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An anti-windup compensator for a rigid-body NDI-based manual attitude control system

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***Abstract*— In this paper, an anti-windup compensation scheme is proposed for the manual mode of a rigid body attitude control system to make the angular velocity dynamics globally asymptotically stable despite actuator saturation. The addressed anti-windup design problem is challenging since the nominal control law includes a nonlinear dynamic inversion element to cancel the nonlinearity in the angular velocity dynamics. The stability of the compensated closed-loop system is proved via the Lyapunov stability criterion appropriately. Moreover, the superiority of the compensated system versus the uncompensated one is demonstrated by simulation.**

I. Introduction

The rigid body attitude control problem has been addressed by researchers for many years [1]–[3]. The problem has an attractive structure, but the dynamics are inherently nonlinear. Although actuator constraints have not been taken into account in most of the studies, there are some works in which these limitations are considered like [4], [5], and references therein. For example, [6] considers the constrained attitude control problem and proves global stability with PD controllers. In [4], [7], a partitioned controller with PD structure is proposed and almost global stability is demonstrated. In [8], an adaptive control scheme is proposed for a spacecraft input-constrained attitude-tracking control problem. An anti-windup approach is proposed in [9] for constrained rigid body attitude stabilization which is different from the aforementioned studies as the gains of its PD-like attitude controller can be chosen independently and are not restricted. However, attitude tracking with bounded error can be achieved by the nominal controller due to attitude dynamics nonlinearities [6].

In this paper, we propose an anti-windup compensation scheme for a rigid body control system in the so-called *manual control* mode, where one attempts to regulate the angular velocity of the rigid body. Our approach is able to guarantee (global) angular velocity stabilization in the presence of both actuator saturation and the nonlinear dynamic inversion (NDI) elements present in the control law. The ideas developed in this paper also form the first steps in a more general treatment of the anti-windup problem for rigid body control with NDI-type control laws.

II. Problem Statement

The rigid body angular velocity dynamics are represented by the following equation:

$$J\dot{\vec{\omega}} = -\vec{\omega}\times J\vec{\omega} + \mathrm{sat}_{\bar{u}}(u) \tag{1}$$

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where $J\in\mathbb{R}^{3\times3}$ is the inertia matrix, $\vec{\omega}$ is the angular velocity vector, $u\in\mathbb{R}^3$ is the control input vector, and the $\mathrm{sat}_{\bar{u}}(u)$ function is defined as

$$\mathrm{sat}_{\bar{u}}(u) := \begin{bmatrix}\mathrm{sat}_{\bar{u}_1}(u_1) & \mathrm{sat}_{\bar{u}_2}(u_2) & \mathrm{sat}_{\bar{u}_3}(u_3)\end{bmatrix}' \tag{2}$$

to represent the actuators' constraints where $\mathrm{sat}_{\bar{u}_i}(u_i) = \mathrm{sign}(u_i)\min\{|u_i|,\bar{u}_i\}$ and $\bar{u}_i>0$ denotes the maximum amount of torque producible by the ith actuator. The nominal controller is the attitude control proposed in [10] which provides almost global exponential stability for the attitude tracking error dynamics of a rigid body with unconstrained actuators. In manual mode, the controller output u_c can be represented as

$$u_c = \vec{\omega}\times J\vec{\omega} + K(\vec{\omega}-\vec{\omega}_d) - J(\vec{\omega}\times\vec{\omega}_d - \dot{\vec{\omega}}_d) \tag{3}$$

where $\vec{\omega}_d$ is the desired angular velocity vector and K is a diagonal 3×3 matrix with negative real elements. Note that $\vec{\omega}_d$ is set to zero in the analysis since the approach aims to achieve global asymptotic stability for angular velocity. Note further, that this control law is *nonlinear* with, in the absence of saturation, the term $\vec{\omega}\times J\vec{\omega}$ precisely cancelling the nonlinear terms in the plant (1).

III. Anti-windup design

The structure of the proposed compensation scheme is depicted in Fig.1. The dynamics of the anti-windup compensator is adopted as

$$J\dot{\vec{\omega}}_a = -\vec{\omega}_a\times J\vec{\omega}_a - \vec{\omega}_a\times J\vec{\omega} - \vec{\omega}\times J\vec{\omega}_a - F\vec{\omega}_a + \mathrm{D_z}(u) \tag{4}$$

where $\vec{\omega}_a$ is the compensator state vector, F is a diagonal 3×3 matrix with positive real elements, and $\mathrm{D_z}(u) := u - \mathrm{sat}_{\bar{u}}(u)$. The output of the anti-windup system u_a is defined as

$$u_a = \vec{\omega}_a\times J(\vec{\omega}+\vec{\omega}_a) + \vec{\omega}\times J\vec{\omega}_a + K\vec{\omega}_a + F\vec{\omega}_a \tag{5}$$

which is added to the original controller output to form the control input torque vector u.

