Trellis Shaping for Joint Communications and Sensing: A Duality to PAPR Mitigation

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***Abstract*—A key challenge in joint communications and sensing (JCS), a.k.a. integrated sensing and communications (ISAC), is the waveform synthesis that needs to modulate the communication messages and achieve good sensing performance simultaneously. A standard communication waveform could be used for sensing, since the sensing receiver, co-located with the sensing transmitter, knows the communication message and thus the waveform. However, the randomness of communications may result in improper waveforms that have high sidelobes masking weak targets. Therefore, the communication waveform needs to be refined in order to improve the sensing performance. This is similar to the peak-to-average power ratio (PAPR) mitigation in orthogonal frequency division multiplexing (OFDM), in which the OFDM-modulated waveform needs to be refined to reduce the PAPR. Motivated by the PAPR issue in OFDM, the approach of trellis shaping, which refines the waveform for specific metrics using convolutional codes and Viterbi decoding, is employed for OFDM-based JCS. In such a scheme, the communication data is encoded and then mapped to the constellation in different subcarriers, such that the time-domain sidelobes are reduced. An interesting observation is that the sidelobe reduction in OFDM-based JCS is dual to the PAPR reduction in OFDM, thus sharing a similar signaling structure. Numerical simulations are carried out to demonstrate the validity of the proposed trellis shaping approach.**

I. Introduction

Joint communications and sensing (JCS), a.k.a. integrated sensing and communications (ISAC), integrates both functions in the same waveform, thus substantially improving the spectral and power efficiencies. It is expected to be a distinguishing feature of 6G wireless communication networks. In JCS, the forward propagation of electromagnetic (EM) wave 'pushes' communication messages to the destination, thus accomplishing the task of communications, and 'pulls' the environmental information in the backward propagation upon reflectors and scatterers, thus achieving the function of sensing.

The major challenge of JCS is the waveform synthesis with two goals. Three possible design criteria could be employed:

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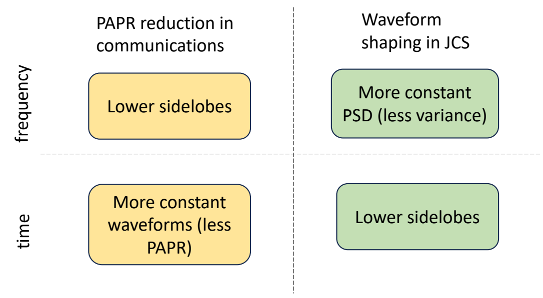

Fig. 1: Duality between PAPR mitigation and JCS waveform shaping

(a) Communication-centric design, where the waveform synthesis stems from an existing communication waveform (such as the orthogonal frequency division multiplexing (OFDM)) and sensing is accomplished by considering the communication waveform as a pseudorandom sequence. The advantage of such schemes is that the communication performance can be well assured, while the disadvantage is that the random communication messages incur uncertainties into the sensing performance. (b) Radar-centric design, where communication messages are embedded in the traditional radar sensing waveforms (say FMCW), such as modulating the radar parameters (e.g., the chirp rate) [1] or using spread spectrum (either direct sequence [2] or frequency hopping patterns [3]). Such schemes are convenient but difficult to achieve high communication data rate due to the lack of an explicit communication signaling structure. (c) Dedicated JCS wave, which is designed from first principles and optimized for achieving a good performance trade-off between communications and sensing, at the cost of more complicated computations.

In this paper, we adopt the communication-centric design and assume the OFDM signaling. On the one hand, the popular communication signal structure of OFDM can guarantee the communication performance and reuse existing communication system hardware and protocols; on the other hand, existing communication waveforms, without any modifications, can indeed be used for sensing (e.g., the enormous work on the Wi-Fi sensing). However, as indicated above, pure communication waveform may bring significant (although not detrimental) performance degradations to sensing (an example of increased sidelobe level can be found in [4]).

