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# Physics-informed machine learning for reliability and systems safety applications: State of the art and challenges

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ABSTRACT

The computerized simulations of physical and socio-economic systems have proliferated in the past decade, at the same time, the capability to develop high-fidelity system predictive models is of growing importance for a multitude of reliability and system safety applications. Traditionally, methodologies for predictive modeling generally fall into two different categories, namely physics-based approaches and machine learning-based approaches. There is a growing consensus that the modeling of complex engineering systems requires novel hybrid methodologies that effectively integrate physics-based modeling with machine learning approaches, referred to as physics-informed machine learning (PIML). Developing advanced PIML techniques is recognized as an important emerging area of research, which could be particularly beneficial in addressing reliability and system safety challenges. With this motivation, this paper provides a review of the state-of-the-art of physics-informed machine learning methods in reliability and system safety applications. The paper highlights different efforts towards aggregating physical information and data-driven models as grouped according to their similarity and application area within each group. The goal is to provide a collection of research articles presenting recent developments of this emergent topic, and shed light on the challenges and future directions which we, as a research community, should focus on for harnessing the full potential of advanced PIML techniques for reliability and safety applications.

## 1. Introduction

Ensuring reliability and system safety is critically important in developing complex engineering systems. Moreover, the ability to develop accurate predictive models for system performances under different designs and operating conditions is vital for effective reliability and safety analysis. Due to the cost effectiveness, predictive models have been employed more frequently in a variety of engineering applications, such as reliability modeling [1–6], degradation analysis [7,8], fault diagnostics [9,10], failure prognostics [11–14], operation and maintenance decision-making [15–17], design and uncertainty quantification [18–20], as well system risk assessment. Traditionally, methodologies for predictive modeling generally fall into two categories, namely physics-based approaches [21] and machine learning (ML)-based approaches [22].

Physics-based approaches, also called forward modeling approaches (e.g., [23,24]), use physical laws of nature to determine the underlying relationship between input parameters and output performances of a complex system. Physics-based approaches are well-suited for representing processes that are conceptually well understood using known scientific principles. By providing a baseline model for the underlying physical relations within the system of interest, physics-based approaches have been widely used for numerous reliability and system safety applications. For example, in structural health monitoring (SHM) [25,26], the physics-based approach provides a calibrated physics-based numerical model that can be used for damage prognosis. However, a critical barrier limiting the in-practice application of physics-based model updating is the modeling error that originates from model simplification and omission [24]. In addition, a wide variety of system modeling tasks, such as optimal control and real-time damage diagnosis, require accurate and timely predictions. In such scenarios, employing physics-based approaches, could be challenging due to computational times required in solving physics-based governing equations and predicting system performances. Furthermore, tasks involve processes that are not completely understood because of the

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inherent complexity of the processes limit the application of physics-based approaches. For example, the prediction of the failure time of complex systems is generally addressed by capturing the physics of failure. While extensive research on physics-based models has been performed (e.g., [27,28]), developing physics-based models for reliability and system safety applications could be challenging, as engineering systems could be complex, physical degradation processes may not be well-understood, or large variability may exist from system to system.

Machine learning approaches [22,29,30] could remedy the challenges on required prior knowledge about the system for physics-based approaches by relying on the data collected from the system of interest. The use of machine learning approaches could be particularly promising in problems involving processes that are not completely understood, or where it is computationally infeasible to implement physics-based approaches at a desired level of accuracy or efficiency. Therefore, machine learning approaches have been implemented to mitigate these drawbacks of physics-based approaches. While these advanced machine learning models could be powerful for modeling complicated systems, there could also be challenges. First, machine learning approaches are often data-driven and require a substantial amount of data, which is usually unreliable or unavailable in engineering practice. For instance, in failure diagnosis applications, machine learning approaches often require training data sets from both undamaged and damaged conditions. However, such data could be limited, especially for those damaged conditions. Additionally, it could be challenging in extrapolating machine learning models or predicting unseen data. Particularly, the "black-box" model highly depends on the representative quality of the labeled data that it is fed in, leading to low accuracy and generalizability outside available data. Research studies on machine learning applications in prognostic and health management (PHM) require a representative data set of run-to-failure degradation trajectories to obtain accurate prognostics models [31,32]. The collection of the representative data set for systems subjected to periodic maintenance interventions can take a long time because failure may be rare, and the system can operate in different environments and follow different mission profiles, resulting in a large range of possible deterioration trajectories.

Generally speaking, physics-based and machine learning approaches both have their own characteristics, and could suffer from certain deficiencies when applied to complex engineering problems in reliability and system safety applications. There is a growing consensus that solutions to complex science and engineering problems require novel methodologies that can integrate prior scientific knowledge (e.g., physics-based modeling approaches) with state-of-the-art machine learning techniques. Physics-informed machine learning (PIML) is an emerging paradigm that aims to leverage the wealth of physical knowledge for improving the effectiveness of machine learning models [33]. By the PIML methods, physical principles are often used as the 'prior' knowledge to enhance the power of the machine learning models. Various approaches have been developed to combine physics-based and machine learning approaches, depending on what type of information is processed and how the pieces of information are combined. Karpatne [34] first formally conceptualized the paradigm of theory-guided data science, where scientific theories are systematically integrated with data science models in the process of knowledge discovery. Karniadakis [35] reviewed some of the prevailing trends in embedding physics into machine learning, present some of the current capabilities and limitations, and discuss diverse applications of physics-informed learning. Rueden [36] provided a definition and propose a concept for informed machine learning, which illustrates its building blocks and distinguishes it from conventional machine learning. A taxonomy that serves as a classification framework for informed machine learning approaches is introduced. It considers the source of knowledge, its representation, and its integration into the machine learning pipeline. Moreover, among physics-informed machine learning methods, the physics-informed neural network has been receiving growing attention due to the potential reduction in computational cost and modeling flexibility. Viana [37] surveyed the developments on Bayesian calibration of computer models and physics-informed neural networks. Besides, a physics-informed machine learning strategy has been widely used in many research areas in recent years. Willard [33] provided an overview on techniques of integrating the traditional physics-based modeling approaches with the machine learning models. This survey focuses primarily on improving the modeling of engineering and environmental systems that are traditionally solved using physics-based modeling. Markidis [38] focused on evaluating the potential of PIML as linear solvers and characterizing PIML linear solvers in terms of accuracy and performance under different network configurations (depth, activation functions, input data set distribution). Rai [39] made a meticulous and systematic attempt at organizing and standardizing the methods of combining machine learning models and physics-based models in the field of cyber–physical systems. Moreover, physics-informed machine learning methods have also been incorporated in some advanced technologies, such as digital twin [40] and additive manufacturing [41–44]. Big data and the industry 4.0 bring opportunities for new hybrid modeling solutions. Sansana [45] provided a review on hybrid modeling techniques, associated system identification methodologies, and model assessment criteria.

While PIML methods have quickly been used in solving differential equations and physical computing, among others, little attention was paid to their method development and application for the enhancement of the reliability and safety of complex technological systems. Developing advanced PIML methods is recognized as an important emerging area of research, which could be particularly beneficial in addressing reliability and system safety related challenges. With this motivation, this paper provides a review of the state-of-the-art of physical-informed machine learning methods in enhancing reliability and safety of complex system from both technical and application perspectives. This survey was conducted in two aspects with the methodology and application focuses. The authors explored physics-informed machine learning (PIML) techniques published in the reliability and system safety-related journals between 2016 and 2022, such as the Journal of Reliability Engineering & System Safety, Mechanical Systems and Signal Processing, Mechanical Design, and Structural and Multidisciplinary Optimization, etc. The focus of this survey is on PIML approaches that incorporate physics knowledge and machine learning models for real-world engineering applications. Moreover, the authors conducted a keyword search for PIML methodologies from google scholar and web of science between 2016 and 2022, such as the words "physics-informed", "physics-guided", "physics-constrained", "physics-aware", or "theory-guided", etc. A collection of research articles presenting recent developments of this emergent topic are provided, and comprehensive analysis on constructing the PIML model is presented focusing on reliability and system safety related methodologies and applications. It also sheds light on the challenges and future directions which we, as a research community, should focus on for harnessing the full potential of advanced PIML techniques for reliability and system safety related techniques and applications. The rest of the paper is organized as follows. Section 2 discusses the existing state of the art methods of the PIML. Section 3 analyzes a multitude of reliability and system safety applications, such as reliability modeling, degradation analysis, fault diagnostics, failure prognostics, operation and maintenance decision-making, as well as system risk assessment. Section 4 explores the limitations and challenges of PIML modeling, and a brief conclusion is provided in Section 5.

## 2. Overview of the PIML methodology

Integration of physics-based models with machine learning techniques to create PIML models has been prevalent recently for engineering applications. This section provides an overview of the past PIML methodology. Section 2.1 provides a brief summary of the past PIML methodology development efforts, especially the usage of PIML in early stages. Section 2.2 presents some recently developed PIML frameworks and provides a summary of the state of the art PIML methodology.

### 2.1. Development of PIML models

While a physical model can be employed initially to establish the essential relationship between system input parameters and output performances for engineering analysis, the field has evolved with more advanced data-driven approaches being incorporated for the development of PIML models in order to achieve a better model prediction performance. For example, Zhu et al. [46] determined the low cycle fatigue of aircraft turbine discs using a physical model, and further combined it with a statistical learning model for uncertainty analysis.

In general, sample data is needed for training and validation of the PIML model, where physics-based simulation models could be used to generate these needed sample data. For example, triple loop Monte Carlo simulation [47] was used for multi-state physics modeling, which was applied to reliability assessment of reactor protection system of a nuclear power plant. This is an early example of a "physics-informed" model that enhances a data driven model. Shi et al. [48] utilized Gaussian process model for reliability based design optimization and applied this technique to optimize crash-impact design of vehicles. Many of the these studies have a primary focus of reducing the computational cost of physics-based simulations by using machine learning based surrogates. In particular, Perrin et al. [49] utilized Gaussian process surrogate models and aims to identify boundaries of failure domain while minimizing computational cost. However, these studies do not utilize the data-driven model to enhance or improve the physics model, instead the physics model is replaced by the data-driven model and further updated given online data measurements.

