Optimal Electric Vehicle Charger Placement as a Congestion Game Problem

Yasin Sonmez, Can Kizilkale, Alex Kurzhanskiy, Murat Arcak
University of California, Berkeley

{yasin_sonmez, cankizilkale, akurzhan, arcak}@berkeley.edu

***Abstract*—We propose an optimization method to place electric vehicle charging stations to minimize total travel time, thereby minimizing additional congestion and detours caused by the chargers. For a tractable optimization scheme, we frame the drivers' route choices as a congestion game that allows us to find equilibrium flows for each candidate set of locations. Our contribution has two primary components. First, we refine the modeling of driver cost functions to account for charging needs as well as travel time, and introduce different agent types based on their unique valuations of charging benefits. Second, we address the exponential growth of the search space of charger locations with a greedy optimization approach. We demonstrate with numerical experiments that: (i) the congestion game formulation allows us to efficiently compute equilibrium flows for each candidate charger placement; (ii) the greedy approach can closely approximate the optimal selection.**

I. Introduction

The number of electric vehicles (EVs) is growing at an unprecedented rate [1]. The United Nations' Paris Declaration on Electro-Mobility and Climate Change set a target of 100 million electric vehicles by 2030 [2]. The United States, in particular, aims for half of new vehicles sold to be zero-emission vehicles by 2030. The recent California legislation [3] stipulates that all passenger vehicle sales be zero emission by 2035, which requires a momentous expansion in the charging infrastructure. California Energy Commission predicts [4] that the state would need nearly 1.2 million public and shared chargers by 2030 to meet the fueling demands of the 7.5 million passenger plug-in EVs anticipated to be on California roads. An additional 157,000 chargers would be needed to support the 180,000 medium- and heavy-duty vehicles anticipated for 2030.

There are many challenges to such dramatic enlargement of the EV charging infrastructure in a short time [5]. First, major investments would be needed for the power distribution system; currently, the grid capacity severely limits where EV chargers can be placed [6]–[9]. In addition, zoning approvals and permits for chargers take a long amount of time. Therefore, for the foreseeable future, the number of chargers are unlikely to match the increased number of EVs on the roads, potentially leading to queues and disruption to nearby traffic. In addition, detours to limited charger locations may add significant travel time.

To mitigate these problems, here we present a method to place EV chargers wihin a set of candidate locations to minimize the total travel time. The travel time depends on equilibrium traffic flows, which are determined by drivers' route choices. For an efficient formulation of route choices, we use the formalism of congestion games, whereby the travel time through each link is modeled as a delay function that depends on the flow on that link. We augment this formulation with additional links to model the travel time through chargers. In addition, we classify the types of drivers according to the benefit they derive from charging and define an appropriate cost function that subtracts the benefit from the travel time on the selected route.

Using this congestion game formalism, we devise a computationally tractable optimization scheme that consists of two layers. The inner layer computes equilibrium flows for given charger locations by solving a convex optimization problem. This layer makes use of a potential function of the game, derived from the cost functions mentioned above, which are shown in the paper to be convex. The outer layer uses the result of the inner layer to compute the total travel time for all drivers corresponding to each candidate set of charger locations and, then, selects the minimum. This layer can become intractable for large networks, as the search space expands exponentially with the size of the set of candidate locations. To address this problem, we pursue a greedy search algorithm where we place one charger at a time. We then show with numerical experiments that the greedy approach can closely approximate the optimal selection.

In the literature, optimal placement of public chargers has been studied from the perspectives of charging stations operators, EV users, and power distribution network operators [10]. The objective functions considered include cost of construction, power loss, and voltage deviation [11]–[17]. What distinguishes our study is the focus on the impact of EV chargers on traffic congestion and detours, which is mitigated by minimizing the total travel time.

The paper also adds to the literature on congestion games [18], [19], [20], [21], [22], [23] – a class of games where payoff of a resource to each player depends on the number of

Yasin Sonmez, Can Kizilkale and Murat Arcak are with the Department of Electrical Engineering and Computer Sciences at the University California, Berkeley. Alex Kurzhanskiy is with the California Partners for Advanced Transportation Technology (PATH) at the University California, Berkeley. This project was sponsored in part by the NSF CPS grant NSF CNS-2135791 and by the California Statewide Research Transportation Program.

players choosing the same resource. These games have been studied extensively for vehicle traffic and communication networks, market competition, and ecosystems. Multiple players seek resources, with each player selecting a subset (e.g., a route in traffic networks) that influences delays on those resources. We contribute a new application to the existing congestion game literature, as well as a paradigm to model chargers within this framework with additional network links and delay functions. We also show that this congestion game remains a potential game with a convex potential function despite our modification of the cost functions.

