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Decentralized game-theoretical approaches for behaviorally-stable and efficient vehicle platooning

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ABSTRACT

Cooperative vehicle platooning enabled by connected automated vehicle (CAV) technology has shown to bring various benefits including energy savings and a reduction in driving effort. Nevertheless, because these benefits vary over different platoon positions, vehicles from different owners may not be willing to platoon together; even if they form a platoon, they may attempt to change positions. To address such a behavioral-instability issue, it is necessary to redistribute the benefits among platoon members. To this end, this study investigates a decentralized multi-agent system where individually rational agents form platoons through peer-to-peer coordination under designated mechanisms that simultaneously determine the benefit reallocation. Depending on whether the scope of coordination is one-to-one or many-to-many, we introduce two types of mechanisms based on the bilateral trade model and one-sided matching. As the privacy of information sharing in the decentralized system is a common concern in practice, we further discuss two settings under each mechanism, differing by whether complete information is or is not known by the other agents. We indicate both theoretically and numerically that the decentralized platooning system is flexible and scalable, and can be implemented in real-time by leveraging the CAV technology.

1. Introduction and motivation

The concept of cooperative vehicle platooning, where virtually-linked vehicles travel together in a string with shorter headway, has been realized with the advent of connected and automated (CAV) technology. Vehicle platooning in general leads to a considerable amount of energy savings and emission reduction, which has been widely validated through theoretical analyses, simulation studies, and real-world experiments (Alam, 2011; Hammache et al., 2002; McAuliffe et al., 2017, 2018). Meanwhile, although the platoon operations in the near future are only partially automated, the complexity of maneuvering the following vehicles will be greatly alleviated, leading to reduced workloads for their drivers.

In view of these promising benefits, a growing number of studies have been conducted to advance the platooning technology from various aspects, including control strategy (Alam, 2011), route planning (Larson et al., 2015, 2016; Larsen et al., 2019), human factors (Zhang et al., 2019a), and impacts on traffic flow (Calvert et al., 2019; Duret et al., 2019). Though their study scopes, research perspectives, and methodologies differ, many of these studies endeavor to answer one fundamental question: how to form and maintain vehicle platooning as much as possible to fully reap its benefits. One line of research, reviewed by Bhoopalam

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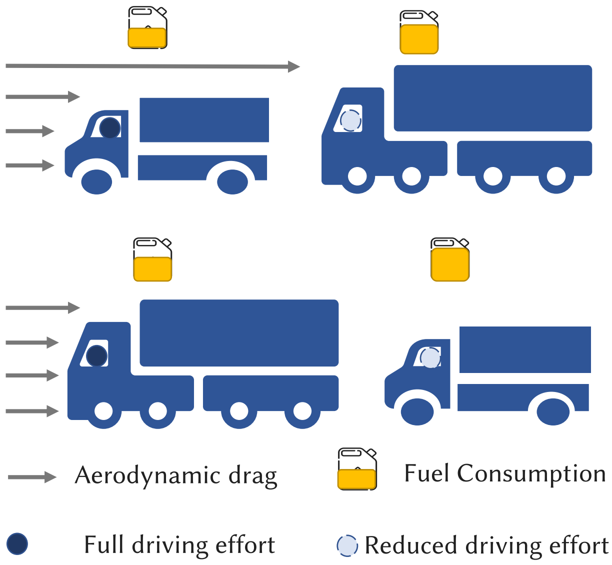

Fig. 1. Fuel efficiency and driving effort reduction are varying among platoon positions.

et al. (2018), targets high-level decision-making on vehicle route planning to maximize platooning opportunities, specifically for more regulated commercial vehicles. Another line of research concentrates on the vehicle controls in a more detailed level when 'platoonable' vehicles are operated within visible distances. By specifying vehicles' speeds, accelerations, and intra-platoon headway, the control policies aim to stabilize the platooning dynamic systems with regard to different traffic and geometric conditions (Duret et al., 2019; Xiong et al., 2021).

Comparatively, studies considering behavioral issues related to cooperative vehicle platooning are lacking. One of such behavioral issues is triggered by an unevenly-distributed benefit among different platoon members originated in the inherent cost-saving mechanisms of platooning, as illustrated in Fig. 1. First, the increase in fuel efficiency from platooning is created by the aerodynamic drag reduction effect when vehicles travel closely together in a platoon, benefiting the vehicles in the middle the most and that in the lead the least (Hammache et al., 2002; Levedahl et al., 2010; McAuliffe et al., 2017). Second, supported by cooperative adaptive cruise control (CACC), current vehicle platooning stipulates the driver in the leading vehicle to take both longitudinal and lateral controls, while those in the following vehicles to conduct only lateral control (Shladover et al., 2015). It would not be a problem if all vehicles in a platoon belonged to one owner. However, when vehicles are owned by different individuals or companies, the sense of unfairness may prevent them from platooning. Even if they happen to form a platoon, some will have the incentive to leave their current positions, yielding a behaviorally unstable platoon with less amount of benefits and more disruptions to surrounding traffic flow. One possible way to settle the behavioral instability is benefit redistribution. With this objective, our previous study structured a benefit reallocation mechanism for a centralized platooning system with multi-brand vehicles (Sun and Yin, 2019). The term multi-brand refers to vehicles with multiple types and configurations, contributing differently to energy savings of platooning. Consequently, a platoon's total utility depends on its vehicle sequence and operating speed. This centralized approach prioritizes system optimality by first arranging the vehicle platoons that maximize the total utility, then fairly redistributing the obtained savings using solution concepts in *cooperative game theory*.

Driven by both practical and theoretical motivations, this paper envisions a decentralized platooning system, where multi-brand vehicles form platoons and redistribute benefits through peer-to-peer coordination. Requiring the existence of a central controller with huge computational power and direct, reliable communications to individual vehicles, the centralized platooning scheme investigated by Sun and Yin (2019) falls short of scalability and flexibility in practice. The decentralized system we attempt to investigate in this paper addresses this issue by adopting vehicle-to-vehicle (V2V) communication and the computational power residing in automated vehicles. From a theoretical perspective, the instruments to redistribute benefits in the centralized approach is limited to those in the domain of cooperative game theory, which may be hard to implement due to the computational complexity and the non-convexity of the system utility function. Another consequent drawback is that the stable solutions in the *core* (Gillies, 1959; Aumann, 1961) may not exist. In contrast, the distributed nature of V2V communication supports the assumption that individual vehicles perform as rational agents. Therefore, one can utilize the more well-studied game-theoretical models, such as Nash equilibrium and stable matching, to characterize the interactions among agents, then implement the benefit redistribution via peer-to-peer monetary transfers.

The main focus of this paper is on the modeling of proper vehicle platooning games and transfer functions from the perspective of a system designer, who aims to achieve behavioral stability for individual vehicles and economic efficiency for the system at the same time. These two aims, however, contradict each other by nature. Another challenge in designing a distributed system stems from information asymmetry due to the absence of a central controller. When vehicles are reluctant to reveal all information or do not bring trustworthy information, customized incentives are difficult to derive and implement, leading to both behavioral instability and loss of efficiency. Moreover, depending on the communication range and traffic densities, a vehicle can coordinate with either one or many individual vehicles, or even previously-formed platoons. Consequently, the complexity for individuals to make optimal decisions may vary from a setting to another tremendously. All of these concerns motivate us to first set key assumptions on the coordination scope and information sharing in Section 2, and then develop specific models tailored to different coordination schemes in Sections 3 and 4. Numerical examples are conducted in Section 5, evaluating the solution qualities and computational efficiencies of the decentralized models, as compared with the centralized approach. Finally, Section 6 concludes the paper.

It should be noted that our models are deliberately as comprehensive as possible to incorporate all possible vehicle platooning scenarios and heterogeneous valuations on driving-effort reduction. Therefore, they are applicable to single-brand vehicle platooning, where vehicles are identical in terms of model and configuration and only differ in driving-effort reduction. Nevertheless, benefit redistribution for single-brand vehicle platooning can be cast into a simpler problem readily solvable through an auction mechanism (Sun and Yin, 2020). In a broader view, the discussions on benefit redistribution mechanisms in this paper are relevant to other formats of shared mobility, including peer-to-peer ride-sharing (Tafreshian and Masoud, 2020), resource allocation in the Mobility-as-a-Service system (Pantelidis et al., 2020), driver team formation and competition for the ride-hailing platform (Zhang et al., 2019b), etc. This paper also enriches the literature of mechanism design, which has been applied to a number of transportation problems, such as parking management in urban areas (Zou et al., 2015; Xu et al., 2016; Xiao et al., 2018), on-demand transportation services (Egan and Jakob, 2016; Bian and Liu, 2019), concession contract design (Shi et al., 2016), public transit regulation (Sun et al., 2020), and logistic services procurement (Liang et al., 2020; Zhang et al., 2018).

2. Premises and key considerations

In a decentralized system, we assume that an individual vehicle or a previously-formed platoon performs as an intelligent agent who tries to maximize their utility via platooning. Note that, in the latter case, the formed platoon collectively behaves as a single agent. If the system is connected, an agent can always find another peer to platoon with via V2V communication. For this reason, we assume all platoons are formed through a two-agent game. Naturally, the whole platooning process cannot be completed within one game, but rather, multiple games. It then generates questions on the agents matched in the games, the information available in the games, and the sequence of the games being played. To settle these concerns, we introduce the system's main setup in Section 2.1 to 2.4.

2.1. The scope of coordination

To begin, we consider *one-to-one coordination* in Section 3, where each agent can directly connect and communicate with another nearby peer agent in their communication range. We refer to these connected agents as *neighbors*. When a new platoon is formed by a pair of neighbors, each agent can either take the leading position or the following position. As being in the following position always benefits more, we assume that agents' actions are expressing their willingness-to-pay for this position. Specifically, we innovate a game based on the classic *bilateral trade model* (Myerson and Satterthwaite, 1983) where the following agent always pays the leading agent, and their position in the platoon is determined by comparing their willingness-to-pay.

The one-to-one coordination describes the interaction between any random pair of neighbors. However, when a rational agent has more than one neighbors, they are likely to platoon with the 'most beneficial' peer instead of a randomly chosen one. A conflict then arises when two or more agents choose the same peer to platoon with, which, however, has not been captured by the one-to-one coordination. In this regard, we extend our model from one-to-one coordination to *many-to-many coordination* structured in Section 4 by taking the communication network topology into consideration. When allowing the agents to rank their neighbors in terms of their self-interests, the many-to-many coordination can be modeled as a *one-sided matching* problem. It is then solved in a decentralized fashion, generating non-conflicting pairs of agents and improving overall utility under the properly designed transfer function.

Both coordination schemes allow multiple platoons to be formed in parallel simultaneously. A new platoon formed by two agents then becomes another agent in the system that is capable of platooning with others using the coordination schemes again. Consequently, the overall platoon formation in the decentralized system is a dynamic process, which is formally described in Section 2.4. By decomposing the whole platoon formation process into a sequence of direct interactions within two agents, the dynamic process overcomes the limitation of the proposed games that they do not permit platoons to be formed by three or more agents simultaneously. The sequence itself is determined by the changing communication network topology, and the coordination scheme applied.

2.2. Complete and incomplete information

One concern of the decentralized system is information asymmetry, since the decentralized system lacks the authority to collect and manage all information compared to the centralized counterpart. The main source of information asymmetry in platooning is agent's utility, which is known by themselves, but not by their neighbors. As mentioned before, energy savings and driving-effort reduction are the two major components of the endogenous benefits of vehicle platooning. Though varying from vehicle to vehicle, the decisive factors for energy savings, including traffic and road conditions, vehicle types and configurations, operating headway, and speed, can be easily discovered by all agents. Therefore, all agents' valuations on energy savings are regarded as

Table 1
System's setup and applied models.

		Information	
		Complete	Incomplete
Coordination	One-to-One	Nash Bargaining Game	Bilateral Trade Model
	Many-to-Many	One-sided Matching	-

public information. Quite the contrary, the valuation of driving-effort reduction is more subjective and is perceived differently by different individuals. An analogy is the reduction in the value of travel time when using fully automated vehicles (van den Berg and Verhoef, 2016). Influential factors include but are not limited to drivers' socioeconomic attributes, travel purposes, and the type and duration of on-board activities (de Almeida Correia et al., 2019; Molin et al., 2020; Pudāne et al., 2018). Consequently, the individual's valuation on driving-effort reduction is regarded as *private information*.

Taking these two different sources of utility into consideration, we conceptualize our game-theoretical models with either *complete* or *incomplete information*. For models with complete information, we assume that every agent is aware of all other agents' exact utility under each possible outcome. It happens when agents share truthful information via direct communication before each game is played. Under this consideration, the agents make decisions depending on the *ex post* utilities, which are realized in the outcome. For models with incomplete information, we assume that every agent does not know others' exact utilities. The uncertainty part of the utility is characterized by the term *type*. Albeit knowing their own type, agents only have a prior belief of the distribution that other agents' types follow. This distribution is common knowledge. Then in each game, every agent makes decisions based on their *interim* utility, which is the expected utility over the other agent's all possible types. The two settings require different treatments for deriving stable outcomes, which will be elaborated in Section 2.3.