Hybrid machine learning physics models were scarce only a few years ago, but data-driven models have been utilized as surrogates to complicated physics of failure models. For instance, Bourinet [50] utilized support vector machine surrogates for the true limit state function, which defines the failure of the given system. In this study, due to lack of data, training points were generated using Metropolis-Hasting algorithm based on the few known data points. Also, machine learning has been utilized for direct estimation of the probability of failure. Gomez et al. [51] implemented an artificial neural network with radial basis function to detect cracks in a rotating shift, where experimental data was used to train the model.

In recent years, there have been advances in combining physical models with machine learning techniques. Specifically, mapping AI structures to experimental data, and merging existing physical models with data driven methods. For instance, Yigit et al. [52] implemented a neural network for adjusting the outputs of frequency response function utilized for torsional vibration damper. In this study, the nodes of the deep neural network are implemented as model elements, including storage loss, stiffness, and damping coefficients. This particular hybrid model is an example of physics enhanced architecture, where the nodes of the network are defined specifically by physical vibration models. In addition, this enables correction to the physical model with observable experimental data. In most applications, the available experimental and simulated data plays a significant role in the performance of the algorithm.

### 2.2. State of the art PIML methodology

Different types of hybrid structures are created for different applications, for example, physical knowledge can be used in the design of model families to restrict the space of models to obtain physically consistent solutions, such as in the selection of response and loss functions or in the design of model architectures. Another way of integrating the physics principles and data is to construct hybrid models, where some aspects of the problem are modeled using physics-based components while other aspects are modeled using data-driven components. Many of these approaches can be applied together in multiple combinations for a particular problem, depending on the nature of physical knowledge and the type of data-driven method. Generally, there are two types of physics-informed machine learning approaches, depending on the roles of physics prior knowledge played in the hybrid model. The first approach enforces known physical constraints into machine learning models, which can be considered as a physics-informed loss function. The second approach, named the physics-informed architecture, incorporates the physics knowledge into the model architecture.

#### 2.2.1. Physics-informed loss function

In this section, we discuss physics-informed loss function strategies that integrate the prior physics knowledge into the loss function of the ML model. Engineering problems often exhibit a high complexity and standard ML models could fail to capture such relationships directly from data, especially when limited data is available for model training. One of the most common techniques to make ML models consistent with physical laws is to incorporate physical constraints into the loss function of ML models as follows

$$Loss = Loss_{ML} + Loss_{Phy} \tag{1}$$

The first term, $Loss_{ML}$, is the loss function in the ML model which is typically defined as a mean-square error (MSE) or a root-mean-square error (RMSE) loss during the training. The addition of physics-based loss, $Loss_{PHY}$, aims to ensure consistency with physical laws. The added physics constraints can be written as the regularization term in the loss function.

Various types of knowledge based upon physical principles, such as those represented by partial differential equations, boundary conditions, physics of failure models, and statistical properties, can be incorporated. The equations can generally be expressed as follows: A generic form of partial differential equations that describe the evolution of a continues value $v(x,t)$:

$$\frac{\partial v}{\partial t} = F\left(t, x, v, \frac{\partial v}{\partial x_i}, \frac{\partial^2 v}{\partial x_i \partial x_j}, \dots\right) \tag{2}$$

Boundary conditions:

$$v(0) = 0, \quad v\left(\frac{\pi}{2}\right) = 2 \tag{3}$$

Statistical (invariance) property that defines a system keeps certain properties under some transformations:

$$g \cdot (h \cdot u) = (gh) \cdot u \tag{4}$$

Physics law:

$$E = m \cdot c^2 \tag{5}$$

Using physics-informed loss function in PIML models has been employed in many recent studies. Physics-informed neural networks [53] were introduced and trained to solve supervised learning tasks while taking into account given laws of physics described by general nonlinear partial differential equations. A neural network-based computational framework Li [54] was established to characterize the finite deformation of elastic plates, where the physical information (PDEs, BCs, and potential energies) was incorporated into the loss function. John [55] proposed a biologically informed neural network framework, which enables the modeler to use domain expertise to include qualitative constraints on the parameter networks by selecting appropriate activation functions and loss terms for the optimization. In addition to neural networks, the physics constraints can also be integrated with other ML models, such as Gaussian Process models. Recent studies have incorporated physical constraints or other priori information within Gaussian process regression (GPR) to supplement limited data and regularize the behavior of the model. Veiga [56] introduced a framework for incorporating constraints in Gaussian process modeling, and extends this framework to any type of linear constraint. The result shows the accuracy of Gaussian process predictions can be enhanced with such constraint knowledge. Lopera [57] extended the method to deal with sets of linear inequalities, and investigate theoretical and numerical properties of a constrained likelihood in the GP model. Swiler [58] gave a survey on constrained GPR, which presents three main opportunities, including transformation to the output, likelihood, and hyperparameter optimization, to enforce constraints for the GP model. Jensen [59] extended the Gaussian process (GP) framework for bounded regression by introducing two bounded likelihood functions that model the noise on the dependent variable explicitly. Bachoc [60] considered covariance parameter estimation for a Gaussian process under inequality constraints. Study shows that the constrained maximum likelihood estimator is generally more accurate on finite samples in simulations.

Overall, incorporating physics knowledge into the loss function often possesses salient features including but not limited to: (1) excellent capability of dealing with less rich data; (2) consistent with physics principles; (3) superior generalizability with robust inference. Moreover, loss function terms corresponding to physical constraints are applicable across many different types of ML frameworks and objectives.

First, one of the obstacles hindering the scaling-up of the initial successes of machine learning is the size of the database that drives the algorithms. Incorporating the already-known physical laws into the training process can significantly reduce the size of the required database. Available physics can provide constraints to the network outputs, alleviate overfitting issues, reduce the need for big training dataset, and thus improve the robustness of the trained model for more reliable prediction. For example, Zhang [61] developed a physics-guided convolutional neural network for data-driven structural seismic response modeling.

Second, the data set could be sparse and noisy. The difficulties of modeling with the noisy data and high-dimensional data can be relieved by integrating the prior scientific knowledge into the loss function. He [62] developed a local convexity data-driven computing method to enhance accuracy and robustness against noise and outliers in the dataset, which also performs well for high dimensional data sets that are relatively sparse in real-world engineering applications. Kissas [63] introduced physics-informed neural networks to solve conservation laws in graph topology with real noisy clinical data for the first time. Sun [64] proposed a physics-constrained Bayesian deep learning approach to reconstruct flow fields from sparse, noisy velocity data, where equation-based constraints are imposed through the likelihood function and uncertainty of the reconstructed flow can be estimated. Patrick [65] used an experimental weakly turbulent fluid flow to demonstrate that combining a data-driven methodology with physical principles enables the discovery of a quantitatively accurate model of a non-equilibrium spatially extended system from high-dimensional data that is both noisy and incomplete. Many other issues lie in the traditional ML methods can also be solved by integrating the scientific knowledge and data-driven methods intelligently. Jun [66] proposed an architecture that combines the deep residual neural network with some underlying physical laws, which overcomes several issues in deep learning methods, such as the dynamic behavior and the gradient explosion.

Third, it could be difficult in practice to obtain the output value (label) of each data, and the dependence on labeling has restricted the application of supervised learning. The ML model integrated with physical principles enables the usages of unlabeled or indirectly labeled data. Chen [67] used variables associated with the label as indirect labels and constructs an indirect physics-constrained loss based on the physical mechanism, so that the model training process does not rely on labels. Zhu [68] proposed a convolutional encoder–decoder neural network for predicting transient PDEs with governing PDE constraint, which expands the work in [69]. The governing equations of the physical model are incorporated in the loss/likelihood functions, resulting in Physics-constrained deep learning models that can be trained without any labeled data. Karumuri [70] used a deep fully connected residual neural network to build a surrogate, eliminating the need for expensive forward model evaluations. A physics-informed loss function was introduced to train the deep neural network for high-dimensional uncertainty propagation. Moreover, due to the multi-scale nature of the physics, such processes can be computationally prohibitive for most real-time applications (e.g., diagnosis and planning) and many-query analyses (e.g., optimization design and uncertainty quantification). Sun [71] proposed a simulation-free, physics-constrained deep learning for surrogate computational fluid dynamics (CFD) model, where the initial and boundary conditions, and the governing PDEs are incorporated into the loss function to drive the training. It shows good promise to geometric variations optimization and uncertainty quantification.

Fourth, physics consistency is another important aspect of model construction. Drgona [72] presented a physics-constrained deep learning method, which encodes physics-based prior knowledge into a structured recurrent neural architecture and uses penalty methods to provide inequality constraints, thereby bounding predictions within physically realistic and safe operating ranges.

Fifth, high accuracy and low error are not only required in the model construction processes but also in the model prediction. Traditional ML methods often lack the ability to extrapolate, which can be solved by the physics-informed ML methods. Zhao [73] proposed a physics constrained ML model, which extrapolates much better than the pure ML model, emphasizing the benefits of combining physics with ML for increased generalizations. Zobeiry [74] developed a Physics-informed neural network to solve heat transfer PDE. The loss function is defined based on errors to satisfy PDE, BCs, and initial conditions. By using physics-informed activation functions, the heat transfer beyond the training zone can be accurately predicted. Moreover, the richness of the information brought by the 'prior' knowledge may also influence the model construction. Sometimes, the scientific principles are incorporated into multiple structures, especially for the incomplete physics knowledge. Zhang [75] introduced a physics-informed deep learning framework for metamodeling of nonlinear structural systems with scarce data, which incorporate incomplete physics knowledge into deep long short-term memory (LSTM) networks. The laws of physics are taken as extra constraints, encoded in the network architecture, and embedded in the overall loss function to enforce the model training in a feasible solution space. The embedded physics can alleviate overfitting issues, reduce the need for big training datasets, and improve the robustness of the trained model for more reliable prediction with extrapolation ability. The details on how to integrate prior knowledge into the architecture of the ML model will be presented in the next section.

