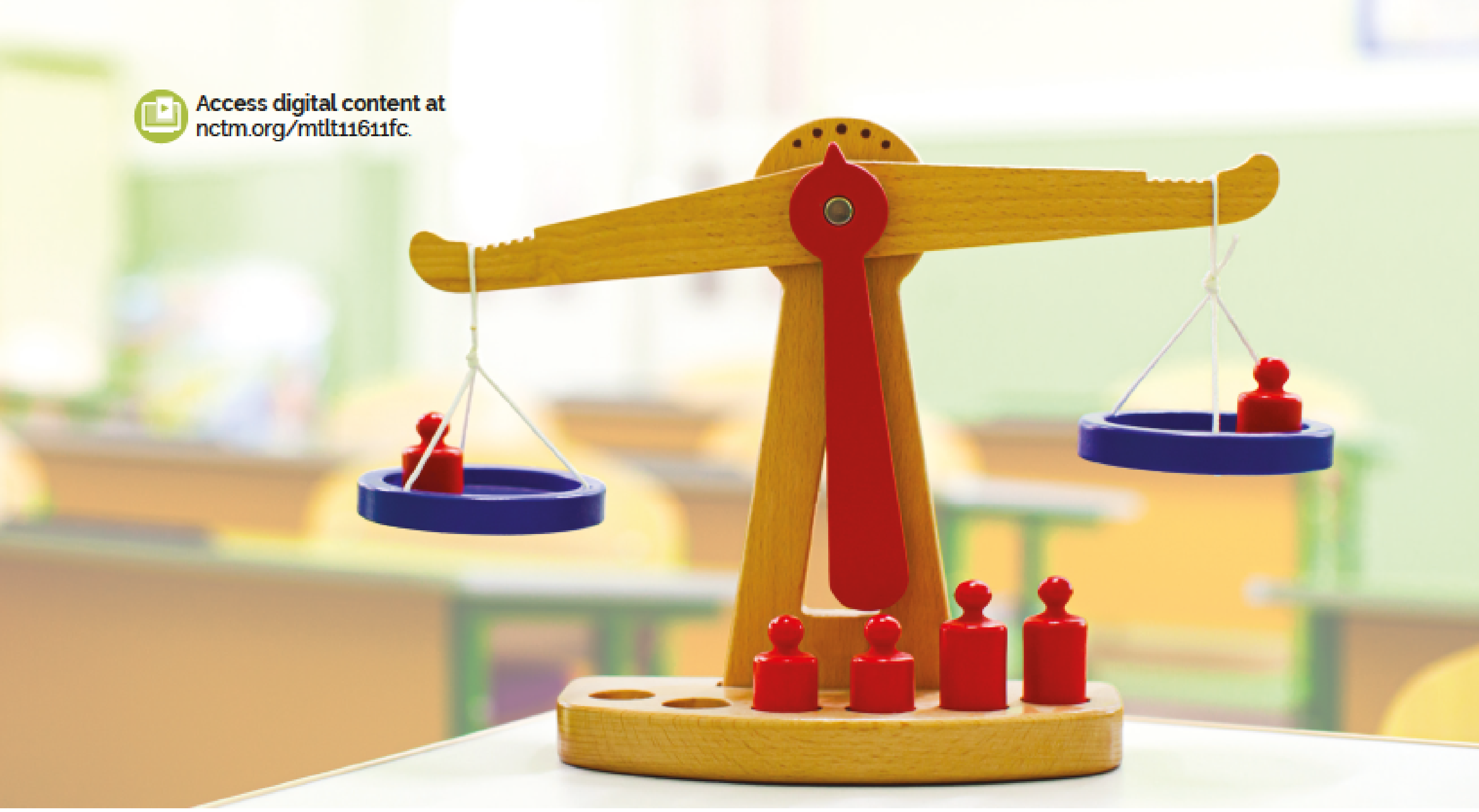

RECONSIDERING Mathematical Authority

Who has mathematical authority in your classroom, and what does authority look like? Find out different ways you can help students gain authority.

Michael D. Hicks, Jessica Pierson Bishop, Christina Koehne, and Mai Bui

Imagine your mathematics classroom. You have posed a task for your students to work on independently. While your students are working at their seats, you notice that one student has some work that you would like to share with the class. How might you share their work? Would you project the student's written work on a document camera? Recreate the student's work on the board yourself? Ask the student to write their work on the board?

Once you have decided how the work will be displayed, who would be involved with the discussion of that work? Do you present the solution to the class yourself so that you can ensure your intended point is made? Do you invite the student to explain the solution in their own words? Do you ask another student to interpret their peer's solution? If something is unclear, how do you respond?