2.3. Required properties

The game-theoretical models imply that the agents are *individually rational*, meaning that forming a platoon with another agent always brings them non-negative utility. As no central controller exists, this system needs to be *budget-balanced*, indicating that monetary transfers only occur among agents, and no external authority compensates or collects profits from them.

The concept of behavioral stability captures a state that once a platoon is formed, vehicles would not leave or change positions with others for a greater individual utility. It is then addressed differently under different coordination schemes. In the one-to-one coordination, we adopt the *Nash equilibrium* if it is under the complete information setting, and *Bayesian Nash equilibrium* if it is under the incomplete information setting. When the equilibrium is achieved, no agents can unilaterally change their decision to achieve a greater utility so that the formed platoon is stable. Since stable matching is applied for the many-to-many coordination, we define *matching stability* using the concept of *blocking pair*, which refers to a pair of agents who have not been matched with, but prefer each other, rather than their current partners.

The existence and *economic efficiency* of the stable solutions are two main concerns, which are closely related to the game format and transfer functions imposed. In the one-to-one coordination with complete information, since the bilateral trade model with linear transfer functions admits no equilibrium solution, we resort to Nash Bargaining game (Nash, 1953), which has a sense of cooperative game as a remedy. Economic efficiency indicates that the total utility achieved under a given solution is a global optimum. As the decentralized system is indirectly controlled by the system designer through the imposed transfer functions, the economic efficiency in each model largely depends on agents' action space. All the aforementioned models are summarized in Table 1.

Finally, the stable solutions are expected to be *incentive compatible*, indicating that every agent's optimal strategy under an equilibrium reflects their true type. It is theoretically not a concern in games with complete information. However, since the complete information in practice is achieved when agents share information via V2V communication, there might be a chance that they misreport for a better utility after learning the mechanism. We hence discuss the *ex post* incentive compatibility of the Nash bargaining solutions and stable matching in Section 3.3 and Section 4.4 respectively. For games with incomplete information, we prove that the Bayesian Nash equilibrium obtained is *interim* incentive compatible.

2.4. Dynamic platoon formation process

We now state the dynamic process of forming platoons. At each step of the process, denoted by t, a game is defined by a tuple $(\mathcal{M}^t, \Theta^t, \mathcal{A}^t, \chi^t, \mathbf{u}^t)$ where

- $\mathcal{M}^t$ is the finite set of current agents. Each agent $i \in \mathcal{M}^t$ represents a previously formed platoon of m_i vehicles, with $m_i \leq \bar{l}$, where $\bar{l}$ is the platoon length limit.
- $\Theta^t = \Theta_1 \times \cdots \times \Theta_{|\mathcal{M}_t|}$ is the type space for all agents, where Θ_i is the type space of agent i. All agents know the common prior distribution ϕ on Θ_i, $\forall i \in \mathcal{M}^t$ that their private information follows.

- $\mathcal{A}^t = \mathcal{A}_1 \times \cdots \times \mathcal{A}_{|\mathcal{M}_t|}$ is the set of actions, where $\mathcal{A}_i = [..., a_i, ...]$ is the set of actions available to agent $i \in \mathcal{M}^t$, and a_i is determined by its realized type θ_i. An action in a one-to-one coordination refers to either taking the leading or following position within a platoon and proposing a bid accordingly. In many-to-many coordination, an agent's action is to generate a preference list of their neighbors, based on which they can sequentially send and receive proposals on pairing.
- χ^t is the space of outcomes. An outcome function $\chi^t : \mathcal{A}^t \to \chi^t$ maps a joint action a to an outcome in χ^t. In this study, it concludes the vehicle sequence in formed platoons. As shown in Sections 3 and 4, the outcome for one-to-one coordination is whom to lead and whom to follow, while for many-to-many coordination, it is whom to pair and the optimal vehicle sequence in paired platoons.
- $\mathbf{u}^t = [u_1, \dots, u_i, \dots, u_{|\mathcal{M}^t|}]$ defines the risk-neutral utility function:

$$u_i(\chi(a), \theta, p) = V_i(\chi(a), \theta) - p_i(a),\ \forall i \in \mathcal{M}^t \tag{2.1}$$

  Here, V_i is the valuation that agent i obtains under outcome $\chi(a)$, given the joint type θ. Transfer functions $p_i : \mathcal{A}^t \to \mathbb{R}, \forall i \in \mathcal{M}^t$ provide the payment for each agent. The utility is defined per unit distance.

  The valuation V_i reflects platooning benefits such as fuel-savings (McAuliffe et al., 2018) and reduced driving effort (Janssen et al., 2015), which can vary with respect to vehicle types, operating speed and headway, road and traffic conditions. We assume the existence of a valuation function that can reflect these factors. Section 5 provides an example of this function. In this example, we consider an arbitrary freeway segment shared by a set of vehicles who are expected to form platoons by coordination. No surrounding traffics interact with these platoonable vehicles. Fuel consumption, the main source of valuation, is modeled as a quadratic function of speed along with parameters reflecting vehicle type, headway, and road conditions. Driving effort reduction, another source of valuation, is modeled as the type of each agent so that it is assumed to be an independent variable. To construct such a valuation function for other specific traffic situations, related studies can be of help. For instance, Liang et al. (2015) simulated the effects of surrounding traffic flow on the timing and speed of platoon merging on a homogeneous multilane highway segment. By utilizing a Markov Decision Process, Xiong et al. (2021) studied the platoon coordination by two streams of vehicles at highway junctions or on-ramp, under which a utility function composed of fuel consumption and travel delay is proposed. Further discussions on integrating practical considerations into the valuation function can be found in Section 6. Nevertheless, we note that the specification of the valuation function does not affect the validity of the game-theoretical framework.

Mathematically, the communication network is expressed as an undirected graph $G(\mathcal{M}^t, E^t)$, $\forall t$. Each agent in the set $\mathcal{M}^t$ is a vertex. If agent j is a neighbor of i and vice versa, there is an edge $e(i, j) \in E^t$. All agents that are neighbors of agent i constitute its neighborhood, $\mathcal{N}(i)$. Clearly, only neighbors can directly interact with each other in the game. The interactions are captured by different models per coordination schemes. The outcome determined by interaction between i and j, denoted as $\chi(a_i, a_j)$, incorporates two aspects: the fact that a new platoon is formed or not and the characteristics of the formed platoon, including vehicle sequence and platoon speed. Agents i and j then merge into a new agent, denoted as k, in time step $t + 1$. In this way, the communication network in time step $t + 1$, $G(\mathcal{M}^{t+1}, E^{t+1})$ bridges $G(\mathcal{M}^t,\ E^t)$ in time step t as follows:

$$\begin{aligned}
&i,\ j \in \mathcal{M}^t;\ i,\ j \notin \mathcal{M}^{t+1}\\
&k \in \mathcal{M}^{t+1};\ k \notin \mathcal{M}^t\\
&e(h,k) \in E^{t+1}; \forall h \in \{h | e(h,j) \in E^t \text{ or } e(h,i) \in E^t\}
\end{aligned}$$

A platoon cannot be infinitely long. Once a platoon is formed, it will not be dissembled in the following steps by principle. An exception is that, when reaching their destinations, individual vehicles leave the formed platoon. In addition, new agents outside of the system are allowed to join the process in each step. As a result, when the number of agents in the system is finite, the dynamic process will always terminate when no new agents can be generated by forming platoons.

A toy example is provided in Fig. 2 to facilitate a better understanding of the dynamic platoon formation process. At the initial step of the process, there are six agents in the system. Two agents that are neighbors of each other are connected by an edge. The edge with red glow indicates the two agents are playing a platooning game. For instance, agent 2 is doing so with agent 1, though it is the neighbor of agent 3 and 5 as well. Step 1 yields three agents (platoons) with two vehicles each. At step 2, agent 1–2 plays a game with agent 3–5 while agent 6–4 does not play any game at all. The outcome of step 2 is another two agents playing a platooning game at step 3.

3. One-to-one coordination

Below we first introduce the general format of the two-agent non-cooperative game-theoretical model used in the one-to-one coordination of vehicle platooning in Section 3.1. In short, we refer to these games generically as vehicle platooning games. Under this framework, we then discuss the existence and implementation of Nash equilibrium in games with complete information in Section 3.2 and those of Bayesian Nash equilibrium in games with incomplete information in Section 3.3, respectively. As we reveal that Nash equilibrium does not exist for most cases, for games with complete information we also discuss the Nash bargaining solution as an alternative.

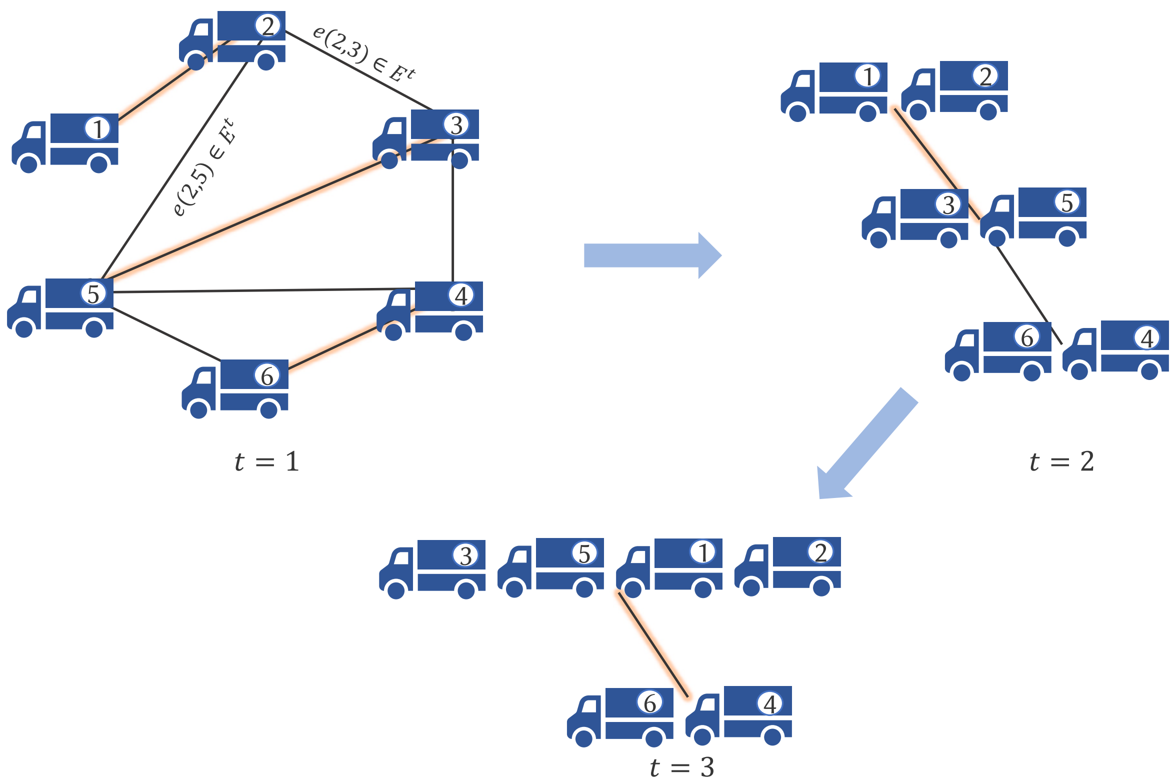

Fig. 2. A toy example of the dynamic process for decentralized vehicle platooning.

3.1. A general model

We assume that the agent who ends up with the following position transfers the positive amount of money to the agent who is leading. In other words, we view the agent at the leading position as the seller of the platooning service, and the other as the buyer of the same service. With this assumption, the dynamic process introduced earlier can be customized as follows:

- In each step t, a random pair of neighbors $(i,j) \in \mathcal{M}^t$ plays the game.
- Before the play, both agents have zero utilities in this step.
- Both agents bid a positive value, i.e., how much they are willing to pay to the leader if they are the follower. For instance, agent i bids b_i, which belongs to their action space A_i.
- The outcome is determined by the relative magnitude of b_i and b_j, and thereby is symbolically expressed as $\chi(b_i, b_j)$. Specifically, the agent with a lower bid automatically becomes the leader and collects the payment, and the other follows and pays. For simplicity, we denote the utility for agent i under the leading position is V_i^L, and that under the following position is V_i^F. If there is a tie in the bid, each agent has a half probability of being the leader. According to vehicle platooning's cost-saving characteristics, we assume that $V_i^F > V_i^L$ without loss of generality.
- We propose a budget-balanced *interdependent payment* rule for the transfer function. Moreover, considering that the game is symmetric, the transaction amount p^* satisfies

$$p^* = \frac{1}{2}(b_i + b_j) \tag{3.1}$$

Then we can derive the utility function $u_i(\chi(b_i, b_j), (V_i, V_j), p*)$ under all three outcomes. In short, we denote they are u_i^L, u_i^F and u_i^T, respectively.

$$u_i^L = V_i^L + \frac{1}{2}(b_i + b_j), \text{ if } b_i < b_j, \tag{3.2a}$$

$$u_i^F = V_i^F - \frac{1}{2}(b_i + b_j), \text{ if } b_i > b_j, \tag{3.2b}$$

$$u_i^T = \frac{1}{2}(V_i^L + V_i^F), \text{ if } b_i = b_j. \tag{3.2c}$$

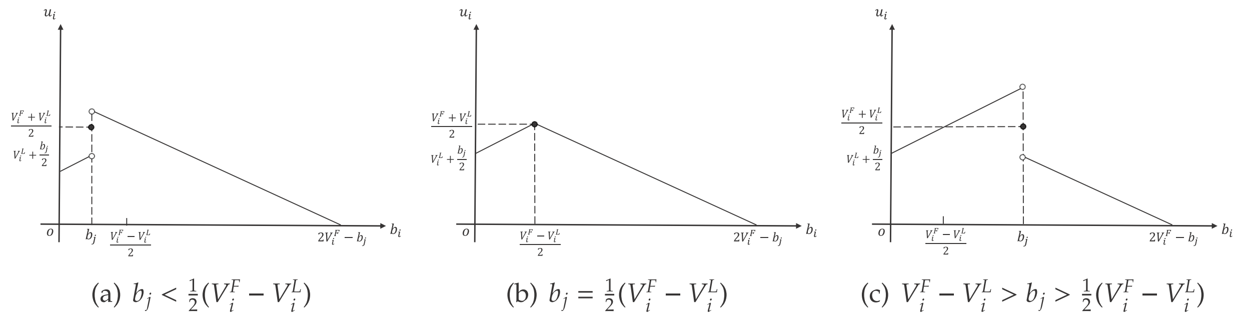

Fig. 3. Utility function of agent i with interdependent payment.