The paper is structured as follows: Section II.A introduces the road network model for the congestion game formulation, and Sections II.B augments it with charging stations. Section II.C present the game theoretic formulation. Section II.D establishes the existence of a Nash equilibrium and details its identification through a potential function. Section II.E outlines a practical, greedy approach to determine optimal EV charging station locations. Section III explains our experimental methodology, and Section IV reports the results of these experiments. Section V reviews the conclusions.

II. EV PLACEMENT AS A CONGESTION GAME

We model EV placement within a congestion game framework, so we can assess the total delay at equilibrium for each candidate placement and choose the placement that minimizes delay. For traffic problems the resource is the road network and the cost to a driver is the time it takes to travel from the origin to destination, minus the benefit of battery charging. Due to the non-atomic nature of the problem, the delay on each road link is a function of total flow on that particular link. We model chargers as additional links in the network, with associated delay.

A. Model of the road network

- The road network is represented as a directed graph $G = (V, A)$ where V is the set of nodes and A is the set of links.
- Time to travel through link $l \in A$ is represented as $d_l(x_l)$, where x_l is the flow on link l. We refer to d_l as the delay function, assumed to be a non-decreasing function on x_l.
- Given a path s, the total delay is additive and equal to $\sum_{l\in s} d_l(x_l)$. We represent the flow for agent of type i on route $s_i \in S_i$ as $x^i_{s_i}$.
- We assume a continuum of agents where the type set is N and p_i represents the proportion of agents of type i; hence $\sum_{i\in N} p_i = 1$.
- Each agent type $i \in N$ is traveling from an origin (represented as O_i) to a destination (represented as D_i).
- S_i represents the set of routes (paths connecting O_i to D_i) for agents of type i.

B. Adding EV charging stations to the model

- We denote by $V_c \subseteq V$ the set of candidate nodes where an EV station can be placed.
- We represent a charger at node $v \in V_c$ as an additional link from v to itself, (v, v). If a route includes the link (v, v) this means that the EV stops at v to charge.
- For the same O/D pair, EV-players' paths will include exactly one charging link if they are charging. We assume that the vehicle does not stop more than once at a charging station. This is reasonable for local road networks where charging once is sufficient.
- The amount of time spent at charging station is represented as a non-decreasing function $d_l(x_{(v,v)})$ and add it to the total delay on the path that includes it.
- The set of the self directed, charging, links is represented by A'.

Note that the road links and chargers are both modeled as links, with associated delay functions. These functions are assumed to be non-decreasing, as increased volumes on road links or chargers imply longer times to get through them.

C. Agent types and cost functions

We consider a non-atomic setup with agents types partitioned into three sets, represented as F_1, F_2, F_3.

- F_1: Agents who do not require charging, e.g., vehicles with gasoline engines or EVs with abundant charge for the trip. These agents's strategy set is the set of paths that do not include a link (v, v) corresponding to a charging station at node v.
- F_2: Agents who need to charge. Their strategy set will be of paths that pass through one charging node v and that includes the link (v, v). We assume that these vehicles will stop at exactly one charging station and will not leave before charging finishes.
- F_3: Agents who will benefit from charging but also can finish their trips without charging. The amount of benefit can change between the types in this set. (The agents in the set F_1 can also be thought as agents in set F_3 that has no benefit from charging).

While agent types in F_1 and F_2 are similar in the sense that they try to pick the route with minimum delay, the agents in the third set decide whether the additional benefit from charging outweighs a longer route delay. We represent the cost for agent type i when taking route $s_i \in S_i$ as $u^i(s_i)$. For agent types in F_1 and F_2 the cost of a route s_i is

$$u^i(s_i) = \sum_{l\in s_i} d_l(x_l).$$

A path for type in F_1 will not include a self directed charging link while, for type in F_2, s_i will contain exactly once such link. For agent types in F_3, if the route s_i does not include a charging link then the cost function will be the same, $\sum_{l\in s_i} d_l(x_l)$; otherwise, it will have an additional term c_i representing the benefit of charging for agent sub-type i hence the cost will be $\sum_{l\in s_3} d_l(x_l) - c_i$ and

$$u^i(s_i) = \sum_{l\in s_i - A'} (d_l(x_l) - \mathbf{1}(l \in A')c_i)$$

where $\mathbf{1}(l \in A')$ represents an indicator function.

