The Supportive Role of Active Learning in a Calculus Course on Low Precalculus Proficiency Students

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College calculus plays an important role in STEM students' degree and career aspirations. One of the key factors considered in assessing a student's ability to be successful in calculus is their proficiency in topics from prior mathematics courses such as algebra and precalculus. This study set out to examine the impact of students' precalculus proficiency on their achievement in introductory calculus based on their classroom environment. Results from the implementation of the Modeling Practices in Calculus (MPC) model, an innovative, active learning approach, are presented. Using a randomized-controlled trial research design, students were randomly assigned to MPC and traditional, lecture-based calculus sections. The Precalculus Concept Assessment inventory was administered to gauge students' precalculus proficiency. We found that students exposed to the MPC model were more likely to be successful in their calculus course, even if they began with low precalculus proficiency. Also, students enrolled in the MPC sections saw significant growth in their precalculus proficiency from the beginning to the end of the semester. Additionally, we observed this model providing support for students in key demographics (low proficiency, female, underclassmen) in terms of the development of their proficiency that they may not receive in traditional classrooms.

Keywords: active learning, calculus, precalculus proficiency, calculus achievement

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Introduction

The success of students in university calculus courses has long been a matter of focus for the academic community (Bressoud et al., 2013; Treisman, 1992). Given its critical role in the science, technology, engineering and mathematics (STEM) curriculum (Bressoud, 2015), students not being able to be successful in their introductory calculus courses leads to them not being able to pursue or continue into a STEM field. Therefore, calculus acts as a filter to STEM degrees and careers (Habre & Abboud, 2006; Judson & Nishimori, 2005; Moore, 2005). The problem of low retention and persistence is coupled with consistent high failure rates that have been incredibly hard for mathematics departments to successfully address (Subramaniam et al., 2008).

Recently, active learning strategies have been utilized in university classrooms as a way to address the struggles of students in introductory STEM courses (Bennett, 2022; Lugosi & Uribe, 2022). Evidence has been found that implementing active learning in undergraduate STEM courses can lead to increases in students' performance and learning (Freeman et al., 2014). These findings have led to the suggestion of including more of these instructional techniques in courses such as introductory calculus in an effort to address the observed issues with performance and persistence in the course (Laursen & Rasmussen, 2019). Although the presence of these strategies has been shown to lead to improved outcomes (Kogan & Laursen, 2014; Lugosi & Uribe, 2022; Springer et al., 1999), not as much is known about the role active learning can play in supporting students who enter these courses with deficiencies in their prior mathematical knowledge.

One of the factors most often attributed to the failure of calculus is precisely these deficiencies, which stem from a lack of prior mathematics preparation. A lack of proficiency in algebra and precalculus skills has been found to have a significantly negative impact on students' performance in their university calculus course (Agustin & Agustin, 2009), while sufficient preparation in previous mathematics courses has been shown to lead to success in calculus (Ayebo et al., 2016). In order to address the issue of students being underprepared for calculus, universities have implemented strategies such as remedial or developmental mathematics courses that must be completed before enrolling in introductory calculus (Bailey et al., 2010). However, it is not clear that these types of courses are succeeding in developing students' proficiency enough to be successful in calculus. One particular study found that students who take university precalculus do not earn higher grades in calculus. In fact, there were signs that prior enrollment in a college precalculus course led to lower calculus performance for particular groups of students (Sonnert & Sadler, 2014). This negative impact could be explained in a couple of ways. Fife (1994) argues that remedial or developmental mathematics courses are usually designed very similarly to high school precalculus, and that this similarity might negatively influence students' motivation. As a consequence, this lack of motivation might limit student learning outcomes. Meanwhile, Rutschow et al. (2017) suggest that these courses are usually too focused on procedural knowledge, limiting students' ability to develop deeper mathematical connections required to make sense of the main calculus ideas.

Despite the reasons, even if students enter calculus with deficiencies in precalculus and algebra skills, it is still important for them to be able to develop their proficiency in these areas. This is especially true if students intend to continue in STEM courses given the skills associated with precalculus and algebra are foundational quantitative and reasoning skills that will continue to be key to the success in future mathematics and science courses.

In an effort to understand more about the impact that active learning can have on students who begin calculus with a low proficiency in precalculus and algebra content, the Precalculus Concept Assessment (PCA), a validated precalculus concept inventory meant to assess students' knowledge on foundational topics for calculus was administered at the beginning and end of the semester in an introductory calculus course implementing the Modeling Practices in Calculus (MPC) model. This model is designed to have students engage in and model the practices of mathematicians by actively doing mathematics in a lecture-reduced classroom with a focus on them working with their peers. Both the inventory and model will be described in detail in later sections.