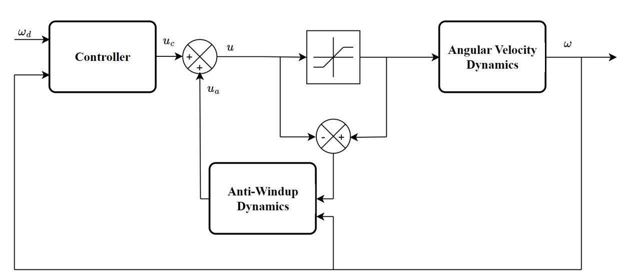


Fig. 1: The Compensated Control System Block Diagram

IV. Stability Analysis

Stability can be proved, as is typical in anti-windup design (e.g. [9]), by examining the stability problem in different coordinates. Letting, $\vec{\omega}_e = \vec{\omega} + \vec{\omega}_a$, allows the $\vec{\omega}_e$ dynamics to be written as

$$J\dot{\omega}_e = K\omega_e \tag{6}$$

which are exactly the dynamics of the system (1) in the absence of saturation and with the control law (3) applied. With an appropriate choice of K, $\vec{\omega}_e$ converges to zero exponentially. Moreover, the closed loop compensated angular velocity dynamics can be represented as

$$J\dot{\vec{\omega}} = -\vec{\omega} \times \ J\vec{\omega} + \mathrm{sat}_{\bar{u}}\left(\vec{\omega}_e \times J\vec{\omega}_e + K\vec{\omega}_e + F\vec{\omega}_e - F\vec{\omega}\right) \tag{7}$$

To prove global asymptotic stability of the angular velocity $V = \vec{\omega}' J\vec{\omega}$ is adopted as the Lyapunov function. Taking its time derivative results in

$$\dot{V} = \vec{\omega}' \,\mathrm{sat}_{\bar{u}}\left(\vec{\omega}_e \times J\vec{\omega}_e + K\vec{\omega}_e + F\vec{\omega}_e - F\vec{\omega}\right) \tag{8}$$

Letting $u_e = \vec{\omega}_e \times \ J\vec{\omega}_e + K\vec{\omega}_e + F\vec{\omega}_e$ and $u_{aw} = F\vec{\omega}$ together with adding and subtracting $\vec{\omega}' u_e$ to the recent equation, we have

$$\dot{V} = \sum_{i=1}^{3} \omega_i u_{ei} - F_i^{-1} u_{awi}\left(u_{ei} \ - \mathrm{sat}_{\bar{u}_i}\left(u_{ei} - u_{awi}\right)\right) \tag{9}$$

Absolute continuity of u_e and exponential stability of $\vec{\omega}_e$ dynamics, imply there exists a finite time $t_1 > 0$ such that

$$|u_{ei}| < \frac{\bar{u}_i}{2} \qquad \forall t > t_1 \quad \forall i \in 1,2,3 \tag{10}$$

Therefore, using lemma 2 from [9] gives

$$\dot{V} \leq \underbrace{\sum_{i=1}^{3} |\omega_i|\,|u_{ei}| - \min\left\{\varepsilon_i\,|\omega_i|\,, F_i\,|\omega_i|^2\right\}}_{W_i} \tag{11}$$

where $\varepsilon_i = \bar{u}_i - |u_{ei}|$. Two cases can be considered for each i:

A) if $|\omega_i| > \frac{\varepsilon_i}{F_i}$ then

$$W_i = -\left(\bar{u}_i - 2\,|u_{ei}|\right)|\omega_i| \tag{12}$$

which implies that $W_i < 0$ as $|u_{ei}| < \frac{\bar{u}_i}{2}$.

B) if $|\omega_i| < \frac{\varepsilon_i}{F_i}$ then

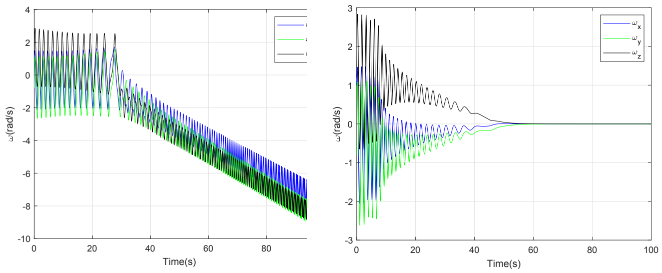
$$W_i = \ -\left(F_i\,|\omega_i| - |u_{ei}|\right)|\omega_i| \tag{13}$$

which is negative if $|\omega_i| > \frac{|u_{ei}|}{F_i}$.

So, after a while, $|\omega|$ will be bounded by $F^{-1}\,|u_e|$ which converges to zero asymptotically. Therefore, the angular velocity closed loop dynamics is globally asymptotically stable.

V. Sample Simulations

Fig.2a and Fig.2b show the uncompensated and compensated control system responses to the same initial condition. As expected, the compensated system can stabilize the angular velocity despite the actuators' constraints while the uncompensated control system cannot, with the angular velocity eventually diverging.



(a) (b)

Fig. 2: (a) Uncompensated system response, (b) Compensated system response

VI. Conclusion

This paper has addressed anti-windup design problem for a nonlinear rigid body attitude control system to attain global asymptotic stability of angular velocity in manual mode even if the actuators are saturating. The proposed method stabilizes the angular velocity dynamics in the considered operational mode successfully. However, the anti-windup compensation problem for this type of attitude control system to achieve global attitude tracking is more challenging and is under study by the authors. The work here forms a first step in the development of more general anti-windup schemes for rigid body control problems involving NDI.

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