

Math matters or maybe not: An astonishing independence between math and rate of learning in chemistry

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Abstract

Research spanning nearly a century has found that math plays an important role in the learning of chemistry. Here, we use a large dataset of student interactions with online courseware to investigate the details of this link between math and chemistry. The activities in the courseware are labeled against a list of knowledge components (KCs) covered by the content, and student interactions are tracked over a full semester of general chemistry at a range of institutions. Logistic regression is used to model student performance as a function of the number of opportunities a student has taken to engage with a particular KC. This regression analysis generates estimates of both the initial knowledge and the learning rate for each student and each KC. Consistent with results from other domains, the initial knowledge varies substantially across students, but the learning rate is nearly the same for all students. The role of math is investigated by labeling each KC with the level of math involved. The overwhelming result from regressions based on these labels is that only the initial knowledge varies strongly across students and across the level of math involved in a particular topic. The student learning rate is nearly independent of both the level of math involved in a KC and the prior mathematical preparation of an individual student. The observation that the primary challenge for students lies in initial knowledge, rather than learning rate, may have implications for course and curriculum design.

1 Introduction

This work explores how large datasets gathered from online learning environments can address long-standing issues in chemical education research, such as the role of math in the learning of chemistry explored here. There is strong evidence from past studies that students with lower preparation in math have lower success rates in introductory college chemistry (Section 2).

Here, analysis of more than 612K student interactions with the Open Learning Initiative (OLI) General Chemistry courseware¹ at a variety of two-year colleges allows us to decompose this link between math and chemistry into two separate components: the initial knowledge of individual students on topics involving a high level of math, and the rate of learning on such topics. The overwhelming result is that it is only the initial knowledge that varies strongly across students and across the level of math involved in a particular topic. The student learning rate is nearly independent of both the level of math involved in a topic and the prior mathematical preparation of an individual student.

A significant feature of the current study is the use of large volumes of data gathered as students learn the content. This rich data has two potential advantages. First, data collected throughout the learning process provides more detail than studies that use pre- and post-tests to assess learning at only two time points. Second, data gathered through an entire semester enables analyses that examine student learning across a broad range of topics. Each practice opportunity in the courseware is labeled with one or more knowledge components (KCs) involved in the learning activity. KCs are an “*acquired unit of cognitive function or structure that can be inferred from performance on a set of related tasks*”.² In this manner, the data may be viewed as including a pre- and post-test for each of the KCs in the first semester General Chemistry Course, along with data gathered in between.

A logistic regression is used to model student learning as a function of the number of interactions an individual student has had with a particular KC. We use the term “opportunities” — in the sense of “learning opportunities taken” rather than “learning opportunities provided” — to represent these interactions. This regression analysis generates estimates of both the initial knowledge and the learning rate for each student and each knowledge component. Here, the learning rate is defined in terms of the log of the odds that a student will complete a task without making an error or requesting

a hint. The learning rate for a KC is the extent to which the log-odds increase each time a student engages with a task associated with that KC.

Another significant feature of this study is that the practice opportunities in the course are highly scaffolded through hints and feedback. Hints provide support upon request by the student, and feedback provides support when the student makes an error. The scaffolding fades as students move through a topic and picks up again with new topics. These scaffolds include support for quantitative reasoning, allowing the course materials to provide just-in-time support for math. This just-in-time support likely has a strong influence on learning rates.

The current study builds on a recent finding from analysis of online practice data gathered within primary, secondary, and post-secondary math, science, and language learning courses, which found that learning rates were “astonishingly similar” across students within these courses.³ Students’ initial knowledge varies widely, but their learning rate varies little. This study adds to these results by considering more nuanced questions related to the degree to which math preparation influences the learning of chemistry content.

2 Literature Background

Mathematics is crucial in learning chemistry; it provides the tools needed to comprehend and manipulate the quantitative aspects of chemical reactions and phenomena. A large and growing body of research spanning nearly 100 years^{4–6} has established correlations between various measures of math preparation and success in general chemistry. In 1958, Kunhart, Olsen, and Gammons⁷ found high school chemistry and algebra grades to have higher correlations with course success ($R=0.26$ and 0.20 respectively) than other more general measures of scholastic aptitude. In 1975, Pickering⁸ found a strong trend line between Math SAT scores and General Chemistry grades, and that a supplemental math/problem-solving course improved grades. In 1979, Ozsogomonyan and Loftus⁹ found that Math SAT scores had the highest correlation with course success ($R = 0.51$) followed by a chemistry pre-test score ($R = 0.42$), high school chemistry grade ($R = 0.38$), and an algebra pre-test score ($R = 0.21$). Predictions based on a combination of these factors showed strong correlation ($R = 0.87$) with course success.

Attempts to identify students who are less likely to successfully complete general chemistry continue to find that measures of math preparation, often along with measures of previous instruction in chemistry, are important factors in statistical models of course success.^{10–20} In 2021, Vyas, Kemp, and Reid²¹ found that a combination of instruments could predict up to 80% of the students who did not pass the course.

Instruments aimed at measuring automaticity of arithmetical skills, such as the Math-Up Skills Test

(MUST), have also found significant correlations with course success.^{22–26} For instance, Williamson et al.²⁴ found that MUST scores correlated with course average ($R = 0.54$, $p < 0.001$) and that a logistic regression model that combined MUST with demographic variables could predict course success with 78% accuracy.

Such correlations prompted Sadler and Tai²⁷ to suggest that “two pillars” underpinning college STEM success are “high school study in the same science subject and more advanced study of mathematics”. Below, we build on this by estimating students’ initial knowledge in both pillars: chemistry initial knowledge and math initial knowledge (Section 4.3).

One possible explanation for the observed correlations between chemistry course outcomes and prior math skills is that prior math knowledge aids the learning of chemistry by reducing the student’s cognitive load in processing chemistry instruction. Indeed empirical results and theory in cognitive science²⁸ support the idea that, to accurately solve problems in the sciences, students should practice fundamental facts and procedures in a variety of mathematical and scientific contexts until they can be recalled and applied fluently and automatically.²⁹ An alternative explanation is that prior math skill predicts course-relevant incoming knowledge and that incoming knowledge is, in turn, highly predictive of course outcomes. In other words, there is ambiguity in explaining this link from prior basic math knowledge to chemistry course outcomes as to whether or not prior basic math predicts learning (i.e., the difference between course-relevant incoming knowledge and outgoing knowledge) or predicts course inputs, which in turn predict outcomes because students with stronger incoming knowledge have less to learn.

The link between course preparation and course success also relates to policies regarding pre- and co-requisite courses. Remedial courses have been widely recommended for students deemed to be underprepared (at-risk) in math and/or chemistry, but evidence for their effectiveness is mixed. Some studies have reported increased success and retention with this approach, such as Donovan and Wheland³⁰ and Stone et al.¹⁹ Others have found remediation efforts produced little to no short-term benefits, and sometimes had negative long-term consequences. For example, a six-year study by Gellene and Bentley³¹ concluded that a “*placement remediation program [was] providing little or no significant academic benefit*,” while further analysis by Jones and Gellene³² showed that this program increased attrition such that overall General Chemistry completion rates decreased. Attewell et al.³³ found that taking remedial courses slightly decreased the likelihood that a four-year college student will complete a degree, but did not affect completion rates for two-year college students. A large-scale study by the U.S. Department of Education³⁴ concluded that remedial courses could help or hinder, depending on students’ level of preparation, with only “*weakly prepared students who successfully completed all remedial courses*” benefitting over-

all. Shah et al.³⁵ found that a parallel General Chemistry course with a corequisite support course improved outcomes for at-risk students in the first semester, but increased achievement gaps in the second semester. Denaro et al.³⁶ reported improved performance on the final exam for students who took General Chemistry with a concurrent preparatory course. Seavian et al.³⁷ found that coenrollment in a course that used an asset-based approach closed assymeteries when a nontraditional curriculum was used but not when a traditional curriculum was used. These mixed results provide further reason to better understand the correlations between math preparation and chemistry outcomes.