- Notice that if an agent represents multiple vehicles, their internal vehicle sequence will not be changed by the game outcome. When a new platoon is formed, the transaction is further allocated to all members evenly within each agent. In this way, every vehicle achieves non-negative utility in each step, thereby satisfying individual rationality. As a result, the dynamic process as a whole is *cross-monotonicity*.

This two-agent game imitates the classic *bilateral trade model*, which describes the bargaining problems between a buyer and a seller for a single object. In the classical model, the trade only occurs when the buyer's bid is higher than seller's valuation. In contrast, a trade always occurs in our game because the roles of buyer and seller are not predetermined, but set by their bids revealed during the play: the one with the higher bid becomes the buyer. Due to this unique feature, well-established results in the classical bilateral trade model cannot be directly applied here.

3.2. Games with complete information

The complete information setting assumes that an agent's valuation is known to their neighbors. Unfortunately, we prove that a Nash equilibrium does not necessarily exist.

Theorem 3.1. *In the vehicle platooning game with complete information between an arbitrary pair of agent (i, j), there is no Nash equilibrium under interdependent payment described in Eq. (3.1), unless*

$$V_i^F - V_i^L = V_j^F - V_j^L \tag{3.3}$$

Proof. To find the equilibrium of a game, we first study an agent's best response to the other's proposed bid.

For agent i, when b_j is fixed, their utility is a function of their own bid, denoted as $u_i(b_i)$ for simplicity. Due to individual rationality, agent i's action space A_i equals to $[0,\ 2V_i^F - b_j]$. Overall, the utility function $u_i(b_i)$ is a piece-wise linear function of b_i. When agent i proposes a bid less than b_j, agent i becomes the leader and $u_i(b_i)$ equals to u_i^L expressed by Eq. (3.2a). Then $u_i(b_i)$ increases with the increase of b_i. When $b_i = b_j$, there is a tie and $u_i(b_i)$ equals to u_i^T expressed by Eq. (3.2c). When $b_i > b_j$, agent i becomes the follower and $u_i(b_i) = u_i^F$, a decreasing function of b_i expressed by Eq. (3.2b). By comparing the value of u_i^T, u_i^L, and u_i^F, it can be seen that if and only if $b_j = \frac{1}{2}(V_i^F - V_i^L)$, $u_i(b_i)$ is a continuous function with the maximum value equal to $\frac{1}{2}(V_i^F + V_i^L)$. Therefore, depending on the values of b_j and $\frac{1}{2}(V_i^F - V_i^L)$, we discuss the best response of agent i, if there is any.

1. When $b_j < \frac{1}{2}(V_i^F - V_i^L)$, the utility function is shown in Fig. 3(a). Agent i will bid a value in the region of $(b_j, \frac{1}{2}(V_i^F - V_i^L)]$ to ensure their following position and a utility at least no less than $\frac{1}{2}(V_i^L + V_i^F)$, the utility they obtained if there is a tie. Meanwhile, since u_i is monotonically decreasing when $b_i > b_j$, agent i will try to bid b_i as small as possible. However, they will not bid $b_i = b_j$ since doing so results in a utility of $\frac{1}{2}(V_i^L + V_i^F)$, which is less than $V_i^F - \frac{1}{2}(b_i + b_j)$. Therefore, agent i has no best response in this case.
2. If $b_j = \frac{1}{2}(V_i^F - V_i^L)$, agent i has a best response: $b_i = b_j$ (Fig. 3(b)).
3. When $b_j > \frac{1}{2}(V_i^F - V_i^L)$, agent i will bid $b_i \in [\frac{1}{2}(V_i^F - V_i^L), b_j)$ to ensure their leading position and a utility at least no less than $\frac{1}{2}(V_i^L + V_i^F)$ (Fig. 3(c)). Since the utility function is discontinuous and monotonically increasing with b_i when $b_i < b_j$, agent i has no best response under this case as well.

Because the game is symmetric, agent j's best response to a given b_i can be analyzed in a similar way. Therefore, the only condition that admits an equilibrium is when $V_i^F - V_i^L = V_j^F - V_j^L = 2p$, where p denotes some constant value that both agents bid. However, the prerequisite for this condition is that the total utility achieved by the two agents is indifferent to their positioning, which only happens for single-brand vehicle platooning and homogeneous drivers. In other words, for the general case of multi-brand vehicle platooning and heterogeneous drivers, no equilibrium solution exists under the interdependent payment function. □

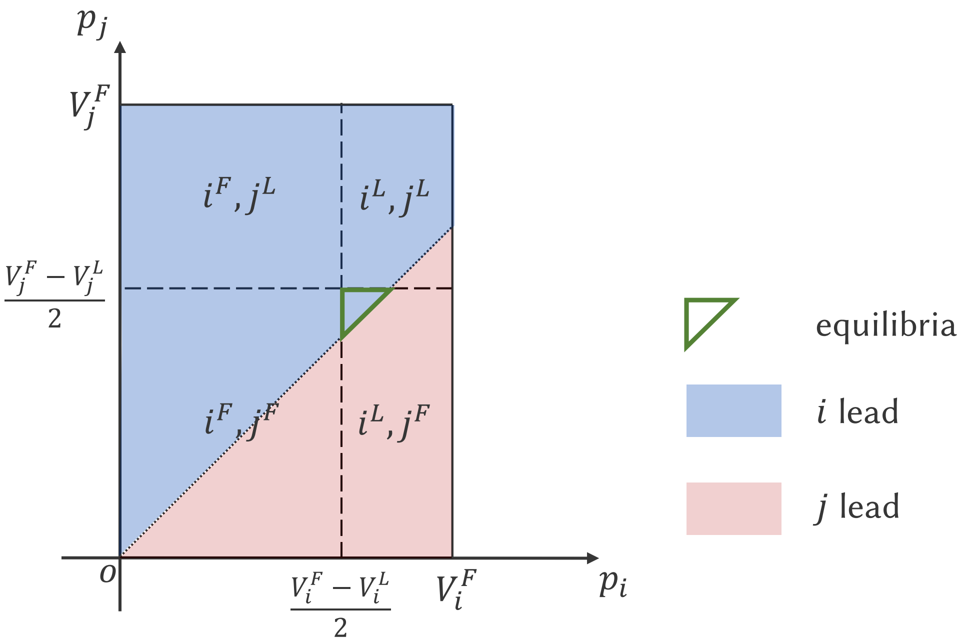

Fig. 4. Agents' best responses and budget-balanced outcomes under given payments. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Remark 3.1 (*Independent Payment*)**.** One may think that the budget balance constraint performs as a hurdle in achieving equilibrium outcomes. However, we deduct another line of analysis on a second side-payment rule, named as *independent payment*, to indicate that the result does not change too much with the new linear payment. Independent payment suggests that each agent collects or pays what they bid. Consequently, the utilities are

$$u_i = V_i^L + p^* = V_i(\chi(b_i, b_j), \theta_i) + b_i, \text{ if } b_i < b_j, \tag{3.4}$$

$$u_i = V_i^F - p^* = V_i(\chi(b_i, b_j), \theta_i) - b_i, \text{ if } b_i > b_j. \tag{3.5}$$

In this way, there is always profit generated from the platooning, making the mechanism weakly budget-balanced. Unfortunately, readers can verify that no equilibrium solution exists either under the independent payment rule for multi-brand platooning with heterogeneous drivers.

3.2.1. A cooperative alternative

Due to the failure of finding an equilibrium solution to stabilize the platoon in the non-cooperative game, we resort to a cooperative-game alternative described as follows. A pair of non-negative payments, p_i and p_j, is given to agents i and j, respectively. The agents then report what positions they would like to take. Still, agent i obtains utility of $V_i^L + p_i$ for being the leader and $V_i^F - p_i$ for being the follower. Based on individual rationality, $p_i \leq V_i^F$ and $p_j \leq V_j^F$.

Without loss of generality, we suppose that $V_i^L + V_j^F \geq V_j^L + V_i^F$. As a result, agent i becomes the leader and j becomes the follower maximizing the total utility of the two. This can easily be seen when

$$p_i > \frac{1}{2}(V_i^F - V_i^L),$$

agent i's best response is to become the leader. Otherwise, their best response is to become the follower.

Accordingly, the agents' best responses to the given p_i and p_j are shown in the two-dimensional diagram in Fig. 4. As an explanation, the notation i^L indicates that the best response for i is being the leader in the dashed rectangle region of the diagram, and the notation i^F indicates that the best response for i is being the follower in the marked region of the diagram. To satisfy at least weak budget-balance, agent j is sought to take the leading position in the lower triangle region that is colored in pink when $p_i \geq p_j$, while agent i is sought to take the leading position in the upper right angle trapezoid region that is colored in blue when $p_j \geq p_i$. Combining these with the best responses, it can be seen that when

$$p_i > \frac{1}{2}(V_i^F - V_i^L),\ p_j < \frac{1}{2}(V_j^F - V_j^L),\ p_j \geq p_i, \tag{3.6}$$

there exist equilibria (outlined in green in Fig. 4) that align with economic efficiency: $V_i^L + V_j^F > V_j^L + V_i^F$. In addition, when enforcing $p_i = p_j = p$ that satisfies

$$\frac{1}{2}(V_i^F - V_i^L) < p < \frac{1}{2}(V_j^F - V_j^L), \tag{3.7}$$

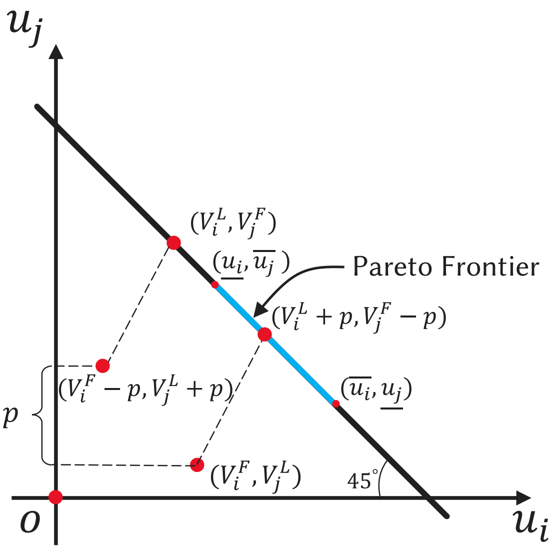

Fig. 5. Pareto optimal solutions.

budget balance is ensured. Therefore, all the efficiency is received by the agents and the equilibria with p satisfying Eq. (3.7) are Pareto optimal. In addition, it is behaviorally stable since given such a payment, no agent would likely switch their positions.

Fig. 5 shows the Pareto frontier in the two-dimension plane of u_i and u_j. It is an open one-dimensional set on the 45 degree oblique line, since the utilities agents received are bounded by Eq. (3.6). In this figure, $(\underline{u_i}, \overline{u_j})$ and $(\overline{u_i}, \underline{u_j})$ confine the boundaries of the Pareto Frontier, where

$$\underline{u_i} = \frac{1}{2}(V_i^F + V_i^L),\ \overline{u_i} = V_i^L + \frac{1}{2}(V_j^F - V_j^L),\ \underline{u_j} = V_j^F - \frac{1}{2}(V_i^F - V_i^L),\ \overline{u_j} = \frac{1}{2}(V_i^F + V_i^L)$$

are generated from Eq. (3.7). The point $(V_i^F - p^*, V_j^L + p^*)$ is considered as the *disagreement point*, or the *threat* of the Nash bargaining game.

Among all the Pareto optimal solutions, we choose a unique *Nash bargaining solution* (Nash, 1953) derived from

$$p^* = \arg\max_p \{(V_i^L + p - (V_i^F - p))(V_j^F - p - (V_j^L + p)) \mid p \geq 0\}$$

which gives

$$p^* = \frac{1}{4}[(V_i^F - V_i^L) + (V_j^F - V_j^L)]. \tag{3.8}$$

It can be used as the payment given to both agents before the play.