This present study aimed to examine the impact of the MPC model on those students who may be considered underprepared for introductory university calculus based on their skills in and understanding of foundational calculus topics - namely those topics native to a precalculus course. So, this study was guided by the following research questions: (1) Does the use of the MPC model in a calculus course affect the likelihood of low precalculus proficiency students being successful in the course? (2) Does the precalculus proficiency of students develop over the course of one semester of being exposed to the MPC model in calculus? If so, how does this development in proficiency compare to the development observed in traditional (non-MPC) classrooms? (3) How does the impact of the MPC model on precalculus proficiency development compare to traditional instruction for students in key demographics such as initial low precalculus proficiency, female, and underclassmen?

Relevant Literature

Mathematics preparation and proficiency in calculus

Students' mathematical preparation prior to entering university calculus and how it affects their performance and other outcomes have long been a point of focus in the mathematics education community. In a nationwide large-scale study of US institutions, Sadler and Sonnert (2018) found mathematics preparation in high school to be a significant predictor of their university calculus grade, including increases of half a letter grade in calculus final grade for students who previously enrolled in high school calculus. Ayebo et al. (2016) confirmed this finding in a mixed-methods study that included calculus professors, teaching assistants, and students, linking adequate preparation in high school precalculus not only to success in college calculus but also to success in future mathematics courses. In terms of the importance of precalculus proficiency in relation to calculus success, a study on professors' views on instructional practices that best prepare students for university calculus found that professors saw a need for stronger algebra and precalculus skills (Wade et al., 2016).

Students having misconceptions or misunderstandings of those topics which are foundational to a calculus course can hinder their success in the course. In fact, it's been found that even though students may be able to comprehend the calculus concepts being taught, inadequacies in algebra and precalculus topics can have a detrimental effect on them being able to solve the actual calculus problems (Agustin & Agustin, 2009). Furthermore, in a study exploring factors that impact achievement in calculus, students' algebraic skills were found to be a significant predictor of course success (Edge & Friedberg, 1984).

Alternatively, it can be argued that some of the fault of students being underprepared for university calculus lies in the lack of previous coursework at the university level. University students labeled as “academically underprepared” in mathematics are typically placed in remedial or developmental mathematics courses or precalculus courses (Hodara, 2013). However, it has been found that students taking a college level precalculus course has no significant impact on their performance in college calculus (Sonnert & Sadler, 2014). So, although the problem of students’ inadequate mathematics preparation in calculus is critical, evidence of efficient strategies that can be implemented to help students overcome this lack of proficiency while enrolled in calculus is limited.

Active learning in mathematics

Along with the perceived underpreparedness of students, another component that has been found to contribute to a student’s performance and persistence in pursuing a STEM degree is the experience they have with the type of instruction in their first-year mathematics course. Research has shown that traditional instruction that mainly emphasizes rote memorization as opposed to actual understanding of the concepts presents one of the main reasons that students fail to persist in STEM (Seymour & Hewitt, 1997). Meanwhile, courses which provide students with opportunities to be actively engaged in the process of learning lead to improved outcomes (Adams & Dove, 2018). In light of this, the implementation of active learning and other student-centered techniques have become a point of focus in improving the outcomes and experiences of students in introductory mathematics courses such as calculus (Laursen & Rasmussen, 2019; Stanberry, 2018). There is evidence of increased student outcomes through the implementation of active learning in precalculus courses (Gruber et al., 2021); however, less is known about how active learning may influence the development of these foundational skills during a calculus course. It may be reasonable to expect that students being actively engaged in the learning and doing of mathematics would lead to improved proficiency in content knowledge and skills, but if its influence would impact particular groups in the same way is less known. More specifically, the potential of active learning techniques benefitting groups who are known to have difficulties in succeeding and persisting in calculus and STEM fields is underexplored.

Key demographics

Studies which have focused on the success and retention in calculus and STEM overall have typically centered on some key populations of students. One group of particular interest for this present study are students who are deemed as not calculus ready, or students who come into introductory calculus underprepared and with a low precalculus proficiency. Given that calculus serves as a gateway course for STEM degrees, mathematically underprepared students face great difficulty in not only being able to be successful in calculus, but also in being able to continue on the STEM path.

Additionally, past studies particularly indicate difficulty in retaining women in college-level mathematics and science courses (Ellis et al., 2016; Steele et al., 2008), which results in women being more likely to leave the STEM major. Since it has been shown that active learning in mathematics can serve as a gender equalizer when it comes to self-confidence for female students (Castillo et al., 2022b), it may be within reason to expect a similar impact of active learning on the proficiency of these students. In terms of students' class standing, it has been reported that while approximately 75% of students enrolled in introductory calculus intend to major in a STEM field (Bressoud et al., 2013), only about 40% of incoming freshmen intending to major in STEM are actually successful in earning a STEM degree (President's Council of Advisors on Science and Technology, 2012). Learning more about if and how active learning impacts first-year students' development of knowledge of foundational concepts is crucial, given that the performance of a university student in their first semester mathematics class has been found to be significantly correlated to their retention in a STEM degree (Gardner et al., 2007).