3 Methods

3.1 Data Collection

We present analyses of datasets collected in General Chemistry I courses at multiple sites across two semesters. We treat these two semesters as separate studies as a form of replication. In the Spring of 2021 (Study 1), data was collected from 183 students at four community colleges with four different instructors. In the following semester, Fall 2021 (Study 2), data collection included 279 students, ten instructors (11 classes), and seven community colleges. These hybrid courses were delivered in a classroom format with lectures and tests given by course instructors and interactive activities with hints and feedback implemented in the Open Learning Initiative (OLI) platform.¹ The analyses presented below are based on the log data collected from the OLI portion of the course. The data and models used for analysis can be found in DataShop,³⁸ a repository for educational data (datasets are ds4856 and ds5939 for Study 1 and 2, respectively). The experimental design and analyses were the same for both studies, though some content improvements were made to the Fall 2021 (Study 2) OLI course materials.

The OLI General Chemistry courseware is based on the OpenStax textbook,³⁹ and is intended to supplant textbook reading and online homework with a single integrated online platform. The content consists of modules that are roughly equivalent to a single textbook chapter. Modules include about 5 to 10 content pages, each of which includes didactic instruction, similar to the text and images in a textbook, “Learn by Doing” activities that typically break problem solving into steps and provide extensive hints and feedback to guide the student to the correct answer, and “Did I Get This?” activities that typically do not break the problem solving into steps and that provide less extensive scaffolding through hints and feedback. “Checkpoints” at the end of each module serve as homework quizzes, where feedback is given only at the end of the quiz. For all activity types, the measure of performance used in the logistic regressions is correct if students provided the correct response without asking for a hint, and incorrect otherwise. (Example activities are included in *Supporting*

Information.)

3.2 Math Coding

Approximately 250 skills or knowledge components (KCs)² were initially identified in the OLI General Chemistry 1 course. Each KC was categorized according to its math demand on a scale from 0 to 3 (see Examples of Math Coding in *Supporting Information*). Level 0 involves no math. Low math (level 1) typically involves a single calculation, such as converting between particles and moles. Medium math (level 2) generally involves several mathematical steps. For example, determining the empirical formula for a compound involves multiple gram-to-mole conversions, making a mole ratio (dividing all mole amounts by the smallest mole amount), then simplifying this to the smallest whole number ratio (which may involve converting a decimal value to a fraction). High math (level 3) usually involves algebra, including logarithms.

Math levels were coded independently by three experienced general chemistry instructors, and the average of their ratings was used as the math level for a given KC. When a coder gave a KC more than one rating, the average rating was used as their math level. For example, one coder rated the KC *Calculate the concentration of ions in solutions* as having both low (math level = 1) and medium (math level = 2) math demand, resulting in a combined math level = 1.5. The final math level code for this KC, in which the other two coders rated it as having a medium math demand (math level = 2), is the average of the 3 coders’ ratings $((2+2+1.5)/3 = 1.833)$.

Since many of the learning activities in the OLI General Chemistry 1 course involve multiple KCs, the KC model also contains about 100 concatenated KCs (i.e., combinations of 2 or more individual KCs). The math level of concatenated KCs is the largest math code of the individual KCs. For example, in a task labeled with the concatenated KC *Convert between mass and moles + Determine the empirical formula of a compound*, the former KC has a math level code = 1, and the latter KC has a math level code = 2; thus the concatenated KC is coded as math level code = 2 (i.e., medium math demand).

Inter-rater reliability (IRR)⁴⁰ was assessed using a two-way mixed, consistency, average-measures intra-class correlation coefficient (ICC)⁴¹ to assess the degree to which coders provided consistency in their ratings of math level across knowledge components. The resulting ICC of 0.934 was in the excellent range,⁴² indicating that coders had a high degree of agreement, suggesting that math level was rated similarly across coders. The high ICC suggests that a minimal amount of measurement error was introduced by the independent coders, and therefore, statistical power for subsequent analyses is not substantially reduced. Math level ratings were therefore deemed to be suitable for use in the present study.

Approximately 10% of the knowledge components did not require any math (0 rating in Table 1), and more than three-fourths of the KCs involved low to medium math knowledge (i.e., 0.01 - 2 in Table 1). Relatedly, only 12% of the unique problem-steps involve no math, and 6% have high math demand (i.e., greater than 2); thus, the remaining 82% involve a low to medium math demand (Table 1).

Table 1: Distribution of math-level ratings for Study 1. Similar distributions were obtained for Study 2 (see *Supporting Information*).

Math rating	0	0.01-1	1.01-2	2.01-3
KCs (%)	33 (10%)	131 (38%)	132 (38%)	48 (14%)
Steps (%)	454 (12%)	1627 (43%)	1494 (39%)	250 (6%)
Opportunities per student KC	5.8	4.8	4.2	2.0
Average performance	0.77	0.72	0.65	0.49
Students	182	181	177	158

3.3 Research Design

To better understand how math influences the learning of chemistry, we explore the following three research questions:

- RQ1** Does lower prior preparation prevent or slow students in learning chemistry?
- RQ2** Are Chemistry problems with high math content harder for students a) to do or b) to learn?
- RQ3** Does lower prior math preparation prevent or slow students in learning chemistry?

To address these questions, we use variations on a logistic regression growth model³ that originated in the Additive Factors Model (AFM).⁴³ AFM models are generalizations of Item Response Theory⁴⁴ that add both a growth or learning element as well as a matrix encoding of a cognitive model⁴⁵ of the underlying knowledge components (KCs) students need for successful task performance.

The growth element included in the logistic regression can be visually represented as a learning curve, shown schematically in Figure 1. The y-axis is the probability that a student will succeed on a practice task, expressed as the log of the odds that the student will successfully complete the task without requesting a hint or making an error. The x-axis is opportunities, the number of times a student has interacted with tasks related to the associated knowledge component. For a student’s first interaction with a KC, the opportunity count is zero, to reflect past opportunities. Note that all students have access to the full courseware. We refer to

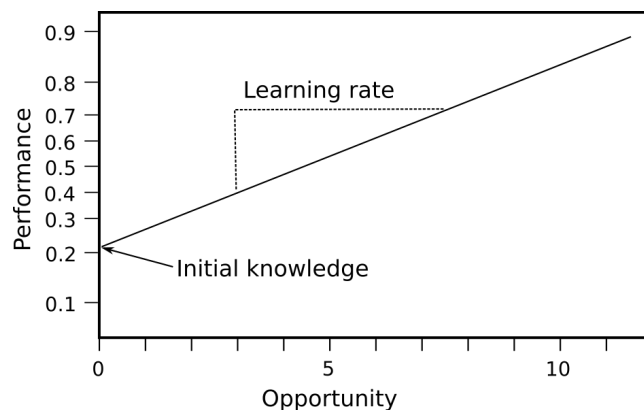


Figure 1: Schematic representation of the learning curves derived from regressions on student data. Predicted performance is on the y-axis, with labels that indicate the probability, p , that a student will succeed on a practice task without making an error or requesting a hint. The scale of the y-axis is log-odds, $\ln(p/(p-1))$, with p shown in the label. The x-axis is the number of practice opportunities to learn the KC the student has engaged with in the past.

the intercept of the regression line as the *initial knowledge*; it is similar to a pretest. We refer to the slope of the regression line as the *learning rate*, as it measures the increase in log odds of success for each opportunity taken. To aid interpretation of learning rate, we also convert this slope to % gain from 50%, i.e. the increase in performance that would be expected from this slope when performance is at 50%. Note that the regressions are performed on the population as a whole, which in effect, averages over various cohorts of students and various groupings of knowledge components.