To address this challenge in the communication-centric JCS, we leverage the lessons learned from mitigating the peak-average power ratio (PAPR) in OFDM systems, which received intensive studied two decades ago and has been employed in 4G and 5G systems. It is well known that one of the major disadvantages of multi-carrier communications is that the peak power could be substantially higher than the average power, thus bringing potential distortions to the power amplifier of the radio frequency (RF) circuits. There have been many algorithms such as the peak clipping [5], selective mapping [6], trellis shaping [7], and so on. A survey can be found in [8]. Here, a high PAPR can be considered as a byproduct of communication modulation. Similarly, the sensing performance degradation (such as the greater sidelobe level) of JCS, compared with traditional radar sensing waveforms, can also be considered as a byproduct of communication modulation. Therefore, many approaches for alleviating the PAPR in OFDM waveforms can be leveraged (not necessarily in all details) for improving the sensing performance in communication-centric JCS.

We propose to adopt the approach of trellis shaping [7] that has been proposed for PAPR, as well as power reduction. The idea is similar to coset coding [9], in which the communication message is modulated in the index of a coset (i.e., a subset of codewords). Within the coset, the codewords having the optimal performance (e.g., the minimal PAPR or the least sidelobes) will be selected for transmission. The larger the coset is, the better sensing performance can be achieved since there are more options, while a reduction in the data rate is certainly incurred. In particular, we will focus on minimizing the sidelobes in the waveform autocorrelation, in order to avoid the confusion between sidelobes and weak targets. The goal of sidelobe reduction in JCS is dual to the PAPR mitigation problem, since in JCS we expect to reduce the sidelobes in the time domain while for PAPR we desire to mitigate the sidelobes in the frequency domain (thus a flatter power profile in the time domain). The challenge to our waveform shaping in JCS is that the manipulation of signal is in the frequency domain due to the OFDM structure while the goal is in the time domain, which makes it more challenging than the trellis-shaping-based PAPR reduction. Note that the possible solutions to waveform design in JCS are not limited to the trellis shaping. The more important lesson is that the experience learned in PAPR mitigation that has been intensively studied for pure OFDM communication systems in the past decades can be leveraged in the context of JCS. Many other methodologies in PAPR mitigation [5]–[7] will be exploited by us, either as motivations or as solutions, in the near future.

The remainder of this paper is organized as follows. Related studies are briefly introduced in Section II. Then, the system model is introduced in Section III. The main part of this paper, namely the algorithm for waveform shaping and the trade-off between communications and sensing, is discussed in Sections IV. Then, numerical results are provided in Section V, and final conclusions are drawn in Section VI.

II. Related Works

There is a boom in the research on JCS in the past five years. Surveys on JCS can be found in [10]–[12]. The spectrum of JCS waveform designs ranges from using traditional radar sensing waveforms (such as frequency modulation and continuous wave (FMCW) [1]) to traditional communication waveforms (e.g., the Wi-Fi waveforms [13]). Plenty of papers work on using pure communication signals for radar sensing [13], while there are much less studies on refining communication signals in order to improve the sensing performance. However, no studies till now have employed the strategies learned from PAPR reduction in OFDM communication systems for the refinement of JCS waveforms, in terms of radar sensing performance (such as alleviating sidelobes). In this section, we briefly introduce the principle of trellis shaping for signal performance improvement in terms of PAPR reduction.

The approach of trellis shaping is proposed by D. Forney in order to reduce signal power [14] while it finds more applications in the PAPR reduction [7]. Consider N_s subcarriers, each being equipped with quadrature amplitude modulation (QAM). The time-domain signal is given by

$$x(t) = \sum_{k=1}^{N_s} X_k e^{j2\pi(f_c+(k-1)\delta f)}, \quad (1)$$

where X_k is the complex QAM symbol over subcarrier k, f_c is the carrier frequency and δf is the frequency spacing between subcarriers. The PAPR is determined by the power profile in the time-domain. It is well known that the time-domain (respectively the frequency-domain) power profile and the autocorrelation of the frequency spectrum (respectively the time domain) form a pair of Fourier transform. Therefore, reducing PAPR is equivalent to reducing the sidelobes of the frequency spectrum autocorrelation defined as