Decisions about how mathematics is communicated—and who gets to communicate that work—influence who has *mathematical authority* in a classroom. When people see the word *authority*, they often think about people in institutional positions of power. For example, a teacher can be viewed as an authority in a classroom, and principals are thought of as authorities for their schools. But what exactly does it mean to have mathematical authority in a classroom, and in what ways can that authority be shared between students and their teacher? In this article, we expand on the idea of what it means to have authority in mathematics classrooms and consider different ways teachers can (1) support students to take on more mathematical authority and (2) purposefully leverage their mathematical authority to create a more productive learning environment for them. This expansion also includes looking beyond the authorship of mathematical ideas when thinking about how authority is distributed in a classroom to include authority for representing and speaking, two ubiquitous forms of communicating mathematics.

WHAT IS MATHEMATICAL AUTHORITY?

Recently, we have been exploring what it means to have mathematical authority in mathematics classrooms. We view mathematical authority as a dynamic and negotiated relationship between people, where one person (or party) agrees to lead and another agrees to follow within a mathematical situation (Bishop et al., 2022). For us, mathematical authority is rooted in the activities within a classroom that contribute directly to the mathematics, as well as who gets to partake in those activities. Referring to the scenario given at the beginning, mathematical authority is present in decisions about how to solve a given problem, who communicates mathematical ideas and solutions publicly, and who decides whether a solution is valid, efficient, or correct. We believe authority is an important feature of productive mathematics discourse, but students may not get to act as a mathematical authority in school contexts very often (Wagner & Herbel-Eisenmann, 2009). When students have mathematical authority, they can take ownership of the mathematics present in the classroom: an observation that is consistent with the mathematical practice of constructing viable arguments and critiquing the reasoning of others (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

We also view mathematical authority as shifting, expanding, and changing hands from one situation to the next, rather than as a fixed entity that is established by institutional power, perceived content expertise, or status. Authority expands based on the situation, so that multiple people can have authority simultaneously without diminishing one another's authority. Thus, when investigating authority in a classroom, we are guided by the following question: Given a mathematical scenario in a classroom, *who* has the authority to do *what*? Two common authoritative activities are authoring mathematical ideas (e.g., Sullivan et al., 2020) that serve as the topic of discussion, and assessing the ideas present during a discussion (e.g., Webel, 2010). In mathematics classrooms, these activities are indicators of who has authority for the mathematics. While authoring ideas has been widely explored as a means for parsing mathematical authority in the classroom, we want to highlight the importance of another activity: *communicating* mathematical ideas.

For us, communication of mathematical ideas means the *public* communication, facilitation, or extension of ideas. In other words, it is the way in which mathematical ideas are propagated or expressed so they are accessible to all. But mathematical ideas can be expressed in different ways. We consider two types of public communication: representing and speaking. Representing refers to visual communication (including written and gestural communication), while speaking refers to oral communication. We believe that who has authority for representing and speaking can influence the overall experience of a mathematical discussion. By focusing on mathematical activities beyond authoring ideas, like communication, we share different ways that teachers can shape mathematical

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authority during whole-class interaction in their classrooms. Below, we present a series of video-based vignettes from a range of K–12 mathematics classrooms that explore the variety of ways that authority might be present in your classroom. In our discussion of the vignettes, we describe how teachers supported students to act with mathematical authority, while also leveraging their own authority as the teacher, to facilitate productive mathematical discourse.

Examples of Authority in Action

We have selected our vignettes to illustrate three different authority structures that teachers have used to increase students' mathematical authority during whole-class interaction: (1) full student authority over the mathematics, (2) sharing authority between students and teachers *across* different mathematical activities, and (3) sharing authority between students and teachers *within* the same mathematical activity. While thinking about these structures of mathematical authority, we invite you to reflect on the potential pros and cons of each and consider the circumstances under which you might use each structure in your own classroom.

FULL MATHEMATICAL AUTHORITY FOR STUDENTS

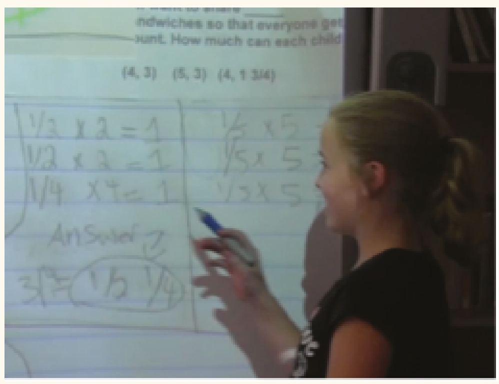
As a teacher, do you always possess some mathematical authority in every discussion? Is it possible for a student to have all the mathematical authority? Is it even desirable for students to have all the mathematical authority? These are questions to consider while watching the following vignette. In this first example, a teacher calls a student named Lily to the front of the room to discuss her understanding of another student's strategy for solving the following problem: "Four children want to share three submarine sandwiches so that everyone gets the same amount. How much can each child have?" As you watch Video 1, we would like you to consider who has authority for which mathematical activities and how the teacher and students shape mathematical authority in this scenario.