Description of the MPC Model

Introductory calculus at the institution where this study took place involves multiple sections with class sizes of 30 – 40 students on average that mainly utilize lecture-based techniques, much like mathematics classrooms at similar large, research-intensive universities (Bressoud, 2015; Keller & Johnson, 2019; Scott et al., 2016). This heavy reliance on lecture can foster passive learning (Bransford et al., 2000) and reduce the opportunities for students to be actively engaged with the material. In light of these shortcomings resulting from lecture-based techniques and in an effort to implement evidence-based instructional practices in calculus courses, the authors of this study created the Modeling Practices in Calculus (MPC) model. At its core, the purpose of the MPC model is to have students learn the concepts of calculus by emulating, or modeling, the practices of mathematicians. These practices include sense making, adaptive understanding and reasoning of mathematical concepts, and developing skills needed to effectively communicate mathematical ideas. This model incorporates recommendations for mathematics and calculus instruction and teaching practices promoted by national mathematics organizations, societies, and reports (Bressoud, 2015; Rasmussen et al., 2019; Sonnert et al., 2015). Overall, the MPC model was designed to have instructors be intentional about reducing the amount of lecture taking place in the classroom and increasing the amount of student engagement with each other and the material. It was expected that the amount of lecturing in a typical MPC classroom, if any, would be brief and not take up more than approximately 20% of class time. The remainder, and majority, of class time is intended to be dedicated to having students actively engage with each other and the material, with the active support and facilitation of the instructors.

Instructors who implemented the MPC model participated in a three-day professional development workshop before the semester began to introduce them to the model. To further support the adoption of the model, instructors also attended weekly planning meetings throughout the semester to facilitate course coordination, set course norms and expectations, and allow them opportunities to provide feedback and reflect on their experiences with the MPC model.

With the intent to promote students' understanding and learning of calculus concepts, the MPC model integrates four central elements: cooperative learning (Johnson et al., 2007; Slavin, 1996), social metacognition (Jost et al., 1998; Stanton et al., 2021), a culturally appropriate learning environment (Ginsberg & Wlodkowski, 2009), and mathematical fluency (National Research Council, 2001). Cooperative learning is fostered by allowing students to accomplish shared goals, develop social skills, and create interpersonal relationships by having them work in small groups on activities that lead them in mathematical investigations. The model promotes social metacognition by these groups not only working together on these activities, but also by asking them to present their work on whiteboards to each other and the whole class. During this process, students are able to observe each other's thinking, identify mistakes, and make suggestions in order to guide their group's problem solving. This opportunity to share leads to students engaging in meaningful discourse while providing justification for their work and taking into consideration others' input.

The MPC model is also intentionally designed as a culturally appropriate learning model as it allows students to test their ideas in a low-stakes, safe environment while receiving continual formative feedback from an instructional team and participating in a community of learners. Rather than only providing lectures, instructors also serve as facilitators of learning with the support of Learning Assistants (LAs), who are trained 'near-peer' undergraduate classroom facilitators, integrated into the classroom to support learning (Otero et al., 2010). The instructor and LAs establish a safe learning environment by messaging to students throughout the semester that it is okay to ask questions and make mistakes. As the demographics of the LAs (age, undergraduates, STEM majors) are that of the students, LAs are seen as natural agents of the culturally appropriate component of the model. They are able to offer insight to students from the point of view of a recent participant of the course and help in the communication between the instructor and the students. Instructors meet weekly with LAs to learn how groups are doing in the course and what struggles students currently face in terms of learning. LAs provide valuable feedback alerting instructors when more clarity is needed. They are an invaluable part in holding the class together as a learning community (Emenike et al., 2020).

The MPC model's elements of cooperative learning, social metacognition, and a culturally appropriate learning environment have been previously described along with their impact on students' attitudes towards mathematics (Castillo et al., 2022b; Duran et al., 2022). However, less emphasis was given to mathematical fluency, which is likely the core element closest linked to precalculus proficiency (Oehrtman et al., 2008). This element involves setting the goal for students to be able to execute mathematical tasks accurately, appropriately, and efficiently (National Research Council, 2001). Fluency in this sense is promoted by asking students to notice mathematical relationships and guiding them to develop strategies to solve problems and present solutions in a manner that is easily understood and justifiable. On a typical day in the MPC model, students are presented with guided notes and a learning activity on a particular topic of coverage. The guided notes consist of an introduction of key concepts, examples, and supporting questions intended to foster discussion among the groups and the whole class. In an effort to promote proficiency with terminology, notation, and symbols, the notes include definitions of key terms and examples that have been solved along with the full solutions. The notes also provide examples for the students to complete in their groups by paying attention to the steps shown and notation used in the completed solutions. The learning activities, which complement the notes, are composed of problems meant to challenge and continue to enhance the students' proficiency and understanding of the topic.