$$R(l) = \sum_{k=1}^{N_s-l} X_k^* X_{k+l}. \quad (2)$$

Now, assume that each subcarrier has N_b bits, denoted by $\{b_{k1}, ..., b_{k,N_b}\}$ for subcarrier k, to transmit. We take $M-1$ bits out of the N_b bits and form M most significant bits (MSBs) that have the most impact on the waveform and consider the remaining $N_b - M$ bits as the least significant bits (LSBs) that have minor impact on the waveform. MSBs will be encoded for waveform shaping while the LSBs will be transmitted as they are. We consider expanding (encoding) the M MSBs into NM bits, namely for each subcarrier k finding a mapping F_k: $\{b_{k1}, ..., b_{kM}\} \rightarrow \{z_{k1}, ..., z_{k,MN}\}$.

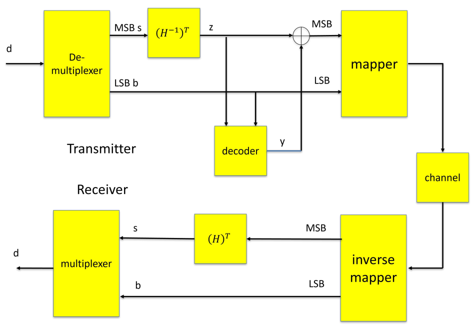

Fig. 2: Trellis waveform shaping

Now, we assume that $N_b = M^2 - 1$ and consider a convolutional code with coding rate $\frac{1}{M}$, whose generating matrix of dimension $1 \times M$ is $\mathbf{G}$ (in the generating polynomial form) and whose parity check matrix of dimension $(M-1) \times M$ is $\mathbf{H}$. For example, the generating matrix could be $\mathbf{G} = (1 + D^2, 1 + D + D^2)$, where D is the unit delay. As illustrated in Fig. 2, we take $M - 1$ raw bits $\mathbf{s}$ at each subcarrier are multiplied with the inverse syndrome former matrix $(\mathbf{H}^{-1})^T$, to form M MSB bits

$$\mathbf{z} = \mathbf{s}(\mathbf{H}^{-1})^T. \tag{3}$$

Then, $\mathbf{z}$ and the $M^2 - M$ LSB bits $\mathbf{b}$ (therefore M^2 bits in $(\mathbf{z}, \mathbf{b})$) are fed into the convolutional decoder, thus generating M MSB bits $\mathbf{y}$ (as a codeword). Then, $\mathbf{z} + \mathbf{y}$ and $\mathbf{b}$ are put into the constellation mapper and sent to the radio frequency (RF) circuits for transmission. At the receiver, the MSB is multiplied by $\mathbf{H}^T$ such that

$$(\mathbf{z} + \mathbf{y})\mathbf{H}^T = \mathbf{s}(\mathbf{H}^{-1})^T\mathbf{H}^T + \mathbf{y}\mathbf{H}^T = \mathbf{s}, \tag{4}$$

where $\mathbf{y}\mathbf{H}^T = 0$ because $\mathbf{y}$ is the outcome of the convolutional decoding, thus being a codeword. Hence, $\mathbf{s}$ is recovered at the receiver.

To reduce PAPR, we carry out the encoding in a subcarrier-by-subcarrier manner, due to the summation structure in (2), which perfectly suits the approach of dynamic programming (or Viterbi decoding in the context convolutional coding). Assume that the decoding of MSBs of the first $k - 1$ subcarriers has been completed. For subcarrier k, the M bits are decoded in the manner of Viterbi decoding. Due to the summation structure in (2), the decoding metric is the power sum of sidelobes. More details can be found in [7].