To determine who had mathematical authority in this scenario, we attended to each of the related but different activities of authoring and communicating mathematical ideas: Who was the *author* of the focal mathematical idea, and who was the *communicator* of the idea? Because the solution to the problem on the board was produced by a fellow student in the class, the students had authority for authorship in this scenario. But not only was it a student who was responsible for authoring the mathematical idea under consideration, it was also a student, Lily, who was responsible for communicating the mathematics to the class. Lily took on the powerful role of interpreting and revoicing the mathematical ideas even though she was not the original author of those ideas. By taking authority for mathematical communication, Lily also took ownership of the mathematics being discussed.

What about the role of the teacher? Although the teacher was clearly present and engaged during Lily's presentation, she did not contribute to the mathematical discussion. Instead, her role was managerial in nature; she facilitated the discussion by managing who had the conversational "floor" during the interaction (e.g., "Lily's telling us. Thank you, ma'am."), but did not directly author or communicate any mathematics. We characterize this authority still held by the teacher in this scenario as *pedagogical authority*, in contrast to mathematical authority. *Pedagogical authority* is the authority teachers have by virtue of their position and includes authority for directing instructional activities such as selecting topics or tasks, opening and closing tasks, nominating speakers, and selecting student work to share (Bishop et al., 2022; see also Wilson & Lloyd, 2000).

This first example shows that it is, in fact, possible for students to be the sole mathematical authority in the classroom. Within this vignette, students were the authority for generating mathematical ideas as well as communicating them, both orally and visually. Thus, students had authority for the activities of authoring, representing,

Video1 Lily Presenting to a Fifth-Grade Class



Watch the full video online.

and speaking. Although students had sole authority for speaking in this example, this is not common. Instead, the authority for speaking is often shared with the teacher. As such, when students are the sole speakers, it is a strong indicator of their overall mathematical authority.

On the surface, it might appear that the ideal mathematics classroom is the one that gives students maximum mathematical authority. However, that is not the position we take in this paper. We suggest that there are many times when it is important and necessary for a teacher to step into the seat of authority to support a more productive mathematical discussion. This raises the following question: What are the different ways that teachers can productively assert mathematical authority? In the vignettes that follow, we share several examples of how teachers might productively insert themselves into a mathematical situation.

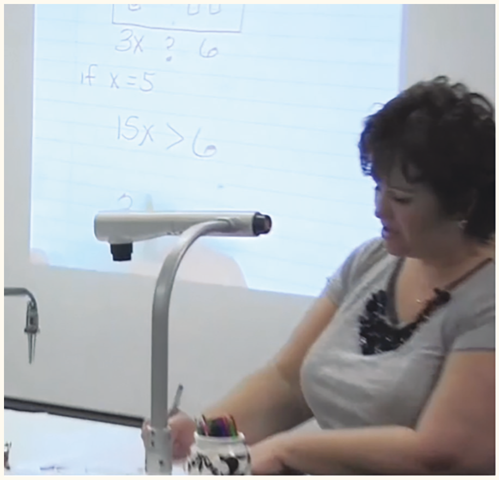
SHARING AUTHORITY ACROSS ACTIVITIES: STUDENT AUTHOR AND TEACHER REPRESENTER

While the previous example shows that students can be the sole authority in a mathematical interaction, we now turn our focus to the many ways that teachers might take up authority. But does this mean that students must lose authority so that the teacher can gain it? We claim that the answer to this question is *no*. Instead, by taking a more nuanced approach to describing mathematical authority beyond identifying who generates the mathematical ideas, we suggest that teachers and students may simultaneously claim mathematical authority for *different* activities. We refer to this authority structure as *sharing across activities* and exemplify this structure in the next vignette.

In this vignette, students in a seventh-grade classroom have just finished working in small groups, using algebra tiles to explore the following problem: What is the relation between the terms $3x$ and 6 when $x = 5$? During the subsequent whole-class discussion, Cody critiques a possible mistake he believes his classmate, Lizzie, has made: she claimed that if $x = 5$ then $3x$ is equivalent to $15x$, rather than 15. As you watch Video 2, attend to who has authority for authorship and who has authority for representing.