### 2.2.2. Physics-informed architecture

In this section, we discuss four physics-informed architectures that integrate the prior physics knowledge and ML techniques. First, new ML architecture can be constructed according to the specific characteristic of physics knowledge. This physics-informed ML architecture incorporates the physics properties into nodes and/or layers of the ML model making the black-box algorithm more interpretable. Second, physics-based models and ML models can also operate simultaneously to enhance the model power, which generally refers to the hybrid physics-informed ML Model. Third, the multi-fidelity framework can be utilized to further improve the efficiency of the PIML strategies. Fourth, the Bayesian framework enables the PIML strategies, especially physics-informed neural networks, to quantify the uncertainties and incorporate various types of prior knowledge for different applications.

- Physics-Informed Machine Learning Architecture

The physics-informed architectures are a powerful approach to enhance the machine learning structure with known physical equations. The design of the architecture often depends on the model structure. For example, neural networks provide opportunities to encode physics prior knowledge in the novel neurons or layers (shown in Fig. 1), where the nodes of the structure are defined by known physical phenomena.

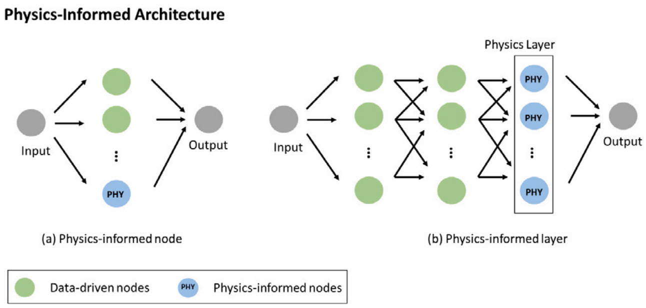


**Fig. 1.** Physics-informed architecture.

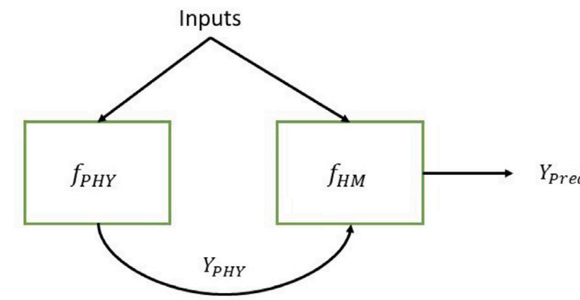


**Fig. 2.** Hybrid PIML model.

Contrasting to the physics-embedded loss function, where the training of the network is constrained by a physical equation; specifically, physics-informed architectures contain nodes for physical equations within the system. For instance, Yucesan [76] embedded a neural network with physics-informed layers which is utilized to estimate bearing and grease damage for wind turbines. In this particular study, there are two main failure modes being determined, grease damage and bearing damage; of which the latter can be determined with physical equations, while the former needs to rely on data-driven models. Still, both of these failure modes have some connection to each other, which is shown within the physics-informed architecture with "physics-informed" layers and data-driven layers. The main takeaway from this reference is that the two different failure modes can be estimated with the combination of known physics equations of one failure while enhancing the data-driven predictions of the other failure mode, and vice versa.

Similar to most hybrid methods, the main challenge is implementing the architecture specifically for a given application. With these different applications, the physics-informed architecture will take on varying structures. Also, the fusion of principles of physics into ML can take on different forms. For example, Cho [77] proposed information index and link functions, which show success in infusing the principles of physics into ML. The information index integrates adjacent information and quantifies the physical similarity between laboratory and reality, enabling ML to see through a complex target system with the perspective of scientists, and the link functions unravel the hidden relations between information index and physics rules. This framework fuses information index, link functions, evolutionary algorithm, and Bayesian update scheme. Zhang [78] introduced a machine learning-based fusion model MIDPhyNet that decomposes, memorizes, and integrates first principle physics-based information with data-driven models. The MIDPhyNet architecture's superiority is most significant when the models are trained over sparse data sets and in general, MIDPhyNet provides a generic way to explore how physical information can be infused with data-driven models. Pawar [79] proposed an architecture that aims to augment the knowledge of the simplified theories with the underlying learning process. The architecture consists of adding certain features at intermediate layers rather than in the input layer. This framework is flexible to be applied to many physical systems. Zamzam [80] proposed a novel learning model that utilizes the structure of the power grid. The proposed neural network architecture reduces the number of coefficients needed to parameterize the mapping from the measurements to the network state by exploiting the separability of the estimation problem. This prevents overfitting and reduces the complexity of the training stage. Lei [81] proposed a physics-informed data-driven optimal power flow (OPF) approach based on the stacked extreme learning machine (SELM) framework. Compared with the deep learning algorithms, the proposed method only requires very few adjustments of the parameters and thus can be easily extended to other systems.

Physics knowledge can also be used as the prior information to enhance the power of machine learning models. Chen [82] proposed a physics-constrained LSTM, in which the physical mechanism behind the geomechanical parameters is utilized as a priori information. This state-of-the-art model is capable of directly estimating geomechanical logs based on easily available data, and it achieves higher prediction accuracy since the domain knowledge of the problem is considered. Raissi [83] proposed a PIML method where Gaussian process priors are modified according to the particular form of parametric linear operators related to the governing equation and are employed to infer parameters of the linear equations from scarce and possibly noisy observations. Moreover, progress has been made at incorporating physical knowledge into kernels by computing kernels for systems governed by linear and weakly nonlinear ordinary and partial differential equations. Such kernels are computed by substituting a GPR approximation of the system's state variables into the governing equation and obtaining a system of equations for the kernel hyperparameters. Swiler [58] discussed a different strategy to design a covariance kernel for the prior of the Gaussian process which enforces the constraint. Such methods are based on derivations of linear transformations of GPs. Considering Gaussian processes as distributions over functions, another strategy is to consider a function space defined by a certain representation such that a global constraint can be translated into a finite set of constraints. This strategy amounts to deriving a specific kernel function related to the representation.

- Hybrid Physics-Informed Machine Learning Model

The previous sections discussed the embedded structure of constructing the PIML model, where the physics are integrated into the loss function and/or architecture of the ML model. This section introduces a separate structure employing and operating the physics-based and ML model jointly, where the ML models and physical models are applied separately to approximate different aspects of the physical system and be connected consistently to perform numerical simulation or meta-modeling. And we name it as the hybrid physics-informed machine learning model. Fig. 2 gives a schematic diagram of this method. Generally, there are mainly three integration strategies to construct the hybrid physics-informed ML model, including but not limited to: (1) operate the physics-based model and ML model simultaneously; (2) feed the output of a physics-based model as input to an ML model. These techniques encapsulate a wide variety of goals like feature extraction, noise removal, data transformation, etc.; (3) use an ML model to replace one or more components of a physics-based model or to predict an intermediate quantity that is poorly modeled using a physics-based model. The structures of the hybrid models have many variations and are widely used in many engineering applications, such as fault diagnosis and performance degradation assessment. Sadoughi [84] proposed a physics-based convolutional neural network (PCNN), for fault diagnosis of rolling element bearings. The proposed approach utilizes spectral kurtosis as well as envelope analysis to extract sidebands from raw sensor signals and minimizes non-transient components of the signals,

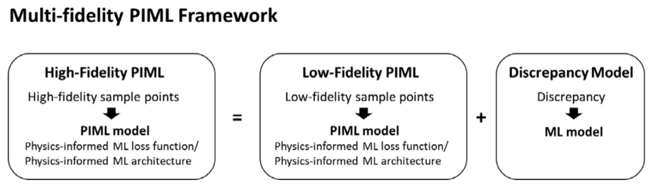


**Fig. 3.** Multi-fidelity PIML framework example.

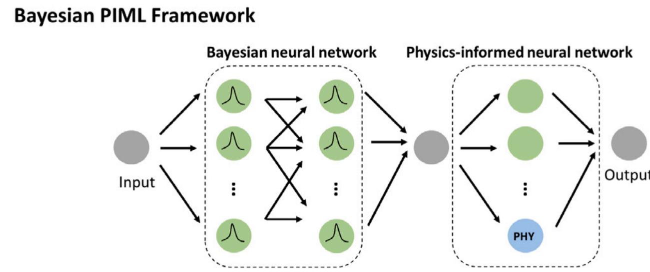


**Fig. 4.** Bayesian PIML framework example: B-PINN.

and then feeds the information about the fault characteristics into the CNN model. Tian [85] presented a method that detects bearing faults and monitors the degradation of bearings in electric motors. Based on spectral kurtosis (SK) and cross correlation, the method extracts fault features that represent different faults, and the features are then combined to form a health index using principal component analysis (PCA) and a semi-supervised k-nearest neighbor (KNN) distance measure. Sun [86] proposed a novel intelligent diagnosis method for fault identification of rotating machines, where the compressed sensing method preprocesses the data and feeds it into deep learning.

In summary, there are several novel aspects achieved by the hybrid structure including but not limited to: (1) the physics-based model and ML model complement each other to create more precise predictive models that respect the underlying physics; (2) the model and parameters are trained and estimated in a symbiotic fashion by using both the physics-based model and acquired data; (3) the robustness of physics model is enhanced by the accuracy of data-driven model.

- A Multi-Fidelity PIML Framework

Although the PIML approaches enhance the power of ML models, the training efficiency is still limited in high-dimensional problems. The concept of multi-fidelity has been explored extensively in surrogate modeling [87–89]. By integrating the data from high-fidelity (HF) and low-fidelity (LF) simulations, the trade-off between efficiency and accuracy for metamodeling can be made, and the model can be constructed with a limited amount of simulation data but achieve a good accuracy of prediction.

The concept of multi-fidelity is introduced to ANNs for the first time by Yan [90]. Multi-fidelity physics-constrained neural network is proposed to reduce the required amount of training data, where physical knowledge is applied to constrain neural networks, and multi-fidelity networks are constructed to improve training efficiency. A simple multi-fidelity PIML framework integrating HF and LF models is shown in Fig. 3. Olleak [44] considered finite element modeling as a physics model and machine learning model to quantify the physical model bias for more accurate additive manufacturing performance prediction with limited real experiment data. Nicholas [91] introduced a novel multi-fidelity deep generative model for the surrogate modeling of high-fidelity turbulent flow fields given the solution of a computationally inexpensive but inaccurate low-fidelity solver. The model is trained with a variational loss that combines both data-driven and physics-constrained learning. Mohammadamin [92] introduced a multi-fidelity neural network (MFNN) architecture for data-driven constitutive metamodeling of complex fluids. The physics-based neural networks developed are informed by the underlying rheological constitutive models through the synthetic generation of low-fidelity model-based data points.