III. System Model

In this paper, we consider an OFDM symbol with N_s subcarriers, each sending averagely N_b bits. We denote by $\{x_k\}_{k=1,...,N_s}$ the samples of time-domain signal. The total transmit power is P_t. For the purpose of sensing, we consider the time-domain autocorrelation function defined as

$$r(l) = \sum_{k=1}^{N_s - l} x_k^* x_{k+l}, \qquad l = 0, ..., N_s - 1. \tag{5}$$

We can use the normalized integrated sidelobe level (ISL) [15] for evaluating the performance:

$$ISL = \frac{\sum_{k=1}^{N_s-1} |r(l)|^2}{|r(0)|^2}. \tag{6}$$

To avoid possible confusion with weak targets, we expect that the amplitudes of sidelobes, namely $r(l)$ when $l > 0$, are as low as possible in the synthesized waveform. Similarly to the PAPR reduction problem, the time-domain autocorrelation and the frequency-domain power spectral density (PSD) are a pair of Fourier transform. Therefore, to reduce the sidelobes in the time domain, we endeavor to make the PSD as flat as possible. One possible goal is to minimize the variance of PSD, namely

$$V(X) = \frac{1}{N_s} \sum_{k=1}^{N_s} \left(|X_k|^2 - E\left[|X_k|^2\right]\right)^2. \tag{7}$$

We assume that the available average transmit power is P_t. A slight deviation from P_t is allowed. Due to the requirement of trellis shaping, a convolutional code is prepared in advance, whose coding rate is $\frac{1}{M}$. The corresponding memory length is ν and the states are denoted by ϕ_1, ..., ϕ_{2^ν}.

IV. Trellis Waveform Shaping

In this section, we employ the technique of trellis shaping in the context of JCS, in order to reduce the sidelobes, similarly to reducing the PAPR in OFDM.

A. *Metric for Waveform Shaping*

We use the same structure of trellis shaping for PAPR reduction in OFDM, as shown in Fig. 2. We fix a convolutional code with codebook $\mathcal{C}$ and data rate $\frac{1}{M}$. Unfortunately, the waveform synthesis in JCS is not completely dual to the PAPR reduction. In JCS, the encoding is in the frequency domain, due to the OFDM signaling structure, and the autocorrelation is in the time domain; in a contrast, both encoding and autocorrelation are in the frequency domain for PAPR reduction. In JCS, the direct goal of encoding is to minimize the variance in (8), which does not have a summation structure when the average power over the subcarriers is kept a constant (otherwise, it makes the variance zero if no signal is transmitted). Therefore, we consider the following relaxed metric:

$$\hat{V}(X) = \frac{1}{N_s} \sum_{k=1}^{N_s} \left(|X_k|^2 - \frac{P_t}{N}\right)^2, \tag{8}$$

where we assume that $E\left[|X_k|^2\right] \approx \frac{P_t}{N}$. This change results in the summation structure for dynamic programming (Viterbi decoding).

B. MSB Decoding

Given the metric in (8), the goal of decoding, mapping from the bits $(\mathbf{z}, \mathbf{b})$ to $\mathbf{y}$, is to minimize the spectrum variance, namely

$$\mathbf{y} = \arg\min_{\mathbf{y}\in\mathcal{C}} \hat{V}(\mathbf{y}), \tag{9}$$

where the approximated variance in (8) is a function of the codeword $\mathbf{y}$ since the final symbol in the constellation is determined by $\mathbf{y}+\mathbf{z}$ and $\mathbf{b}$. Then, we consider the schemes of sign-bit signaling ($M = 2$) and high-dimensional signaling.