As we move through the above research questions, we extend the logistic regression model of AFM to allow both initial knowledge and learning rate to depend on the level of math involved in a KC.

RQ1 investigates students’ initial knowledge and learning rate across all knowledge components to investigate if lower initial knowledge is associated with slower learning rates. This investigation is done using a modified AFM model called individual AFM or iAFM.⁴⁶ The iAFM model includes a factor for individual student learning, δ_i of Eq. 1, that is not present in an AFM model. The logistic regression formula for iAFM is

$$P_{i,j} = \left(\theta + \theta_i + \sum_k q_{j,k} \beta_k \right) + \left(\sum_k q_{j,k} (\delta + \delta_i + \gamma_k) T_{i,k} \right) \quad (1)$$

where performance $P_{i,j}$ is the log-odds, $\ln(p/(p-1))$, of the probability, p , that student i is correct on task j . The first term in parentheses models initial knowledge, and the second models learning rate. For initial

knowledge, θ is the mean across all students and KCs, θ_i is the deviation from this mean for student i , and β_k is the deviation for the k^{th} KC. The summation over KCs is included because each task can, in general, be associated with more than one KC through the matrix $q_{j,k}$, which is 1 if task j involves KC k and 0 otherwise. However, here we instead treat tasks involving multiple knowledge components by labeling them with a new KC that concatenates the individual KCs (Section 3.2). The learning rate is the slope of performance with respect to $T_{i,k}$, where $T_{i,k}$ is the number of past opportunities student i has had to interact with a task involving KC k . The learning rate is modeled with an overall mean, δ , and deviations for an individual student, δ_i , and KC, γ_k .

For research question 2 (RQ2), we explore the impact of math difficulty on initial knowledge and learning rate. Here, we add math level (based on the coding of KCs described above) as an independent variable to the iAFM model to estimate the dependence of initial knowledge and learning rate on math level,

$$P_{i,j} = \left(\theta + \theta_i + \sum_k q_{j,k} (\beta_k + \mathbf{M}\mathbf{L}_k) \right) + \left(\sum_k q_{j,k} (\delta + \delta_i + \gamma_k + \mathbf{N}\mathbf{L}_k) T_{i,k} \right) \quad (2)$$

where L_k is a continuous variable, with a range of 0 to 3, describing the math level of the k^{th} KC (Section 3.2), while M and N describe, respectively, the dependence of initial knowledge and learning rate on L_k .

In research question 3 (RQ3), we examine the effects of prior math preparation on learning chemistry. To do so, the dependence of initial knowledge on math level is allowed to vary by student,

$$P_{i,j} = \left(\theta + \theta_i + \sum_k q_{j,k} (\beta_k + (M + \mathbf{M}_i) L_k) \right) + \left(\sum_k q_{j,k} (\delta + \delta_i + \gamma_k + N L_k) T_{i,k} \right) \quad (3)$$

where M_i describes the degree to which the initial knowledge of student i depends on math level, L_k . We take M_i as a measure of an individual student's math preparation because larger magnitudes indicate that the i^{th} student's initial knowledge on a KC depends strongly on the level of math associated with that KC. We refer to θ_i from Eq. 3 as an individual student's chemistry preparation, as it measures the i^{th} student's initial knowledge extrapolated back to math level $L_k = 0$.

An additional investigation for RQ3 involves math preparation and learning opportunities: Do students with higher math preparation take greater advantage of learning opportunities by doing more problem steps? Or, conversely, do students with lower math preparation, who need more practice, do fewer problem steps?

The regressions are performed using the generalized

linear mixed-effect model (glmer) function in the R Statistical Software.⁴⁷ The regression formulas and other details are in the *Supporting Information*.

4 Results

We applied regression analyses as indicated by Eqs. 1-3 to the two datasets of Section 3.1. For each research question, we first discuss the detailed results from the Study 1 dataset. We then assess the extent to which the analysis of the Study 2 dataset aligns with these findings.

4.1 RQ1 Does lower prior preparation prevent or slow students in learning Chemistry?

To address RQ1, we use learning curves to visualize the parameter estimates resulting from applying the logistic regression of Eq. 1 to the dataset for Study 1. The regression parameters can be used to generate a learning curve for each student, with $\theta + \theta_i$ for initial knowledge and $\delta + \delta_i$ for learning rate (left panel of Figure 2). The large spread of intercepts on the y-axis indicates wide variation in students' initial knowledge (interquartile range, IQR = 54%, 69%). In contrast, the nearly parallel lines seen in the learning curves indicate only small differences in student learning rates. Thus, the less prepared students (lines that start lower on the y-axis) are learning chemistry at a rate similar to that of the better prepared students (lines that start higher on the y-axis have about the same slope).

The right panel of Figure 2 shows learning curves for each knowledge component in the course. These curves are generated from the parameter estimates from Eq. 1 using $\theta + \beta_k$ for initial knowledge and $\delta + \gamma_k$ for learning rate. Here, the regression essentially averages over all students to generate a learning curve for each knowledge component. As seen above for students, there is a large variation in the difficulty of knowledge components as indicated by the wide range of initial knowledge (IQR = 48%, 70%). However, unlike students, there is also substantial variation in learning rates, as indicated by the non-parallel lines in the right panel of Figure 2. Some initially hard KCs are learned more quickly and others are learned more slowly. For example, the KC *Predict differences in melting points of ionic substances* has an initial starting point (i.e., intercept) of 34%, thus quite difficult. It improves at 0.11 log odds (about 2.6% from 50%) per opportunity. However, the (also) hard KC *Calculate changes in vapor pressure based on concentration of solution*, with an intercept of 37%, only improves 0.036 log odds (about 0.90% from 50%) per opportunity. In other words, both KCs are hard, but *Predict differences in melting points of ionic substances* is learned more quickly than *Calculate changes in vapor pressure based on concentration of solution*. Similarly,