In addition to incorporating the multi-fidelity framework with the physics-informed neural networks, the multi-fidelity framework can also be adapted and used for the physics-informed Kriging model. Yang [93] proposed a physics-informed Kriging (PhIK). In PhIK, the mean and covariance function are computed from realizations of available stochastic models. To reduce the computational cost of obtaining stochastic model realizations, Yang [94] proposed a multilevel Monte Carlo estimate of the mean and covariance functions, and further present an active learning algorithm that guides the selection of additional observation locations. The main drawback of PhIK is that it is highly dependent on the physical model, because the prior mean and covariance are determined entirely by the model and are not informed by data. Therefore, the convergence of PhIK to the true solution with the increasing number of available observations is slower than in the data-driven GPR if the physical model is incorrect. The multi-fidelity framework can be used to relieve these issues. Yang [95] proposed a new GPR-based multi-fidelity method CoPhIK, which is a modified version of the recently developed physics-informed Kriging (PhIK) method to improve its accuracy.

- A Bayesian PIML Framework

Traditional neural networks and PINN are not probabilistic in nature resulting in Bayesian extensions of these models to quantify the underlying uncertainty in black-box algorithms. There are many sources of uncertainty in data-driven PDE solvers, including aleatoric uncertainty associated with noisy data, epistemic uncertainty associated with unknown parameters, and model uncertainty associated with the type of PDE that models the target phenomena. Geneva [96] proposed a deep auto-regressive dense encoder–decoder for predicting transient PDEs with the physics-constrained learning algorithm that enables the model to learn dynamics without training data. This model was extended to a Bayesian framework using the recently proposed stochastic weight averaging Gaussian algorithm [97] to quantify both epistemic and aleatoric uncertainty. The model is implemented for a chaotic/turbulent system and extends Zhu's method [68]. Yang [98] proposed a Bayesian physics-informed neural network (B-PINN) (as shown in Fig. 4) where the Bayesian neural network (BNN) combined with a PINN for PDEs serves as the prior while the Hamiltonian Monte Carlo (HMC) or the variational inference (VI) could serve as an estimator of the posterior. B-PINNs make use of both physical laws and scattered noisy measurements to provide predictions and quantify the aleatoric uncertainty arising from the noisy data in the Bayesian framework. Compared with PINNs, in addition to uncertainty quantification, B-PINNs obtain more accurate predictions in scenarios with large noise due to their capability of avoiding overfitting.

## 3. PIML for reliability and system safety applications

Enhancing reliability and system safety is critically important in today's industrial environment, where increasingly complex technological systems are utilized. The ability to model, characterize and predict the behavior and performance of a product, component, or system provides a significant competitive advantage, especially critical in estimating the reliability, safety and risks inherent in a system. Physics-informed ML strategies improve the power of these abilities and are employed more frequently in a variety of engineering applications, such as uncertainty quantification, reliability analysis and risk assessment, prognostics and health management (including degradation analysis, fault diagnostics, failure prognostics, operation and maintenance decision-making) to adequately represent and understanding complex behavior of real-world systems. In this section, we review the use of PIML methods in recent literature for reliability and system safety applications. Subsections are organized by the application area, and the last subsection summarizes the previous subsections and discusses the influence of prior knowledge properties on PIML model construction.

### 3.1. PIML for uncertainty quantification

Uncertainty Quantification (UQ) refers to strategies of quantifying, characterizing, tracing, and managing uncertainty in computational and real world systems (e.g., [99]). UQ seeks to address the problems associated with incorporating real world variability and probabilistic behavior into engineering and systems analysis. Uncertainty quantification methods play a pivotal role in reducing the impact of uncertainties during engineering modeling, design, optimization, and decision-making processes. They have been applied to solve a variety of real-world problems. Bayesian approximation and ensemble learning techniques are two widely-used types of uncertainty quantification (UQ) methods. Abdar [100] provided an extensive review of uncertainty quantification methods in deep learning.

In UQ applications, several types of knowledge, including physics equations, theories, and simulation data, often provide useful information that characterize the system. To incorporate those types of knowledge enhancing the power of the model, three types of integration methods, including physics-informed loss function, hybrid physics-informed machine learning model, and Bayesian framework are commonly used.

The integration of prior physic knowledge into ML for UQ aims to provide a better characterization of uncertainty. Yang [69,101] considered the application of deep generative models in propagating uncertainty through complex physical systems (e.g., randomness in inputs or noise in observations), where physics-informed constraints provide a regularization mechanism for effectively training deep probabilistic models. Xie [102] proposed an implementation framework of quantification of margins and mixed uncertainties, based on evidence theory and Kriging model with adaptive sampling. The confidence factor (CF) for the quantification of margins and uncertainties (QMU) calculation is then evaluated by integrating the surrogate model and the Dempster–Shafer theory of evidence (DSTE) analysis.

Despite the success of physics-informed machine learning methods in solving various real-world problems, they cannot provide information about the uncertainties of their predictions. In a deterministic physics-informed machine learning, a loss function is defined by combining both the data mismatches and residuals of governing equations of a physical model. However, the measurement noise associated with data and model-form uncertainties due to model inadequacy cannot be considered since the models are formulated in a deterministic way. To solve these issues, probabilistic physics-informed Bayesian learning methods are proposed, where the physics-constrained training is formulated in a Bayesian way. Instead of defining the loss, a physics-informed likelihood function is constructed, where the measurement noise and equation residual are modeled as random variables with specified distributions. Sun [64] formulated the equation-constrained training in a Bayesian manner. The confidence of the physical constraints is modeled in a probabilistic way, being combined with data uncertainty to form the likelihood function. The proposed PC-BNN can accurately predict the mean flow field, meanwhile reasonably estimating the prediction uncertainties corresponding to different data noise levels. Compared with a previous work [71] showing that the flow solutions can be obtained from physics-constrained deep learning even without any labeled data if the boundary conditions are imposed properly, adding some labeled data would further improve the equation-constrained learning. Wang [103] proposed an approach that integrates data from a simulation model, with real-time traffic data streams and available physical constraints to predict aircraft trajectory, using the Bayesian-Entropy information fusion methodology. There have also been attempts to approximate Bayesian processes in neural networks. These approximate methods make the training process more feasible for certain applications. One popular method is Monte Carlo dropout (MC dropout) proposed by Gal and Ghahramani [104] which proposed dropout at testing to approximate parameter uncertainty. This method has been used to more accurately identify uncertainty in surrogate models for infrastructure damage, response, and safety predictions [105–107].

Under the uncertainty quantification field, sensitivity analysis (SA) [108–114] is an important branch, which aims to provide a quantitative uncertainty assessment of the relative contribution of each uncertainty source to the model response. When physics-based or ML models are used for the sensitivity analysis of engineering systems, the sensitivity estimate is affected by the accuracy and uncertainty of the model. Purely data-driven ML models do not explicitly consider physical laws and may produce physically inconsistent results. Thus, incorporating prior scientific/physics knowledge with ML models is needed to improve the accuracy and efficiency of SA computations. Kapusuzoglu [115] considered global sensitivity analysis (GSA) for situations where both a physics-based model and experimental observations are available, and investigates physics-informed machine learning strategies to effectively combine the two sources of information in order to maximize the accuracy of the sensitivity estimate. Two strategies for incorporating physics knowledge within the deep neural networks and GP models are investigated. One is incorporating loss functions in the ML models to enforce physics constraints, and the other one is pre-training and updating the ML model using simulation and experimental data respectively.

### 3.2. PIML for prognostic and health management

This section discusses the physics-informed ML strategies and applications in Prognostic and Health Management (PHM) field, which includes many topics, such as prognostic, diagnostic, degradation modeling, remaining useful life (RUL) prediction, health management, and maintenance.

The PHM field aims to predict and forecast failures of a given system. Incorporating PHM methods [116] can potentially decrease the probability of a catastrophic event; and in addition can reduce maintenance costs. In general, the PHM methods contain mostly data-driven approaches. Mainly, for PHM methods, the failure state is modeled by a data-driven model, which in some literature has been machine learning models, and in recent literature, these models are enhanced by physical models. Physics-based approaches evaluate structural conditions through updating a representative physics-based model of the target structure, such as a finite element (FE) model, by minimizing the discrepancy of its predictions from the measured data. Physics-based and data-driven models for PHM typically suffer from two major challenges that limit their applicability to complex real-world cases: (1) incompleteness of physics-based models and (2) limited representativeness of the training dataset for data-driven models. Integrating physics knowledge into the machine learning model can relieve the above issues.

In PHM applications, different branches of applications encompass different types of information. For instance, in the branch of diagnosis, physics-based model or numerical simulation models are integrated with the ML model by using physics-informed loss function approach or the hybrid physics-informed ML methods; in the branch of SHM, physics law reflecting the relationships of the system properties are often added to the ML architecture enhancing the physics consistency; in the branch of prognosis and decision-making, expert knowledge plays an important role and can be integrated by the hybrid physics-informed ML strategy. In general, there are four types of knowledge and three types of integration methods are commonly used in the PHM applications. Four types of prior knowledge includes the physics models (e.g., PDEs, governing equations, physics-of-failure, empirical models, etc.), data (e.g., experimental data, sensor data, simulation data, etc.), theory and rule (e.g., empirical rule, predefined rule, temporal logic, etc.), and expert knowledge. Physics-informed loss function method utilizing the prior knowledge as the physics constraints, physics-informed architecture methods integrating prior knowledge into the layers and/or nodes of the neural networks, and hybrid physics-informed machine learning methods are used as the integration methods. In the following, we discuss different branches of PHM applications that take advantage of different types of prior knowledge and integration methods.

- Fault Diagnosis

Physics-informed loss function and physics-informed architecture are employed corresponding to available knowledge in the diagnosis applications.