D. Nash Equilibrium computation

In this subsection we discuss how we obtain the Nash Equilibrium for fixed EV charger locations. In the next subsection we use these equilibria to estimate the total delay for each candidate location.

Since the players choose paths to minimize their cost, s^* is a Nash Equilibrium (NE) if

$$u^i(s_i^*) \geq u^i(s_i)$$

for all agent types i and feasible paths $s_i \in S_i$.

To find the equilibrium points in our model we use the following potential function:

$$\sum_{l \in A} \int_0^{\sum_i x_l^i} d_l(x)\, dx - \sum_{l \in A'} \sum_{i \in F_3} c_i x_l^i \tag{1}$$

This is an extension of the well known Beckmann potential and the optima of the following formulation are the Nash Equilibria. We minimize (1) wrt. flow constraints that is, for each agent type, the total amount of flow over the set of feasible paths must carry the whole agents of type i:

$$\text{minimize} \sum_{l \in A} \int_0^{\sum_i x_l^i} d_l(x)\, dx - \sum_{l \in A'} \sum_{i \in F_3} c_i x_l^i \tag{2}$$

$$\text{subject to} \sum_{s_i \in S_i} x_{s_i}^i = 1, \forall i \in N \tag{3}$$

The following theorem uses the convexity of this extended potential to ascertain the correspondence between the minimizers and Nash equilibria:

Theorem 1: Extended potential function (1) is convex and every minimizer s^* of (2) is a Nash Equilibrium.

Proof 1: Given the delay functions d_l are non-decreasing, the summation over their integrals, $\sum_{l \in A} \int_0^{\sum_i x_l^i} d_l(x)\, dx$, is convex. Since $\sum_{l \in A'} \sum_{i \in F_3} c_i x_l^i$ is linear, 1 is convex. For some agent type i let $s_i \in S_i$ improves over s_i^* (path that optimizes the potential). If i's type is in $F_1 \cup F_2$ then $\sum_{l \in s_i} d_l(x_l) < \sum_{l \in s_i^*} d_l(x_l)$, if the agent's type is in F_3 then $\sum_{l \in s_i} d_l(x_l) - c_i \sum_{l \in A' \cap s_i} x_l < \sum_{l \in s_i^*} d_l(x_l) - c_i \sum_{l \in A' \cap s^*} x_l$. But then for small enough δ increasing flows by δ in s_i while decreasing by the same amount in s_i^* we can decrease the potential function, which is a contradiction. Hence every minimizer of 2 is a Nash Equilibrium.

Lemma 1: There exists an equilibrium for the congestion game.

Proof 2: Given the flows being bounded, 2 always has a minimizer. From proposition 1 all the minimizers are NE which concludes the proof.

Since (2) can be solved as a convex optimization problem, this allows us to find NE points efficiently.

E. Optimal EV charger placement

Let $S \subseteq V_c$ be a selection of nodes for placing the EV stations and $x^*(S)$ be the Nash Equilibrium equilibrium flow for that selection. The optimal EV placement problem can be defined as follows.

$$\text{minimize} \sum_l (x_l^*(S) d_l(x_l^*(S))) \tag{4}$$

$$\text{subject to } S \subseteq V_c, \|S\| = c \tag{5}$$

In words, minimize the total delay experienced by the users over all of the possible selection of c ev charging station locations from the candidate set.

Finding the optimal subset can be a challenging task, particularly because the search space expands exponentially with the size of the set V. However, for the purposes of this paper, we introduce a greedy approach given in algorithm1. Our approach consists of adding EV nodes incrementally, one at a time. At each step, we identify the location that yields the greatest improvement in the objective at the equilibrium point, and add it to the set of nodes. We should mention that this is the scalable approach, but for smaller networks, alternative methods can be explored.