1) Sign-bit Signaling: For the simplest sign-bit signaling scheme, we set $M = 2$. Therefore, each subcarrier has $M - 1 = 1$ MSB bit, denoted by s_1, ..., s_{N_s}, and $M(M-1) = 2$ LSB bits, denoted by $b_{11}, b_{12}, ..., b_{N_s1}, b_{N_s2}$. Note that, when the generating matrix is $\mathbf{G} = (1 + D^2, 1 + D + D^2)$, the matrix $(\mathbf{H}^{-1})^T$ may not be unique. One possible selection is $(D, 1 + D)^T$. The bit vector $\mathbf{s}$ used to generate the MSB $\mathbf{z}$ is given by the polynomial (instead of a single bit) $\mathbf{s}(D) = \sum_{i=1}^{N_s} s_i D^{i-1}$. Similarly, the MSB $\mathbf{z} = \mathbf{s}\mathbf{G}$ is a vector of polynomails consisting of $2N_s$ bits given by

$$\mathbf{z} = \left(\underbrace{z_0, z_1}_{\text{subcarrier } 1}, \underbrace{z_2, z_3}_{\text{subcarrier } 2}, \cdots, \underbrace{z_{2N_s-2}, z_{2N_s-1}}_{\text{subcarrier } N_s} \right). \tag{10}$$

The corresponding polynomial form of $\mathbf{z}$ can be written as $\mathbf{z}(D) = (\mathbf{z}_1(D), \mathbf{z}_2(D))$, where $\mathbf{z}_j(D) = \sum_{i=1}^{N_s} z_{ji} D^{i-1}$, for $j = 1, 2$.

We can consider the MSB bits $\mathbf{s}$ as the symdrone of the bits $\mathbf{z}$. The bits $\mathbf{z}$ and $\mathbf{b}$, together of dimension M^2=4, are put into the convolutional decoder, in order to find a codeword $\mathbf{y}$ of dimension $M = 2$. Moreover, the codeword $\mathbf{y}$ should make the MSB $\mathbf{z}+\mathbf{y}$ generate N_s symbols (over the N_s subcarriers) that yield a flat PSD (thus less sidelobes in the time-domain autocorrelation for a better sensing performance). To this end, the convolutional decoding procedure is carried out by the following Bellman equation:

$$V(\phi_t) = \arg \min_{\phi_{t+1}\sim\phi_t} V(\phi_{t+1}) + \left(|X_t|^2 - \frac{P_t}{N_s} \right)^2, \tag{11}$$

where ϕ_t is the state of the trellis for subcarrier t, the notion $\phi_t \sim \phi_{t+1}$ means that the two states are connected in the trellis, and X_t is determined by the communication symbol determined by the transition from ϕ_t to ϕ_{t+1}. The boundary condition for the Bellman's equation is $V(s_{N_s+1}) = 0$. Then, the Viterbi decoding algorithm (or equivalently dynamic programming) is applied recursively to solve the equation in (11).

The convolutional decoding output is given by

$$\mathbf{y} = \left(\underbrace{y_0, y_1}_{\text{subcarrier } 1}, \underbrace{y_2, y_3}_{\text{subcarrier } 2}, \cdots, \underbrace{y_{2N_s-2}, y_{2N_s-1}}_{\text{subcarrier } N+s} \right). \tag{12}$$

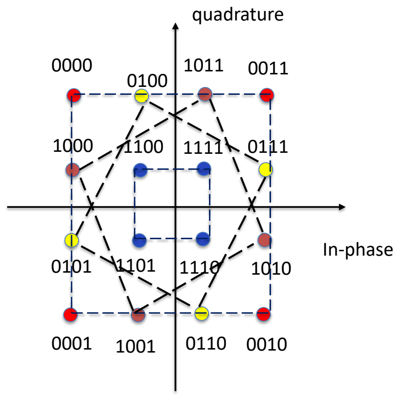

Fig. 3: QAM constellation for trellis shaping in JCS

Then, it is (modular 2) added to $\mathbf{z}$ to form two bits over each subcarrier.