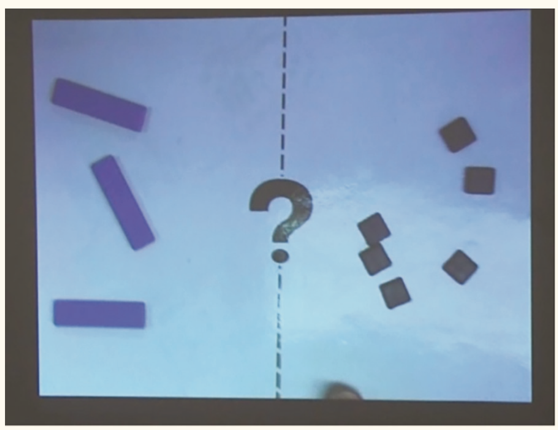
The focal mathematical idea in this video is a student-driven argument about the related meanings of $3x$, 15, and $15x$, when $x = 5$. If authorship is seen as the only indicator of mathematical authority in the classroom, we might conclude that the students are the sole authority in this interaction. However, by expanding our conceptualization of authority to include authority for communication, we see that the teacher is actually playing a critical role in the oral, gestural, and written forms of communication. As seen in Figure 1 below, the teacher is supporting the class to understand and engage with Cody's critique of Lizzie's solution. The teacher publicly displays the algebra tiles Cody referenced privately at his desk during his explanation for why 15 was the solution.

Video 2 A Seventh-Grade Class Comparing $3x$ and 6



Watch the full video online.

Figure 1 Using Algebra Tiles to Publicly Represent Students' Thinking



In particular, the teacher visually represented Cody's argument that Lizzie was not replacing the x in the expression with a 5 to obtain 15, but was multiplying all of $3x$ by 5 to obtain $15x$. Without the teacher's public use of the algebra tiles to represent the quantities involved and her accompanying written text, Cody's argument may have been difficult for others to follow along with verbally.

Even though students had authority for authoring the focal mathematical ideas in this video, the teacher had authority for representing those ideas by creating a public visual and written record of their thinking. Thus, we describe the authority structure in this vignette as *sharing across activities*. By taking authority for representing in this vignette, the teacher was able to highlight important aspects of Cody's critique and assist with a potential confusion about the meaning of variables, substitution, and notation by visually representing Cody's solution for other students in the class. We also highlight that unlike in Vignette 1, both the teacher and students spoke in this vignette. In fact, this shared authority for speaking is the norm for most whole-class mathematics interaction. Because of this norm, we do not include the activity of speaking in our conceptualization of sharing authority across activities. Instead, we focus on the mathematical activities of authoring and representing when identifying an authority structure of sharing across.

SHARING AUTHORITY WITHIN THE SAME ACTIVITY: REPRESENTING

In the previous example, the teacher leveraged her authority for representing to both amplify and clarify the competing mathematical ideas that her students authored. We conceptualized the authority structure in that interaction as sharing mathematical authority across different activities. But sharing authority between students and the teacher does not always entail distributing activities so that the teacher is the sole authority for one, while students are the sole authority for another. This leads us to our final authority structure: sharing mathematical authority *for the same activity*, rather than sharing authority across different activities. Indeed, there are times when the teacher may choose to either co-author or co-represent mathematics alongside a student. We note here that because of the frequency at which students and teachers both speak during whole-class interaction, the authority for speaking is most often shared within, as was the case with the previous example. Thus, although in each the following vignettes authority for speaking is shared within that activity, we only focus upon sharing authority for the activities of authoring and representing.

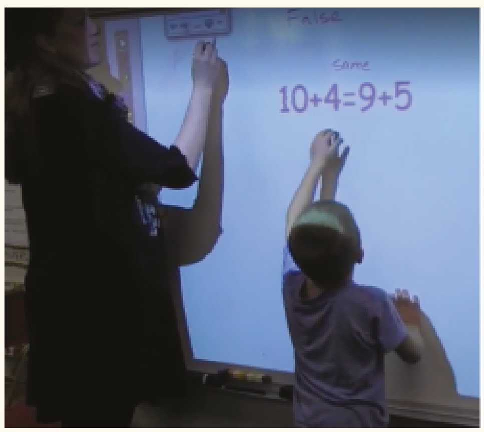
In the example that follows, a first-grade class is reasoning about the following problem:

True or False: $10 + 4 = 9 + 5$

One student, Julio, argued that the statement is false because the "numbers aren't the same." James disagreed and was invited to explain his thinking about why the equation is true. James argued that by taking 1 from the 10 and adding it to the 4, you would have $9 + 5$, which matches the right side of the equation. As you watch Video 3, consider who has authority for representing the mathematics during the discussion. (We note that this classroom is an inclusion classroom for students with hearing impairments, so a microphone is used during presentations and an interpreter is present.)