The first type is the physics-informed loss function. Zhang [117] extended the original modal-property based features with the damage identification result of finite element model updating. A physics-based loss function is designed to evaluate the discrepancy between the neural network model output and that of finite element model updating. With the guidance from the scientific knowledge contained in finite element model updating, the learned neural network model has the potential to improve the generality and scientific consistency of the damage detection results. Xu [118] proposed the first multiple source domain adaptation framework for building damage diagnosis without any labels of the target building. This study design a new physics-guided weight in the loss function based on the physical similarities of buildings. The new physics-guided loss provides a tighter upper bound for the damage prediction risk on the target domain. Shen [119] proposed a physics-informed deep learning approach that consists of a simple threshold model and a deep convolutional neural network (CNN) model for bearing fault detection. A loss function is designed for training and validating the CNN model that selectively amplifies the effect of the physical knowledge.

The second type is the physics-informed ML architecture. Lai [120] exploited a new direction of structural identification by means of Physics-informed Neural Ordinary Differential Equations (Neural ODEs), which involve a physics-informed term, that stems from possible prior knowledge of a dynamical system, and a discrepancy term, captured by means of a feed-forward neural network. Physics-informed Neural ODEs comes with the benefits of direct approximation of the governing dynamics, and a versatile and flexible framework for discrepancy modeling in structural identification problems. When the sensor data are available, a new ML structure can be constructed. Liu [121] proposed a two-stage physics-based statistical approach for modeling the cooling efficiency based on the thermodynamic law that governs the relationship between system internal states, operating conditions and cooling efficiency. Moreover, multiple failure modes can be connected by the physics-informed architecture. Yucesan [76] embedded a neural network with physics-informed layers which is utilized to estimate bearing and grease damage for wind turbines. The bearing fatigue damage portion consists of known physics formulations, and unknown grease degradation is represented with deep neural networks. The two different failure modes are connected by the physics-informed architecture and successfully estimated with the combination of known physics equations of one failure while enhancing the data-driven predictions of the other failure mode, and vice versa.

Besides the diagnosis applications, the hybrid physics-informed ML models have a wide variety of applications in PHM, including prognosis, degradation modeling, RUL prediction, maintenance, as well as monitoring and decision-making.

- Degradation Modeling and RUL Prognosis

A multitude of prognosis problems, such as fatigue prognosis, damage prognosis, and many other types of prognosis problems, often employ hybrid physics-informed ML strategies to provide a physics consistent and interpretable degradation model. Blancke [122] took into account the complexity of failure mechanisms as a system and integrates both expert knowledge and diagnostic information. Model assumptions are first proposed by experts and then formalized using graph theory and stochastic models. Jiang [9] presented a dynamic model correction framework for a simplified degradation model using strain measurements. This framework integrates data-driven model with physics-based degradation model and improves the accuracy of model-based failure prognostics. Arias Chao [123] proposed a novel hybrid framework for prognostics of complex safety-critical systems. The framework combines deep learning and physics-based performance models. The physics-based performance models infer unobservable model parameters related to a system's components health by solving a calibration problem. These parameters are subsequently combined with sensor readings and used as input to a deep neural network, thereby generating a data-driven prognostics model with physics-augmented features. The proposed framework outperforms equivalent purely data-driven approaches. Vega [124] developed a novel hybrid damage prognosis framework for miter gate component of navigational locks, by mitigating effects of human errors on the condition assessment and integrating the highly abstracted inspection data with the SHM. It overcomes two main challenges, namely (1) there is no physical or empirical model available to model the loss-of-contact degradation in the gate, and (2) the mismatches between the inspection data and the SHM system due to data abstraction. Prakash [125] developed a hybrid approach with the fusion of model–based and data–driven approaches for the prognosis of dynamical system components whose degradation may follow different nonlinear trends. The degradation levels of different components are identified by using bond graph model–based distributed prognosis approach. Artificial neural network based degradation models learned from the run–to–failure data of the components are used to predict the future degradation patterns and RUL of the components. Chang [126] proposed a hybrid prognostic scheme with the capability of uncertainty assessment, which combines particle filter (PF) and relevance vector machine (RVM). The proposed prognostic method can provide accurate and stable RUL prediction.

Successful modeling and quantification of the degradation performance is important for engineering systems, as it will help improve failure prediction accuracy and facilitate subsequent decision-making at design, operation, and maintenance stage. Various types of prior information can be utilized with the hybrid physics-informed ML strategies to improve the model performance. Chiachio [127] developed a knowledge-based prognostics approach by fusing on-line data for track settlement with a physics-based model for track degradation within a filtering-based prognostics algorithm. Sun [128] proposed a physical–statistical modeling approach to evaluating and quantifying the latent degradation performance of corroding aluminum alloys at both individual and population levels. Guo [129] proposed an improved inverse Gaussian process which considers the dependency between degradation increments and prior degradation states. Whiteley [130] proposed a method for problem with complex internal operational mechanisms impacting on degradation phenomenon where a combined physics-based and stochastic model can offer more accurate and valuable information about the operation of such systems, with impacts to future design, manufacture and operation. Integration of a physics based model with the stochastic Petri net has enabled accurate lifetime analysis. Liu [131] proposed a reliability estimation and degradation modeling method, where Wiener process is combined with evidence theory by applying evidential variable to describe model parameters. The proposed method can be used to evaluate population reliability without increasing parameter size, resulting in stochastic process becoming more practical under small sample conditions. Following the degradation modeling, an accurate prediction of the future degradation patterns and RUL of the components are essential indicators to guarantee system safety. And filtering-based methods and physics-based models are commonly employed by the hybrid physics-informed ML approaches for the RUL prediction. Baptista [132] proposed a Kalman-based data-driven approach to prognostics. The model uses Kalman filtering to fuse the estimates of remaining useful life. Results suggest Kalman-based models are better in precision and convergence. Jain [133] mathematically modeled the tool degradation progression via a new, adaptive, and hybrid stochastic degradation model. The methodology is conceptually unique as the challenges allied with time-variant operating profiles were explicitly addressed by integrating its physics, capturing the uncertainty in the evolution of dynamic operating profiles in real-time. Consequently, the approximated RUL taps past as well as the future characteristics of operating profiles. Li [134] proposed an interacting multiple model framework with particle filter and support vector regression to realize multi-step-ahead estimation of the capacity and remaining useful life of batteries. Polynomial and exponential model are chosen to describe the capacity degradation. Support vector regression is used to predict future measurements online. Zang [135] proposed a hybrid method that balances model-based and data-driven methods. Particle filtering method and feedforward neural network are used to predict RULs. Hybrid method predicts more accurate RUL predictions under different stresses.

- Condition Monitoring, Operations and Maintenance Decision Making

Optimal inspection and maintenance planning play an important role in engineering problems. Chahrour [136] integrated physics-based and dependability models for monitoring the state evolution of protection structures and improving maintenance decision-making processes. The modeling approach proposed is based on (1) physics-based modeling for identifying the probabilistic laws of the transition times between the defined states of the structure depending on its behavior over time and (2) a decision aiding method based on Petri nets, which helps in choosing the best maintenance strategy while considering budgetary constraints. Sometimes there is no clear physical understanding of how damage progresses in time; for example, it is not clear how the bearing gaps change in time in the quoin blocks of a miter gate. Therefore, Vega [2] developed a new hybrid condition-based maintenance (CBM) approach that integrates high-fidelity (physics-based) FE model-based SHM with inspection data-based transition matrix for effective diagnosis, prognosis, and maintenance planning.

With the increasing complexity of engineering systems that contain various component dependencies and degradation behaviors, there has been increasing interest in on-line SHM capability to continuously monitor (via sensor and other methods of observation) system components for detection and diagnostic of safety-critical systems. The ability to have accurate on-line system monitoring improves maintenance and decision-making to reduce cost and avoid possible critical failure. Pan [137] integrated information from the monitoring data and expert knowledge. The integrated model with a three-layer structure is constructed that incorporates both expert knowledge and data. Arcosjimenez [138] employed an approach that considers advanced signal processing and machine learning to determine the thickness of the dirt and mud in a wind turbine blades (WTB). A condition monitoring system based on non-destructive tests by ultrasonic waves was used to analyze wind turbine blades. Favaro [139] integrated model-based hazard modeling/monitoring with the verification of safety properties expressed in Temporal Logic (TL). This expanded framework leverages tools and ideas from Control Theory and Computer Science. Iamsumang [140] proposed a hybrid Dynamic Bayesian Network (DBN) to represent complex engineering systems with underlying physics of failure by modeling a theoretical or empirical degradation model with continuous variables. Hong [141] provided new opportunities for integrating data from massive IoT sensors and devices to enhance the accuracy of simulation results, which are used to inform decision-making on energy retrofits and efficiency improvements of existing buildings. Yang [142] proposed a risk assessment guided development process using machine learning techniques and multi-source data. A predefine safety-critical attributes have been identified from major accident scenarios to guide machine learning process to define operational limits based on multi-source data. The attributes that can represent the risk factor are established based on domain knowledge and further feed into machine learning process.

### 3.3. System FMEA and reliability analysis

This section discusses the physics-informed machine learning approaches and applications in system failure mode and effect analysis (FMEA) and reliability analysis (RA) fields.

In the RA applications, various types of knowledge provide the understandings of the system. Integrating one or more of those prior knowledge requires different ML structures and strategies. Four types of knowledge are typically available in the reliability assessment applications, containing: (1) physics models (e.g., physics-o-failure, limit-state functions, failure mechanism, etc.); (2) experimental data and simulation data; (3) theory and rule (e.g., empirical rule, belief reliability theory, uncertainty theory, predefined rule, logic-based model, etc.); and (4) expert knowledge. To incorporate one or more of those prior knowledge, three types of integration methods are commonly utilized including the physics-informed loss function, physics-informed architecture, and hybrid physics-informed machine learning methods.

Based on different objectives of the application and types of available knowledge, different integration methods are utilized. A significant effort has been made in the last few years to combine the advances in RA data and RA modeling. Xi [143] provided strategies for more accurate reliability analysis based on different data availability (e.g., relatively sufficient, and insufficient) and the combination with a physics model. Tan [144] proposed an algorithm to combine data-driven and domain knowledge condition, where the loss function and alternative training scheme of component models are specified for harnessing the information from sensor readings and empirical rules to serve the modeling. Sakurahara [145] developed a simulation-informed method for Common Cause Failure (CCF) Analysis. Physical failure mechanisms are explicitly incorporated by simulation models. Simulation-based CCF data is integrated with existing CCF data by Bayesian method. Pence [146] introduced the Data-Theoretic module of Integrated Probabilistic Risk Assessment, which build a theoretical framework equipped with reliable modeling techniques and data analytic to quantify the influence of organizational performance on risk scenarios. Kim [147] proposed an analysis based on the Bayesian logistic regression method that incorporates empirical data with prior knowledge, which overcomes the limitations of the traditional regression technique. Sakurahara [148] initiated a line of research to integrate renewal process modeling with probabilistic models of underlying mechanisms associated with physical degradation and maintenance. The methodology integrates Markov modeling with Probabilistic Physics-of-Failure models of degradation, while maintenance is treated by a data-driven approach. The methodology explicitly incorporates the underlying spatiotemporal causes of failure into the renewal model, allowing to rank the criticality of causal factors to improve maintenance and mitigation strategies.