The procedure until now is similar to the PAPR reduction, except for the goal function in (8). However, the modulator that maps the bits $(\mathbf{y}, \mathbf{b})$ to the point in the QAM constellation needs to be different. For PAPR, the MSB bits are used to partition the complex plane into quadrants. In the complex plane, one bit in MSB identifies the side with respect to the vertical axis, while the other bit in MSB determines the side with respect to the horizontal axis. Therefore, the MSB bits can control the symbol phase, such that the symbols can better cancel each other in the sidelobes. In a contrast, in the context of waveform for JCS, the goal is to control the power of each subcarrier in order to obtain a uniform PSD for the purpose of less sidelobes in the time domain. Therefore, the MSB bits need to distinguish different power allocations. Hence, in this paper, we propose the bit allocation to 16-QAM (suppose that 2 LSB bits are allocated to each subcarrier) illustrated in Fig. 3, where 16 signal points are categorized into 4 subsets (marked by different colors), each corresponding to 2 MSB bits. The signals within each subset have the same power, regardless of the phase. Therefore, the 2 MSB bits control the power while the LSB bits determine the phase.

2) High-dimensional Constellation: An alternative approach is the high-dimensional constellation, namely using the symbols over M_s subcarriers to form a high-dimensional symbol, where M_s should divide N_s. The convolutional decoding output of $2N_s$ bits is given by

$$\mathbf{y} = \left(\underbrace{y_0, , ..., y_{M_s-1}}_{\mathbf{y}_0}, \cdots, \underbrace{y_{N_s-M_s}, \cdots, y_{N_s-1}}_{\mathbf{y}_{\frac{N_s}{M_s}-1}} \right), \tag{13}$$

where each y_k is a complex number over subcarrier k. Note that, when $M_s = 1$, we attain the above sign-bit scheme.

Then, the communication symbols are formed for every M_s subcarriers. For simplicity, we assume that $M_s = 2$, namely y_0 (2 bits) and y_1 (2 bits) form a 4-dimensional symbol (or a 2-complex-dimensional symbol). Similarly to the sign-bit signaling, we use the MSB bits to control the power. We assume that the 4 cases of 2 bits in subcarrier 1 form 4 levels of amplitudes over subcarrier 1, and so is subcarrier 2. The amplitude levels are not necessarily the same for the two subcarriers. During the decoding procedure, determined by the Bellman equation (11), the optimization is carried out for subcarriers 0 and 1 in a joint manner. The same procedure is repeated for the remaining $\frac{N_s}{M_s} - 1$ subcarrier groups.

C. LSB Constellation

In the trellis shaping for PAPR reduction, the LSB bits allocation in the constellation has a significant impact on the performance, since the decoding procedure in the trellis shaping mainly optimizes the symbol phase, in order to cancel out the sidelobes in the autocorrelation. In the context of JCS, since the goal is equivalent to minimizing the PSD variance, the phase does not have much impact. Therefore, the LSB plays a much less important role in the trellis shaping in JCS than in PAPR reduction. According to our numerical results, random allocations of the LSB bits achieve similar performances in terms of sidelobes.

D. Communication-Sensing Trade-off

Obviously, the performance of sensing is improved by the trellis shaping at the cost of lowering communication data rate. Take the sign-bit scheme for instance, the raw data for MSB is 1 bit per subcarrier and is extended to 2 bits by multiplying $\left(\mathbf{H}^{-1}\right)^T$. Meanwhile, 2 LSB bits are used for modulation directly. Therefore, each 16-QAM symbol encodes 3 bits, thus making the data rate $\frac{3}{4}$ of the original rate. As we will see, the substantial gain in the sensing performance justifies the drop in data rate.

V. Numerical Results

In this section, we use numerical simulation results to demonstrate the performance of the proposed trellis-shaping-based waveform synthesis for JCS.

A. Convolutional Codes

We adopt the convolutional code with generating matrix $(1+D^2, 1+D+D^2)$ of data rate $\frac{1}{2}$. For comparison, we also tested another code with generating matrix $(1+D, 1+D+D^2)$.