3.3. Games with incomplete information

Though a Nash bargaining solution provides an equilibrium with Pareto optimality, it is not *ex post incentive compatible*. Plainly speaking, *ex post incentive compatibility* (EPIC) assures that both agents achieve their maximum utilities if they reveal their valuations truthfully. However, if agent i reports a false valuation, $\hat{V}_i^F$, which is greater than the true valuation V_i^F, they can collect more from agent j. In fact, since all Pareto optimal payments depend on agents' valuations, there is no way to guarantee EPIC for platooning games with complete information.

For this reason, the assumption of complete information is relaxed to achieve a Bayesian Nash equilibrium that is hopefully *interim incentive compatible* (IIC). With incomplete information, every agent's valuation is refined as functions of their types, $\theta_i, \forall i \in M^t, \forall t$, which is the private information intangible to others. Therefore, *interim incentive compatibility* in general means that before any outcome is realized, every agent can maximize their *expected utility* over all other agents' possible types by reporting their own type truthfully, given that all other agents perform in the same way.

3.3.1. Utility functions

More specifically, agents' type in vehicle platooning games represent the valuation of driving-effort reduction sensed by different agents. Using δ_i to represent their energy-saving benefit, agent i thereby has valuation

$$V_i^L = \delta_i^L$$

if they are the leader, and valuation

$$V_i^F = \delta_i^F + \theta_i$$

if they are a follower. When θ_i is the private information enclosed to agent i themselves, agent j and other agents only know that $\theta_i \in \Theta = [\underline{\theta}, \overline{\theta}]$, and it follows a cumulative distribution function $F : \Theta \to [0,1]$, with probability density function $f : \Theta \to \mathbb{R}$. Moreover, functions F and f are assumed to be continuous and differentiable. Similarly, agent i only knows that $\theta_i \in \Theta$, but not the exact value. Consequently, each agent proposes the bid to maximizes their interim expected utility. In this way, their bid can be represented as a one-to-one mapping from their own type. When both agents bid by following the same mapping, the expected Bayesian Nash equilibrium can be derived.

3.3.2. The Bayesian Nash equilibrium

Our exploration for the games with incomplete information finds that under some certain feasibility conditions, it is possible to achieve a Bayesian Nash equilibrium. The main result is formally stated as follows:

Theorem 3.2. *Suppose that agents i and j participate in the two-agent non-cooperative game with incomplete information, if their types both follow uniform distribution on $[\underline{\theta}, \overline{\theta}]$ and satisfy*

$$\underline{\theta} - \frac{3}{4}\Delta e \leq \theta_i \leq \overline{\theta} - \frac{3}{4}\Delta e, \tag{3.9a}$$

$$\underline{\theta} + \frac{3}{4}\Delta e \leq \theta_j \leq \overline{\theta} + \frac{3}{4}\Delta e \tag{3.9b}$$

with Δe being defined as

$$\Delta e = (\delta_i^F - \delta_i^L) - (\delta_j^F - \delta_j^L), \tag{3.9c}$$

there exists a unique Bayesian Nash equilibrium under which agent i has the optimal linear strategy of bidding

$$b_i = \sigma_i(\theta_i) = \frac{1}{3}\theta_i + \frac{3}{8}(\delta_i^F - \delta_i^L) + \frac{1}{8}(\delta_j^F - \delta_j^L) + \frac{\underline{\theta} + \overline{\theta}}{12}, \tag{3.10a}$$

and agent j has the optimal linear strategy of bidding

$$b_j = \sigma_j(\theta_j) = \frac{1}{3}\theta_j + \frac{3}{8}(\delta_j^F - \delta_j^L) + \frac{1}{8}(\delta_i^F - \delta_i^L) + \frac{\underline{\theta} + \overline{\theta}}{12}. \tag{3.10b}$$

The proof is given in Appendix A.1.

Remark 3.2 (*Different Uniform Distributions on Types*)**.** The theorem above can be extended to the case when agents' types follow different uniform distributions. For instance, agent i's type uniformly lies in $[\underline{\theta}_i, \overline{\theta}_i]$, and agent j's type uniformly lies in $[\underline{\theta}_j, \overline{\theta}_j]$. In this case, when θ_i and θ_j satisfy

$$\underline{\theta}_j - \frac{3}{4}\Delta e + \frac{1}{24}[(\overline{\theta}_i + \underline{\theta}_i) - (\overline{\theta}_j + \underline{\theta}_j)] \leq \theta_i \leq \overline{\theta}_j - \frac{3}{4}\Delta e + \frac{1}{24}[(\overline{\theta}_i + \underline{\theta}_i) - (\overline{\theta}_j + \underline{\theta}_j)],$$

$$\underline{\theta}_i + \frac{3}{4}\Delta e - \frac{1}{24}[(\overline{\theta}_i + \underline{\theta}_i) - (\overline{\theta}_j + \underline{\theta}_j)] \leq \theta_j \leq \overline{\theta}_i + \frac{3}{4}\Delta e - \frac{1}{24}[(\overline{\theta}_i + \underline{\theta}_i) - (\overline{\theta}_j + \underline{\theta}_j)],$$

there exists a Bayesian Nash equilibrium under which agents' optimal linear strategies of bidding are

$$b_i = \sigma_i(\theta_i) = \frac{1}{3}\theta_i + \frac{3}{8}(\delta_i^F - \delta_i^L) + \frac{1}{8}(\delta_j^F - \delta_j^L) + \frac{\underline{\theta}_i + \overline{\theta}_i}{48} + \frac{\underline{\theta}_j + \overline{\theta}_j}{16}, \tag{3.11a}$$

$$b_j = \sigma_j(\theta_j) = \frac{1}{3}\theta_j + \frac{3}{8}(\delta_j^F - \delta_j^L) + \frac{1}{8}(\delta_i^F - \delta_i^L) + \frac{\underline{\theta}_j + \overline{\theta}_j}{48} + \frac{\underline{\theta}_i + \overline{\theta}_i}{16}. \tag{3.11b}$$

Remark 3.3 (*General Distributions on Types*)**.** In the above, we only focus on the scenario with uniform distributions. Our analyses on general distributions reveal that it is hard to derive a closed-form solution because agents adopt asymmetric bidding strategies considering their differences in energy-saving benefits. We leave the general distribution case as an open question for further study.

3.3.3. Discussions

Fig. 6 illustrates the strategies applied by each agent specified by their types. In the *feasible region* colored in blue, both agents adopt the linear bidding strategies specified by Eq. (3.10) and end up with the Bayesian Nash equilibrium. Here, each agent can retrieve the other agent's true type by their bid, making the whole mechanism interim incentive compatible. However, in regions colored in red, one or more agents adopt the extreme strategies, $\sigma_j(\underline{\theta})$ or $\sigma_i(\overline{\theta})$, to maximize their expected utility. The derivation of optimal strategies in red regions is provided in Appendix A.1. Therefore, the whole mechanism is not interim incentive compatible: when an agent are using extreme strategies, the other can only tell the upper and lower bounds of this agent's type, but not the exact value.

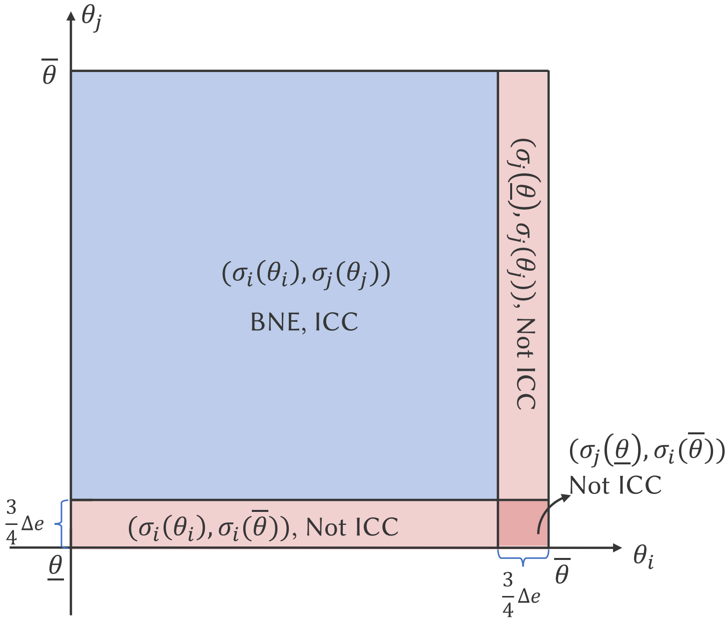

Fig. 6. Optimal bidding strategies. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

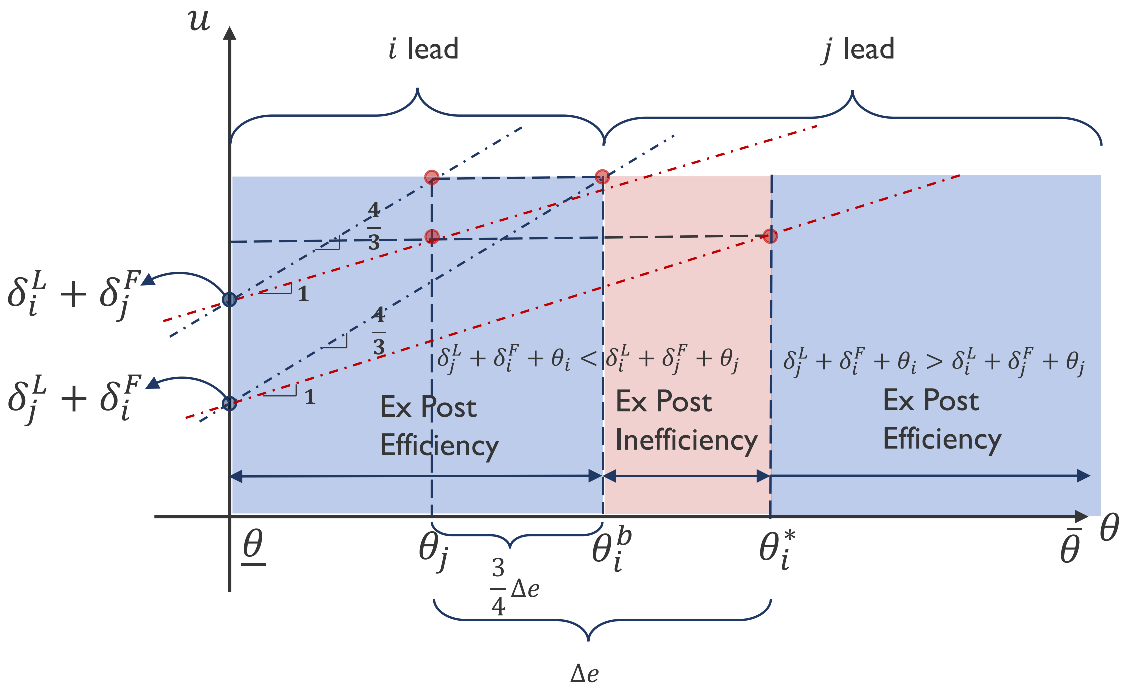

Fig. 7. Ex post economic efficiency.

Nevertheless, the space for misreporting, equivalently, the infeasible regions for Bayesian Nash equilibrium, can be negligible when Δe approaches to zero. This happens when agents are identical in terms of their engine performance, so that changes in the total energy-saving benefit is insignificant by switching the leading and following positions. Furthermore, if the driving-effort reduction weighs more heavily in the total valuation than the energy-saving benefit, Δe can be ignored.

We then evaluate the ex post economic efficiency from the Bayesian Nash equilibrium. Without loss of generality, assume that the total utility is maximized when agent i becomes the follower, that is to say,

$$\delta_j^L + \delta_i^F + \theta_i > \delta_i^L + \delta_j^F + \theta_j, \tag{3.12}$$

If agent i wins the bid under a Bayesian Nash equilibrium, we know that

$$\sigma(\theta_i) \geq \sigma(\theta_j) \Longleftrightarrow \delta_i^F + \delta_j^L + \frac{4}{3}\theta_i \geq \delta_j^F + \delta_i^L + \frac{4}{3}\theta_j. \tag{3.13}$$

Comparing the two conditions above, one can conclude that when

$$\theta_i - \theta_j \in (\frac{3\Delta e}{4}, \Delta e),$$

Eq. (3.13) is satisfied but Eq. (3.12) is not, indicating that the outcome is ex post inefficient under this Bayesian Nash equilibrium.

The efficiency is further illustrated in Fig. 7, where **x** axis indicates agents' type, and **y** axis indicates their utility. When agent j has the type θ_j, θ_i^b is the lowest type of agent i that ensures their following position, and θ_i^* is the agent i's lowest type that ensures the efficient outcome is achieved when they are the follower. If $\theta_i < \theta_i^b$, the bidding outcome is ex post efficient with agent i being the leader. If $\theta_i \geq \theta_i^*$, the bidding outcome is ex post efficient with agent i being the following. Thus when $\theta_i^b < \theta_i < \theta_i^*$, there is no ex post efficient.