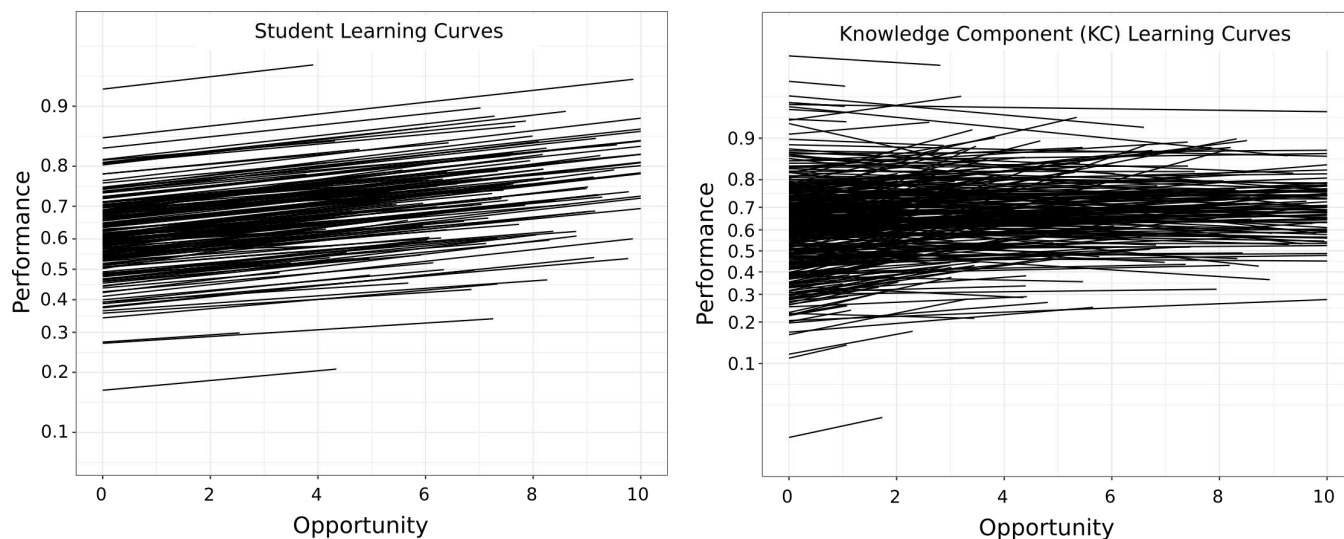


Figure 2: Student learning curves (left panel) show a large variation in initial knowledge, as indicated by the wide range of intercepts, but small differences in learning rate, as indicated by the mostly parallel slopes. The range of intercepts on the KC learning curves (right panel) shows a large variance in initial knowledge, and the slope variance suggests some KCs are learned faster than others. The axes are as in Figure 1.

some initially easier KCs are learned more quickly and others are learned more slowly.

Similar results are found when this analysis is applied to the data set for Study 2 (see *Supporting Information*). The learning curves for individual students show large variation in initial knowledge (IQR = 54%, 71%) with nearly parallel lines indicating small variations in learning rates. The learning curves for individual KCs shows large variations in both initial knowledge and learning rates.

The results from both studies suggest that lower prior preparation does not hinder or slow students' ability to learn chemistry. Instead, all students tend to learn at nearly the same rate. However, there are significant differences in the rates at which different KCs are learned.

4.2 RQ2 Are chemistry problems with high math content harder for students a) to do or b) to learn?

Having established that initial knowledge varies significantly among students while learning rates remain largely consistent, we now turn our attention to examining how the mathematical complexity of a knowledge component (KC) impacts both initial knowledge and learning rate. To address this, we extend the logistic regression to that of Eq. 2, which adds a dependence on the level of math involved in a particular KC when estimating both initial knowledge (M of Eq. 2) and learning rate (N of Eq. 2). The results in Table 2 for the Study 1 dataset indicate a statistically significant dependence of initial knowledge on math level (M ; $p < 0.001$). The dependence of learning rate on math level is only marginally significant (N ; $p = 0.083$), but it is notable that the trend is for content with a high math level to be learned faster than content with a low math

level.

Table 2: Results of the impact of math on initial knowledge and learning rate from the regression analysis of Eq. 2.

	Estimate (Std Error)	P value
Study 1		
Initial knowledge (θ)	1.172 (0.11)	<2e-16 ***
Learning rate (δ)	0.041 (0.02)	0.01 *
Initial knowledge by math level (M)	-0.582 (0.07)	<2e-16 ***
Learning rate by math level (N)	0.021 (0.01)	0.083
Study 2		
Initial knowledge (θ)	1.198 (0.10)	<2e-16 ***
Learning Rate (δ)	0.081 (0.02)	6.39e-05 ***
Initial knowledge by math level (M)	-0.574 (0.07)	3.61e-16 ***
Learning rate by math level (N)	0.011 (0.01)	0.446

(Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1)

Table 3 uses the regression parameters of Table 2 to summarize the impact of math level (column 1) on initial knowledge (column 2) and learning rate (columns 3 and 4). Initial knowledge has a strong and statistically significant dependence on math level, with initial knowledge for the typical student being about 76% correct for chemistry KCs with no math (math level = 0), dropping to 36% correct for KCs with the highest math level (math level = 3). The overall learning rate of $\delta = 0.041$ log odds gain per opportunity is statistically significant ($p = 0.011$). Table 3 also shows a marginally significant increase in learning rate with math level, reaching 0.105

log odds per opportunity for a math level of 3.

Figure 3 provides a complementary visualization by using the regression parameters to generate learning curves for math levels 0 through 3. As illustrated, performance on KCs with higher math levels starts off lower but the learning rate is somewhat faster than for lower math levels (marginally significant, $p = 0.083$). Thus, students are learning chemistry skills and concepts involving harder mathematics (levels 2 and 3) at least as readily, if not more, than they are learning skills and concepts involving little or no math.

Table 3: Impact of the math-level of knowledge components on student initial knowledge and learning rate. Note that the statistical significance of the increase in learning rate with math level is marginal ($p = 0.083$ in Study 1 and 0.446 in Study 2).

Math Level	Initial % correct	Log odds gain per opportunity	% gain from 50%*
Study 1			
0	76%	0.041	1.0%
1	64%	0.062	1.6%
2	50%	0.084	2.1%
3	36%	0.105	2.6%
Study 2			
0	77%	0.081	2.1%
1	65%	0.092	2.2%
2	51%	0.104	2.6%
3	37%	0.115	2.9%

* We convert log odds gain to percent correct from. 50% (e.g., 0.041 to 51.0% in row one) and then subtract 50%

The lower sections of Tables 2 and 3 present the results from applying this analysis to the Study 2 dataset. Consistent with Study 1, initial knowledge shows a strong and statistically significant dependence on math level (M ; $p < 0.001$). Similarly, the learning rate is estimated to increase with math level, as observed in Study 1. However, in Study 2, this increase in learning rate with math level is not statistically significant (N ; $p = 0.446$), whereas it was marginally significant in Study 1 (N ; $p = 0.083$). In both studies, we found no evidence that the increased math level of a KC slows the learning rate when averaged across all students. Next, we explore how individual differences in math preparation might influence student learning rates.