B. ISL Reduction

We first tested the sign-bit trellis shaping scheme, for both convolutional codes, with respect to different numbers of subcarriers (ranging from 32 to 352). 16-QAM is used for modulation. The resulting ISL is normalized by that of 16-QAM modulation without trellis shaping. The numerical results, averaged over 1000 realizations of random data, are plotted in Fig. 4. The following observations are made:

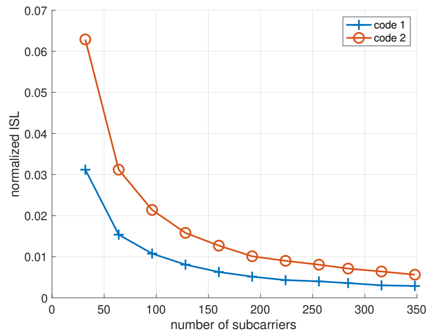

Fig. 4: ISL versus different numbers of subcarriers

- The ISL is substantially mitigated. Even for the worse case, the ISL is reduced by more than 90%. This demonstrates that the performance gain in sensing is worthy of the reduction in data rate.
- The performance is improved by using more subcarriers.
- The performance of the code $(1+D^2, 1+D+D^2)$ is remarkably better than the alternative code $(1+D, 1+D+D^2)$.

To further explore the mechanism of ISL reduction, we plotted the cumulative distribution functions (CDFs) of the 1000 values of ISL resulted from the trellis shaping, for the case of 32 subcarriers and 128 subcarriers. We observe that the ISL assumes only a few values with different probabilities. Further exploring into the simulation outcome, we found that, in certain situations, the cost function can be reduced to zero; namely all the decoding output (the codeword $\mathbf{y}$) has MSB 01 or 10, thus making the PSD constant. Then, the LSB bits modulate the phase. In other situations, due to the requirement of starting state 00, the first subcarrier cannot output 01 or 10, thus incurring a small power variance and a small but nonzero ISL. This demonstrates that the reduction in the data rate (from 1 to 1/2) substantially enlarges the signal space such that the waveform can be shaped for a very good performance. Meanwhile, it motivates us to study the simple scheme of phase shift keying (PSK) for communications and subcarrier power control for sensing.

We also tested the performance of the high-dimensional modulation and found that the performance is similar to the sign-bit trellis shaping.

C. Demodulation

The original bits in $\mathbf{s}$ can be recovered by removing $\mathbf{y}$ with $\left(\mathbf{H}^{-1}\right)^T$. However, this depends on the assumption that the demodulation is free of errors. In noisy channels,

