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Optimal Lending Contracts with Retrospective and Prospective Bias†

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A growing theoretical and empirical literature studies how individuals exhibit biases in processing information and forming beliefs and explores the impact of such distortions on markets.[1] The model misspecification framework, in which an individual uses an incorrect model to learn from a signal, is a common approach to modeling such distortions.[2]

In this paper, we consider an entrepreneur who borrows to invest in a project. She learns about her project quality from a signal and interprets the signal with a misspecified model. This analysis builds on Bohren and Hauser (2023), who establish that a misspecified model can be decomposed into the two key classes of distortions that it induces: prospective biases and retrospective biases. Prospective biases correspond to distortions in forecasting future beliefs, while retrospective biases correspond to distortions in interpreting information after it is observed. We explore how these two types of distortions impact the structure of optimal lending contracts.

In our setup, the entrepreneur first decides whether to pay an up-front fee to originate a line of credit. She then observes a signal of project quality and decides how much to borrow and invest in her project. Finally, she receives the earnings from her project and pays back the loan at the specified interest rate. Prospective bias impacts the entrepreneur's decision to originate credit, as this decision depends on her forecast of her future belief about project quality, while retrospective bias impacts her decision on how much to borrow, as this decision is made after she observes the signal. A lender is aware of the entrepreneur's bias and chooses a contract (i.e., an interest rate and origination fee) to maximize his expected profit.

Our main result shows that each form of bias has a distinct and intuitive impact on the structure of the optimal contract. We characterize the optimal contract as a function of the retrospective and prospective biases and then use this result to examine each class of bias in isolation. When the entrepreneur only exhibits retrospective bias, then the lender manipulates the origination fee to take advantage of the bias but charges the same interest rate that he would charge an unbiased entrepreneur. In contrast, when the entrepreneur only exhibits prospective bias, the lender manipulates both the interest rate and origination fee to capitalize on the bias. He charges a high up-front fee and low interest rate when the entrepreneur is overconfident about the precision of future information and therefore overestimates the benefit of low interest. In contrast, he charges a low up-front fee and high interest rate when the entrepreneur is underconfident and, hence, underestimates the future cost of high interest.

Our simple lending framework demonstrates the benefits of linking the literature on model misspecification with the theoretical and empirical literature on analyzing specific biases in information processing.

I. The Entrepreneur's Borrowing Problem

A. *Setup*

Consider a setting in which a lender offers an entrepreneur access to credit. The entrepreneur has a project that is either low or high quality, $\omega \in \{L,H\}$, drawn with equal probability. Neither the lender nor the entrepreneur observe

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[1] See Benjamin (2019) for a survey.

[2] Recent papers in this area include Esponda and Pouzo (2016); Fudenberg, Romanyuk, and Strack (2017); Heidhues, Koszegi, and Strack (2018); Frick, Iijima, and Ishii (2020); and Bohren and Hauser (2021).

this quality. The entrepreneur first chooses whether to open a line of credit, $a \in \{y,n\}$. Opening a line of credit is associated with an origination fee $c > 0$.

Next, the entrepreneur observes a signal $z \in \mathcal{Z} \subseteq [0,1]$ about the quality of her project. Let μ^{ω} denote the Borel probability measure describing the signal's distribution in state ω, and let $\mu = (\mu^L + \mu^H)/2$ denote the unconditional signal measure. The entrepreneur believes the signal is distributed according to subjective measure $\hat{\mu}^{\omega}$ in state ω, with analogous unconditional measure $\hat{\mu}$, where $\hat{\mu}^L \neq \mu^L$ or $\hat{\mu}^R \neq \mu^R$ captures a misspecified model. Assume that $(\mu^H, \mu^L, \hat{\mu}^H, \hat{\mu}^L)$ are all mutually absolutely continuous.[3] The entrepreneur uses Bayes' rule with respect to her subjective signal distribution to update her belief about project quality.

After observing the signal, if the entrepreneur opened a line of credit, then she chooses an amount $I \geq 0$ to borrow at rate $r > 0$. If the entrepreneur did not open a line of credit, then she cannot borrow, $I = 0$. The entrepreneur invests all of the money she borrows in the project.

When the project is of low quality, it yields revenue $g(I,L) = 0$ for any level of investment I. When the project is of high quality, it yields revenue $g(I,H) = 2\sqrt{I}$ that is increasing in the level of investment by the entrepreneur. After receiving this revenue, the entrepreneur repays her loan plus interest. The entrepreneur's payoff is

$$g(I,\omega) - (1+r)I - c \times \mathbf{1}_{a=y}. \tag{1}$$

Note that the entrepreneur repays her loan regardless of the realized project revenue.