4.3 RQ3 Does lower prior math preparation prevent or slow students in learning Chemistry?

In addressing RQ2, the logistic regression included a dependence of initial knowledge on the level of math involved in a particular KC through the term M of Eq. 2. This M was an average over all students. To address RQ3, we extend this to allow dependence on individual students via the term M_i of Eq. 3. This additional factor allows us to extrapolate initial knowledge back

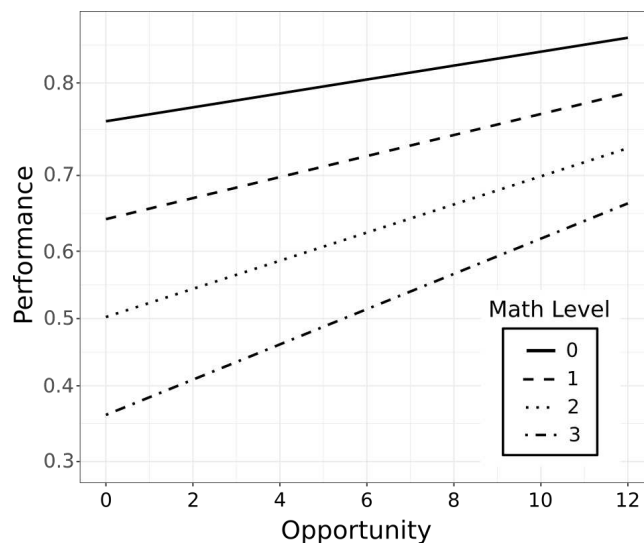


Figure 3: Learning curves generated from the regression parameters for study 1 in Table 3. Initial knowledge decreases dramatically with math level. The learning curves are steeper for higher math level but this increase in learning rate is only marginally statistically significant (see Table 3). The results do however indicate that students are learning high math content at least as readily as they are learning low math content. The axes are as in Figure 1.

to math level zero, $L_k = 0$ in Eq. 3, obtaining an estimate, $\theta + \theta_i$, for initial knowledge of the i^{th} student on knowledge components that involve no math. We refer to this extrapolated value, $\theta + \theta_i$, as the *chemistry initial knowledge* of that student. The dependence of the i^{th} student's initial knowledge on math level is estimated in Eq. 3 as $M + M_i$, which we refer to as *math initial knowledge*. Chemistry and math initial knowledge are strikingly uncorrelated, $R = 0.063$ for the Study 1 dataset (Figure 4). Next, we separately examine correlations of math and chemistry initial knowledge with learning rates and other factors.

To determine whether lower math preparation prevents or slows student learning in chemistry, we computed the correlation between individual student learning rate, $\delta + \delta_i$ of Eq. 3, and math initial knowledge, $M + M_i$ of Eq. 3. We found no correlation, $R = -0.034$, $p = 0.65$ (Figure 5 left panel). In other words, students with lower prior math preparation learn chemistry at a rate equivalent to students with higher math preparation.

We also investigated whether chemistry initial knowledge is associated with individual student learning rates, $\delta + \delta_i$ of Eq. 3. Here, we found a statistically significant positive correlation, $R = 0.30$, $p < 0.0001$ (Figure 5 right panel). Students with lower chemistry initial knowledge (below median) do learn at a somewhat slower rate (slope = 0.062 log odds or about 1.5% gain per opportunity) than students with higher incoming chemistry knowledge (above median: slope = 0.070 log odds or about 1.9% gain per opportunity). This signif-

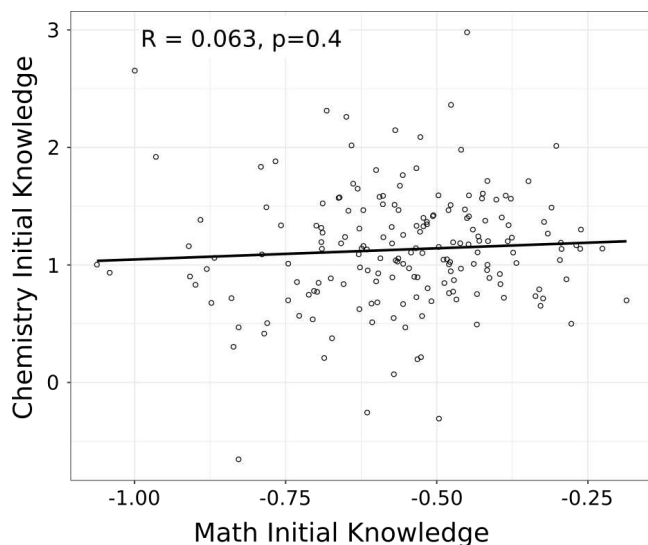


Figure 4: Study 1 estimates of students' Math Initial Knowledge ($M + M_i$ of Eq. 3) and Chemistry Initial Knowledge ($\theta + \theta_i$ of Eq. 3) are quite independent of each other ($R = 0.06$) in contrast to the expectation that students with more math preparation are likely to have more chemistry preparation.

icant correlation helps confirm that individual learning rates are picking up true student-level variability, not random noise. Thus, it lends confidence to the inference that a lack of prior math knowledge does not slow chemistry learning (i.e., the lack of correlation between math initial knowledge and learning rate cannot be attributed to a lack of variability in learning rate). The faster learning rate observed in students with higher initial chemistry knowledge may be attributed to their familiarity with closely related content from previous chemistry courses, allowing them to relearn the material, whereas students with lower initial knowledge may be encountering this content for the first time.

We give a sense of the practical significance of the correlation between chemistry initial knowledge and learning rate by inspecting differences between the typical less prepared student (i.e., the first quartile) and the typical more prepared student (i.e., the third quartile). A typical less prepared student is estimated to start at about 57% correct on math level 1 tasks and has a learning rate (based on the correlation in Figure 5) of 0.063 log odds (about 1.5% gain from 50%) per opportunity. At this learning rate, they need about 17 opportunities to reach a reasonable mastery level of 80%. If these students had the slightly higher learning rate of the typical more prepared student, namely, 0.068 log odds (about 1.7% gain from 50%), they would need about 16 opportunities, that is, only about one less opportunity. Thus, the challenge for less prepared students is not the rate at which they are learning but the fact that they have more to learn.

We next examine the relation between initial knowledge and the number of opportunities taken. Recall

that the opportunity variable refers to the number of interactive learning opportunities students engage in, not the total available (which is the same for all students). There is a small and significant correlation between math initial knowledge and learning opportunities ($R = 0.21$, $p = 0.005$). A similar small and significant correlation exists between chemistry initial knowledge and opportunities ($R = 0.24$, $p = 0.001$). This effect of initial knowledge on opportunities is larger than the effect of initial knowledge on learning rate. Similar to the above, we inspect differences in opportunities taken between the typical less prepared student (i.e., the first quartile) and the typical more prepared student (i.e., the third quartile). A student at the first quartile in math initial knowledge ($M + M_i = -0.65$ log odds; 34%) engages in about 1300 practice opportunities, whereas a student at the third quartile (-0.44 log odds; 39%) in math initial knowledge engages in about 1570 opportunities. In other words, the top half of students in math initial knowledge do about 270 (21%) more opportunities than the bottom half of students in math initial knowledge.

Likewise, a student at the first quartile in chemistry initial knowledge (0.84 log odds; 69.9% for level 0 math content) is estimated to take 1280 opportunities, whereas a student at the third quartile (1.41 log odds; 80.4% for level 0 math content) in chemistry initial knowledge does about 1560 opportunities. Thus, the top half of students in chemistry initial knowledge complete about 280 (22%) more opportunities than the bottom half of students in chemistry initial knowledge.