Remark 3.4 (*Independent Payment*)**.** In the end, we provide the equilibrium result when independent payment is applied. The optimal bidding strategies adopt a similar linear format with parameters defined below:

$$\begin{aligned}
a_i &= a_j = \frac{1}{4},\\
c_i &= \frac{1}{3}(\delta_i^F - \delta_i^L) + \frac{1}{6}(\delta_j^F - \delta_j^L) + \frac{\underline{\theta} + \overline{\theta}}{8},\\
c_j &= \frac{1}{3}(\delta_j^F - \delta_j^L) + \frac{1}{6}(\delta_i^F - \delta_i^L) + \frac{\underline{\theta} + \overline{\theta}}{8}.
\end{aligned}$$

The feasible region is given by

$$\begin{aligned}
&\underline{\theta} - \frac{2}{3}\Delta e \leq \theta_i \leq \overline{\theta} - \frac{2}{3}\Delta e,\\
&\underline{\theta} + \frac{2}{3}\Delta e \leq \theta_i \leq \overline{\theta} + \frac{2}{3}\Delta e.
\end{aligned}$$

Unlike that in games with complete information, applying independent payment changes the result in games with incomplete information. Obviously, independent payment leads to a larger feasible region. As a trade-off, we can prove that the ex post inefficient region is enlarged from $\theta_i - \theta_j \in (\frac{3\Delta e}{4}, \Delta e)$ to $\theta_i - \theta_j \in (\frac{2\Delta e}{3}, \Delta e)$. As a conclusion, interdependent payment is more favorable from the system's perspective and will be further studied numerically in Section 5.

4. Many-to-many coordination

Below we first introduce the one-sided matching model and its feasibility to be used in the many-to-many coordination in Section 4. Section 4.2 introduces a benefit redistribution mechanism that secures the matching stability. Section 4.3 then describes a decentralized algorithm that generates the stable matching. Finally, the underlying optimization problem of the decentralized algorithm is provided and discussed in Section 4.4.

4.1. A general model

The one-sided matching model is originated in *roommate matching* (Gale and Shapley, 1962), which is a classic economic model describing how graduate students choose each other as their roommate. Comparatively, readers from the transportation community might be more familiar with bipartite matching or two-sided matching problems. A typical example is the matching of passengers and drivers in a ride-hailing market. In bipartite matching, agents are divided into two sides, and no agent can pair with another from the same side. In contrast, agents in the roommate matching problem are not distinguishable so that they are regarded as on one side. Accordingly, the dynamic process described in Section 2.4 can be customized as follows:

- In each time step t, agents broadcast their information within their neighborhoods.
- Based on the information received and the benefit mechanism specified, agents calculate the utilities their neighbors can bring to them and rank their neighbors accordingly, generating a preference list denoted as $P(i), \forall i \in \mathcal{M}^t$. Moreover, the most preferable neighbor of agent i is denoted as $P(i,1)$, the second most is denoted as $P(i,2)$, and so on.
- Each agent then proposes the pairing requirement by following the order of their preference list. In the meantime, they are allowed to receive proposals from their neighbors. If both agents agree to pair with each other, they become a pair in the matching and are referred to as each other's *partners*. An agent is also allowed to not form platoons with others, either because there are an odd number of agents, or because matching with others deteriorates their own utility.
- When no agent can pair with another, the whole matching process stops.

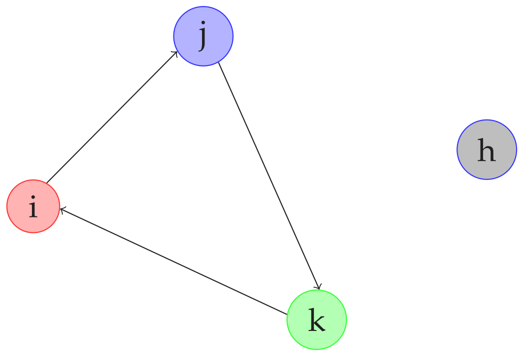

Fig. 8. Stable matching.

4.2. Matching stability

The core solution concept in all kinds of matching problems is *stability*. Generally speaking, stability ensures each agent has no incentive to change its current partner, either because the current partner is the most-preferable one or because their more preferred ones prefer others. Contrary to the fact that stable matching always exists in bipartite matching problems, Irving (1985) has shown that the stable matching in the one-sided matching problems does not always exist. For example, Fig. 8 shows four agents i, j, k, h who can be matched into two pairs. Assume that agent i prefers agent j the most, agent j prefers agent k the most, agent k prefers agent i the most, and agent h is least preferred by agents i, j, k. If agents i pairs with agent j and agent k pairs with agent h, agent j and k will prefer each other rather than their current partners, forming a *blocking pair* that makes the current matching unstable. Indeed, it can be easily discovered that other matchings of the four also possess blocking pairs, indicating that no stable matching exists in this example.

Fortunately, since agents in vehicle platooning use cardinal utilities as references to generate ordinal preferences of their neighbors, we prove that the undesirable blocking pairs can be eliminated by a simple benefit redistribution mechanism, thereby ensuring the existence of stable matching. Theorem 4.1 provides the formal statement.

Theorem 4.1. *Assume that in step t of the dynamic process, a random pair of neighbors i and j achieve the maximum total utility, $r_{i,j}$, under an optimal platoon formation $\chi^*_{i,j}$. Their individual utilities are $w_i(r_{i,j})$ and $w_j(r_{i,j})$, respectively. If $w_i(r_{i,j})$ is a strictly increasing function of $r_{i,j}$, $\forall i \in \mathcal{M}^t$, $\forall e(i,j) \in E^t$, then there exist stable matchings in this step.*

To prove this, we first introduce the concept of *odd ring* in agents' preferences. As an example, blocking pairs always exist in Fig. 8 because agents i, j, k's preferences form an odd ring:

$$i \succ_k j \succ_i k \succ_j i.$$

Here, the notation $i \succ_k j$ states that agent k *strictly* prefers agent i over agent j. Therefore, the three agents' preferences contribute a loop, which starts from agent i and ends at agent i. An odd ring also permits the existence of weak preference: e.g., agent k may *weakly* prefer agent i over agent j, denoting as $i \succeq_k j$. However, there must exist at least one strict preference in an odd ring. In other words, an odd ring does not have the following format:

$$i \succeq_k j \succeq_i k \succeq_j i.$$

Then the relationship between odd rings and stable matchings are given by the following lemma.

Lemma 4.1. If agents' preferences generate no odd rings, there exist stable one-sided matching.

We skip its proof for the sake of space. Readers who have an interest may refer to Chung (2000). Based on Lemma 4.1, we now provide the proof for Theorem 4.1.

Proof. Assume that there exists an odd ring on agents' preferences. Therefore, it is sufficient to say that there are agents $1\ldots,n$ such that for each agent i, $1 \le i \le n$,

$$i-1 \succeq_i i+1 (\text{mod } n),$$

and n is a odd number.

Since preferences are generated from utilities, and w_i is a monotonically increasing function, one can further conclude that

$$w_i(r_{i,i-1}) \ge w_i(r_{i,i+1}) \Longleftrightarrow r_{i,i-1} \ge r_{i,i+1} \quad (\text{mod } n).$$

By definition, there must exist some agent k in the ring, who strictly prefers agent $k-1$ over agent $k+1$, meaning that $r_{k,k-1} > r_{k,k+1}$. In sum, we get

$$r_{n1} \geq r_{12} \geq \cdots \geq r_{k,k-1} > r_{k,k+1} \cdots \geq r_{n-1,n} \geq r_{n1} \Rightarrow r_{n1} > r_{n1},$$

a clear false statement. Therefore, the assumption must be incorrect. Using Lemma 4.1, we can conclude that stable matchings exist, when using utilities $w_i(r_{i,j}), \forall i \in M^t, \forall e(i,j) \in E^t$ described above. □

For any arbitrary pair of neighbors i and j, once the optimal platoon $\chi^*_{i,j}$ is known, each agent is aware of their valuations $V_i(\chi^*_{i,j}, \theta_i, \theta_j)$, $V_j(\chi^*_{i,j}, \theta_i, \theta_j)$, and their utilities $w_i(r_{i,j})$, $w_j(r_{i,j})$. Let

$$w_i(r_{i,j}) = w_j(r_{i,j}) = \frac{1}{2} r_{i,j}, \tag{4.1}$$

and the transfer functions be

$$p_i = \frac{1}{2} r_{i,j} - V_i(\chi^*_{i,j}, \theta_i, \theta_j),$$
$$p_j = \frac{1}{2} r_{i,j} - V_j(\chi^*_{i,j}, \theta_i, \theta_j),$$

the mechanism is budget-balanced and will be used in the further numerical examples. In this way, agents' transfers are determined by the final utilities. It is quiet contrary to the games in one-to-one coordination, where transfer functions are given at first to induce the final utilities. In practice, the optimal formation of a single platoon formed from two neighbors can be easily calculated by themselves without using the computational power of the centralized controller. However, agents must share their complete information to serve this purpose. It then leads to the question on incentive compatibility, which we will discuss in Section 4.4.

4.3. The decentralized dynamic process

Based on the previous result, this section discusses agents' pairing procedure that results in a stable matching. Under the one-sided matching environment, each agent can propose to and receive proposals from their neighbors simultaneously. Moreover, agents' behaviors not only affect their neighbors, but also affect those who are indirectly connected with them, as long as the underlying communication network is a connected graph. For this reason, depending on the moment that the new agent influences the rest of the agent's pairing procedure, different dynamic processes can be generated.

Suppose that when two agents successfully pair with each other, all their neighbors will be notified via V2V communication. In one method, we can require neighbors to reproduce their preferences by taking the newly-generated agent into consideration, regardless of whether they have paired with others or not. Consequently, an agent is likely to reproduce their preference several times before a pair happens. Because the dynamic process is conducted distributively and asynchronously, this method provokes a stochastic and unpredictable final result.

```
Data: Vehicle Information
Result: Platoon Formation and Individual Payoffs
while |𝓜^t| > 1 do
    for ∀i ∈ |𝓜^t| do
        Update their preference list P(i).
    end
    while ∃ i ∈ |𝓜^t| s.t. P(i) ≠ ∅ do
        if Receive a rejection then
            Update P(i) : n ← reject agent, P(i,m) = P(i,m+1), ∀n ≤ m ≤ |P(i)| − 1.
        else if Receive a proposal then
            Update P(i) : n ← proposed agent, P(i,m) =null,  ∀n+1 ≤ m ≤ |P(i)|;
            if n = 1 then
                Accept.
        else if Receive acceptance from P(i,1) then
            P(i) = ∅;
            pair with P(i,1) into a new agent i′, let P(i′) = ∅;
            𝓜^t = 𝓜^t / {i, P(i,1)} ⋃ {i′}.
        else
            Propose to agent P(i,1);
        end
    end
    t = t + 1
end
```

Algorithm 1: *The Deterministic Matching Procedure*

Another method enforces that neighbors only consider the newly-generated agent once they complete the current pairing. Therefore, only if a matching for all current agents is completed can the dynamic process move to the next step. This method actually applies the first phase of Irving's algorithm (Irving, 1985) in each step of the dynamic process, and is formally stated in

Algorithm 1. The whole process is then illustrated by the flowchart provided in Fig. 11. We call it the *Deterministic Matching Procedure* (DMP) to emphasize that it leads to a fixed outcome. The proof is given in Proposition 4.1. Before that, two related properties are stated and proved.

Lemma 4.2. *A matching is finished when all agents have empty preference lists, indicating whether the agent is paired with another agent, or stays single.*

Proof. This can be simply proved by contradiction. If agent i with a nonempty preference list $P(i)$ is unmatched, then all agents on $P(i)$ are unmatched. Otherwise, if an arbitrary agent j on $P(i)$ has been matched with someone else, see k, j would broadcast the matching to i and i would delete j. Therefore i must have an empty preference list. □

Since the number of agents is finite in the dynamic platoon formation process, Lemma 4.2 ensures that DMP will terminate in finite steps.