There are a couple of reasons why an active learning approach such as the MPC model could have a significant impact on the achievement of students with low precalculus proficiency. First, active learning classrooms increase opportunities for instructors and LAs to provide formative feedback tailored to more specific needs. Fear of judgement or ridicule due to gaps in prior knowledge often discourage students with low precalculus proficiency to pose questions, ask for help, and get timely feedback from instructors and LAs in lecture-based classrooms (Kalinec-Craig, 2017). Second, the more familiar setting of small groups and establishment of appropriate social norms in well-designed active learning approaches facilitate a sense of community in MPC classrooms. This sense of community could lead to increased social cohesion (Moiseyenko, 2005), naturally linked to enhanced peer-to-peer support and better distribution of skills in the classrooms when compared to lecture-based sections. Lastly, gains in self-confidence found in active learning calculus classrooms (Castillo et al., 2022b) might be associated with a low-stakes environment where these students are more likely to overcome their limitations to ask for help. Despite these theoretical expectations, direct evidence of the impact of the MPC model, and active learning strategies in general, on students with low precalculus proficiency is still limited. The present study was intended to address this gap in the literature.

Methods

This study took place at a large, research-intensive (R1), Hispanic-serving institution in Calculus I courses during the fall of 2019 and was conducted under the approval of the Institutional Review Board. In order to assess the effectiveness of the MPC model on student outcomes, we utilized a randomized control trial (RCT) research design with students being assigned to Calculus I sections using either the MPC model or sections using traditional instructional techniques (non-MPC). In the non-MPC sections, the instructors were not directed to use any specific instructional practices, but the use of lecture-based methods are the norm for these sections. In the fall of 2019, a total of 527 students were randomly assigned to 16 sections (8 MPC and 8 non-MPC) of a Calculus I course. Given the focus of this study, participants only included those who responded to at least one item of one administration of the survey that was used to determine their precalculus proficiency.

Participants

Of the students who were assigned to the MPC and non-MPC sections, we collected a total of 387 surveys during the initial administration (pre) and a total of 271 for the final administration (post). Demographic data was reported by students to the university and collected at the time of course enrollment. A breakdown of those participants who took at least one administration of the survey and their demographics can be seen in Table 1.

Measures and procedure

Students' precalculus proficiency was measured using the Precalculus Concept Assessment (PCA) instrument (Carlson et al., 2010). The PCA is a 25-item multiple choice inventory designed to assess a student's knowledge on those topics native to precalculus courses and those which are foundational to calculus courses. It was developed based on the PCA taxonomy which compiles several studies intended to characterize essential reasoning abilities and understandings for learning introductory calculus. The PCA survey was validated through clinical interviews and included satisfactory measures of reliability (Carlson et al., 2010). Since each item in the PCA survey was worth one point and students were not necessarily required to answer each item, the scale values ranged between 0 to 25.

In terms of the administration of the survey, students were asked to complete the PCA survey at the beginning (first week of classes) and end of the semester (last two weeks of classes). Surveys were distributed and collected by the instructors, who followed a protocol in which students were ensured their participation was voluntary and was not going to influence their grade in any way. Additionally, students were never informed of their score on either administration of the survey.

Data analysis

To confirm the well-balanced sample design of the RCT study, we conducted a two-tailed independent samples Welch's t-test assessing the significance of differences in mean score on the initial administration of the PCA inventory between the MPC and non-MPC sections.

An optimal cutpoint analysis using the Youden index metric (Perkins & Schisterman, 2006) was carried out on the non-MPC sections as a baseline to identify two proficiency levels: Low and High. Scores below the cutoff found in this analysis were associated with students in the Low precalculus proficiency level group considered more likely to fail the class.