In a follow-up regression analysis (see *Supporting Information*) we found that students' chemistry and math initial knowledge both have a significant and independent association with students' total practice opportunities. This result is consistent with the idea that students coming into the course less well-prepared in either math or general chemistry end up pursuing fewer opportunities. This result is perplexing given the high similarity in learning rates: *Lower initial knowledge students pursue fewer opportunities even though they are making progress at about the same rate as students with higher prior chemistry and math preparation.*

Similar results are obtained when the analyses for RQ3 are applied to the Study 2 dataset. As in Study 1, chemistry initial knowledge and math initial knowledge show only a weak and insignificant correlation ($R=0.079$, $p=0.19$). Also as in Study 1, we find math initial knowledge is not associated with learning rate ($R=0.045$, $p=0.46$), but chemistry initial knowledge is positively correlated with learning rate ($R=0.40$, $p=4.2e-12$). The difference in learning rate for students in the lower half of chemistry initial knowledge versus their counterparts in the upper half are 0.089 versus 0.099 log odds gain per opportunity. These learning rates in more intuitive terms are 2.2% gain versus 2.5% gain from 50%, respectively.

Also as in Study 1, Study 2 finds a small and significant correlation between math initial knowledge and

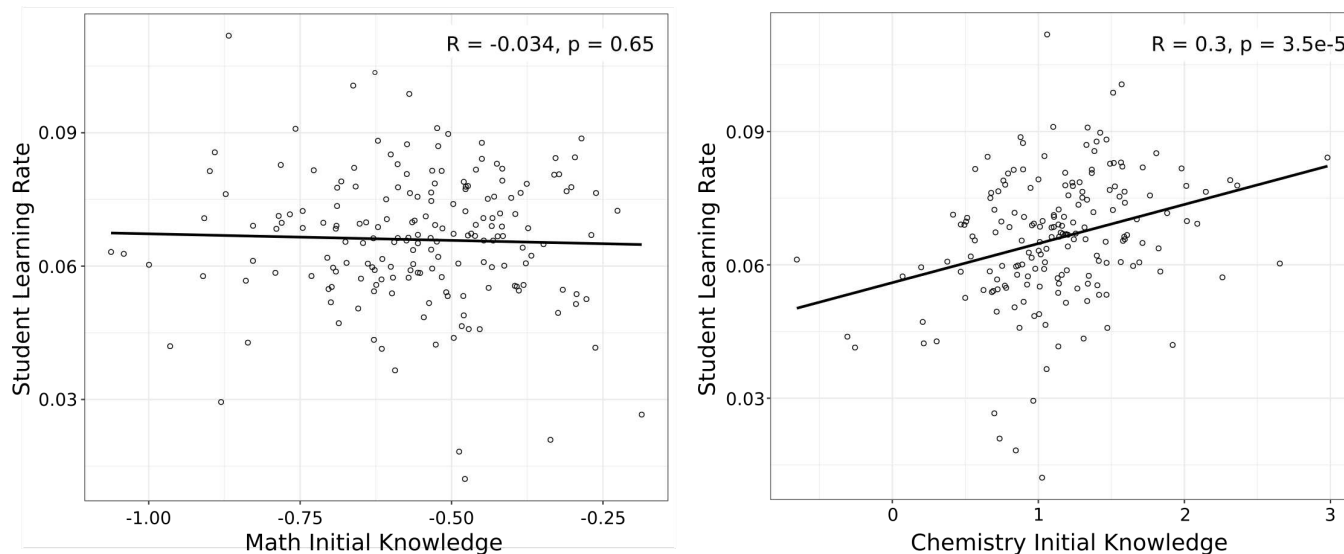


Figure 5: The left scatterplot shows that math initial knowledge is not associated with student learning rate (flat slope), while the right scatterplot indicates that chemistry initial knowledge is associated with student learning rate (upward slope).

the number of learning opportunities students chose to engage with ($R = 0.171$, $p = 0.004$). We make these relationships more concrete as follows. A typical student at the first quartile in prior math ($M + M_i$, which is -0.66 log odds; 34.1%) is estimated to engage in about 1180 opportunities whereas a student at the third quartile in prior math (-0.47 log odds; 39.6%) engages in about 1401 opportunities. In other words, the top half of students in math preparation do 221 (18.7%) more opportunities than the bottom half of students in math preparation.

In Study 1, the correlation between chemistry initial knowledge and opportunities taken was similar to that of math initial knowledge. In Study 2, we instead find that the correlation between chemistry initial knowledge and opportunities taken is not significant ($R = 0.003$, $p = 0.97$). A typical student at the first quartile in prior chemistry ($\theta + \theta_i$ which is 0.86 log odds; 70.4% for level 0 math content) takes an estimated 1276 opportunities whereas a typical student at the third quartile (1.52 log odds; 82.0% for level 0 math content) takes an estimated 1280 opportunities. Consistent with the lack of correlation, there is no meaningful difference (at 4 opportunities and 0.3%) in opportunities taken.

From both studies, we find the challenge for less prepared students is not that they have trouble learning chemistry – they are learning at essentially the same rate as better prepared students. The challenge is that they need more learning opportunities to reach a given performance level. Both studies also find a small but statistically significant correlation between math initial knowledge and number of opportunities taken, while only Study 1 finds a significant correlation of chemistry initial knowledge with opportunities taken.

5 Discussion

Unquestionably, student prior math preparation has a substantial influence on their final success in introductory college chemistry courses. Here, we use data gathered as students engage with online learning materials to first examine initial knowledge and learning rates for both students and knowledge components (RQ1) and then decompose this link between preparation and course success into five components: initial knowledge in math and chemistry, learning rate for math and chemistry, and learning opportunities taken. We first summarize our findings and then consider possible implications for course and curriculum design.

5.1 Summary

Our results suggest that the challenge for less prepared students is not the rate at which they are learning, but rather the fact that they come in with less initial knowledge (Section 4.1). The analysis shows that initial knowledge, averaged over the various knowledge components (KCs) of the course, varies substantially among students. The learning rates are, however, surprisingly similar across students. These results are consistent with findings in 27 other datasets of student learning in course-embedded use of online practice in science, math, and language courses where student learning rates were found to be surprisingly similar.³ As we observed here, this similarity in learning rate is particularly striking in comparison to the high variability in student initial knowledge.

The similarity in learning rate across students is also striking in comparison to the high variability in learning rate across knowledge components (KCs). Because student learning rate is relatively constant, the difference

in learning rates among KCs is not a result of better prepared students learning harder KCs more quickly and less prepared students learning them more slowly. The differences are likely due to either: differences in the complexity of the KC that impact its acquisition (e.g., the Clausius-Clapeyron equation is a more complex formula than Boyle's law) or imperfections of the labeling of tasks with KCs (e.g., some of the tasks labeled with the same KC may have other knowledge requirements that have not been considered and when these tasks are in later opportunities they flatten the learning curve). In either case, identifying KCs with slow learning provides a direct means to guide iterative improvement in the learning resources.^{46,48}

We also investigated whether Chemistry problems with high math content are harder for students to do or to learn (RQ2). Our results suggest the level of challenge for students increases substantially with the level of math involved in a particular KC, which is consistent with past studies showing a link between math preparation and course success. However, we again find that this challenge is related to initial knowledge not learning rate. Initial success is estimated at about 75% accuracy on no-math tasks and at about 36% on the highest math level tasks. Surprisingly, higher math level does not seem to inhibit learning rate. If anything, students may learn chemistry content involving harder math at a slightly higher rate (marginally significant in Study 1 but not statistically significant in Study 2).