B. *Aside: Decomposing Misspecified Models*

Bohren and Hauser (2023) establish that any misspecified model can be decomposed into two objects—a forecast and an updating rule—which are defined as follows (tailored to this setting). An updating rule specifies how the entrepreneur forms beliefs after observing each signal realization. It maps each signal realization into a probability that the project is of high quality.

DEFINITION 1 (Updating Rule): *An updating rule $h : \mathcal{Z} \to [0,1]$ is a measurable function that maps each signal realization to a posterior belief that the state is H and is not constant μ-almost everywhere.*

A special case of an updating rule is Bayesian updating with respect to the correct model. Let h_B denote this updating rule.

A forecast is the entrepreneur's prediction of how she will form beliefs about the quality of the project after observing the signal. That is, it is a probability distribution of her posterior belief that the project is of high quality.

DEFINITION 2 (Forecast): *A forecast $\hat{\rho}$ is a c.d.f. over the posterior belief x that the state is H with support* $\mathrm{supp}(\hat{\rho}) \subset [0,1]$ *and for which there exists a measurable function $\alpha : \mathcal{Z} \to [0,1]$ such that $\hat{\rho}$ and $\mu \circ \alpha^{-1}$ are mutually absolutely continuous.*

The latter part of the definition is a technical requirement to ensure that the forecast is compatible with the signal. A special case of a forecast is the accurate forecast with respect to updating rule h, denoted by $\rho_h(x) \equiv \mu\{z : h(z) \leq x\}$. Let ρ_B denote the accurate forecast with respect to Bayes' rule h_B.

The updating rule and the forecast each capture a different form of informational distortion. The updating rule captures the retrospective bias, in that it describes how the entrepreneur reasons about information after it is realized. The forecast captures the prospective bias, in that it describes how the entrepreneur reasons about information before it is realized.

Bohren and Hauser (2023) explore when an updating rule and forecast can be jointly represented by a misspecified model, in the sense that the misspecified model prescribes posterior beliefs that coincide with the updating rule after each signal realization and predicts posterior beliefs that coincide with the forecast ex ante. The key requirement is that the forecast is "plausible," in that its expected value is equal to the

[3] This implies that no signal perfectly reveals the state, that the entrepreneur believes that no signal perfectly reveals the state, that no set of signal realizations that arise with probability zero under the misspecified model occur with positive probability under the correctly specified model, and that the misspecified model does not place positive probability on sets of signal realizations that occur with probability zero under the correctly specified model.

prior, $\int_0^1 x\,d\hat{\rho} = 1/2$. They show that any plausible forecast and updating rule has an essentially unique representation as a misspecified model. Further, any misspecified model $(\hat{\mu}^L, \hat{\mu}^H)$ pins down a unique updating rule via Bayes' rule,

$$h(z) = \frac{1}{1 + d\hat{\mu}^L/d\hat{\mu}^H(z)} \tag{2}$$

for any $z \in \mathcal{Z}$, and a unique forecast via the unconditional subjective signal measure,

$$\hat{\rho}(x) = \hat{\mu}\{z : h(z) \leq x\} \tag{3}$$

for any $x \in [0,1]$. This allows us to decompose a misspecified model into its prospective and retrospective distortions of the signal. Note that the correctly specified model induces updating rule h_B and forecast ρ_B.

C. *Optimal Borrowing*

With this decomposition in mind, let h and $\hat{\rho}$ denote the updating rule and forecast induced by the entrepreneur's misspecified model. Suppose the entrepreneur has posterior belief $x \in [0,1]$ that the return is high after observing the signal. Then she chooses an investment level to maximize her ex post expected revenue minus the loan repayment,

$$\max_{I \geq 0} 2x\sqrt{I} - (1+r)I. \tag{4}$$

This yields optimal investment strategy $I^*(x;r) = x^2/(1+r)^2$. Therefore, when the entrepreneur uses updating rule h to form her posterior belief, she chooses investment level $I^*(h(z);r) = h(z)^2/(1+r)^2$ following signal realization z.

The entrepreneur chooses to open a line of credit if, given her optimal investment strategy, her ex ante expected revenue minus the loan repayment exceeds the origination fee. Substituting $I^*(x;r)$ into equation (4) and taking the expectation with respect to the entrepreneur's prediction of her future posterior belief $\hat{\rho}$, the entrepreneur opens a line of credit when

$$E_{\hat{\rho}}[x^2]/(1+r) \geq c. \tag{5}$$

Therefore, the entrepreneur's updating rule influences her chosen investment level after observing the signal, whereas her forecast influences her origination decision before observing the signal.