The likelihood of low precalculus proficiency students being successful in the course was modeled using a logistic regression predicting the categorical binary variable of course success (1: Pass; 0: DFW) for those students who scored below the cutoff previously mentioned. Three independent categorical binary variables were considered in this model: Treatment (1: MPC; 0: non-MPC), Gender (1: Female; 0: Male), and Class Standing (1: Freshman/Sophomore; 0: Junior/Senior). Gender was represented as a binary variable given the available institutional data. This description of gender was considered a limitation of this study since it precluded the model from better representing all students. Predictors were chosen based on a priori expectations of interactions with the dependent variable, as explained in previous sections. Additional relevant predictors such as race/ethnicity, high-school GPA, and socioeconomic status were not included in the study due to data collection limitations. Since this study was exploratory, and there were a priori reasons to assume initial variables were all relevant, a full model was used for variable selection. Stepwise methods were also avoided to minimize bias in parameter estimation and error inflation (Harrell, 2001).

The development of precalculus proficiency was examined using two different analyses. First, precalculus proficiency development was measured using PCA score gains (difference between post- and pre-test raw scores). Gains were initially preferred over other measures of change (such as post-test scores or normalized gains) because the RCT design suggested a reasonable baseline comparison to control for prescore bias, and it was also easier to interpret than normalized gains (Coletta & Steinert, 2020). Gains provided a direct interpretation in terms of the difference in the number of items students responded to correctly between treatments. Descriptive statistics were used on prescores to illustrate baseline comparisons. The significance of these gains (whether they were different than zero) in each treatment group was assessed using two-tailed Welch's t-tests. In addition to gains, as an alternative measure of proficiency development, differences in frequency of students categorized by initial and end-of-semester precalculus proficiency levels were also examined. Comparisons of these differences between MPC and non-MPC sections were carried out using Pearson's Chi-square tests (McHugh, 2013).

Comparisons of proficiency development by demographics groups (gender, class standing, and initial proficiency) were carried out by assessing differences in PCA normalized gains between treatments. The significance of these differences was assessed using two-tailed Welch's t-tests. Normalized gains were preferred over raw gains in this analysis since the research question was focused on the impact of the MPC model on groups of students, and normalized gains provided a better basis of comparison and measure of effect size (Coletta & Steinert, 2020).

Since the nature of this study is exploratory, corrections for multiplicity were not conducted to prevent error type II inflation (Streiner, 2015; Streiner & Norman, 2011). Cohen's d effect sizes were calculated in calculated gains to facilitate comparisons with other studies (Cohen, 1988).

Considering the usual drop rate that occurs over the course of a semester and the lower daily attendance rate during the end of the semester, it was expected that fewer students would complete the post administration of the survey. However, the overall unweighted unit non-response rate in both MPC and non-MPC groups was less than 38%, which is still considered appropriate in academic studies in general (Baruch, 2016). Item non-response rates were considered small (less than 5%) to address item missingness with multiple imputation (Jakobsen et al., 2017). Hence, missing items were treated as incorrect answers.

Results

Sample comparison

In the fall of 2019, the mean score on the initial administration of the PCA (out of 25) for the non-MPC group was 9.79 ($n = 177$, $SD = 4.15$) and 9.51 ($n = 210$, $SD = 4.28$) for the MPC group. This difference between the two groups was not statistically significant [$t(377.6) = 0.644$, $p = .5197$]. Such similar mean prescores between groups are consistent and expected with the RCT design of the study. We note that these pre-test scores were slightly lower than, but still similar to, the mean score found in a previous study (mean = 10.2, SD = 4.1) conducted by Carlson et al. (2010). However, that particular study was situated in a different context that included a sample of over 900 college students who took the PCA inventory at the end of a precalculus course.

Cutoff score

The optimal cutpoint analysis identified a PCA score of 11 as the cutoff for Low and High precalculus proficiency levels. Although the accuracy of this cutoff score was fairly reasonable, an equal frequency bins analysis was also conducted finding consistent results. Using this cutoff score, we found that 58.2% of non-MPC students and 59.5% of MPC students were classified as having low initial precalculus proficiency. The difference between the percentage of students identified as Low or High in the initial administration of the assessment was not significantly different when comparing the MPC and non-MPC sections ($\chi^2 = 0.0261$, $p = .8716$).

Course achievement

In order to learn about the impact of the MPC model on students' achievement who begin calculus with low precalculus proficiency, we only considered data from the initial administration of the PCA inventory. So, a logistic regression analysis was conducted to investigate the extent to which the use of the MPC model in a calculus course affects the likelihood of achievement for Low precalculus proficiency students. No violation of linearity of the logit of the course success variable was found. This regression revealed that not only are students exposed to the MPC model more likely to pass the course, but also the odds of passing the course increased by 85% (95% CI: [1.05, 3.24], $p = .0328$) for Low proficiency students in the MPC sections compared to those in the non-MPC sections, when controlling for gender and academic level.

