ &= \|\mathbf{a}_r\|^2 \mathbf{a}_t^H \mathbf{u} x(t-\tau) + w_r(t), \end{aligned} \tag{5}$$

[1]There are other possibly ways to integrate communication and sensing signals, such as using dedicated sensing sequences to spread the communication signals, similarly to the code division multiple access (CDMA).

[2]The phase differences due to the antenna distances are incorporated into the vector $\mathbf{a}_r$.

where the scalar noise $w_r(t) = \mathbf{a}_r^H \mathbf{w}_r(t)$. Note that the determinations of $\mathbf{a}_t$ and $\mathbf{a}_r$ require the information of the target position. Due to the far field assumption, only the incident angle information is needed, which is assumed to be known from previous sensing measurements.

Similarly, the received signal at the communication receiver is given by

$$\mathbf{y}_c(t) = \mathbf{a}_t^H \mathbf{u} x(t-\tau_{c1})\mathbf{a}_c + \mathbf{H}_{LOS}\mathbf{u} x(t-\tau_{c2}) + \mathbf{w}_c(t), \quad (6)$$

where $\mathbf{a}_c$ is the N_c-dimensional signature waveform to the communication receiver, $\mathbf{H}_{LOS}$ is the channel matrix of the line of sight (LOS) propagation and τ_{c1} and τ_{c2} are the time delays of the non-LOS (NLOS) and LOS paths, respectively. The steering vector for the communication receiver is denoted by $\mathbf{r}_c$.

C. Ambiguity Function

A useful performance characterization for radar sensing is the Ambiguity Function (AF) proposed by Woodward [17], [18]. For time-domain signal $s(t)$, the corresponding AF is defined as

$$\chi(\tau, \nu) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) e^{-j2\pi\nu(t-\tau)} dt, \quad (7)$$

where τ is the round-trip time due to reflection and ν is the Doppler-shift due to the mobility of the target. It is desirable that the peak $\chi(0,0)$ is dominant, and other sidelobes in the τ-ν plane, which incur confusions in ranging or Doppler estimation, are weak.

For simplicity, we consider only the AF along the τ-axis, namely only the performance of ranging, or equivalently the autocorrelation function r:

$$r(\tau) = \int_{\tau}^{T_p} x(t) x^*(t-\tau) dt, \quad (8)$$

where T_p is the period of each communication symbol (sensing pulse). We define the chip period $T_c = \frac{T_p}{M}$. To avoid the variational analysis on the continuous-time signal $x(t)$, we sample the autocorrelation function at $\tau = kT_c$, $k = 0, 1, 2, 3, ...$, and obtain the discrete-time samples $r[k] = r(kT_c)$. It is desirable for the peak at $k = 0$ (the main lobe) to dominate the sidelobes ($k \neq 0$), which improves the resolution of nearby radar targets. Therefore, in this paper we use the integrated sidelobe level (ISL) [18] for the radar sensing performance metric:

$$\xi = \sum_{k=-(N_c-1), k\neq 0}^{N_c-1} |r[k]|^2 = 2\sum_{k=1}^{N_c-1} |r[k]|^2. \quad (9)$$

However, as a performance metric of sensing, ISL does not incorporate the noise power into account. Therefore, in this paper, we consider $r^2[0]$ as the signal power and ISL as the self-interference, and thus define the signal-to-interference-and-noise ratio (SINR) as $\frac{r^2[0]}{\xi + N_0^2}$, where N_0 is the noise power.

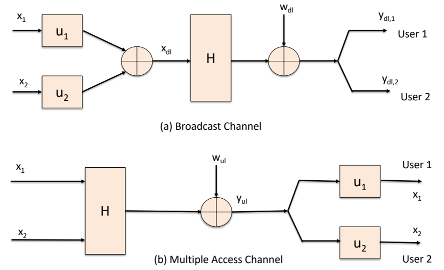

Fig. 2: BC-MAC duality [19]

IV. Information-theoretic Feasible Region

In this section, we use the BC framework in information theory to analyze the feasible region of JCS performance metrics (communication channel capacity and sensing MSE), which characterizes the trade-off between communications and sensing and provides intuition for the subsequent waveform design.