Algorithm 1: Greedy EV station Placement

Input: Set of candidate locations V, required number of EV stations n_{ev}, road network (V, A), link delay functions $d_l()$, agent types/strategy sets/utility functions.

Output: Selection of EV stations $V_o \in V$, $\|V_o\| = n_{ev}$.

$V_o \leftarrow \emptyset$;
while $\|V_o\| \leq n_{ev}$ **do**
  for $e \in V \setminus V_o$ **do**
    $\hat{x}_e \leftarrow$ minimizer of 2 for $V_o \cup \{e\}$ as the set of ev locations;
    $c_d(e) \leftarrow$ cumulative delay at $\hat{x}_e$;
  end
  $e^* \leftarrow$ minimizer of $c_d(e)$ over all $V \setminus V_o$;
  $V_o \leftarrow V_o \cup \{e^*\}$
end
return V_o;

III. Experiments

We conducted numerical experiments using a grid network with bidirectional links connecting the nodes. In each experiment, we selected a subset of possible locations for charger placements, and positioned the chargers as self-links, meaning that the charger's start and end points were the same.

To determine the potential routes for a given Origin-Destination (OD) pair, we employed four distinct paths, all of which represented the shortest paths with slight variations. The first path follows an up-down trajectory until it aligns with the same vertical position, and then proceeds with a right-left movement until reaching the target. The second path uses the same algorithm but in reverse order, commencing with right-left and concluding with up-down. The remaining two routes are constructed similarly, but they follow a zigzag pattern instead of proceeding all the way to the same vertical or horizontal position. In these zigzag paths, the route advances one step at a time, changing direction until it reaches the target.

We considered two types of cars for each OD pair: those requiring charging and those that do not. In our simulations, we maintained a ratio of one car needing charging for every two cars that did not. Those in need of charging had to find a charger en route before reaching their destination, resulting in different routes compared to those who did not require charging.

To determine the potential routes involving chargers for a given OD pair, we introduced an additional route generation method. This involved identifying all connecting pairs between the origin and the charger, as well as finding the paths linking the charger to the destination. We then combined all possible combinations of these two distinct routes to generate all potential routes between the origin and destination. This process was repeated for all chargers, and the resulting routes were added to the corresponding OD pairs' route options.

For our experiments, we exclusively created OD pairs among the first and last rows, and each route was bidirectional, meaning for each Origin-Destination pair (OD), there was a corresponding Origin-Destination pair (OD*) where the roles of origin and destination were reversed compared to OD. However, to simulate traffic patterns during rush hours, we specifically focused on scenarios where a significant portion of the population traveled from the top row to the bottom row. The ratio of those traveling from top to bottom compared to those traveling from bottom to top was set at 10. We generated random demands for each OD pair and scaled this demand according to the ratios mentioned above for the corresponding OD pairs. For the delay function we use the Bureau of Public Roads (BPR) function [24], defined as

$$d_l(x_l) = 1 + a\left(\frac{x_l}{b}\right)^c \tag{6}$$

For the experiments we conducted on grid network, we chose $a = 1, b = 1, c = 2$ for simplicity. In our experiments, two types of optimizations were implemented. The inner optimization aimed to find the Nash equilibrium of the route flows by optimizing the sum of the objective function of the link flows. The outer optimization, on the other hand, sought to determine the optimal charger locations once the Nash equilibrium was reached for the link flows, thanks to the inner optimization. As a result, the inner optimization was performed multiple times.

For the inner optimization algorithm, we utilized the "Trust-Region Constrained Algorithm" implemented in the Scipy [25] optimize package which uses "Byrd-Omojokun Trust-Region SQP method" which is described in [26]. We also provided the Jacobian and Hessian of the objective function to enhance numerical convergence. This method was chosen because it is recommended for linear equality-constrained and large-scale problems. Our optimization problem featured linear equality constraints since total flow on possible routes of a given OD pair needed to match the demand of the corresponding OD pair, and the objective function (1) is a convex function. In our experiments, as shown in Table I, the problems were high-dimensional, with the decision variable, route flows, having around 300 dimensions.