Considering the actual pass rates, we found that 72.0% of students in the MPC sections who were designated as having low precalculus proficiency at the beginning of the course were able to complete the course successfully, compared to 59.2% in the non-MPC sections. This difference in pass rates was significant ($\chi^2 = 3.57$, $p = .0445$) and translated to 30 more MPC students in the Low precalculus proficiency group being able to pass the course than those in the non-MPC sections.

Also of note is that the Low precalculus proficiency students in the MPC sections were passing at a rate comparable to the overall pass rate of the whole MPC group (76.8%). This difference in pass rates between the Low proficiency group and the whole group is more emphasized in the non-MPC sections where the overall pass rate was 68.1%.

Development of precalculus proficiency

To learn more about the impact of the implementation of the MPC model on students' precalculus proficiency development, we analyzed the data from both the initial and final administrations of the PCA to determine if there were any significant gains in scores or development of proficiency.

The mean scores of students who took both the administrations (pre and post) of the PCA inventory are illustrated in Figure 1. Of those students who completed both administrations, we observed a significant gain ($t(166) = 5.42$, $p < .001$) in the performance of students enrolled in MPC sections of calculus on the PCA between the beginning and end of the semester. As seen in Table 2, on average, the MPC sections scored about 1.5 points higher on the post-test. Given the PCA was scored with each question being worth one point, the sections of MPC answered almost two more questions correctly between the pre- and post-test. The students in the non-MPC sections displayed a significant decrease in scores ($t(92) = -2.52$, $p = .0135$) on the assessment, averaging a little over a point less on the post-test. This translates to the non-MPC sections answering one less question correctly on the post-test. We also notice that the post score of students in the MPC section is on average 1.54 points higher ($t(177.35) = -2.70$, $p = .0075$) than the post scores of those in the non-MPC sections. This difference translates to the MPC sections answering about 2 more questions correctly on the post-test than the non-MPC sections.

It should be noted that in the matched data (students who took both the pre and post assessments), the non-MPC sections of calculus had a significantly higher average score than the MPC group on the pre-test ($t(211.68) = 2.04$, $p = .0425$). But we recall that in the unmatched data (all students who took the pre-test), there were no significant differences on the prescore.

In order to add more to the picture of the gains we observed in the raw scores of the PCA, we also looked at the development over time in terms of proficiency level. Using the cutoff of a score of 11 or higher, Figure 2 illustrates the percentage of students in both groups who fell into the Low and High proficiency levels for the pre-test and post-test.

We first noticed that when considering the frequency of students in the Low and High levels for the post assessment, the MPC group has a smaller percentage in the Low proficiency level and a larger percentage in the High proficiency group when compared to the non-MPC group. This imbalance is even more compelling given the fact that the percentage of students classified as Low or High in the initial administrations of the assessment are significantly different when comparing the MPC and non-MPC groups ($\chi^2 = 4.45$, $p = .0348$). A primary reason why the Low proficiency group was initially larger in the MPC sections is likely due to the higher drop rate found in the non-MPC sections. A higher number of students dropping the course in non-MPC sections translated into a higher number of students missing the post assessment. Since the precalculus proficiency of those students dropping the course was lower on average, the final percentage of students in the Low proficiency group was then likely underrepresented for the non-MPC sections. At the same time, since the pre-tests of these students were discarded in the pre-post matching process, the initial percentage of non-MPC students with Low proficiency was also presumably underrepresented.

Now, focusing on the movement between the levels from the initial to final administrations, we did observe some significant differences in the movement, or lack thereof, between levels when comparing the MPC and non-MPC groups. Table 3 gives the percentages of students in each group who changed proficiency level from the beginning to end of the semester, or remained at the same level. Of the students who began in the Low proficiency group, over twice as many students in the MPC group were able to move up into the High proficiency level by the end of the semester when compared to the non-MPC group. This shift in levels is mirrored in the High pre-level group, with two times as many non-MPC students moving down into the Low proficiency group for the post-assessment. These differences in movement are both significant, however the movement within the Low proficiency pre-level group is quite notable. This movement between levels of proficiency is also illustrated in Figure 3 where the proficiency development within the MPC group is noticeable. We can see a significant number of MPC students in the Low proficiency pre-level group moving into the High proficiency post-level.

Development of precalculus proficiency by key demographics

To understand more about the gains observed from the pre-test to post-test, normalized gains were used to compare these differences across key demographics in both MPC and non-MPC sections. These gains are presented for each subgroup in Table 4 and Figure 4. As expected, based on the gains we observed in the raw scores, the MPC group had a significantly higher normalized gain overall ($t(148.6) = -3.93$, $p < .001$) when compared to the non-MPC group with a medium effect size ($d = 0.553$). Comparisons across the demographics of interest behaved in a similar manner as the overall, with the normalized gains being significantly higher for the MPC group in each subgroup.