A. BC-MAC Duality

For BC in traditional downlink communications, the duality of BC and MAC has been identified in [13]. Here we follow the introduction in [19] to briefly explain the duality by considering two users, as illustrated in Fig. 2. In the BC (in Fig. 2 (a)), where $\mathbf{u}_i$, $i = 1, 2$, is the vector of weighting factor for different antennas ($\mathbf{u}_i = \mathbf{u}$ in JCS), and $\mathbf{H}$ is the channel matrix, the SINR of user k is given by

$$SINR_k^{dl} = \frac{P_k \|\mathbf{H}_k \mathbf{u}_k\|_2^2}{N_0 + \sum_{j\neq k} P_j \|\mathbf{H}_j \mathbf{u}_j\|_2^2}, \; k = 1, 2, \quad (10)$$

where N_0 is the noise power, $\mathbf{H}_k$ is the k-th sub-matrix in $\mathbf{H}$, and P_k is the power allocated to user k. Note that in the context of JCS, the channel matrix is given by (when the delays are omitted)

$$\mathbf{H}_k = \begin{cases} \mathbf{a}_r \mathbf{a}_t^H, & k = 1 \\ \mathbf{a}_c \mathbf{a}_t^H + \mathbf{H}_{LOS}, & k = 2 \end{cases}. \quad (11)$$

Similarly, the SINR of user k in the MAC (in Fig. 2 (b)) is given by

$$SINR_k^{ul} = \frac{Q_k \|\mathbf{H}_k \mathbf{u}_k\|_2^2}{N_0 + \sum_{j\neq k} Q_j \|\mathbf{H}_j \mathbf{u}_j\|_2^2}, \; k = 1, 2, \quad (12)$$

where Q_k is the transmit power of user k. We observe that the expressions of SINRs in both the BC and MAC, in Equations (10) and (12), are identical. Therefore, for Gaussian signal and noise, where the SINR determines the performance, the performance of BC can be obtained from that of MAC.

Since the channel capacity region of generic signaling of MIMO BC is still unknown, we can assume that Gaussian signaling is used for all the users. Then, the signals of different users can be layered, such that DPC [5] can be used to remove the interference from higher layers (with higher priority) to lower layers (with lower priority). This is very similar to the successive interference cancellation (SIC) in MAC, where messages encoded or decoded later receive less interference due to the mitigation of interference via DPC or SIC. For the context of two users, the capacity region of BC, given the DPC scheme, is given by [13]

$$C_{BC}^{DPC}(P_t, \mathbf{H}) = Co\left(\cup_{\pi,\mathbf{p}} R(\pi, \mathbf{p})\right), \qquad (13)$$

where Co means convex hull, P_t is the total transmit power in BC, π is the permutation of $\{1, 2\}$, $\mathbf{p} = (P_1, P_2)$ is the vector of power allocations to the two users in MAC, such that $P_1 + P_2 = P$.

B. Communication-Sensing Trade-off

As mentioned in the introduction, we can consider the JCS as a broadcast to a virtual user of sensing and a concrete user of communications. Although this analogy provides insight and motivation for understanding and designing new JCS waveforms, JCS is different from traditional downlink broadcast communications. Based on the above similarity and distinction, we propose the following two layered signaling schemes for JCS, based on the BC framework, as illustrated in Fig. 3:

- Communication-first priority (CFP): In this scheme, communication is laid at the top layer and thus has the higher priority. The sensing waveform x_s will be synthesized first, independent of the realization of communication messages (but could be dependent on the corresponding statistics). Then, the communication signal x_c will be generated with DPC, with respect to the sensing signal x_s, such that the interference from x_s is completely eliminated at the communication receiver. At the JCS transceiver, the whole signal $x_s + x_c$ is used for the target information inference.
- Sensing-first priority (SFP): In this scheme, sensing is laid at the top layer and thus has the higher priority. The communication signal x_c is generated first without DPC, subject to the interference of the sensing signal. Then, the sensing signal is optimized with respect to the realization of communication signal. Again, the JCS receiver will use the entire signal $x_s + x_c$ for sensing.

Given the above two layered signaling schemes, we obtain an inner bound[3] for the feasible performance region, when the powers for x_s and x_c are P_s and P_c, respectively, where $P_c + P_s = P_t$. For simplicity, we consider the reciprocal of sensing error as the performance of sensing, which is expected to be large and is proportional to the allocated power and time. The performance of communications is represented by the data rate R. The region is very similar to that of MAC, as illustrated in Fig. 4. We first fix the performance points for

[3]It is inner bound, since the DPC scheme could be sub-optimal.