This vignette exemplifies another way in which a teacher can productively influence the mathematical authority relations within the class. James was given the empowering role of presenting his own solution in such a way that he had authority for both the authorship of the focal idea, as well as the communication of that idea. However, James's verbal and gestural explanations may not have been accessible to other students in the class. To help clarify James's oral and gestural communication, the teacher represented his strategy of taking 1 from 10 and adding it to the 4 by writing and gesturing on the board (see Figure 2). By assuming

Video3 A First-Grade Class Working on $10 + 4 = 9 + 5$



Watch the full video online.

shared authority for representing and creating a permanent record of James's thinking on the board, the teacher elucidated aspects of his explanation that his classmates may not have initially understood.

Moreover, this vignette also shows how teachers and students can represent mathematics with physical objects. The teacher introduced a new representation—a stack of 10 and a stack of 4 Unifix cubes—to visually model and clarify James's idea for his classmates. She then invited Julio to the front to represent James's strategy with Unifix cubes, and they connected the physical decomposition of 10 to the original equivalence relationship and other writing on the board. Julio is seen counting blocks in Figure 3.

This example illustrates how teachers can productively leverage their mathematical authority without diminishing or limiting students' mathematical authority. Although the teacher introduced the mathematical representations, we do not view her contributions as adding new mathematical ideas, so the teacher is not sharing authority for authoring with James. Instead, we view her mathematical actions in this vignette as supporting the communication of James's thinking. By assuming shared authority for representing, the teacher clarified James's explanation by writing, gesturing, and using physical objects so that others could engage more productively with the student-generated ideas put forward in the class.

We suggest that sharing authority for representing may be particularly important when students struggle to precisely articulate their mathematical thinking clearly to their peers. We also highlight that shared mathematical authority in this vignette is consistent with the communication standard of the National Council of Teachers of Mathematics (2000) in that "students in lower grades [may] need help from teachers to share mathematical ideas with one another in ways that are clear enough for other students to understand" (p. 61). Thus, shared authority for representing may be especially pertinent for younger children. In this first-grade class, we believe that sharing mathematical authority for representing was beneficial and helped the class to understand and engage with multiple complex ideas, including number decomposition, equivalence, and using mathematical notation, in ways that might not have been possible without the teacher's intervention.

SHARED AUTHORITY WITHIN THE SAME ACTIVITY: AUTHORING

Our final text-based vignette describes a tenth-grade geometry classroom as they work on developing a proof for Thale's theorem: If A, B, and C are distinct points on a circle, and $\overline{AB}$ is the diameter of the circle, then the angle ACB is a right angle. In this vignette, students were exploring relationships between inscribed angles, circles, and diameters. Using homemade circular Geoboards, students worked in groups to create different triangles with the stipulation that one of the sides

Figure 2 A Student and Teacher Share Authority

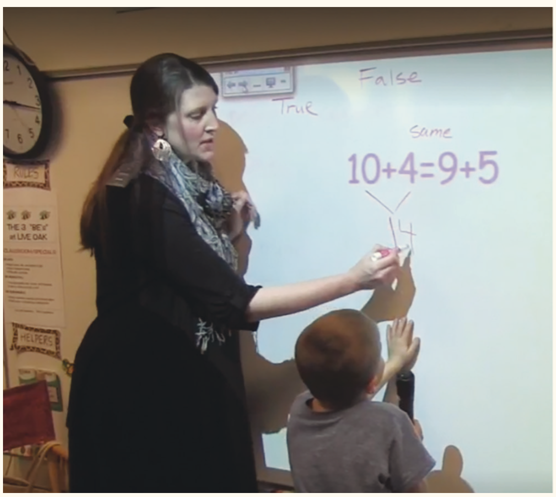


Figure 3 The Teacher and Julio Represent James's Strategy

had to be the diameter of the circle. Several groups of students shared their triangles on a document camera and discussed what they noticed about the different triangles. (See Figure 4 below for a work sample.) In the following vignette, the class formalizes their primary observation—that the triangles they made are right triangles—and attempts to prove this conjecture. While reading this vignette, we invite you to reflect on how authority is shared in this vignette, with a particular focus on authorship. (Note that there is no accompanying video for this vignette.)

Teacher: *Okay, you all seem to agree that we have right triangles. What conjecture can you make?*

Claudia: *If you have a circle, and you draw a triangle and one side is the diameter, then you have a right triangle.*

Teacher: *Can you write that on the board?*

[Claudia writes the verbal conjecture on board.]