II. The Optimal Contract

We next derive the contract that maximizes a risk-neutral lender's earnings. A contract consists of an interest rate $r \in \mathbb{R}$ and an origination fee $c \in \mathbb{R}$. We allow both the interest rate and the origination fee to be negative, which corresponds to an ex post or up-front subsidy, respectively. The lender earns return $rI + c$ when the entrepreneur originates a loan and borrows I, and has zero earnings if the entrepreneur does not originate a loan. The lender has a correctly specified model of the signal process and a correct model of the entrepreneur's model. This induces the accurate forecast $\rho_h(x) = \mu\{z : h(z) \leq x\}$ over the entrepreneur's posterior belief. In the optimal contract, the lender chooses an interest rate and origination fee to maximize expected earnings subject to the constraint that the entrepreneur originates credit,

$$\max_{c,r \in \mathbb{R}} c + rE_{\rho_h}[I^*(x;r)] \tag{6}$$

subject to

$$E_{\hat{\rho}}[x^2]/(1+r) \geq c.$$

Note that it is never optimal to choose (c,r) such that the entrepreneur does not open a line of credit. This is because the lender can guarantee positive earnings by offering an interest rate of $r = 0$ and setting $c = E_{\hat{\rho}}[x^2] > 0$, where the inequality follows from $\hat{\rho}$ plausible. Therefore, the solution to equation (6) characterizes the optimal contract. Also note that the expectation of the belief in the investment strategy is taken with respect to the lender's forecast ρ_h, as this is the lender's expectation of his earnings, whereas the expected belief in the constraint is taken with respect to the entrepreneur's forecast $\hat{\rho}$ as derived in equation (5).

From the decomposition theorem discussed above, we know that a misspecified model is fully pinned down by its induced updating rule and forecast. We next show that the optimal contract can be described as a function of the expectation and variance of these two objects. Let $m_h \equiv \int_0^1 h(z)\,d\mu$ denote the actual expectation of the entrepreneur's posterior belief, let

$V_h \equiv \int_0^1 h(z)^2 d\mu - \left(\int_0^1 h(z)\, d\mu\right)^2$ denote the actual variance of the entrepreneur's posterior belief, and let $V_{\hat{\rho}} \equiv \int_0^1 x^2 d\hat{\rho} - 1/4$ denote the entrepreneur's prediction of the variance of her posterior belief (i.e., the variance of her forecast). The entrepreneur's expectation of her posterior belief is $m_{\hat{\rho}} = 1/2$ since the forecast is plausible. These statistics summarize the retrospective and prospective components of the signal distortion relevant for determining the optimal contract, as shown in the following proposition.

PROPOSITION 1 (The Optimal Contract): *Given updating rule h and forecast $\hat{\rho}$, the optimal interest rate is*

$$(7) \qquad r^*(h, \hat{\rho}) = \frac{V_h - V_{\hat{\rho}} + m_h^2 - 1/4}{V_h + V_{\hat{\rho}} + m_h^2 + 1/4},$$

and the optimal origination fee is

$$(8) \quad c^*(h, \hat{\rho}) = \left(V_{\hat{\rho}} + 1/4\right) / \left(1 + r^*(h, \hat{\rho})\right).$$

PROOF:

The optimal origination fee satisfies the participation constraint with equality, $c = E_{\hat{\rho}}\left[x^2\right] / (1 + r)$. Plugging this and the expression for $I^*\left(h(z); r\right)$ into equation (6), the lender's problem simplifies to

$$\max_{r \in \mathbb{R}} \frac{E_{\hat{\rho}}\left[x^2\right]}{(1 + r)} + \frac{rE\left[h(z)^2\right]}{(1 + r)^2}$$
$$= \frac{V_{\hat{\rho}} + 1/4}{(1 + r)} + \frac{rV_h + rm_h^2}{(1 + r)^2}.$$

Taking the first-order condition and setting it equal to zero yields equation (7). Plugging equation (7) into $c = E_{\hat{\rho}}\left[x^2\right] / (1 + r)$ yields equation (8). ■

Fixing an updating rule h, the impact of the prospective bias on the optimal contract is summarized by the variance of the forecast. As the entrepreneur exhibits more overconfidence in how informative she expects her signal to be (as measured by a higher variance of her forecast), she expects to have more precise information before making an investment decision. Therefore, she has a higher value for the lending product, and the lender can charge a higher origination fee. Further, the lender finds it optimal to charge a lower interest rate: the entrepreneur's perceived benefit from a lower interest rate is increasing in her investment, and the investment strategy is convex in the posterior belief (recall $I^*(x; r) = x^2 / (1 + r)^2$). Therefore, when the entrepreneur expects more extreme posterior beliefs, she overestimates the value of a low interest rate and is willing to pay an even higher origination fee to enter such a contract.