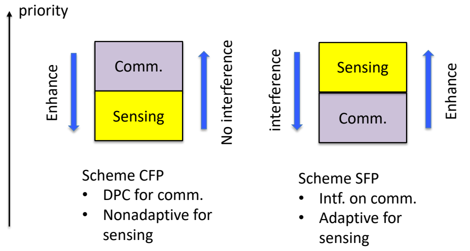

Fig. 3: Two layered signaling schemes

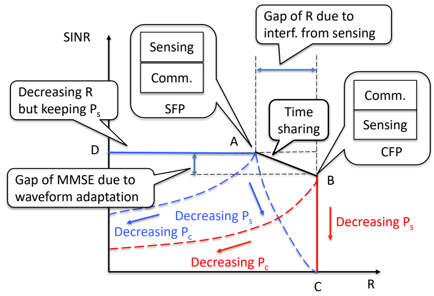

Fig. 4: Feasible performance region for JCS

the CFP and SFP schemes. Since in SFP communications experience interference from the sensing signal, while the sensing waveform is optimized with respect to the generated communication signals, the performance point of SFP is in the upper left of that of CFP. The horizontal boundary AD is obtained from decreasing the data rate R without changing P_s and P_c. The vertical boundary BC is obtained by decreasing P_s while keeping P_c. The boundary AB is then obtained from time-sharing between SFP and CFP. The trajectories of decreasing P_c or P_s from A, and decreasing P_c from B, are all plotted and found to be within the region.

V. Waveform Synthesis

In this section, we study the waveform synthesis for MIMO JCS, with the layered structure for communications and sensing.

A. Communication-First Priority

For the CFP case, the sensing waveform x_s is first optimized over the statistics of the communication signal x_c; then, x_c is encoded using DPC, adaptively to x_s, thus eliminating the interference from x_s. Then, the optimization of the sensing waveform is formulated as follows, where the objective function is the expectation of the ISL (over the randomness of the communication signal x_c) and the constraint is the power allocated to the sensing signal:

$$\begin{aligned} &\min_{x_s} E[ISL(x_c + x_s)] \\ s.t. \quad & E[|x_s|^2] \leq P_s. \end{aligned} \qquad (14)$$

The optimization problem can be solved using numerical approaches (e.g., gradient descent similar to [18]), since an explicit solution is prohibitive. In this paper, we propose an intuitive approach, whose validity will be demonstrated using numerical results. We first notice that the Fourier transform of the autocorrelation function $r(\tau)$ is the power spectral density (PSD) $P(\omega)$. Then, the Parseval's identity states

$$\int_0^\infty r^2(\tau) \propto \int_{\omega_c+\frac{W}{2}}^{\omega_c-\frac{W}{2}} P^2(\omega)d\omega. \quad (15)$$

where the left hand side can be approximated by

$$\int_0^\infty r^2(\tau) \approx T_c(ISL + r[0]) = T_c(ISL + P_t), \quad (16)$$

and the right hand side equals

$$\int_{\omega+\frac{W}{2}}^{\omega-\frac{W}{2}} P^2(\omega)dw = WVar(P(\omega)) + \frac{P_t^2}{W}. \quad (17)$$

Therefore, it is reasonable to minimize the variance of the PSD, in order to minimize the ISL, despite the approximation. Since the sensing waveform x_s is synthesized before the formation of communication signal x_c, x_s is designed adaptively to the statistics of x_c. In this paper, we propose a simple water-filling approach summarized in Algorithm 1. The philosophy is that the water-filling can effectively reduce the variance of the PSD, thus reducing the ISL, as disclosed in (16) and (17). The optimality of the water-filling scheme for minimizing the PSD variance is established in Prop. 1 and the proof is omitted in this paper.

Proposition 1: The water-filling scheme in Algorithm 1 minimizes the PSD variance, and thus the ISL, given the constraint of sensing power P_s.

Algorithm 1 Sensing waveform synthesis in CFP

1: Given the average subcarrier power allocation of communication signals $\{P_m^c\}_{m=1,...,M}$.
2: Set the initial value of Lagrange multiplier λ, and the threshold γ and step ϵ.
3: **while** $\tilde{P}_s < P_s - \gamma$ **do**
4: Set $P_m^s = (\lambda - P_m^c)^+$, for $m = 1, ..., M$.
5: Calculate $\tilde{P}_s = \sum_{m=1}^M P_m^s$.
6: Set $\lambda = \lambda + \epsilon$.
7: **end while**
8: Set the sensing signals X_m^s according to the power P_m^s with random phases, $m = 1, ..., M$.