We initiated the decision variables (route flows) for each inner optimization process by evenly distributing the given demand for an OD pair across all possible routes connecting the OD pair. Our expectation was to see more evenly distributed link flows after optimization by optimal utilization of each link and mitigating congestion after the optimization. In Figure 1, we have plotted the variance of the link flows for each potential charger combination before and after the optimization. It is evident from the graph that the optimization process reduces variance and effectively spreads the flow throughout the network, thereby mitigating congestion.

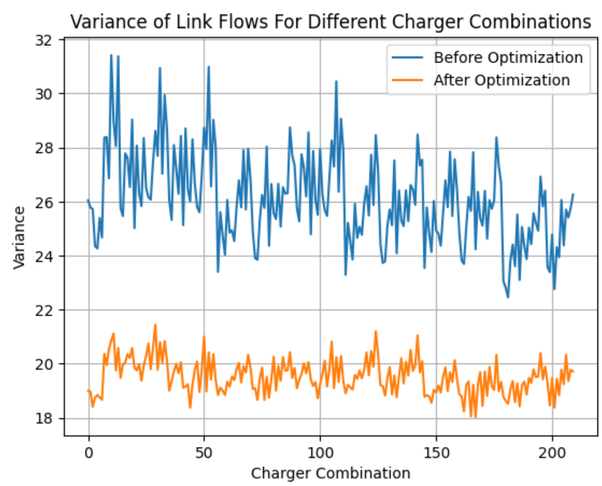

Figure 1: Variance of the link flows before and after the inner optimization. Note that the link flows are more evenly distributed after the optimization.

In contrast, the outer optimization considered all potential charger locations. In Table I we present the results of complete optimization and the greedy optimization approach, where we initially optimized the use of a single charger and iteratively fixed the charger's position based on the best results, repeating this process until all necessary chargers were placed. After the greedy optimization was completed, we continued to compute travel times for all possible charger locations for reference.

We executed our experiments using Google Colab, and you can find the code, plots, and videos of the optimization process at the following link: [1]

IV. Results

The experimental results demonstrate that the greedy approach closely approximates the global optimum. For the grid networks utilized for the experiments, employing the