Among the reliability analysis applications, human reliability analysis (HRA) has been widely recognized as an important activity of probabilistic safety assessments, which was conducted to identify significant mechanisms of human error probabilistic for significant tasks. Different hybrid physics-informed ML architectures are widely employed to integrate multiple information and improve HRA.

Santhosh [149] presented an integrated approach to predict the lifetime and reliability of I&C cables by ANN from the accelerated aging data and Weibull theory. Study demonstrates that by an appropriate training algorithm with suitable network architecture, it is possible to predict the reliability of I&C cables by ANN with the minimal accelerated life testing. Qian [150] proposed a time-variant reliability method for an industrial robot rotate vector reducer. Firstly, the limit state functions of the industrial robot RV reducer are built by considering time-variant load and material degradation based on the failure physic method. Secondly, a time-variant reliability analysis method for multiple failure modes is proposed based on a double-loop Kriging model. Wu [151] constructed a new uncertain accelerated degradation model based on uncertainty theory and belief reliability theory, and presents the uncertain statistical method for parameter estimations. Cai [152] developed a novel real time methodology for reliability estimation with cubic spline interpolation by combining cluster-system theory and k-means clustering. Moradi [153] proposed the integration of deep learning methods and logic-based models and explore how to systematically draw together the advances in PHM and PRA to provide a more forward-looking, model- and data-driven approach for assessing and predicting the risk and health of CES. Sohoin [154] presented a novel approach to deal with early reliability estimation of upgraded automotive components. The key idea is to combine reliability analysis based on efficient surrogate models and time transformation function principle. Vasilyev [155] developed a novel model for dynamic reliability analysis of a polymer electrolyte membrane fuel cell to account for multi-state dynamics and aging.

Moreover, multiple different types of knowledge can be integrated and utilized simultaneously for HRA. Groth [156] defined a comprehensive hybrid algorithm that uses causal models built from and parameterized by a combination of data from cognitive literature, systems engineering, existing Human Reliability Analysis (HRA) methods, simulator data, and expert elicitation. Fusing data and models can seem as a cognitive version of hybrid "physics of failure" modeling which overcomes limitations of data, models, and literature by fusing multiple evidence. The hybrid modeling strategy enhances the traceability and credibility of the qualitative and quantitative aspects of HRA. Furthermore, Guo [157] investigated the Bayesian melding method (BMM) for system reliability analysis by effectively integrating various available sources of expert knowledge and data at both subsystem and system levels. Additionally, Bayesian networks are used for probabilistic risk assessment, which represent variables and dependencies using nodes and arcs where the outputs can be smoothly connected to the neural networks. Abrishami [158] proposed a Bayesian network based physics-informed ML (BN-SLIM) model which is based on both the available data and the predefined rules, in general, outperforming the rule-based methods. The proposed method was assessed and compared with the performance of some of the well-known Bayesian Network Human Reliability Analysis (BN-HRA) techniques. Zywiec [159] proposed a new methodology for training a neural network metamodel and incorporating it into a Bayesian network-based probabilistic risk assessment. The neural network metamodel is based on structural design and physics-informed metamodel research, which use metamodels to reduce the computational burden of performing direct, simulation-based analysis of physical systems. The main benefit of this methodology is that it combines the interpretability and sampling algorithm of a Bayesian network with the high-dimensional, latent variable modeling capability of a neural network metamodel.

### 3.4. Surrogate modeling for reliability and system safety

This section discusses the physics-informed machine learning strategies and applications in surrogate modeling for reliability and safety of complex systems. Surrogate modeling is an important strategy to replace the computationally intensive physics model to accelerate the evaluation process. Therefore, it would be highly desirable to construct a metamodel that complies with the expected physical behavior/constraints. Veiga [160] introduced a framework for incorporating any type of linear constraints in Gaussian process modeling, this including common bound and monotonicity constraints. Ray [161] proposed a general framework for systematically executing efficient modeling and simulation studies. Empirical model does not really describe the underlying physics-based mechanisms, causation, and relationships between variables. Many of the challenges and limitations can be mostly overcome by synthesizing outputs and data from both computer models and experimentation. Wang [162] introduced a novel framework based on the Bayesian-Entropy (BE) principle. BEGP uses Bayesian-Entropy method to encode constraints into GP regression for enhanced prediction and extrapolation. BEGP serves as an information fusion tool to enhance the extrapolation behavior of the GP model by incorporating additional knowledge about the problem, such as physical constraints, boundary conditions, and empirical knowledge.

## 4. Discussion of PIML in reliability and system safety applications

The recent PIML related studies for reliability and system safety applications are summarized (in Figs. 5 and Table 1) from two perspectives: (1) the knowledge type and (2) the integration methods. Considering the knowledge types, the existing studies are divided into scientific equations, experimental data, theory/rule, and expert knowledge. Scientific equations refer to the PDEs, physics-of-failure equations, governing equations, and statistical constraints. Experimental data refers to the data from physical experiments, sensor data, and simulated data. Theory and rule refer to the scientific theory and/or law, empirical rules, predefined rules, etc. Concerning the integration methods, the existing studies are categorized as: physics-informed loss function, physics-informed architecture, and hybrid physics-informed machine learning models. Physics-informed loss function approaches incorporate the prior knowledge as the penalty term to the loss function or as the extra constraints. Physics-informed architecture approaches include (1) incorporating the prior knowledge in the node of the neural networks, (2) adding/constructing layers corresponding to the prior knowledge, and (3) choosing appropriate activation functions based on prior knowledge. Hybrid physics-informed ML models include integrating with physical models, adding physics preprocessing, and a Bayesian framework. In addition, Fig. 6 summarizes the relationships among the application area, knowledge type, and integration method from the PIML studies in reliability and system safety application areas (see Table 1).

The means of physics-informed machine learning model construction are highly affected by the properties of the available knowledge, such as the knowledge type, quantity, and quality. In this section, we will provide an analysis for PIML construction with different knowledge properties (summarized in Fig. 7) based on the literature discussed in the previous sections.

- Knowledge Type

Different from scientific problems where the systems are mainly captured by the PDEs and governing equations, various types of knowledge are available in the reliability and safety applications including the physics equations, experimental data, theories and laws, predefined rules, expert knowledge, etc. However, several types of knowledge cannot directly be integrated into the ML model in the same manner as the PDEs and governing equations. Therefore, varying hybrid models with diverse structures are investigated to utilize different types of knowledge.

- Knowledge Quantity

Distinct knowledge quantities, such as the amount of data and the amount of equations, may guide our choices and decisions on PIML model construction. The amount of data is a main element that influences the choice of model types and integration methods. Data availability is a key factor in model selection and construction. When only limited experimental data or unlabeled data are available, the