Finally, we explored whether lower prior math or chemistry preparation prevents or slows student learning (RQ3). Our results suggest that students' math initial knowledge also has little influence on their learning rate. This investigation was done by decomposing initial knowledge into math initial knowledge, as measured by the dependence of a students' initial knowledge on the level of math involved in a particular KC, and chemistry initial knowledge, as measured by estimating initial knowledge on KCs with math difficulty factored out. It is interesting that these two components of initial knowledge show little correlation with one another. It is also interesting that student learning rate shows a statistically significant correlation with chemistry initial knowledge but not math initial knowledge. This positive correlation of learning rate with initial chemistry knowledge is, however, quite small, with initial knowledge remaining the primary source of challenge for less well-prepared students.

Small correlations were found between the number of opportunities taken and initial math knowledge ($p = 0.005$ for Study 1 and $p = 0.004$ for Study 2) and the number of opportunities and chemistry initial knowledge (only Study 1 was significant with $p = 0.001$).

Even though less prepared students make progress at almost the same rate, they tend to engage with somewhat fewer learning opportunities. Although students with lower initial knowledge *can and do succeed*, some *may believe they cannot* and therefore participate less.

5.2 Implications

The observation that the student challenge lies primarily in initial knowledge, as opposed to learning rate, may have implications for course and curriculum design. If low math preparation slowed learning, then it would seem advisable to have less well-prepared students take a math preparation course before enrolling in chemistry. However if, as observed here, low math preparation does not slow learning, then providing math support before entering chemistry may not be necessary. Rather, providing support through math-specific instructional feedback and hints on tasks involving math, as is done in the courseware employed in this study, appears a highly viable alternative.

Although no link is observed here between initial knowledge and learning rate, this does not imply that students with lower preparation do not need additional support to achieve course success. Even if learning rates are the same, less well-prepared students will require substantially more opportunities, and thus time, to succeed. A systemic approach to providing this time is not consistent with most current curricula, which assign credit hours based on content and without consideration of student needs. The need for educational systems that include support for this extra learning time is further supported by the results reported here: Students with less preparation tend to take somewhat fewer opportunities to engage with the material. The reasons for this remain to be investigated. We suggest exploration of three candidate hypotheses. First, individual opportunities may take less well-prepared students longer to complete, because the scaffolding provided in the opportunities takes time to engage with. Second, the life circumstances that lead some students to enter the course with lower initial knowledge may also lead to less available time to engage with learning materials during the course. Third, some students may find it discouraging to experience errors, even if they are learning from them⁴⁹ and thus decide to disengage. This third reason suggests putting more attention to supporting student motivation and perhaps more explicit statements that every learner makes errors and it is a natural part of the learning process.

In general, we advocate efforts to free up time for students by making their learning more efficient. The observation that learning rates are similar across all students does not imply that improvements in instruction cannot speed learning. Learning rate does vary substantially across KCs (Figures 2 and 6), with many KCs showing slow learning. We recommend applying methods for identifying KCs with slow learning and modifying the relevant instruction (e.g., by tailoring new practice tasks and instruction that directly address cognitive challenges) to improve learning efficiency and effectiveness.^{46,48}

More generally, this effort illustrates the potential for analysis of fine-grained longitudinal data, gathered in authentic educational settings, to address century-

old research questions in the learning of chemistry. A large and expanding collection of such data is available through LearnLab’s DataShop along with tools to support analysis.⁵⁰

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Supporting Information

The Supporting Information is available free of charge at <http://pubs.acs.org>.

- Example instructional activities and associated math coding; equations in R used for the logistic regressions; regression analyses of math and chemistry initial knowledge effects on opportunities (Section 4.3); and additional results from study 2. (PDF).

References

- (1) Carnegie Mellon University Open Learning Initiative (OLI) General Chemistry I. <https://oli.cmu.edu/courses/general-chemistry-1/>.
- (2) Koedinger, K. R.; Corbett, A. T.; Perfetti, C. The knowledge-learning-instruction framework: bridging the science-practice chasm to enhance robust student learning. *Cogn. Sci.* **2012**, *36*, 757–798.
- (3) Koedinger, K. R.; Carvalho, P. F.; Liu, R.; McLaughlin, E. A. An astonishing regularity in student learning rate. *Proc. Natl. Acad. Sci. U. S. A.* **2023**, *120*, e2221311120.
- (4) Cornog, J.; Stoddard, G. D. Predicting performance in chemistry. *J. Chem. Educ.* **1925**, *2*, 701.
- (5) Scofield, M. B. An experiment in predicting performance in general chemistry. *J. Chem. Educ.* **1927**, *4*, 1168.
- (6) Fay, P. J. The history of chemistry teaching in American high schools. *J. Chem. Educ.* **1931**, *8*, 1533.
- (7) Kunhart, W. E.; Olsen, L. R.; Gammons, R. S. Predicting success of junior college students in introductory chemistry. *J. Chem. Educ.* **1958**, *35*, 391.
- (8) Pickering, M. Helping the high risk freshman chemist. *J. Chem. Educ.* **1975**, *52*, 512.
- (9) Ozsogomonyan, A.; Loftus, D. Predictors of general chemistry grades. *J. Chem. Educ.* **1979**, *56*, 173.
- (10) Hovey, N. W.; Krohn, A. An evaluation of the Toledo chemistry placement examination. *J. Chem. Educ.* **1963**, *40*, 370.
- (11) Craney, C. L.; Armstrong, R. W. Predictors of grades in general chemistry for allied health students. *J. Chem. Educ.* **1985**, *62*, 127.
- (12) Spencer, H. E. Mathematical SAT Test Scores and College Chemistry Grades. *J. Chem. Educ.* **1996**, *73*, 1150.
- (13) McFate, C.; Olmsted, J., III Assessing student preparation through placement tests. *J. Chem. Educ.* **1999**, *76*, 562.
- (14) Wagner, E. P.; Sasser, H.; DiBiase, W. J. Predicting students at risk in general chemistry using pre-semester assessments and demographic information. *J. Chem. Educ.* **2002**, *79*, 749.
- (15) Kennepohl, D.; Guay, M.; Thomas, V. Using an Online, Self-Diagnostic Test for Introductory General Chemistry at an Open University. *J. Chem. Educ.* **2010**, *87*, 1273–1277.
- (16) Cooper, C. I.; Pearson, P. T. A Genetically Optimized Predictive System for Success in General Chemistry Using a Diagnostic Algebra Test. *J. Sci. Educ. Technol.* **2012**, *21*, 197–205.
- (17) Nakakoji, Y.; Wilson, R.; Poladian, L. Mixed methods research on the nexus between mathematics and science. *International Journal of Innovation in Science and Mathematics Education* **2014**, *22*.
- (18) Pyburn, D. T.; Pazicni, S.; Benassi, V. A.; Tappin, E. E. Assessing the relation between language comprehension and performance in general chemistry. *Chemistry Education Research and Practice* **2013**, *14*, 524–541.
- (19) Stone, K. L.; Shaner, S. E.; Fendrick, C. M. Improving the Success of First Term General Chemistry Students at a Liberal Arts Institution. *Education Sciences* **2018**, *8*, 5.
- (20) Towns, M. H.; Bain, K.; Rodriguez, J.-M. G. How did we get here? Using and applying mathematics in chemistry. In *ACS Symposium Series*; ACS symposium series. American Chemical Society; American Chemical Society: Washington, DC, 2019; pp 1–8.