[1] https://github.com/YasinSonmez/Grid-Network-Charger-Placement

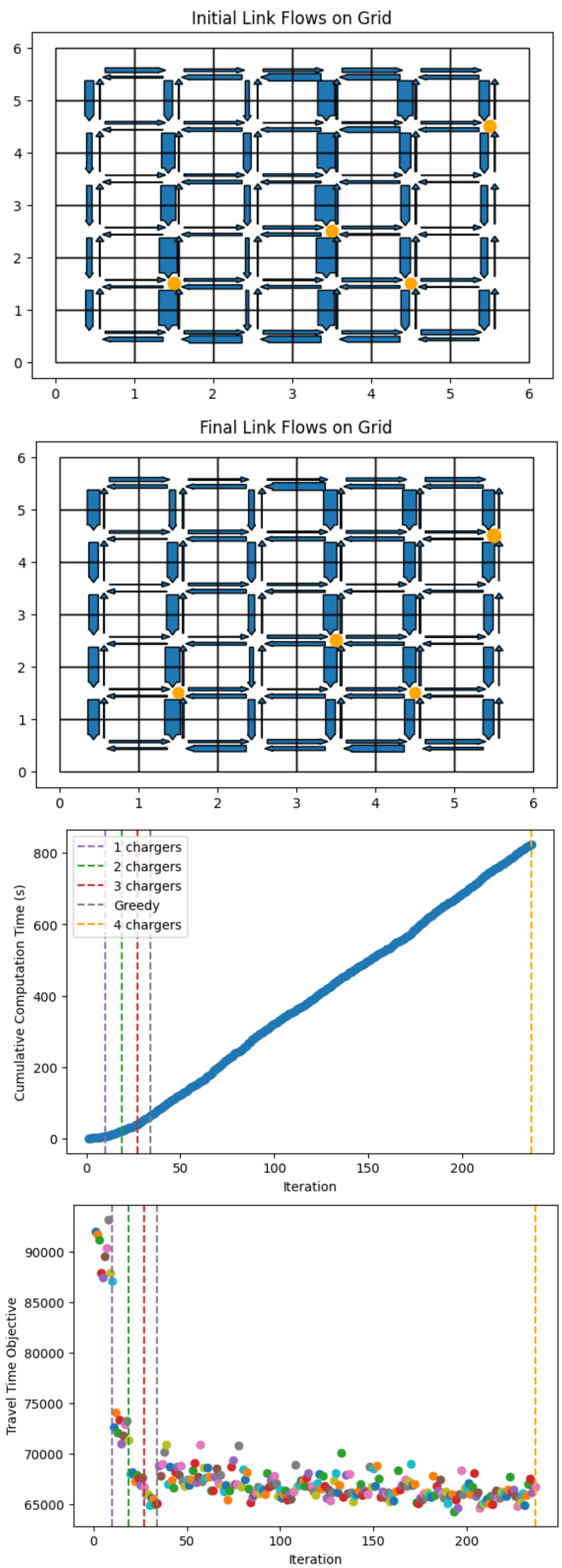

Figure 2: Optimization results. Top two plots illustrate the inner optimization to find equilibrium flows. The bottom two plots show the the outer optimization results.

Grid Size(n)	6×6	6×6	6×6	7×7
#Links	120	120	120	168
#OD Pairs	4	4	8	8
#Possible Charger Locations p	8	10	10	10
#Chargers c	3	4	4	4
#Combinations $\binom{p}{c}$	56	210	210	210
#Routes $\mu \pm \sigma$	120 $\pm$ 18	158 $\pm$ 26	305 $\pm$ 17	347 $\pm$ 19
Greedy Opt. ↓ Result (Time)	95.5% (22s)	95.8% (49s)	97. 6% (65s)	96.3% (166s)
Full Opt. Result ↓ $\mu \pm \sigma$ Min (Time)	100% $\pm$ 3.0% 94.1% (86s)	100% $\pm$ 2.7% 95.1% (436s)	100% $\pm$ 1.8% 96.3% (785s)	100% $\pm$ 2.0% 95.6% (2005s)

Table I: The optimization outcomes as well as the mean and standard deviation of the results among all possible selection of c charging locations are presented. All travel times are normalized to the mean travel time over all possible selections of charging locations.

greedy EV station placement method resulted in an average reduction of approximately four percent in the total travel time caused by traffic congestion (Table I). Notice that the overall computation time is primarily influenced by the number of routes, as this determines the dimension of the optimization problem for the extended potential function, and the number of potential combinations, which in turn dictates how many times the inner optimization is executed.

Table I presents the results and computation time for both the greedy optimization algorithm and the exhaustive search over all possible selections of EV charging stations to evaluate how effectively the greedy algorithm performs. Additionally, the mean and standard deviation of the results over all selections of c chargers, along with the minimum cost, are provided for the family of selection sets. All travel times are normalized to the mean travel time over all possible selections of ev charging stations. We see around 1% difference between the greedy optimization and the global minimum, all while significantly reducing computation time to just around one-tenth when employing the greedy optimization approach.

We further observe in Figure 1 that the greedy selection of charging locations lead to a more even distribution of the flow. This outcome is unsurprising, as the delay on each link experiences exponential growth as the flow on each link approaches the link's capacity. Therefore, in order to minimize overall congestion, it is crucial to distribute traffic flows as uniformly as feasible. We further see that the greedy approach scales well with the network size and the number of OD pairs.

In Figure 2 we report the results of an experiment we conducted. The top two graphs illustrate the alteration in link flows within the grid before and after the inner optimization. The thickness of the arrows represents the link flows and the thickness of orange circles represents the number of cars that use that specific charger. The third graph depicts the temporal progression of the outer optimization, with vertical lines denoting the completion of optimization for specific charger quantities. The last plot showcases the cumulative travel time for all combinations tested.

V. Conclusion

We formulated EV charging station placement as an optimization problem that aims to minimize total travel time, thereby mitigating the impact of chargers on congestion and detours. We framed the drivers' route choices as a congestion game, which enabled us to compute equilibrium flows efficiently in the inner optimization layer. The outer layer uses the delays at equilibrium for each candidate placement to find the optimal placement. Next, to ensure scalability, we proposed a greedy optimization algorithm for the outer loop. In numerical experiments the greedy approach was able to closely approximate the true optimal. It should be noted, however, that this comparison was possible for small-scale networks where we can compute the actual optimum.

The next task will be to test the results on realistic traffic networks using microscopic traffic simulators and available demand data. Another research direction is to derive theoretical bounds on the closeness of the greedy optimization to the true optimum.

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