As seen in Table 4, we observed a positive gain in each key demographic for the MPC group as opposed to negative gains for each non-MPC subgroup. Also, Figure 4 indicates that each subgroup of interest in the MPC group had a normalized gain significantly above zero. In both the MPC and non-MPC groups, the gains for the initial Low precalculus proficiency subgroups were larger than the other subgroup gains. Even more compelling is the fact that the Low proficiency group in the MPC sections had the largest overall normalized gain when comparing within and across groups. We also noticed the largest gap between the MPC and non-MPC group occurred in the

Freshmen/Sophomore subgroup with the MPC group seeing a larger, positive impact on their precalculus proficiency.

Discussion

In this study, we set out to learn more about the effectiveness of utilizing the MPC model in an active learning calculus course. Specifically, we were interested in learning about how the model influenced the performance of students beginning the course with low precalculus proficiency and how this model impacted the development of students' precalculus proficiency. The results showed students enrolled in MPC sections were more likely to pass the course ($p = .0445$) even if they began with a low precalculus proficiency. Furthermore, students exposed to the MPC model in calculus were able to significantly develop their precalculus proficiency over the course of the semester ($p < .001$). When compared to students in the non-MPC sections of calculus, students in MPC sections showed higher gains in their proficiency ($p < .001$) and were able to move up to a high proficiency level in greater numbers ($p = .0071$). This significantly higher development in precalculus proficiency for students in the MPC sections was also observed among each key demographic: initial low precalculus proficiency ($p < .001$), female ($p = .0022$), and underclassmen ($p < .001$).

We believe the results of this study emphasize the importance of designing university calculus courses which present students a path to success even if they enter the course with inadequate preparation. This study also adds to existing evidence showing that the act of revising the types of instructional techniques in these introductory mathematics courses, as opposed to revising the actual content of the course, can lead to improved and desired outcomes.

The MPC model and low precalculus proficiency students

The results in this study are consistent with the findings by Carlson et. al. (2010) that a students' initial precalculus proficiency has a significant impact on students' ability to receive a passing grade in college calculus. This was true for students in both MPC and traditional sections of calculus. However, we found that students in the MPC sections were more likely to succeed in the course even if they come into the course with a lower precalculus proficiency. So, having a low precalculus proficiency did not preclude students enrolled in MPC from successfully completing first semester calculus. Therefore, the MPC model is effective in helping students who may be deemed as "not calculus ready" to be able to successfully complete calculus. This also points to opportunities for universities to alter the ways in which they view and determine students' readiness for calculus. The results from this study suggest that by providing certain types of instructional techniques and environments in these courses, students' prior knowledge or preparation will have less of an impact on their ability to succeed.

The positive impact of the MPC model on the achievement of students with low precalculus proficiency is significant, and well-aligned with our initial theoretical expectations. The more familiar setting of small groups, individualized feedback from instructors and LAs, and social norms in MPC classrooms were expected to promote a low-stakes environment where students with low precalculus proficiency were more confident to overcome their limitations. Additionally, increased social cohesion in MPC classrooms could have facilitated a better distribution of skills in the classrooms. For example, students with higher precalculus proficiency could be providing support to students with lower proficiency in addition to the support found in the instructors and LAs.

The MPC model and precalculus proficiency development

We also observed that students enrolled in MPC sections were more likely to improve their precalculus proficiency, as measured by the PCA survey, from the beginning to the end of the term. This finding is extremely encouraging for various reasons. For one, it implies that active learning is playing a primary role in supporting the precalculus proficiency development of university calculus students. This is a compelling finding because it shows that active learning is not only positively impacting students' achievement in calculus, but could also be providing more opportunities for students to develop skills by being exposed to an environment where consistent practice of these skills is encouraged and necessary. These opportunities are likely to be minimized in a traditional, lecture setting. Furthermore, it suggests that active learning could be helping students' preparation for subsequent coursework in STEM. This is also a particularly relevant outcome, considering the need for foundational quantitative skills found in these courses. This enhanced preparation could have a positive impact on a student's choice to continue to pursue a STEM major and career, one of the primary concerns in STEM education (President's Council of Advisors on Science and Technology, 2012).

The increases found in students' precalculus proficiency is an especially significant result for those in particular demographics that have been the focus of past studies on persistence and retention in STEM, namely, female students, underclassmen, and those with low precalculus proficiency. Each subgroup in the MPC sections saw similar or better gains in precalculus proficiency when compared to the overall group. When looking at gender, we found that both male and female students are benefiting from the MPC model in similar ways in terms of proficiency development. Also, when comparing the normalized gains on the PCA inventory between the MPC and non-MPC groups, we saw a large effect size among Freshmen students. This, again, implies that the MPC model is having a particularly positive influence on this group. Furthermore, since the majority of the students in our study are underclassmen and STEM majors, our results indicate that active learning could help to better address the quantitative needs of their curriculum, as their understanding of the process view of functions, covariational reasoning, and proficiency in computational abilities, measured by the PCA inventory, have previously been identified as key needs by multiple studies (Ganter & Barker, 2004; Marshall & Durán, 2018).