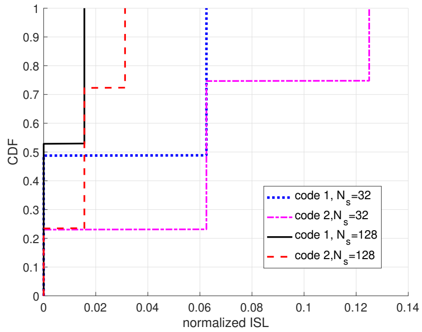

Fig. 5: CDFs of ISL for different coding schemes

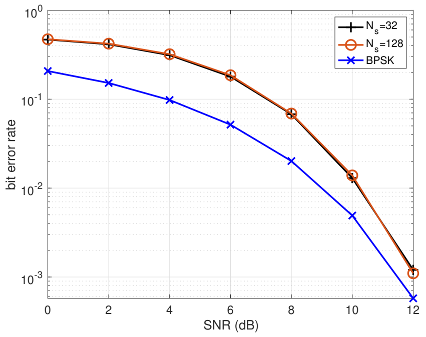

Fig. 6: Bit error rate of MSB in trellis shaping

when there exist errors, it is not clear whether $\mathbf{s}$ can still be recovered (despite some errors) as if $\mathbf{y}$ does not exist. Therefore, we tested the performance of bit error rate at the receiver, by assuming Gaussian noise with different signal-to-noise ratios (SNRs). We use the code with $G = (1 + D^2, 1 + D + D^2)$. The corresponding parity check matrix is given by $H^T = (1 + D + D^2, 1 + D^2)^T$, with the generalized inverse $\left(H^{-1}\right)^T = (D, 1 + D)$. The bit error rates of cases $N_s = 32$ and $N_s = 128$ are calculated for different values of SNR. We also calculated the bit error rate of Binary Phase Shift Keying (BPSK) for comparison. The results are plotted in Fig. 6. We observe that there is a significant gap between the bit error rates (BERs) of the trellis shaping and BPSK. Therefore, the increased BER also reduces the channel capacity, in addition to the lower data rate.

VI. Conclusions

In this paper, we have discussed the scheme of waveform synthesis using trellis shaping. Similarly to and also motivated by the PAPR reduction in OFDM waveforms, we consider standard communication modulation and coding schemes in OFDM, and refine the waveform for a better sensing performance using the trellis coding, which has been employed in PAPR reduction in OFDM communication systems. Being dual to the PAPR reduction in OFDM, the trellis shaping for improving the sensing performance in JCS has been formulated using the metric of PSD variance. We have used numerical simulations to demonstrate the validity of the proposed JCS scheme.

References

[1] Y. Fan, J. Bao, M. S. Aljumaily, and H. Li, "Communications via frequency-modulated continuous-wave radar in millimeter wave band," in *Proc. IEEE Global Communication Conference (Globecom)*, 2019.

[2] H. Li, "Interference mitigation in joint communications and sensing-Part I: Correlation and collision," in *Proc. IEEE International Symposium of Joint Communications and Sensing*, 2023.

[3] E. Aboutanios, W. Baxter, and Y. D. Zhang, "Improved implementation of the frequency hopped code selection DFRC scheme," in *Proc. IEEE Radar Conference*, 2023.

[4] H. Li, "Performance trade-off in inseparable joint communications and sensing: A Pareto analysis," in *IEEE International Conference on Communications (ICC)*, 2022.

[5] H. O'Neil and L. B. Lopes, "Performance of amplitude limited multitone signals," in *Proc. IEEE Vehicular Technology Conference*, 1994.

[6] P. W. J. van Eetvelt, G. Wade, and M. Tomlinson, "Peak to average power reduction of OFDM schemes by selective scrambling," *Electornics Letters*, vol. 32, pp. 1863–1964, 1996.

[7] H. Ochiai, "A novel trellis-shaping design with both peak and average power reduction for OFDM systems," *IEEE Trans. on Communications*, vol. 52, no. 11, 2004.

[8] S. Litsyn, *Peak Power Control in Multicarrier Communications*. Cambridge University Press, 2007.

[9] G. D. F. Jr, "Coset codes - Part I: Introduction and geometrical classification," *IEEE Trans. on Inform. Theory*, vol. 34, pp. 1123–1151, 1988.

[10] L. Zheng, M. Lops, Y. C. Eldar, and X. Wang, "Radar and communication co-existence: An overview," *arXiv preprint arXiv:1902.08676*, 2019.

[11] F. Liu, C. Masouros, A. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: Applications, state-of-the-art, and the road ahead," *IEEE Transactions on Communications*, 2020.

[12] D. Ma, N. Shlezinger, T. Huang, Y. Liu, and Y. C. Eldar, "Joint radar-communications strategies for autonomous vehicles," *IEEE Signal Processing Magazine*, vol. 37, no. 4, pp. 85–97, 2020.

[13] Y. Ma, G. Zhou, and S. Wang, "Wi-Fi sensing with channel state information," *ACM Comput. Surv.*, vol. 52, no. 3, 2019.

[14] G. D. F. Jr, "Trellis shaping," *IEEE Trans. on Inform. Theory*, vol. 38, pp. 281–300, 1992.

[15] J. L. H. He and P. Stoica, *Waveform Design for Active Sensing Systems: A Computational Approach*. Cambridge University Press, 2012.