Lemma 4.3. *In each step of DMP, the agent pair that achieves the maximum total utility must be matched.*

Proof. If the maximum total utility is achieved by pair i, j, meaning that $r_{i,j} \geq r_{k,l}, \forall k, l \in \mathcal{M}$, according to Eq. (4.1) the following inequalities must stay true as well:

$$w_i(r_{i,j}) \geq w_i(r_{i,k}), \forall k \in \mathcal{N}(i)$$
$$w_j(r_{i,j}) \geq w_j(r_{i,k}), \forall k \in \mathcal{N}(j)$$

Equivalently, i is the first preference on j's list and vise versa. By the proposed sequence, both of them will first propose to each other and they will be matched accordingly. □

Proposition 4.1. *DMP always results in a deterministic outcome.*

Proof. In each step t, Lemma 4.3 ensures that the pair of agents with the maximum total utility will be matched and then removed. Among the rest of the agents, the pair with the maximum total utility will be matched. As the number of agents is finite and the total utility can be ordered, the matching will always terminate at a finite number of steps, say m steps. The total utility is then the summation of the m largest total utilities generated by m disjointed pairs of agents. Thus the outcome at the end of each step is deterministic. When no agents outside join the matching system during the dynamic process, the final outcome will always be fixed. □

4.4. The optimization perspective

As introduced previously, the purpose of matching is to allow agents to pair with better partners to improve the total system utility. Therefore, the matching procedure can be regarded as decentralized heuristic algorithms that solve the following optimization problem in each step:

$$\max_{z} \sum_{e(i,j)\in E} w_{i,j} z_{i,j} \tag{4.2a}$$

$$\sum_{j\in\mathcal{N}(i)} z_{i,j} \leq 1, \ \forall i \in \mathcal{M} \tag{4.2b}$$

$$z_{i,j} = z_{j,i}, \ \forall e(i,j) \in E \tag{4.2c}$$

$$\max\{ \sum_{k:\, e(i,k)\in E,\, k\neq j} w_{i,k} z_{i,k} - w_{i,j}(1 - z_{i,j}), \sum_{k:\, e(j,k)\in E,\, k\neq i} w_{j,k} z_{j,k} - w_{j,i}(1 - z_{j,i})\} \geq 0, \ \forall e(i,j) \in E \tag{4.2d}$$

$$z_{i,j} \in \{0, \ 1\}, \forall e(i,j) \in E \tag{4.2e}$$

Here, agents' communication network is represented by a general graph $G(\mathcal{M}, E)$. If $i, \ j \in \mathcal{M}$ are neighbors, there is an edge $e(i,j) \in E$. The objective (4.2a) maximizes the system total utility by pairing two agents with each other. Parameter $w_{i,j}$ indicates the predetermined utility of agent i if they pair with agent j. A binary variable $z_{i,j}$ indicates whether i pairs with j. Hence constraint (4.2b) states that each agent can match with at most one other agent in its neighborhood and constraint (4.2c) regulates that the matching is reciprocal: if i pairs with j, j pairs with i as well. Constraint (4.2d) mathematically describes matching stability, under which no blocking pair is allowed. Furthermore, the solution quality of DMP can be concluded in Theorem 4.2:

Theorem 4.2. *By using the utility function Eq. (4.1), DMP in each step generates the matching with the maximum system utility that guarantees stability, equivalently, DMP provides the optimal solution of Problem (4.2).*

The proof is provided in Appendix A.3.

Remark 4.1 (*Incentive Compatibility and Incomplete Information*)**.** By assuming all agents broadcast truthful information in their neighborhoods, we set the incentive compatibility issue aside and derive the one-sided stable matching. Theoretically, no mechanism implements stable matchings in which truth-telling is the dominant strategy for all agents, even in two-sided matching (Roth, 1982). Consequently, one-sided matching cannot guarantee the incentive compatibility by nature. Studying stable matchings with incomplete information is meaningful in this sense. However, when information asymmetry is involved between any candidate pair of agents, no consensus on their optimal platoon formation can be achieved by this pair. And optimal platoon formation is indeed the basis for achieving a stable one-sided matching for vehicle platooning. Therefore, we only focus on the matching with complete information and leave those with incomplete information for a future study.

5. Numerical examples

In this section, we consider an arbitrary freeway segment shared by a set of vehicles who are expected to form platoons by coordination. Cases with the number of vehicles varying from 7 to 25 are studied. In each case, a vehicle has a type belonging to either heavy-duty, medium-duty or light-duty vehicles, whose parameters are identical as those in Sun and Yin (2019). The platoon size limit is set to be seven for all cases. The solution quality and computational efficiency of the decentralized coordination schemes are evaluated by comparing them with solutions achieved in the centralized platoon formation approach developed in Sun and Yin (2019). The centralized approach employs a mixed integer program and a column-generation-based algorithm to find the optimal platoon formation for all vehicles in the system, considering their heterogeneity in fuel efficiency, vehicle type, and speed preference. The optimal platoon formation specifies the number of platoons, the vehicle sequence in each platoon, and the platoon operating speed. We consider two scenarios. The first scenario implies that all agents possess complete information on others' utilities, so that Nash bargaining solutions are used in the one-to-one coordination. The second scenario allows agents to have incomplete information on others' utilities. Therefore, Bayesian Nash equilibrium solutions are adopted in the one-to-one coordination.

5.1. Games with complete information

In this scenario, we assume that fuel-savings is the main source of vehicle platooning' utility. By referencing the fuel consumption function developed in Sun and Yin (2019), we provide u_i^L and u_i^F, the utility functions for agent i when platooning with agent j under the leading and the following positions respectively, in the one-to-one coordination:

$$\begin{aligned} u_i^L &= V_i^L + p^* \\ &= \gamma \sum_{m \in M(i)} [F_m(v_{i1}, n_i, \mathbf{P(i)}) - F_m(v_{i1}, n_i + n_j, \begin{bmatrix} \mathbf{P(i)} & 0 \\ 0 & \mathbf{P(j)} \end{bmatrix})] + p^* \end{aligned} \tag{5.1}$$

$$\begin{aligned} u_i^F &= V_i^F - p^* \\ &= \gamma \sum_{m \in M(i)} [F_m(v_{i1}, n_i, \mathbf{P(i)}) - F_m(v_{j1}, n_i + n_j, \begin{bmatrix} \mathbf{P(j)} & 0 \\ 0 & \mathbf{P(i)} \end{bmatrix})] \\ &+ \alpha_i \sum_{m \in M(i)} [(v_{i1} - v_m)^2 - (v_{j1} - v_m)^2] - p^* \end{aligned} \tag{5.2}$$

In Eq. (5.1), p^* represents the side-payment determined by the Nash bargaining solution. The valuation V_i^L is given by the monetary value of fuel-savings. The parameter γ is the gas price. The ordered set of vehicles contained in agent i is denoted as $M(i)$, which has a size of $|M(i)|$. For the mth vehicle in the set $M(i)$, its individual fuel consumption before joining with agent j is $F_m(v_{i1}, n_i, \mathbf{P(i)})$, a convex function of the leader vehicle's speed v_{i1}:

$$F_m(v_{i1}, n_i, \mathbf{P(i)}) = \beta_{m3}(n_i, \mathbf{P(i)}) v_{i1}^2 + \beta_{m2} v_{i1} + \beta_{m1} + \frac{\beta_{m0}}{v_{i1}}$$

Here, β_{mk}, $k = 0, 1, 2, 3$ are vehicle-specific parameters, and β_{m3} is determined by the number of vehicles n_i and the vehicle sequence $\mathbf{P(i)}$ in platoon i. More specifically, $\mathbf{P(i)}$ is a $n_i \times n_i$ matrix with entries p_{lm} defined as follows.

$$p_{lm} = \begin{cases} 1, \text{ if } m = l, \\ 0, \text{ otherwise.} \end{cases} \quad \forall 1 \le m,\ l \le |M(i)|.$$

When the mth vehicle is in the platoon composed by agent i and j, its individual fuel cost is determined by the leading vehicle's speed v_{i1}, number of vehicles $n_i + n_j$, and the vehicle sequence $\begin{bmatrix} \mathbf{P(i)} & 0 \\ 0 & \mathbf{P(j)} \end{bmatrix}$, with $\mathbf{P(j)}$ representing the vehicle sequence in agent j. Therefore, the large matrix $\begin{bmatrix} \mathbf{P(i)} & 0 \\ 0 & \mathbf{P(j)} \end{bmatrix}$ indicates that agent i leads agent j with their inner vehicle sequences remaining unchanged.

Similarly, the term $\gamma \sum_{m \in M(i)} [F_m(v_{i1}, n_i, [\mathbf{P(i)}]) - F_m(v_{j1}, n_i + n_j, \begin{bmatrix} \mathbf{P(j)} & 0 \\ 0 & \mathbf{P(i)} \end{bmatrix})]$ in Eq. (5.2) defines the monetary value of fuel-savings when agent i is the follower. Since agent j is leading the whole platoon now, the fuel consumption of mth vehicle under platooning is determined by the operating speed v_{j1}, the number of vehicles $n_i + n_j$, and the vehicle sequence $\begin{bmatrix} \mathbf{P(j)} & 0 \\ 0 & \mathbf{P(i)} \end{bmatrix}$. An additional term,

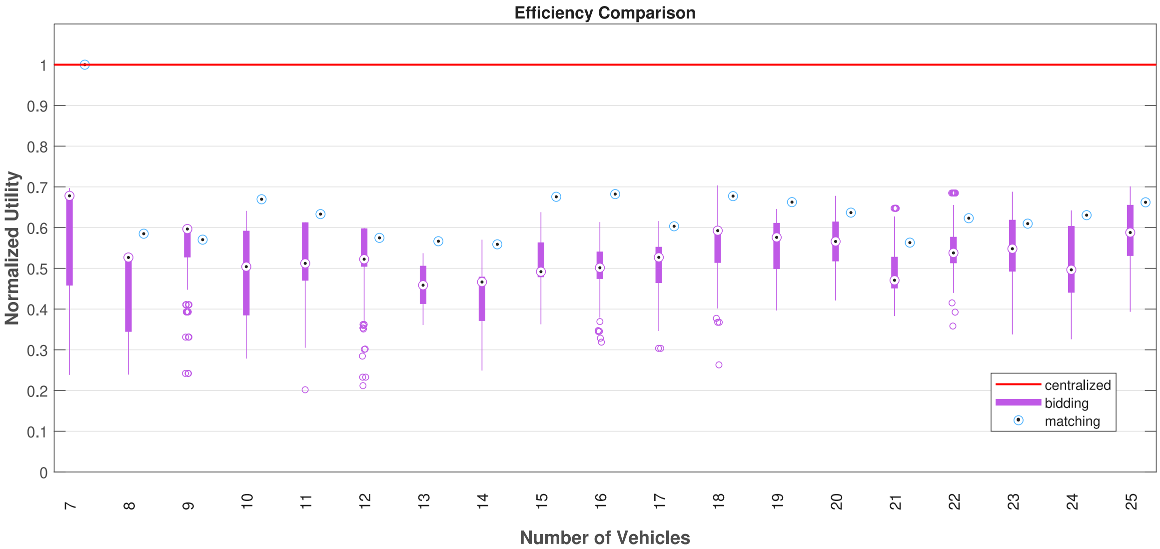

Fig. 9. Efficiency comparisons of the centralized approach, Nash bargaining solution, and one-sided matching for utilities without driving-effort reduction. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$\alpha_i \sum_{m\in M(i)}[(v_{i1}-v_m)^2-(v_{j1}-v_m)^2]$, is then introduced to describe the disutility in speed deviation and the sensitivity term α_i is assumed to be known.

In the one-sided matching problem, the maximum utility of agent i and j being in one platoon is provided by

$$r_{i,j} = \max_{v,\mathbf{P(i,j)}} \sum_{m\in M(i,j)} [C_m^0 - F_m(v, |M(i,j)|, \mathbf{P(i,j)})] - \sum_{m\in M(i)} \alpha_i (v - v_m^0)^2 - \sum_{m\in M(j)} \alpha_j (v - v_m^0)^2$$
$$s.t. v \in [\underline{v}, \overline{v}]$$

Here, $M(i,j)$ refers to the set of vehicles from both agent i and j, which has a size of $|M(i,j)|$. For simplicity, we use C_m^0 to indicate vehicle m's current cost, and v_m^0 to indicate its current speed $\forall m \in M(i,j)$. The maximum utility $r_{i,j}$ is derived by optimizing the platoon speed v and vehicle sequence $\mathbf{P(i,j)}$, with v subject to the speed limits $\underline{v}$ and $\overline{v}$. By optimizing $\mathbf{P(i,j)}$, the previous vehicle sequences within agent i and j will be changed.

It should be noted that all the utilities are derived when agent i and agent j together has a vehicle size that is less that $\bar{l}$, the platoon size limit defined in Section 2.4. Otherwise, they two will not form platoon together.

Fig. 9 presents the utilities under the Nash bargaining solution and one-sided matching in all 19 cased by comparing them with the optimal utilities achieved by the centralized approach. For a better illustration, all utilities have been normalized, so that those achieved under the centralized approach equals 1 (red line), regardless of the number of vehicles tested. As indicated previously, one-to-one coordination generates stochastic solution due to random pairing, so that we conduct 200 times of the dynamic process, resulting in a range of random outcomes shown in the purple boxes in Fig. 9. More specifically, each vertical purple line indicates the full range of the obtained total utilities under the corresponding case; the box represents the first quartile to the third quartile of the utilities; the dot in the middle is the median value of the utilities, while the purple circles are outliers plotted individually.

On average, the generated total utility achieves 51.8% of the global optimality generated by solving the optimal platoon formation in the centralized approach. It is an acceptable solution quality for several reasons. First, one-to-one coordination does not optimize platoon operating speed and intra-agent vehicle sequence, while the centralized approach does so. Then one-to-one coordination could only find solutions in a much smaller feasible region than the centralized approach, limiting its performance in solution optimality. Second, the trade-off between solution optimality and computational efficiency is inevitable when shifting from the centralized approach to the decentralized approach. But the loss in solution optimality is compensated by the gain in computational efficiency in the latter approach. A 50% loss is well accepted in the literature. For example, a decentralized consensus building to perform robot task allocation can only guarantee 50% optimality at best (Choi et al., 2009). Lastly, decentralized approaches only generate stable solutions, where the optimal solutions from the centralized approach are hard to prove to be stable.