The findings in our study also help to highlight the emphasis of mathematical fluency as a key principle of the MPC model. We believe that providing students with multiple and consistent opportunities to not only practice problems involving a combination of calculus and precalculus skills, but to also focus on using logical and correct mathematical notation and representations, serves as a way to reinforce and strengthen these foundational skills and to develop in proficiency (McNicholl et al., 2021). In addition, we claim that constantly placing an emphasis on these skills will not only translate to success in a student's introductory calculus course, but will also present itself as useful in future mathematics and science courses.

Future research and limitations

An analysis of the specific items and topics, as characterized by Carlson et al.'s PCA taxonomy (2010), that are leading to increases in precalculus proficiency would provide more depth to our results indicating specific reasoning abilities, understandings, and skills in which students are improving. Additionally, in light of a recent study exploring LA perceptions in an active learning calculus course (Castillo et al., 2022a), a further analysis of how the near-peer support of LAs is impacting not only student performance in the course and attitudes about the subject, but also their development of quantitative skills, reasoning abilities and precalculus proficiency will further support the need for LAs in these types of introductory, gateway courses. Along with focusing more on the interaction between the LAs and the students, a study on how the overall learning community impacts students in this particular environment may be warranted in order to fully understand how this model influences the engagement and learning of students.

Some of the limitations in this study are related to the difficulty found in trying to carry out an RCT research design in postsecondary settings and implementing an active learning approach. First, this study was limited in its ability to capture what instruction looked like in both MPC and non-MPC sections. In terms of alignment between written and implemented curriculum materials, this study did not include direct measures of instructor fidelity of implementation. However, this limitation was partially addressed in the MPC sections by the three-day workshop before the start of the semester and weekly planning meetings throughout the semester to support a more reliable curriculum adoption. Additionally, although the use of LAs in the non-MPC sections was not directly examined, it can be assumed that their role was largely limited due to the predominance of lecturing in these sections, thus significantly reducing the opportunities for the LAs to meaningfully interact with the students.

Second, a slight imbalance in gender distribution when comparing MPC and non-MPC sections developed due to the way in which participants were randomly assigned from a pool of enrolled students. Students in post-secondary educational settings must have the ability to change their enrollment status and so in some cases students will move from section to section for reasons that are not related to the study. Although the statistical tools used in this study were robust against unbalanced subpopulations, a follow-up study would take an in-depth look at those students switching sections.

Lastly, the underlying mechanism of precalculus proficiency development observed in the MPC sections cannot be fully explained with the data collected in this study. However, students' mastering of key foundational concepts, particularly those with low precalculus proficiency, could be at least partly explained by increased opportunities provided by active learning for clarifying misconceptions. Instructors, LAs, and peers, especially those with high precalculus proficiency, are able to offer contrasting views and explanations that help to promote conceptual change (Asterhan & Schwarz, 2009; Driver, 1987). Since precalculus proficiency skills are foundational, these discussions would naturally present themselves throughout the semester and as a result, would offer students with low precalculus proficiency multiple opportunities to clarify misconceptions. Furthermore, structured learning activities, similar to those offered in the MPC model, provide multiple occasions for students to negotiate multiple perspectives. These opportunities are known to lead to a deeper understanding of prior concepts than group members could acquire alone (Voiklis & Corter, 2012). Qualitative studies documenting students' interactions in the classroom with peers and learning facilitators would help to further clarify this issue.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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Figures

Figure 1. Mean PCA scores of MPC and Non-MPC sections for students who took both pre- and post-administrations. Note: Error bars represent ±1 standard error.

Figure 2. Pre and post levels (High vs. Low) of precalculus proficiency for students who took both pre- and post-administrations of the PCA.

Figure 3. Precalculus proficiency level movement for MPC and Non-MPC sections for students who took both pre- and post-administrations of the PCA.

Figure 4. Normalized gains on the PCA by key demographics. Note: Error bars represent ±1 standard error.

Tables

Table 1. Participant demographics by group (MPC and non-MPC) and survey administration (pre/post/matched)

Table 2. Gains in PCA scores for MPC and Non-MPC sections

Table 3. Proficiency level movement between pre and post administrations of PCA inventory

Table 4. Normalized gains in PCA scores by key demographics for MPC and Non-MPC sections