Two aspects of the retrospective bias are relevant for the determining the optimal contract. When the entrepreneur is more optimistic, in that her actual interpretation of the signal is more slanted toward state H (as measured by a higher average posterior belief), or she exhibits more overreaction, in that her beliefs move to more extreme values after observing the signal (as measured by a higher variance of her posterior belief), the optimal interest rate is higher and the optimal origination fee is lower. This is because the entrepreneur's investment strategy is increasing and convex in her posterior belief. Therefore, a higher average belief or, fixing the average, a higher variance leads to higher expected investment (where the expectation is from the lender's perspective) and, hence, higher revenue from interest. In turn, the lender reduces the origination fee in order to be able to charge a higher interest rate.

From this result, we see that the retrospective and prospective biases induced by a misspecified model have a fundamentally different impact on borrowing decisions and contract design. Therefore, decomposing a misspecified model into these two components provides a crucial tool for understanding how the different classes of distortions induced by a misspecified model impact economic behavior.

Optimal Contracts with Retrospective Bias.—Given an updating rule h, Bohren and Hauser (2023) argue that if the accurate forecast ρ_h is plausible, then it is a natural forecast to select when pinning down a misspecified model representation. An accurate forecast can be represented by a misspecified model with the appealing property that the predicted distribution over signals matches the true unconditional signal distribution. Therefore, misspecification with such a forecast is in some sense "undetectable."

When the misspecified model has an accurate forecast, then the entrepreneur does not exhibit prospective bias: the only form of bias is retrospective. It follows from Proposition 1 that in this case, the lender charges the entrepreneur the same interest rate that he would charge a correctly specified entrepreneur. This is because the entrepreneur correctly anticipates the distribution of her posterior belief and, therefore, the value of a given interest rate, as is the case for a correctly specified entrepreneur. However, when the entrepreneur exhibits retrospective bias in that $h \neq h_B$, then the optimal origination fee differs from that for the correctly specified entrepreneur. This stems from different posterior beliefs leading to different investment decisions and, hence, different valuations for a given contract. This insight is summarized in the following corollary; a brief proof is in the online Appendix.

COROLLARY 1: *When the entrepreneur has an accurate forecast and retrospective bias*, $\hat{\rho} = \rho_h$ *and* $h \neq h_B$, *then the optimal interest rate is the same as that charged to a correctly specified entrepreneur*, $r^*(h, \rho_h) = r^*(h_B, \rho_B) = 0$, *but the optimal origination fee differs*, $c^*(h, \rho_h) \neq c^*(h_B, \rho_B)$ *(provided* $V_{\rho_h} \neq V_{\rho_B}$*). The lender's expected profit is increasing in* V_{ρ_h}.

Whether the retrospective bias raises or lowers the lender's expected profit depends on the updating rule. If h induces more extreme beliefs than h_B, as measured in terms of the variance V_{ρ_h}, then the lender earns higher profit in expectation, relative to an entrepreneur with no retrospective bias. Otherwise he earns lower expected profit. Hence, a bias such as overreaction increases the lender's profit, while underreaction decreases the profit.

Optimal Contracts with Prospective Bias.—We next consider a misspecified model in which the entrepreneur exhibits prospective bias in the form of under- or overconfidence in forecasting her future beliefs. We parameterize this bias with the following family of forecasts, where $d\hat{\rho}_\theta$ denotes the probability density function of the forecast:

$$d\hat{\rho}_\theta(x) = \frac{x^{\theta-1}(1-x)^{\theta-1}}{\Gamma(\theta)^2/\Gamma(2\theta)} \tag{9}$$

for $\theta > 0$ and $x \in [0,1]$. This corresponds to the family of beta distributions with mean $1/2$ (any such forecast is plausible). Suppose that the accurate forecast with respect to Bayes' rule is uniform— that is, $d\rho_B = 1$. Then $\theta = 1$ corresponds to the accurate forecast.