When applying the one-sided matching model for many-to-many coordination, every candidate pair of agents must optimize their vehicle sequence and speed to determine the associated utilities and preferences first. Theorem 4.2 ensures a deterministic result under each case. However, the solution performance is unavoidably influenced by the communication network topology: if an agent can connect to at most four other agents, DMP generates an average of 64.3% of the global optimality. When all agents are assumed to be connected, DMP achieves 68.2% of the global optimalities on average, which is a 16.4% improvement compared to the Nash bargaining solution. The blue dots in Fig. 9 illustrate the deterministic result when all agents are connected.

It can be seen that when all information is public, many-to-many coordination improves both individuals' and system's utilities. However, compared to the centralized approach, there is still a 31.8% optimality gap on average. The optimization problem (4.2)

Table 2
Solutions for cases under utilities without driving-effort reduction.

Case	Normalized Utility			Computational Time (s)		
	One-to-One		Many-to-Many	Centralized	One-to-One	Many-to-Many
	Mean	SD				
7	0.564	0.130	0.739	5.70	0.17	0.47
8	0.456	0.088	0.831	13.40	0.12	0.30
9	0.550	0.080	0.645	22.30	0.12	0.29
10	0.488	0.109	0.806	28.37	0.10	0.24
11	0.530	0.070	0.556	39.08	0.11	0.28
12	0.525	0.073	0.824	39.09	0.12	0.27
13	0.462	0.054	0.696	85.94	0.13	0.23
14	0.436	0.072	0.636	110.00	0.12	0.26
15	0.517	0.070	0.628	121.35	0.12	0.51
16	0.501	0.058	0.608	157.79	0.12	0.24
17	0.514	0.068	0.610	154.00	0.13	0.24
18	0.579	0.087	0.790	168.02	0.17	0.29
19	0.555	0.068	0.823	185.40	0.16	0.30
20	0.565	0.066	0.586	231.53	0.13	0.25
21	0.488	0.056	0.611	295.47	0.13	0.52
22	0.554	0.064	0.705	322.41	0.13	0.32
23	0.551	0.076	0.725	288.22	0.12	0.30
24	0.512	0.089	0.518	281.24	0.15	0.27
25	0.588	0.071	0.624	314.81	0.14	0.29

reveals that the dynamic platoon formation process offers a heuristic algorithm framework for solving the original platoon formation problem. At each step of the process, one-sided matching generates the optimal solution with the constraint that one agent can only match with another one agent in their neighborhood. Comparatively, the centralized approach allows many-to-many matching all at once. The loss of economic efficiency is compensated in two aspects. First, the behavioral stability is theoretically ensured by matching stability, which is in general impossible in the centralized approach. Second, one-sided matching can be performed very fast, releasing the computational burden from the centralized approach as we expect.

More detailed numerical results are provided in Table 2. For one-to-one coordination, we provide the mean value and the standard deviation of the normalized utilities under each case. For many-to-many coordination, we provide the deterministic results. The computational time Nasof all three approaches are then compared in seconds. It can be seen that the two decentralized approaches significantly improve computational efficiencies, making them suitable to be implemented in real-time for large-scale systems.

5.2. Games with incomplete information

To intimate the one-to-one coordination with incomplete information, each agent's exact valuation on driving-effort reduction is assumed to be known only by themselves. Therefore, when agent i is the leader, their utility remains the same as that in Eq. (5.1). When agent i is following agent j, their utility is provided as follows:

$$\begin{aligned}
u_i^F &= V_i^F - p^* \\
&= \delta_i^F + s_i\theta_i - p^* \\
&= \gamma \sum_{m\in M(i)} [F_m(v_{i1}, n_i, \mathbf{P(i)}) - F_m(v_{j1}, n_i + n_j, \begin{bmatrix} \mathbf{P(j)} & 0 \\ 0 & \mathbf{P(i)} \end{bmatrix})] \\
&+ (\sum_{m\in M(i)} \beta[(v_{i1} - v_m)^2 - (v_{j1} - v_m)^2] + 1)\theta_i - p^*
\end{aligned} \tag{5.3}$$

Here, δ_i^F is agent i's monetary fuel-savings at the following position, and $s_i\theta_i$ is their driving-effort reduction deteriorated by the deviation disutility. In this way, we assume the agent-specific sensitivity α_i appeared in Eq. (5.2) is a production of their driving-effort reduction θ_i and a positive constant β. By assumption, agent j knows the term $s_i = \beta[(v_{i1} - v_m)^2 - (v_{j1} - v_m)^2] + 1$ but does not know the term $s_i\theta_i$. Therefore, agent j assumes that $s_i\theta_i$ follows a uniform distribution on $[s_i\underline{\theta}, s_i\overline{\theta}]$. Accordingly, the optimal bidding strategy elaborated in Eq. (3.11) is applied. It is also worth mentioning that $\beta[(v_{i1} - v_m)^2 - (v_{j1} - v_m)^2]$ is usually relatively small so that the value of s_i is around 1.

Contrarily, we assume that the valuations on driving-effort reduction are known when conducting the optimal platoon formation in the centralized approach and the one-sided matching. For the latter case, agent i and j jointly determine the their optimal utility by solving the following problem:

$$\begin{aligned}
r_{i,j} = \max_{v,\mathbf{P(i,j)}} \{ &\sum_{m\in M(i,j)} C_m^0 - F_m(v, |M(i,j)|, \mathbf{P(i,j)})] \\
- &\sum_{m\in M(i)} (1 - s_i(v - v_m^0)^2)\theta_i - \sum_{m\in M(j)} (1 - s_j(v - v_m^0)^2)\theta_j | v \in [\underline{v}, \overline{v}]\}
\end{aligned}$$

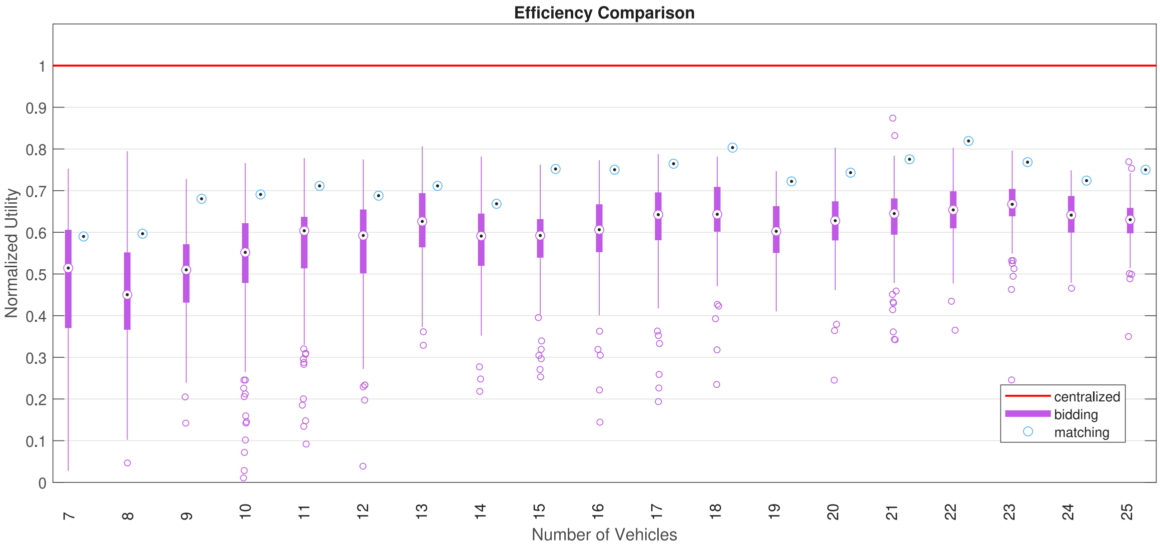

Fig. 10. Efficiency comparisons of the centralized approach, Bayesian Nash equilibrium solution, and one-sided matching for utilities with driving-effort reduction. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 3
Solutions for cases under utilities with driving-effort reduction.

Case	Normalized Utilities			Computational Time (s)		
	One-to-One		Many-to-Many	Centralized	One-to-One	Many-to-Many
	Mean	SD				
7	0.474	0.179	0.590	89.85	0.03	0.05
8	0.440	0.162	0.597	76.26	0.03	0.06
9	0.494	0.124	0.680	209.45	0.03	0.08
10	0.523	0.162	0.691	345.94	0.03	0.09
11	0.569	0.121	0.711	311.40	0.03	0.11
12	0.563	0.132	0.688	687.93	0.03	0.11
13	0.622	0.091	0.711	441.67	0.03	0.15
14	0.582	0.106	0.668	1085.09	0.03	0.16
15	0.579	0.083	0.752	423.76	0.03	0.20
16	0.602	0.092	0.750	619.51	0.04	0.21
17	0.630	0.095	0.765	905.23	0.04	0.24
18	0.642	0.087	0.803	956.19	0.04	0.30
19	0.604	0.075	0.722	1138.77	0.04	0.29
20	0.624	0.077	0.743	1584.62	0.04	0.34
21	0.634	0.077	0.775	423.84	0.04	0.37
22	0.652	0.066	0.819	813.43	0.04	0.41
23	0.667	0.063	0.768	1252.06	0.04	0.45
24	0.638	0.060	0.724	2085.00	0.04	0.50
25	0.628	0.056	0.750	2292.65	0.04	0.54

As for the result, the one-to-one coordination achieves 58.8% of the global optimalities on average (purple box plots in Fig. 10), which is a 7.0% optimality improvement compared to the previous scenario. Moreover, the standard deviation of the normalized utilities is decreasing with the increase of number of vehicles. When all vehicles are connected with each other, one-sided matching achieves 72.1% of the economic efficiency on average (blue dots in Fig. 10), a 3.9% optimality improvement from that under the first scenario. The solution quality and computational time can be found in Table 3.

The results reveal that many-to-many coordination via one-sided matching is a better coordination scheme in terms of economic efficiency. However, when driving effort is considered, one-to-one coordination via bidding brings relatively greater efficiency improvement. In the numerical examples, the parameters are selected in a way that the driving-effort reduction is of the same magnitude as energy-savings: each occupies about half of the total utility. While finding the formation with the most energy-saving benefit is a complicated combinatorial optimization problem, finding the platoon formation with the most driving-effort reduction is much less complicated. Therefore, when the ratio of driving-effort reduction to the total utility increases, the advantage of using the one-to-one coordination over the many-to-many coordination appears. In addition, as we indicated previously, the ex post inefficiency by using Bayesian Nash equilibrium diminishes when the energy-saving benefit is negligible.

6. Discussions and conclusions

As an early adopter of CAV technology, cooperative vehicle platooning has drawn great attention from both industry and academia in recent years. Among the many research questions and practical problems, the behavioral instability issue can be a crucial barrier for facilitating platooning by different vehicle owners, limiting the vehicle platooning to only a small scale.

This study has proposed a decentralized platooning system for general, multi-brand vehicles with heterogeneous drivers. Compared to the previous study where vehicles are formed into platoons by a central controller all at once (Sun and Yin, 2019), this decentralized system embraces a dynamic platoon formation process. Individual vehicles and previously formed platoons are regarded as rational agents in the process, who can only platoon with another agent at each step. Based on whether agents evaluate one or more peer agents before platooning, two schemes, one-to-one and many-to-many coordination, are developed.

In practice, the platoon formation process can be restricted by the meeting locations, e.g., whether vehicles meet in a hub or in motion, and their road and traffic conditions. The dynamic decentralized system can be tailored to these practical considerations by changing the agents in the system, the communication network topology, and their valuation functions. When vehicles meet at hubs or parking lots, their platooning process is separate from traffic so that their costs in forming platoons are negligible. Therefore, their valuations are mainly composed by benefits and costs in the formed platoons. Meanwhile, all vehicles can participate in the system as long as they are connected via V2V communication. However, depending on their meeting distance, speeds, surrounding traffics, road conditions, including the number of lanes and road gradient, vehicles' fuel and time consumption to form platoons in motion can vary significantly, which should also be included in the valuation. For instance, when a vehicle intends to platoon with its precedent vehicle, it may cost too much fuel energy and time to conduct lane-changing and overtaking due to the surrounded heavy traffic (Liang et al., 2015), making the opportunity to be the lead vehicle less attractive.

Theoretically, the one-to-one coordination extends the classical bilateral trade model in the economics community. This is inspired by the fact that a platoon's leading vehicle always gains less benefits than its followers while contributing the most to the overall fuel efficiency and taking the most driving effort. Therefore, it is reasonable to view the leader as a seller of platooning service and the follower as a buyer of it, whose bidding strategies are well studied in the bilateral trade model. However, the difficulty of vehicle platooning is that the two agents are 'born equal': they are both capable of being the leader or the follower. With this uniqueness, we have showed that when using the extended bilateral trade model as a two-agent platooning game, there is no Nash equilibrium in general when agents possess complete information. However, when randomness on others' utilities is involved, it is possible to achieve Bayesian Nash equilibrium in certain cases. Moreover, when one's utility as a whole is random to all but themselves, Bayesian Nash equilibrium together with ex post efficiency holds in general.