Teacher: *Can anyone draw something to visualize it?*

[Samuel volunteers to go to the board and draws Figure 5*].*

Samuel: *[points at the top "corner"] We want to show that this is a right triangle.*

Teacher: *Let's name the points on your drawing so that it is easier to communicate. And I also think we need to be more specific about the vertex at the top. Right now, Claudia's conjecture states that one side of the triangle is a diameter. But it doesn't say anything about the other vertex. Samuel go ahead and label the points.*

[Samuel labels the circle center with O, and three vertices of the triangle as A, B, C. See Figure 6.*]*

Claudia: *I don't know what you mean about the other vertex. I think my conjecture is true.*

Teacher: *But can point C be outside of the circle? Or inside of the circle?*

Figure 4 An Example of a Student's Geoboard

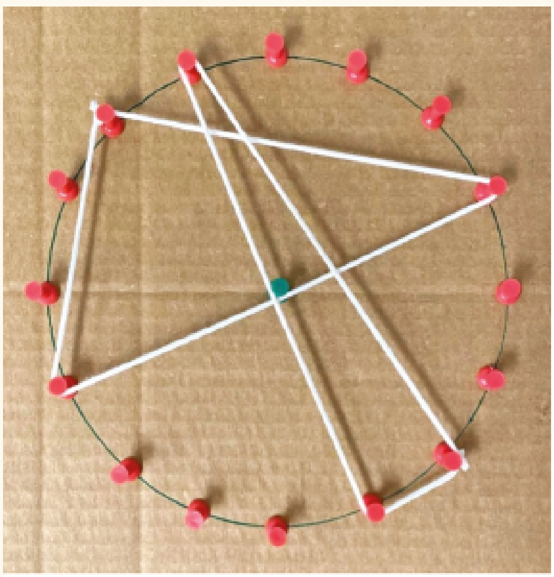


Figure 5 A Public Drawing of a Circumscribed Triangle

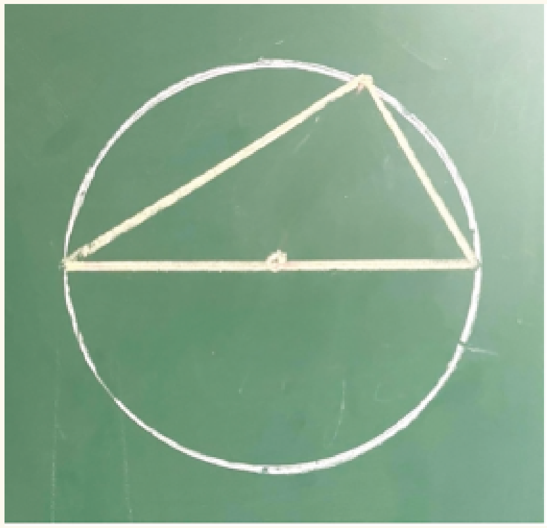
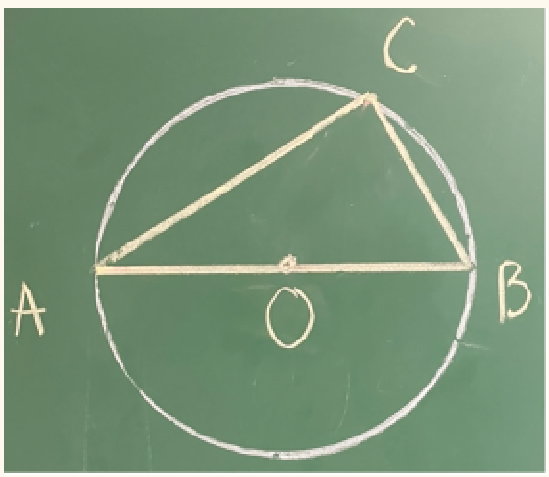


Figure 6 Labeling *A*, *B*, and *C* on the Triangle



Claudia: *Well no, but that's kind of obvious from the geoboards.*

Teacher: *Even so, we need to say that in the conjecture. Since the triangle is inscribed, point C has to be on the circle. Samuel, can you revise Claudia's conjecture*

Helena: *Why don't we just say that all points have to be on the circle. Then we don't have the same problem with A and B too.*

Teacher: *That's a good point. We could say that, but Claudia's conjecture already guarantees that A and B will be on the circle. Do you see how?*

Helena: *No.*

Teacher: *Well by saying that one side is a diameter, then each endpoint is on the circle by definition. It's not necessary to say all vertices are on the circle. But adding that statement would be one way we can make sure the other vertex is on the circle. So how should we revise Claudia's conjecture?*

[Samuel records the new conjecture, which now reads: If you have a circle, and you draw a triangle and one side is the diameter, all points are on the circle, then you have a right triangle.]