B. Sensing-First Priority

When sensing is of the first priority, the communication signal x_c will be first constructed according to the communication data, and then the sensing signal is formed adaptively to x_c. It can be formulated as the following optimization problem.

$$\min_{x_s} ISL(x_c + x_s)$$
$$s.t. \quad E[|x_s|^2] \le P_s \quad (18)$$

Compared with (14), we observe that the only difference is the missing expectation in the objective function, since x_c is deterministic for the SFP case. Similarly to the CFP case, the adaptive design of x_s is also to minimize the PSD variance. The corresponding algorithm is very similar to Algorithm 1, except that we update the power of each subcarrier using $P_m^s = (\lambda - |X_m^c|^2)^+$.

VI. Numerical Results

In this section, we provide numerical simulation results to demonstrate the proposed methodologies.

A. Simulation Setup

We consider an OFDM-based JCS with 2048 subcarriers, starting frequency of 6GHz and frequency spacing of 240kHz (thus the total bandwidth approximately equals 550MHz). We assume that the distances between the JCS transceiver and target, and between the target and communication receiver, are 40 and 20 meters, respectively, while the LOS path between the JCS transceiver and communication receiver is 50 meters. The pathloss model is assumed to be $48 + 20\log_{10} d(dB)$, where d is the distance in meters. The PSD of noise is -194dBm/Hz, while the total transmit power is 20mW. The reflection coefficients of directions to the JCS transceiver and communication receiver are assumed to be 1 and 0.2, respectively. The communication power allocated to different subcarriers is obtained from water-filling, since the two propagation paths result in a frequency-selective channel.

B. Simulation Results

We implemented the simulations for both the CFP and SFP strategies, where the proportion of communication power P_c/P_t ranges from $\frac{1}{20}$ to 1. The corresponding SINR and channel capacity, obtained from the bit error rate of QAM and the assumption of symmetric binary channel, are plotted in Fig. 5. We observe that, in terms of sensing performance (SINR), the SFP scheme is only marginally better than the CFP in Algorithm 1. Meanwhile, in terms of the communication channel capacity, the CFP scheme substantially outperforms the SFP scheme; in particular, when the proportion of communication signal power is small, the channel capacity of SFP is close to 0, which means that the interference from the sensing signal is detrimental. This does not imply that the SFP strategy be discarded, since it has not been optimized.

Based on the performance metrics in Fig. 5, we plot the feasible performance region of JCS in Fig. 6. We observe

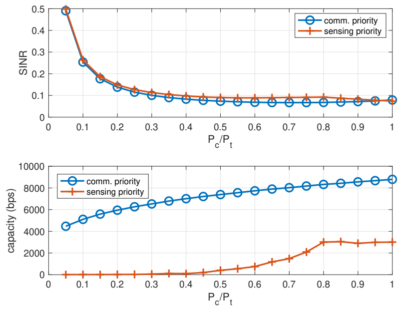

Fig. 5: Sensing SINR and communication channel capacity versus the proportion of communication power P_c/P_t

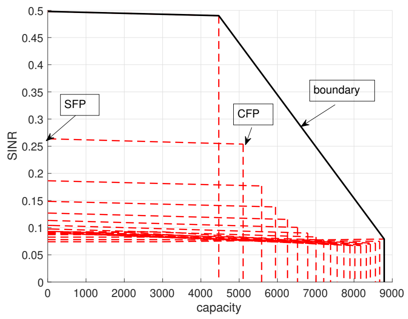

Fig. 6: Region of feasible performance of JCS

that the original feasible regions formed by the time sharing of CFP and SFP construct a non-convex region. The time sharing between the power allocation schemes forms a convex region of performance. Then, we change the distances in the setup to 150, 120 and 60 meters, correspondingly. We observe a significant change in the performance region, while the basic features remain the same.

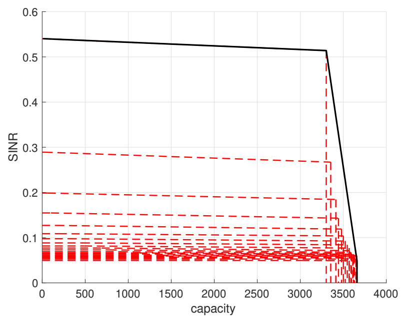

Fig. 7: Region of feasible performance of JCS with larger distance

VII. Conclusions

In this paper, we have studied the feasible performance region of JCS, based on the framework of broadcast channel and DPC coding scheme. Concrete algorithms for waveform synthesis have been proposed for the CFP and SFP schemes of JCS, which have been demonstrated by numerical results.

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