- (21) Vyas, V. S.; Kemp, B.; Reid, S. A. Zeroing in on the best early-course metrics to identify at-risk students in general chemistry: an adaptive learning pre-assessment vs. traditional diagnostic exam. *Int. J. Sci. Educ.* **2021**, *43*, 552–569.
- (22) Leopold, D. G.; Edgar, B. Degree of Mathematics Fluency and Success in Second-Semester Introductory Chemistry. *J. Chem. Educ.* **2008**, *85*, 724.
- (23) Albaladejo, J. D. P.; Broadway, S.; Mamiya, B.; Petros, A.; Powell, C. B.; Shelton, G. R.; Walker, D. R.; Weber, R.; Williamson, V. M.; Mason, D. ConfChem Conference on Mathematics in Undergraduate Chemistry Instruction: MUST-Know Pilot Study—Math Preparation Study from Texas. *J. Chem. Educ.* **2018**, *95*, 1428–1429.
- (24) Williamson, V. M.; Walker, D. R.; Chuu, E.; Broadway, S.; Mamiya, B.; Powell, C. B.; Robert Shelton, G.; Weber, R.; Dabney, A. R.; Mason, D. Impact of basic arithmetic skills on success in first-semester general chemistry. *Chemistry Education Research and Practice* **2020**, *21*, 51–61.
- (25) Powell, C. B.; Simpson, J.; Williamson, V. M.; Dubrovskiy, A.; Walker, D. R.; Jang, B.; Robert Shelton, G.; Mason, D. Impact of arithmetic automaticity on students' success in second-semester general chemistry. *Chemistry Education Research and Practice* **2020**, *21*, 1028–1041.
- (26) Shelton, G. R.; Mamiya, B.; Weber, R.; Rush Walker, D.; Powell, C. B.; Jang, B.; Dubrovskiy, A. V.; Villalta-Cerdas, A.; Mason, D. Early Warning Signals from Automaticity Diagnostic Instruments for First- and Second-Semester General Chemistry. *J. Chem. Educ.* **2021**, *98*, 3061–3072.
- (27) Sadler, P. M.; Tai, R. H. The two high-school pillars supporting college science. *Science* **2007**, *317*, 457–458.
- (28) van Merriënboer, J. J. G.; Sweller, J. Cognitive load theory and complex learning: Recent developments and future directions. *Educ. Psychol. Rev.* **2005**, *17*, 147–177.
- (29) Hartman, J. R.; Nelson, E. A. Automaticity in Computation and Student Success in Introductory Physical Science Courses. *arXiv [physics.ed-ph]* **2016**,
- (30) Donovan, W. J.; Wheland, E. R. Comparisons of success and retention in a general chemistry course before and after the adoption of a mathematics prerequisite. *Sch. Sci. Math.* **2009**, *109*, 371–382.
- (31) Gellene, G. I.; Bentley, A. B. A Six-Year Study of the Effects of a Remedial Course in the Chemistry Curriculum. *J. Chem. Educ.* **2005**, *82*, 125.
- (32) Jones, K. B.; Gellene, G. I. Understanding Attrition in an Introductory Chemistry Sequence Following Successful Completion of a Remedial Course. *J. Chem. Educ.* **2005**, *82*, 1241.
- (33) Attewell, P.; Lavin, D.; Domina, T.; Levey, T. New Evidence on College Remediation. *J. Higher Educ.* **2006**, *77*, 886–924.
- (34) Chen, X. Remedial coursetaking at U.S. public 2- and 4-year institutions: Scope, experiences, and outcomes. Statistical analysis report. *National Center for Education Statistics* **2016**, *NCES 2016-405*.
- (35) Shah, L.; Butler Basner, E.; Ferraro, K.; Sajan, A.; Fatima, A.; Rushton, G. T. Diversifying Undergraduate Chemistry Course Pathways to Improve Outcomes for At-Risk Students. *J. Chem. Educ.* **2020**, *97*, 1822–1831.
- (36) Denaro, K.; Lo, S. M.; Holton, A. J. Effect of a Concurrent Enrollment Preparatory Course on Student Achievement and Persistence in General Chemistry. *J. Chem. Educ.* **2021**, *98*, 2820–2828.
- (37) Sevian, H.; King-Meadows, T. D.; Caushi, K.; Kakhoidze, T.; Karch, J. M. Addressing Equity Asymmetries in General Chemistry Outcomes Through an Asset-Based Supplemental Course. *JACS Au* **2023**, *3*, 2715–2735.
- (38) Koedinger, K. R.; Baker, R. S.; Cunningham, K.; Skogsholm, A.; Leber, B.; Stamper, J. A data repository for the EDM community: The PSLC DataShop. *Handbook of educational data mining* **2010**, *43*.
- (39) Flowers, P., Theopold, K., Langley, R., & Robinson, W. *Chemistry 2e* (<https://openstax.org/details/books/chemistry-2e>); OpenStax, 2019.
- (40) Hallgren, K. A. Computing Inter-Rater Reliability for Observational Data: An Overview and Tutorial. *Tutor. Quant. Methods Psychol.* **2012**, *8*, 23–34.
- (41) McGraw, K. O.; Wong, S. P. Forming inferences about some intraclass correlation coefficients. *Psychol. Methods* **1996**, *1*, 30–46.
- (42) Cicchetti, D. V. Guidelines, criteria, and rules of thumb for evaluating normed and standardized assessment instruments in psychology. *Psychol. Assess.* **1994**, *6*, 284–290.
- (43) Cen, H.; Koedinger, K.; Junker, B. Learning Factors Analysis – A General Method for Cognitive Model Evaluation and Improvement. *Intelligent Tutoring Systems*. 2006; pp 164–175.

- (44) De Boeck, P.; Wilson, M. Descriptive and explanatory item response models. In *Explanatory item response models: A generalized linear and nonlinear approach*; De Boeck, P., Wilson, M., Eds.; Statistics for Social and Behavioral Sciences; Springer: New York, NY, 2011; pp 43–74.
- (45) Barnes, T. The Q-matrix method: Mining student response data for knowledge. 2005; pp 1–8.
- (46) Liu, R.; Koedinger, K. R. Closing the Loop: Automated Data-Driven Cognitive Model Discoveries Lead to Improved Instruction and Learning Gains. *Journal of Educational Data Mining* **2017**, *9*, 25–41.
- (47) R Core Team R: A Language and Environment for Statistical Computing. 2021.
- (48) Koedinger, K. R.; Stamper, J. C.; McLaughlin, E. A.; Nixon, T. Using Data-Driven Discovery of Better Student Models to Improve Student Learning. In *Artificial Intelligence in Education*; Hutchison, D., Kanade, T., Kittler, J., Kleinberg, J. M., Mattern, F., Mitchell, J. C., Naor, M., Nierstrasz, O., Pandu Rangan, C., Steffen, B., Sudan, M., Terzopoulos, D., Tygar, D., Vardi, M. Y., Weikum, G., Lane, H. C., Yacef, K., Mostow, J., Pavlik, P., Eds.; Lecture Notes in Computer Science; Springer Berlin Heidelberg: Berlin, Heidelberg, 2013; Vol. 7926; pp 421–430.
- (49) Deslauriers, L.; McCarty, L. S.; Miller, K.; Callaghan, K.; Kestin, G. Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom. *Proc. Natl. Acad. Sci. U. S. A.* **2019**, *116*, 19251–19257.
- (50) Zhou, M.; Xu, Y.; Nesbit, J.; Winne, P. A data repository for the EDM community: The PSLC DataShop: Methodology and applications. In *Handbook of Educational Data Mining*, 1st ed.; CRC Press, 2010; pp 65–78.