Teacher: *Does everyone agree with this? [Class murmurs* yes]. *Okay. So, what do we want to show to prove this?*

Brandon: *Angle* ACB *is a right angle.*

[Samuel records this on the board WTS $\angle ACB = 90°$*]*

Brandon: *Oh, a triangle is 180°. And if you divide it by 2, you get 90°.*

Samuel: *But how do you know to divide it by 2?*

Teacher: *Samuel has asked an important question. So let's think about these three points, A, B, and C? What is special about them and where they are located?*

Claudia: *They are all on the circle.*

Helena: *Oh, the radius, so they have the same distance to the center.*

Teacher: *Samuel, please use the pink chalk to draw the equal distances.*

[Samuel uses pink chalk to draw OA, OB, OC. *See* Figure 7.*]*

Samuel: *I still don't see why you would divide by two.*

Teacher: *Look at the diagram now with the three different radii shown in pink. What do you notice?*

Claudia: *We have isosceles triangles?*

Teacher: *Yes, exactly! Okay. So, I think we have enough here for you all to get back in groups and work on proving that $\angle ACB = 90°$. Huddle up with your group and see if you can use the fact that triangles* AOC *and* COB *are isosceles to reason about the measure of angle* ACB.

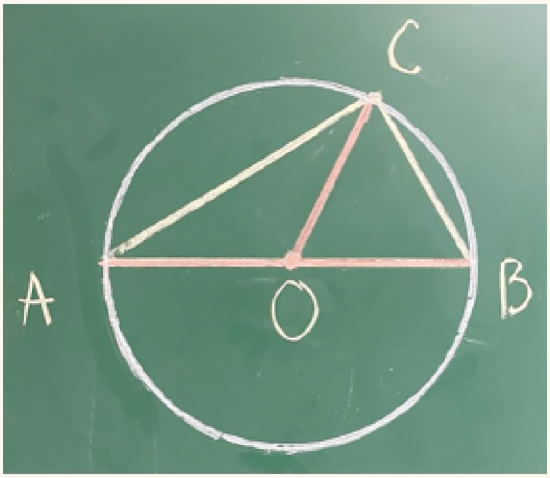
In this example, students generated a conjecture that was taken up and explored by the class. However, students were not the sole authors. As this vignette illustrates, there are times when it is productive for teachers to support their students by strategically co-authoring ideas with them, thus sharing authority within the activity of authorship. By choosing to provide critical details during the class discussion, the teacher assisted the students with developing the mathematical ideas. The teacher not only helped clarify what the conjecture was by specifying the location of the triangle's vertices, but also helped to formalize their discussion using mathematical symbols and notation by asking students to label the public drawings on the board. In this way, the teacher's contributions pushed the mathematical conversation beyond what the students were generating on their own.

Unlike the previous examples, the students in this vignette needed help formulating the focal mathematical idea rather than articulating their mathematical thinking to the class. Although the teacher interjected to help co-author the mathematical ideas present in the situation, the students still maintained authority over the written and visual communication of mathematics, as seen by Samuel scribing mathematics at the board and Claudia recording the original conjecture.

AN EXPANDED VIEW OF MATHEMATICAL AUTHORITY: AUTHORING, REPRESENTING, AND SPEAKING

The video-based vignettes illustrate the interrelated nature of three key mathematical activities for which

Figure 7 Labeling *OA*, *OB*, and *OC*



teachers and students negotiate authority: authoring, representing, and speaking. As we have shown in this article, determining who has mathematical authority is not just a matter of identifying who produced the ideas under consideration (the activity we refer to as authoring.) Instead, our understanding of mathematical authority is made richer when we disentangle other mathematical activities, such as communication, from that of authorship. We believe that the different modes of mathematical authority shared here are a productive lens to reflect on practice. Each of the vignettes illustrates a different combination of people (teacher, student, both) with authority for the activities of authoring, representing, and speaking: Table 1 lists who has authority for each of these activities by vignette. We also categorize the vignettes in Table 1 by their more general authority structures: full student authority, shared authority across activities, and shared authority within the same activity. Although we included the activity of speaking in the table, we remind the reader that we focused on the activities of authoring and representing to determine whether a vignette shared authority within activities or across activities because of the frequent occurrence of students and teachers both speaking.

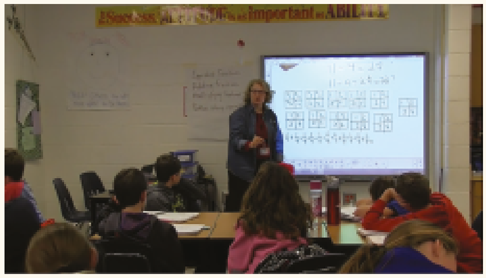
Our examples also implicitly reveal that there may be benefits to choosing certain authority structures over others, depending on the situation. For example, could the mathematical discussion have been improved in the first vignette (where students had full mathematical authority) had the teacher intervened in some way? When is it more productive to assist students with representing an idea versus letting them represent all on their own? While each of these authority structures has their pros and cons, we suggest it can be fruitful to intentionally diversify the authority in mathematics classroom.