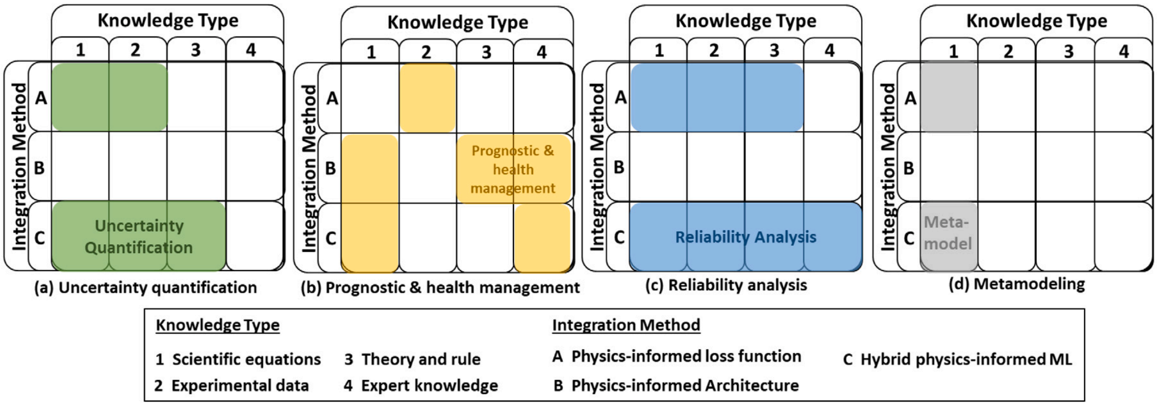


**Fig. 5.** Summary of application areas, knowledge types, and integration methods.

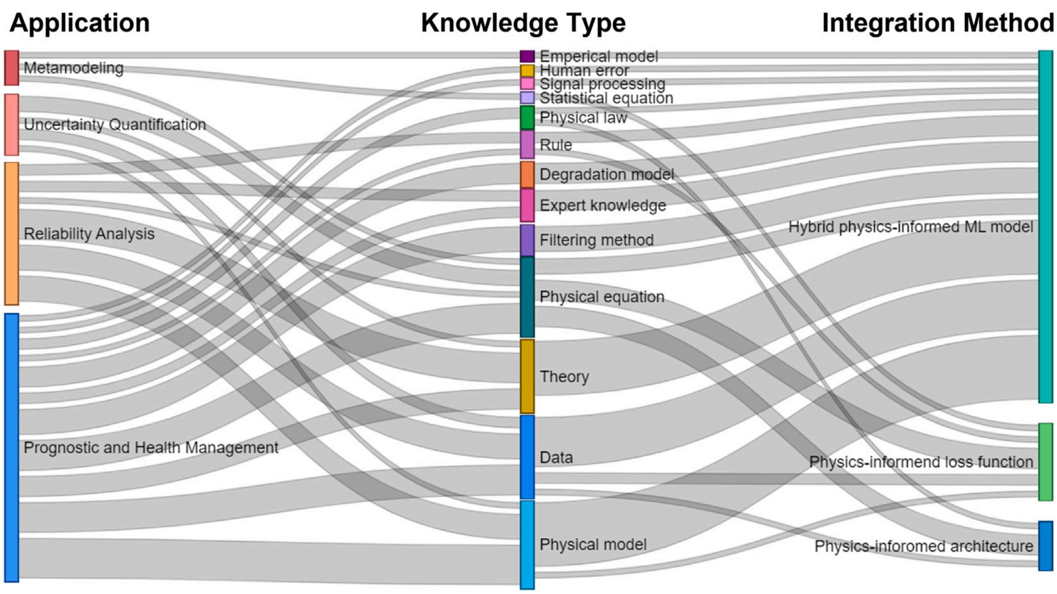


**Fig. 6.** Relations among application, knowledge type, and integration method.

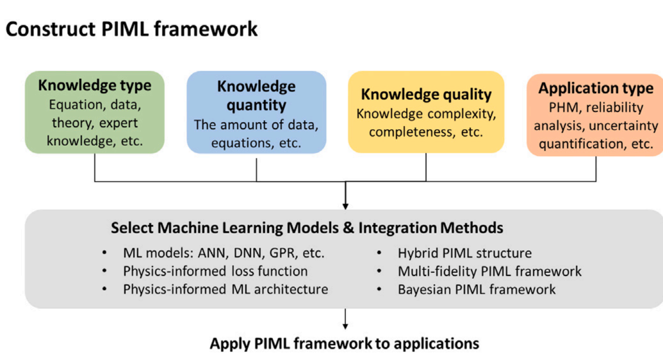


**Fig. 7.** Knowledge properties direct model construction.

physics-informed loss function approaches are usually used to integrate that physical knowledge and improve the physics consistency and training convergence. The number of equations is another crucial factor in model selection and construction. For complex engineering systems, scientific equations are usually used to represent underlying physics or dynamics. Generally, those underlying relationships can be represented by a few scientific/physics equations, and the equations and boundary conditions are often incorporated into the loss function of the ML model. For problems with multiple failure modes, a certain amount of physics equations are used to reflect different aspects of the system. Incorporating multiple equations into the loss function brings the complexity for solving the problem and challenges in finding the right weight for each physics equations. In this case, incorporating multiple equations into the architecture of the ML model, such as nodes and/or layers of the neural network, is preferred.

- Knowledge Quality

Knowledge quality refers to the complexity and the completeness of the knowledge/information, which will trigger our different options on model construction. Equation complexity affects our decision on model selection and construction. In engineering applications, various prior knowledge, such as scientific equations and laws, may possess different complexities, varying from statistics properties to complex PDEs. Generally, complex equations enjoy rich physics information, but they will also bring limitations to model construction. Most of the studies incorporate complex equations or laws using the physics-informed loss function approach. Unlike complex equations, knowledge with low complexity like statistic constraints possess more choices and are more flexible on model construction, where several approaches can integrate prior knowledge into the loss function, the nodes and/or layers of architecture, the activation function of the ML model. Completeness of the knowledge determines the usage of the models and

**Table 1**
Characteristics of PIML for reliability and system safety related literature.

| Year | Author | Area | Method | | PIML objectives |
|---|---|---|---|---|---|
| 2017 | Liu [121] | PHM | 1 | B | Propose a two-stage physics-based statistical approach |
| 2018 | Xie [102] | UQ | 3 | C | Add adaptive sampling to PIML |
| 2018, 2019 | Yang [69,101] | UQ | 1 | A | Train a deep probabilistic model |
| 2018 | Guo [157] | RA | 2, 4 | C | Propose a bayesian melding method for system reliability analysis |
| 2018 | Santhosh [149] | RA | 3 | B | Predict reliability with minimal accelerated life testing |
| 2018 | Blancke [122] | PHM | 4 | C | Take into account the complexity of failure mechanisms |
| 2018 | Guo [129] | PHM | 1 | C | Consider the dependency between degradation increments and prior degradation states |
| 2018 | Sun [128] | PHM | 1 | C | Quantify the latent degradation performance at both individual and population level |
| 2018 | Iamsumang [140] | PHM | 1 | C | Represent complex system with underlying physics of failure |
| 2018 | Favaro [139] | PHM | 3 | C | PIML framework leverages tools and ideas from control theory to computer science |
| 2019 | Tan [144] | RA | 2, 3 | A | Loss function and alternative training schemes are specified for harnessing the information from sensor data and empirical rule |
| 2019 | Pence [146] | RA | 2, 3 | C | Build a theoretical framework to quantify the influence of organizational performance on risk scenarios |
| 2019 | Sakurahara [145,148] | RA | 1, 2 | C | Initiate a line of research to integrate renewal process with physical degradation and maintenance; simulation-informed method for common cause failure analysis |
| 2019 | Groth [156] | RA | 2, 4 | C | Overcome limitation of data, models, and literature by fusing multiple evidence |
| 2019 | Baptista [132] | PHM | 1 | C | Obtain better precision and convergence |
| 2019 | Chang [126] | PHM | 1 | C | Provide accurate and stable RUL prediction |
| 2019 | Chiachio [127] | PHM | 1 | C | Fuse on-line data for the PIML |
| 2019 | Whiteley [130] | PHM | 1 | C | Offer more accurate information about the system operation and accurate lifetime analysis |
| 2019 | Hong [141] | PHM | 2 | C | Enhance the accuracy of simulation results |
| 2019 | Arcosjimenez [138] | PHM | 1 | C | PIML for condition monitoring systems |
| 2020 | Sun [64,71] | UQ | 1 | A | Improve the learning process by adding some labeled data to unlabeled data sets |
| 2020 | Abriishami [158] | RA | 2, 3 | C | PIML outperform rule-based methods |
| 2020 | Moradi [153] | RA | 3 | C | Systematically draw together the advances in PHM and PRA |
| 2020 | Qian [150] | RA | 1 | C | Propose a time-variant reliability method using PIML |
| 2020 | Wu [151] | RA | 3 | C | Construct a new uncertain accelerated degradation model using PIML |
| 2020 | Cai [152] | RA | 3 | C | Develop a real-time reliability estimation method |
| 2020 | Veiga [160] | SM | 1 | C | Introduce a PIML framework for incorporating any type of linear constraints in GP model |
| 2020 | Ray [161] | SM | 1, 2 | C | Combine computer models with experimentation to enhance prediction |
| 2020 | Liu [131] | PHM | 3 | C | Evaluate population reliability without increase parameter size |
| 2020 | Jain [133] | PHM | 1 | C | Approximated RUL taps past and future characteristics of operating profiles using PIML |
| 2020 | Yang [142] | PHM | 2 | C | Risk factor are established based on domain knowledge and multi-source data via PIML |
| 2020, 2021 | Vega [2,124] | PHM | 1,2,4 | C | Consider condition based maintenance problem; successful model the loss-of-contact degradation; solve the mismatch between the inspection data and the SHM system |
| 2021 | Kapusuzoglu [115] | UQ | 1 | A, C | Improve global sensitivity analysis accuracy |
| 2021 | Wang [103] | UQ | 1, 2 | A | Use bayesian-entropy information fusion to integrate data |
| 2021 | Sohoin [154] | RA | 1 | C | Deal with early reliability estimation |
| 2021 | Vasilyev [155] | RA | 1 | C | Account multi-state dynamics and aging |
| 2021 | Zywiec [159] | RA | 1 | C | Combine the interpretability and sampling algorithm of a bayesian network |
| 2021 | Wang [162] | SM | 1, 4 | C | Use bayesian-entropy method to encode constraints into GP model to enhance prediction and extrapolation |
| 2021 | Zang [135] | PHM | 1 | C | Provide more accurate RUL prediction |
| 2021 | Li [134] | PHM | 1 | C | Propose an interacting multiple model framework with the online prediction |
| 2021 | Prakash [125] | PHM | 1 | C | Predict future degradation patterns and RULof the components |
| 2021 | Jiang [9] | PHM | 1 | C | Propose a dynamic model correction framework to improve the accuracy of failure prognostic |
| 2021 | Lai [120] | PHM | 1 | B | Directly approximate the governing dynamics |
| 2021 | Xu [118] | PHM | 1 | A | Propose a multiple source domain adaption framework with unlabeled data; provide tighter bound for prediction |
| 2021 | Yucesan [76] | PHM | 1 | B | Identify bearing and grease damage for wind turbines |
| 2021 | Shen [119] | PHM | 1 | A | Bearing fault detection |
| 2021 | Zhang [117] | PHM | 1 | A | Improve generality and scientific consistency for damage detection |
| 2021 | Pan [137] | PHM | 4 | C | Realize on-line system monitoring |
| 2021 | Chahrour [136] | PHM | 1 | C | Improve maintenance decision-making processes |
| 2022 | Arias Chao [123] | PHM | 1 | C | PIML outperform purely data-driven approaches |

integration methods. The quality of empirical and/or physics models generally relies on our understanding of the system, and the completeness of our understanding of the system affects our decision on model selection and construction. If the physical equations/models own a high completeness, they can be operated independently to capture the physics mechanism of the system, and a hybrid/precessing model can be constructed to integrate the physics into ML. Contrary, if the model can only reflect part of the system, then other knowledge, such as experimental data, physics rules, and expert knowledge, are needed to capture the character of the system. In this case, hybrid/fusing models tend to be utilized to obtain a comprehensive understanding of the system and construct a complete model.

- A General PIML Framework is Needed for Reliability and System Safety

The challenges for physics-informed machine learning are often specific to its applications. In particular, for reliability applications, if there are multiple failure modes to be determined, the structure of the model can take on different forms. Applications can be divided into different levels of understanding. For instance, Yucesan [52] has a failure mode that is well-defined by physical equations (bearing fatigue); while for determination of grease damage, a pure data-driven model is utilized. For processes that have less known variables, data-driven models and random variable modeling is implemented. The data-driven models can then be utilized to enhance existing known physics models, or vice versa. The main difference between enhancing the physical equations compared to enhancing the data-driven models are within the observables. If one can observe the outputs of the data-driven model, then the data-driven model should enhance existing physics equations; and the reverse should be done for enhancing the data-driven model. Moreover, the design of effective physics-informed ML architectures is generally done empirically by users in the present literatures, which could be very time-consuming. All these problems may severely limit the applicability of the PIML methodology. Therefore, a general PIML framework that can flexibly integrate prior knowledge and ML models is required for reliability and system safety applications.