For $\theta > 1$, as θ increases, the entrepreneur is increasingly underconfident about the precision of her information, in that she places more mass on intermediate posteriors and less mass on extreme posteriors relative to the accurate forecast. For $\theta < 1$, the opposite holds, as θ decreases and the entrepreneur is increasingly overconfident. To isolate the impact of the prospective bias, we assume the entrepreneur has no retrospective bias, $h = h_B$.

When the entrepreneur is overconfident, she believes she will have very precise information to utilize when choosing how much to borrow in the future. This leads her to overestimate the value of a lower interest rate, and she is willing to pay a higher up-front fee for such a contract. In contrast, the lender knows that the entrepreneur overestimates the frequency of signal realizations for which the she will borrow a large amount (i.e., the realizations for which the negative interest rate is very costly to the lender). The lender takes advantage of this forecasting bias by charging a high upfront price for a very favorable interest rate.

In contrast, when the entrepreneur is underconfident, she underestimates the frequency of the signal realizations that induce her to borrow a large amount, and therefore she underestimates the future cost of a high interest rate. The lender takes advantage of this by offering a low up-front fee in order to induce the entrepreneur into a contract with a high interest rate. The following corollary summarizes these insights; a brief proof is in the online Appendix.

COROLLARY 2: *When the entrepreneur is overconfident*, $\theta < 1$, *then the optimal interest rate is smaller and the optimal origination fee is larger than those charged to an entrepreneur with no bias*, $r^*(h_B, \hat{\rho}_\theta) < r^*(h_B, \rho_B) = 0$ *and* $c^*(h_B, \hat{\rho}_\theta) > c^*(h_B, \rho_B)$. *When the entrepreneur is underconfident*, $\theta > 1$, *then the optimal interest rate is larger and the optimal origination fee is smaller than those charged to an entrepreneur with no bias*, $r^*(h_B, \hat{\rho}_\theta) > r^*(h_B, \rho_B) = 0$ *and* $c^*(h_B, \hat{\rho}_\theta) < c^*(h_B, \rho_B)$. *The lender's expected profit is decreasing in* θ.

Overconfidence raises the lender's expected profit relative to an entrepreneur with no prospective bias, while underconfidence lowers

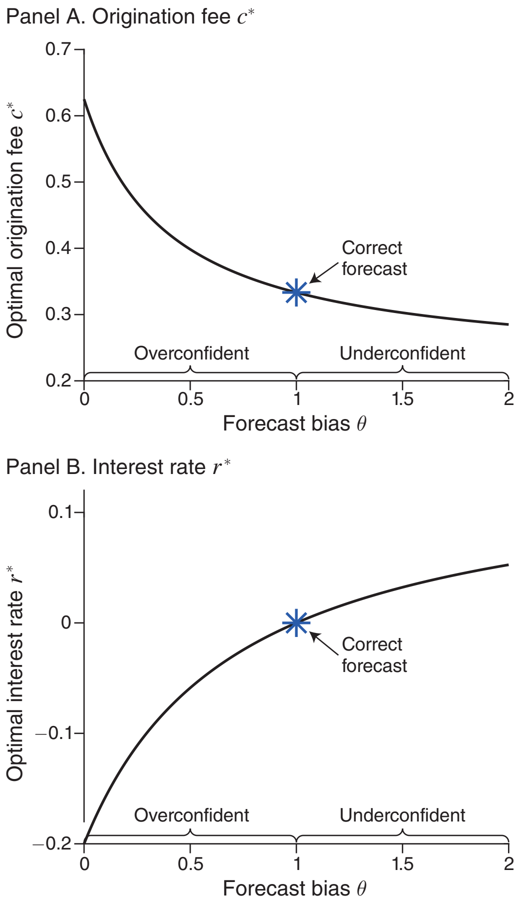

Figure 1. Optimal Contract

expected profit. Figure 1 plots the optimal interest rate and origination fee as a function of θ.

Corollary 2 demonstrates that the same updating rule can lead to very different origination and borrowing costs depending on the forecast—on its own, the updating rule does not significantly restrict the range of optimal contract terms. Therefore, the induced forecast is a crucial property of a misspecified model and has as important behavioral implications as the more oft-studied updating rule.

III. Conclusion

We explore how the two classes of distortions induced by a misspecified model of a signal about project quality impact the structure of optimal lending contracts. Specifically, we disentangle the impact of prospective biases in forecasting future beliefs about project quality from retrospective biases in interpreting information after it arrives. The lender leverages each form of bias in different ways via the optimal interest rate and origination fee.

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