TRY IT OUT AND TEST YOURSELF!

Now that we have shared and discussed the different authority structures in each of the vignettes, we invite you to try identifying authority in a classroom for yourself. Videos 4–6 depict three different fifth-grade mathematics classrooms, each illustrating a different combination of authority for authoring, representing, and speaking mathematical ideas.

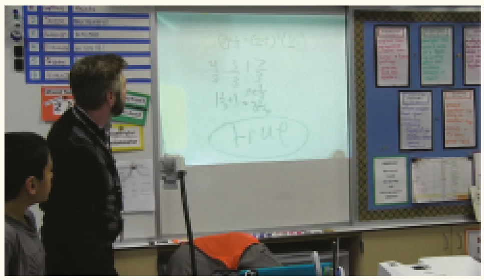
As you watch Videos 4–6, decide who you think has authority for each mathematical activity, and consider watching, discussing, or debating these videos with a colleague to get their perspective. You can see our interpretation of each video in the supplemental material (link online). We also encourage you to consider when and how you might intentionally plan for and use some of the different authority structures shown in these videos in your own classroom.

Video4 A Fifth-Grade Class Working on an Equal-Sharing Problem



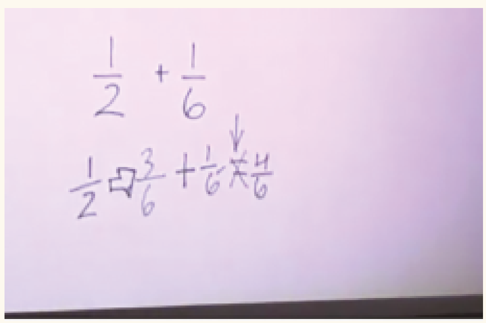
Watch the full video online.

Video5 A Fifth-Grade Class Working on a True/False Equivalence Relation



Watch the full video online.

Video6 A Fifth-Grade Class Discussing the Use of the Equal Sign



Watch the full video online.

CLOSING THOUGHTS

As all of our vignettes and videos showed, di erent classroom situations call for di erent distributions of mathematical authority, depending on instructional goals, time, and other factors. At times, it can be productive for students to have full authority for mathematical activities, as seen in Video 1. More likely though, students will need targeted support from their teacher as they develop and communicate mathematical ideas. By identifying mathematical activities beyond authorship to include the activities of representing and speaking, we believe the idea of *productively sharing authority* is made clearer: sharing authority is not equivalent to the teacher abdicating their mathematical authority; instead it is about using one's own authority as a teacher to best assist students with their learning (Oyler, 1996). Our examples of sharing authority across activities and sharing authority within the same activity show how shared authority can amplify, clarify, and build upon students' authored ideas, thereby positioning them as doers of mathematics.

While it appears that authority for speaking is typically shared between students and teachers in most classrooms, this Front & Center article highlights the various ways that authority for representing can be productively distributed between students and teachers as well. Therefore, we encourage teachers to consider ways they can provide students more opportunities not only to author and speak, but also—and importantly—to represent mathematics. We believe that sharing authority for representing creates critical opportunities for teachers to sca old students' learning of discipline-speci c ways of writing, representing, and communicating mathematically: that is, to become pro cient with mathematical discourse. Although it may be clearer or more e cient for the teacher to maintain authority for representing, students pro t greatly when we hand them the chalk, dry erase marker, or pointer and support them to publicly write, represent, and communicate mathematical ideas.

Imagine your classroom again. You want to share a student's work with the class. Would you ask the student to project their written work on the document camera, recreate the student's work on the board yourself, or ask the student to write their work on the board? And who would discuss that work? Would you present the solution to the class yourself to ensure your intended point is made? Would you invite the student to explain their work, or ask another student to interpret their peer's solution? We believe that by focusing on representing as a marker of mathematical authority, we can make decisions that encourage our students to take on new and di erent authoritative roles in the mathematics classroom.

Table1 Identifying who has Authority for Each Activity

Authority Component/Activity	Full Student Authority	Shared Across Activities	Shared Within the Same Activity	
	Vignette 1	*Vignette 2*	*Vignette 3*	*Vignette 4*
Authoring: Generating the mathematical idea that is the focus of the interaction.	Student	Student	Student	Both
Representing: Visual communication of mathematics that is publicly accessible and mathematically meaningful. Includes both writing and gestures.	Student	Teacher	Both	Student
Speaking: Oral communication of mathematics that is publicly accessible.	Student	Both	Both	Both

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