## 5. Challenges and future directions

Introducing PIML into reliability and safety applications come with limitations and challenges despite the numerous advantages of PIML outlined in previous sections. In this section, we first identify some common limitations and challenges regarding information characteristics and model properties in PIML implementation, and then discuss key future research directions in developing advanced PIML technology for reliability and system safety applications.

### 5.1. Limitations and challenges of PIML

#### 5.1.1. Limitation and challenges in information characteristics

Data availability is an essential assumption of ML-based models. However, many reliability and system safety applications have sparse, unavailable, or bias data. The lack of high quality data has often forced researchers and practitioners to rely on overly simplified models which may lead to inaccurate results [163]. However, using PIML models which are generally highly parameterized can lead to overfitting and a lack of generalizability.

Even though the era of big data has led to more available data, this data may not be high quality and be in different or new forms. This new data may not be useful in its existing form or create noisy data sets that are difficult to use in training applications. Moreover, these new data sources need to be validated. Much of this validation is dependent on human analysis and expertise which may be too costly or infeasible [164]. Finally, the increase in data can also provide data sources that are highly complex or highly variable in format and type. Highly variable data can be very difficult to format and process leading to limitations in speed and usability. Large amounts of data can quickly lead to limitations in storage and processing speed. Therefore, researchers and decision makers may prefer simplified models that can provide quick predictions even if the PIML model may improve overall accuracy.

#### 5.1.2. Limitations and challenges in model properties

Some key limitations and challenges of PIML in model property aspects are summarized into five types: model selection, model structure, model parameter, model optimizer, and model prediction.

First, reliability and system safety problems are often complex and can be multi-dimensional. Many of these dimensions (or levels) have different properties and may be correlated in space and/or time [165]. Creating nested models which can accurately account for uncertainty, material properties, and correlations is very challenging. Moreover, these features make it difficult to perform tasks such as feature or model selection which may be necessary for reliability and system safety applications. Model selection for PIML is the process of choosing one final model among a collection of candidate models and strategies for a predictive modeling problem, which can be applied both across different types of models (e.g. neural networks, Gaussian process model, SVM etc.) and across models of the different types of structure with different model integration strategies (e.g. physics-informed loss function, architecture, and hybrid model). The challenge of PIML model construction is how to choose among a range of different models and integration strategies that you can use for your problem. There may be many competing concerns when performing model selection beyond model performance, such as complexity, efficiency of the model, and properties of available resources. Even though model selection in physics-based and machine learning domains [29] are extensively investigated and well-developed, there is no clear guideline for model selection in the PIML modeling space so far. And a certain extent experience is required to establish a PIML model properly. For example, the design of effective PIML architectures is currently done empirically by users, which often brings limitations for applying the PIML methods. Therefore, a well-defined set of guidelines for PIML model selection and construction is indispensable, which will improve the performance of hybrid models and broaden their scope of applications.

Second, lots of restrictions still located in the model construction process. Different types of prior knowledge and ML models often yield challenges for knowledge integration and model construction. For example, logic rules are encoded in the architecture of the neural network with only a few layers in existing PIML studies, and the feasibility and functionality of this approach for deep machine learning models are under-investigated. Another example is that, currently, the GP model can integrate equality and inequality equations, and little attention has been paid to the physics-informed GP model with other types of knowledge, such as the knowledge fusion framework.

Third, we see a potential challenge in finding the right weights for prior knowledge that arise from the recent PIML studies. When multiple equations/constraints are integrated into the loss function of the ML model, the weights for each physics penalty term corresponding to each equation are usually predefined by the user and effectuate the PIML model learning process. Moreover, in the hybrid/preprocessing model structure, the weight for physics-based model and ML model balances the proportion of the physics meaning in the ML model. There is no universal approach in determining the right weight and the value of it is problem dependent.

Fourth, adding prior knowledge and/or constraints into the ML model generates additional complexities into the optimization and model training process. Consequently, this highly complex problem may bring challenges to traditional optimizers. However, advanced optimizers/algorithms have not been paid much attention to PIML methods. Thus, advanced optimizers are urgently needed to improve the performance of physics-informed models and keep up with the rapid development of PIML technologies.

Fifth, the main challenges of using PIML for online prediction are convergence and computational efficiency. For an algorithm to run and make accurate predictions in real time, it must have low computational complexity to provide fast estimations. Additionally, if there is a significant amount of uncertainty in the existing model (which is common for PHM and reliability applications), the initial parameters of the model may not be an accurate representation of the system. This means the parameters need to be updated within the algorithm, and they must converge quickly for the framework to provide accurate predictions. Currently, there does not exist substantial literature on combining PIML models and updating operators for online applications. There is an opportunity for future research in employing PIML for online prediction, as studies have shown that PIML structures can significantly reduce convergence time, particularly with the physics-informed loss function.

### 5.2. Future directions

Several promising research directions on PIML methodology are summarized from two aspects: information characteristics and model properties.

#### 5.2.1. Future directions in model construction with different information characteristics

Data collection is often an expensive process in engineering applications. Collecting data from the area that we are interested in is an important step in discovering the underlying behavior of complex systems. As discussed in Sections 2 and 3, partial differential equations are widely adopted to explain a variety of phenomena due to their ability to model and capture the behavior of complex systems. Collecting data samples from appropriate locations has a great impact on convergence and results of modeling training. Traditionally, the data collection process relies heavily on expert knowledge and the understanding of the system. Recently, Zheng [166] proposed a framework using the idea of adaptive sampling strategy to iteratively estimate the future dynamical behavior and select sample points based on the form of PDEs. Although the PIML strategies are widely used to model the behavior of complex systems, little attention has been paid to data collection and adaptive sampling strategies.

For reliability and safety applications, in order to develop a complete model of the system, new data shall be collected from the regions of the system's space that are far from the training set, in which the model is likely to fail. Such data acquisition will be instrumental for a system where the consequences of failure can be catastrophic. The model shall be trained on such data for reducing its bias and until its accuracy is saturated. Different from the adaptive sampling strategies [167–175] developed in the ML area, the data collection process for PIML methods is more complex, where not only the model learning target and performances but also the multiple types of prior knowledge need to be considered for the data collection. In summary, there is an opportunity for future research in developing new adaptive sampling strategies to speed-up the PIML development. Moreover, in many reliability and safety applications, labeling is an expensive procedure that may require domain-specific expertise. Thus, developing hybrid models that learn with fewer labels opens up a new avenue of research and practical applications.

Besides the data collection, the training efficiency of PIML models is still limited in complex systems since the cost of obtaining enough samples for achieving reasonable accuracy is high. Multi-fidelity PIML frameworks, therefore, have been developed to compensate for expensive high fidelity samples with cheap low fidelity samples. In reliability and system safety applications, available knowledge often come from multiple different sources with varying fidelities and multi-fidelity modeling is a common approach to employ in this type of resource-expensive computationally demanding problems. How to utilize those multi-fidelity data comprehensively becomes a key question that needs to be investigated. Nowadays, bi-fidelity [176] and multi-fidelity frameworks [177,178] for ML models are well-developed, while the investigation of multi-fidelity frameworks for PIML approaches is in the preliminary stage of development. Only limited bi-fidelity [94,95,179] and multi-fidelity [90–92] structures for PIML methods are proposed for PIML models so far, and further researches on it will enhance the applicability of the PIML methods.

#### 5.2.2. Future directions in model construction with different model properties

As discussed in Section 5.1.2, at least five types of grand challenges in PIML models are yet to be addressed: model selection, model structure, model parameter, model optimizer, and model prediction.

For model selection, a well-defined set of guidelines for PIML model selection among a range of different models and integration strategies is needed, which will improve the performance of hybrid models and broaden their scope of applications. In addition, in recent years, PIML-related researches have developed rapidly, and how to measure and evaluate the performances of emerged PIML approaches become a critical challenge in the PIML methods development. Benchmark problems ease the evaluation and comparison of different algorithms and are essential for the growth of physics-informed model structures and algorithms. Consequently, new metrics that reveal the test accuracy and learning capability need to be brought up to evaluate how well-posed the physics-informed structures.

For model construction, methods for integrating some types of prior knowledge into ML models have not yet been explored. New approaches to unexplored spaces will broaden PIML applications. Moreover, simulation results may bring up the challenge of mismatch between real and simulated data. New Hybrid models that combine ML and simulation in more sophisticated ways are needed to enable the calibration capability.

For model parameter estimation, there is no universal approach in determining the right weight and the value of it is problem dependent. Strategies for tuning and optimizing model parameter require further attention.

For model optimization, highly complex model structures bring challenges to traditional optimizers, and advanced optimizers design for PIML are urgently required to improve the model performance and keep up with the rapid development of PIML technologies.

For the model prediction, there is an opportunity for future research in employing PIML for online prediction, as studies have shown that PIML structures can significantly reduce convergence time.

## 6. Conclusion

This paper presented a literature review for the state-of-the-art of physical-informed machine learning methods for the reliability and system safety applications. The study highlights different efforts towards aggregating physical information and data-driven models as grouped according to their similarity and application area within each group. Moreover, the challenges of the applications of physics-informed machine learning methods to address practical reliability and system safety problems and future research needs have also been discussed. It is the authors' intention to provide a collection of research articles presenting recent development of this emergent topic, and shed light on the challenges and future directions which we, as a research community, should focus on for harnessing the full potential of advanced physics-informed machine learning techniques for reliability and safety applications.

## CRediT authorship contribution statement

**Yanwen Xu:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Visualization, Writing – original draft. **Sara Kohtz:** Conceptualization, Investigation, Validation, Writing – review & editing. **Jessica Boakye:** Conceptualization, Writing – review & editing. **Paolo Gardoni:** Conceptualization, Writing – review & editing, Supervision. **Pingfeng Wang:** Conceptualization, Methodology, Resources, Writing – review & editing, Supervision, Project administration, Funding acquisition.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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