The many-to-many coordination is characterized by a one-sided matching problem. Similarly, the reason to use one-sided matching instead of a two-sided one is that all agents are 'born equal' and cannot be distinguished into the leader side and follower side before the play. By designing the proper transfer function and the matching algorithm, we have ensured that the most economically-efficient stable matching can be easily derived in a decentralized manner. However, one-sided matching could not be conducted with an incomplete information setting, and the incentive compatibility is difficult to check theoretically.

The numerical studies, together with the theoretical analyses, reveal that these two coordination schemes achieve different performances regarding different scenarios: one could be more fitting in one scenario than the other, and vice versa. If information sharing is the main concern, one-to-one coordination with the Bayesian Nash equilibrium solution can better address utilities' uncertainty. The matching mechanism works better when complete information sharing is possible. Both schemes have been verified to be flexible, scalable, and computationally efficient.

A few extensions based on this current model framework can be anticipated. The 'lightweight' property of the decentralized system makes it possible to easily integrate with decision-making at higher levels, e.g., routing and departure-time decisions in path planning. The multi-agent system studied here implies that each vehicle belongs to one agent, however, it is possible that one agent in the game owns multiple vehicles in different platoons. For instance, a truck company can invest the platooning technology on its own fleet and allows them to platoon with others as well. Beyond the context of platooning, the models developed in this study are relevant to shared mobility services, which often require reciprocal coordination among peers. But the fact that individuals are benefited differently, either by the nature of coordination or unreasonable operating mechanisms, prevents the widespread deployment of these services. As the essence of our models settles the controversy via benefit redistribution, they are applicable for other shared mobility services.

CRediT authorship contribution statement

Xiaotong Sun: Conceptualization, Methodology, Formal analysis, Writing – original draft. **Yafeng Yin:** Conceptualization, Formal analysis, Writing – review & editing, Supervision.

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Appendix

A.1. Proof of Theorem 3.2

Proof. Under interdependent payment, the ex post utility function of agent i can be written as

$$u_i(\chi(b_i,b_j),(\theta_i,\theta_j),p^*) = \begin{cases} \delta_i^L + \frac{1}{2}(b_i+b_j), & \text{if } b_i < b_j, \\ \frac{1}{2}(\delta_i^L + \delta_i^F + \theta_i), & \text{if } b_i = b_j, \\ \delta_i^F + \theta_i - \frac{1}{2}(b_i+b_j), & \text{if } b_i > b_j. \end{cases}$$

Hence, agent i's expected utility conditioning on their own type θ_i can be expressed as follows:

$$\begin{aligned} \mathbf{E}[u_i(\chi(b_i,b_j),(\theta_i,\theta_j),p^*)|\theta_i] =\; & Pr(b_i > b_j)[\delta_i^F + \theta_i - \frac{1}{2}(b_i + \mathbf{E}[b_j|b_i > b_j])] \\ & + \frac{1}{2}Pr(b_i = b_j)(\delta_i^F + \theta_i + \delta_i^L) \\ & + Pr(b_i < b_j)[\delta_i^L + \frac{1}{2}(b_i + \mathbf{E}[b_j|b_i < b_j])] \end{aligned}$$

When F is a uniform distribution on $[\underline{\theta},\overline{\theta}]$, we make a few assumptions on b_j in advance and verify them later:

(1). Agent j's strategy b_j is a function of their type θ_j, denoted as $\sigma_j(\theta_j)$.
(2). σ_j is differentiable and strictly increasing to θ_j.
(3). Moreover, σ_j is a linear strategy, meaning that

$$\sigma_j(\theta_j) = a_j\theta_j + c_j.$$

Assumption (2) indicates that $a_j > 0$. Non-negativity of bid leads to $a_j\underline{\theta} + c_j \geq 0$. With determined a_j and c_j, the range of b_j is bounded in $[a_j\underline{\theta}+c_j,\ a_j\overline{\theta}+c_j]$.

With the aforementioned assumptions, we specify agent i's conditional expected utility and denote it as $\mathbf{E}[u_i(\theta_j;b_i)|\theta_i]$ for short:

$$\begin{aligned} \mathbf{E}[u_i(\theta_j;b_i)|\theta_i] =\; & (\delta_i^F + \theta_i - \frac{1}{2}b_i)F(\sigma_j^{-1}(b_i)) - \frac{1}{2}\mathbf{E}[b_j|b_i > b_j]F(\sigma_j^{-1}(b_i)) \\ & +(\delta_i^L + \frac{1}{2}b_i)(1 - F(\sigma_j^{-1}(b_i))) + \frac{1}{2}\mathbf{E}[b_j|b_i < b_j](1 - F^{-1}(\sigma_j^{-1}(b_i)) \end{aligned} \tag{A.1a}$$

where $\sigma_j^{-1}(b_i)$, $F(\sigma_j^{-1}(b_i))$, $\mathbf{E}[b_j|b_i > b_j]$ and $\mathbf{E}[b_j|b_i < b_j]$ takes the following formats, respectively:

$$\sigma_j^{-1}(b_i) = \frac{b_i - c_j}{a_j} \tag{A.1b}$$

$$F(\sigma_j^{-1}(b_i)) = \begin{cases} 0, & \text{if } b_i \leq a_j\underline{\theta} + c_j, \\ \frac{b_i - c_j - \underline{\theta}a_j}{(\overline{\theta}-\underline{\theta})a_j}, & \text{if } a_j\underline{\theta} + c_j < b_i < a_j\overline{\theta} + c_j, \\ 1, & \text{if } b_i \geq a_j\overline{\theta} + c_j\ . \end{cases} \tag{A.1c}$$

$$\begin{aligned} \mathbf{E}[b_j|b_i > b_j] &= a_j\mathbf{E}[\theta_j|\frac{b - c_j}{a_j} > \theta_j > \underline{\theta}] + c_j \\ &= \frac{1}{2}(b_i + c_j + \underline{\theta}a_j) \end{aligned} \tag{A.1d}$$

$$\begin{aligned} \mathbf{E}[b_j|b_i < b_j] &= a_j\mathbf{E}[\theta_j|\frac{b - c_j}{a_j} < \theta_j < \overline{\theta}] + c_j \\ &= \frac{1}{2}(b_i + c_j + \overline{\theta}a_j) \end{aligned} \tag{A.1e}$$

Under uniform distribution θ_j and linear strategy, it is easy to check that $\mathbf{E}[u_i(\theta_j;b_i)|\theta_i]$ is a concave quadratic function of b_i. If it is maximized when $a_j\underline{\theta} + c_j < b_i < a_j\overline{\theta} + c_j$, the first order conditions leads to

$$b_i = \frac{1}{3}\theta_i + \frac{1}{3}(\delta_i^F - \delta_i^L) + \frac{1}{3}c_j + \frac{a_j}{6}(\underline{\theta} + \overline{\theta}) \tag{A.2}$$

Therefore, the optimal strategy for agent i under the given σ_j would be

- bidding $b_i = a_j\underline{\theta} + c_j$ if $\frac{1}{3}\theta_i + \frac{1}{3}(\delta_i^F - \delta_i^L) + \frac{1}{3}c_j + \frac{a_j}{6}(\underline{\theta} + \overline{\theta}) < a_j\underline{\theta} + c_j$,

- bidding b_i indicated in Eq. (A.2) if $a_j\underline{\theta}+c_j \le \frac{1}{3}\theta_i+\frac{1}{3}(\delta_i^F-\delta_i^L)+\frac{1}{3}c_j+\frac{a_j}{6}(\underline{\theta}+\overline{\theta}) \le a_j\overline{\theta}+c_j$,
- bidding $b_i = a_j\overline{\theta}+c_j$ if $\frac{1}{3}\theta_i+\frac{1}{3}(\delta_i^F-\delta_i^L)+\frac{1}{3}c_j+\frac{a_j}{6}(\underline{\theta}+\overline{\theta}) > a_j\overline{\theta}+c_j$.

Similarly, by assuming that the maximum of $\mathbf{E}U_j$ is achieved in the interior of $[a_i\underline{\theta}+c_i, a_i\overline{\theta}+c_i]$, we have

$$b_j = \frac{1}{3}\theta_j+\frac{1}{3}(\delta_j^F-\delta_j^L)+\frac{1}{3}c_i+\frac{a_i}{6}(\underline{\theta}+\overline{\theta}) \tag{A.3}$$

Existence of both Eq. (A.2) and Eq. (A.3) generates an equilibrium, under which

$$a_j = \frac{1}{3}, \quad c_i = \frac{3}{8}(\delta_j^F-\delta_j^L)+\frac{1}{8}(\delta_i^F-\delta_i^L)+\frac{\underline{\theta}+\overline{\theta}}{12}, \tag{A.4a}$$

$$a_i = \frac{1}{3}, \quad c_j = \frac{3}{8}(\delta_i^F-\delta_i^L)+\frac{1}{8}(\delta_j^F-\delta_j^L)+\frac{\underline{\theta}+\overline{\theta}}{12}. \tag{A.4b}$$

The equilibrium holds when

$$\sigma_j(\underline{\theta}) \le b_i \le \sigma_j(\overline{\theta}),$$
$$\sigma_i(\underline{\theta}) \le b_j \le \sigma_i(\overline{\theta}),$$

Taking the values in Eq. (A.4) into consideration, the above inequalities can be expressed as

$$\underline{\theta}-\frac{3}{4}[(\delta_i^F-\delta_i^L)-(\delta_j^F-\delta_j^L)] \le \theta_i \le \overline{\theta}-\frac{3}{4}[(\delta_i^F-\delta_i^L)-(\delta_j^F-\delta_j^L)], \tag{A.5a}$$

$$\underline{\theta}+\frac{3}{4}[(\delta_i^F-\delta_i^L)-(\delta_j^F-\delta_j^L)] \le \theta_i \le \overline{\theta}+\frac{3}{4}[(\delta_i^F-\delta_i^L)-(\delta_j^F-\delta_j^L)], \tag{A.5b}$$

which are exactly the same as the feasibility condition Eq. (3.9). Therefore, we complete the proof of the existence and the feasible region for the Bayesian Nash equilibrium.

It should be mentioned that we do not take the interim individual rationality into consideration when deriving the best response. This is not a concern because it can be automatically satisfied under equilibrium solutions. To see this, note that expected utility $\mathbf{E}[u_i(\theta_j;b_i)|\theta_i]$ takes a concave quadratic form over b_i and the boundary solution $\mathbf{E}[u_i(\theta_j;\sigma(\underline{\theta}))|\theta_i] = \delta_i^L+\frac{1}{2}(b_i+a_j\frac{(\underline{\theta}+\overline{\theta})}{2}+c_j)$ is non-negative and is no greater than the maximum expected utility $\mathbf{E}[u_i(\theta_j;\sigma(\theta_i))|\theta_i]$.

Next we discuss the optimal strategy for agents that do not satisfy Eq. (A.5), or in other words, outside of the feasible region. Without loss of generality, suppose $\Delta e > 0$. Then for agent i with $\theta_i \in [\overline{\theta}-\frac{3}{4}\Delta e, \overline{\theta}]$, their bid under Eq. (A.4a) exceeds the highest possible bid of b_j if agent j's bid follows Eq. (A.4b), making them the follower regardless of b_j's exact value. Therefore, their optimal strategy is bidding $\sigma_j(\overline{\theta})$, the lowest bid that secures their following position. Notice that agent j does not know the exact value of θ_i in the interim stage. If they have a type θ_j within their feasible region $[\underline{\theta}+\frac{3}{4}\Delta e, \overline{\theta}]$, they bid according to Eq. (A.4b). Otherwise, they bid $\sigma_i(\underline{\theta})$, the highest bid that secures their leading position. As a result, agent i becomes the follower and pays, agent j becomes the leader and collects the payment.

From this analysis, we can see that even if a Bayesian Nash equilibrium cannot be achieved by all pairs of agents, the associated linear bidding strategies can still be applied to achieve some platooning outcomes. The underlying reason is that the decision is made during the interim stage when agents assume their peers' types follow the common distribution and they have no way to detect more precise information other than bidding. The only possible reason for them to quit this game is their expected utility being negative. Nevertheless, this situation can only happen for agent i with $\theta_i \in [\overline{\theta}-\frac{3}{4}\Delta e, \overline{\theta}]$ in the above example, since the payment might be higher than the valuation. □

A.2. Flow chart of deterministic matching procedure

A.3. Proof of Theorem 4.2

Proof. Denote the stable matching from DMP is m. By contradiction, suppose that there exists another matching m', which is also stable but owns a greater system utility than that in m. It suggests that

$$u(m') > u(m).$$

Comparing m and m', there must exist some pair (i, j) matched in m', but unmatched in m:

$$(i, j) \in m', \ (i, j) \notin m \tag{A.6}$$

Those pairs that are both matched and unmatched under m and m' contribute the same portion of utility, and thereby are not of interest in this proof. Assume that agents i and j are matched with $p(i)$ and $p(j)$, respectively, the possible relationships among $r_{i,j}$, $r_{i,p(i)}$ and $r_{j,p(j)}$ can be discussed:

1. For all $(i,j) \in m'$ and $(i,j) \notin m$, $r_{i,p(i)} \ge r_{i,j}$ and $r_{j,p(j)} \ge r_{i,j}$.
   Such a relationship is impossible since it leads to

   $$u(m) \ge u(m'),$$

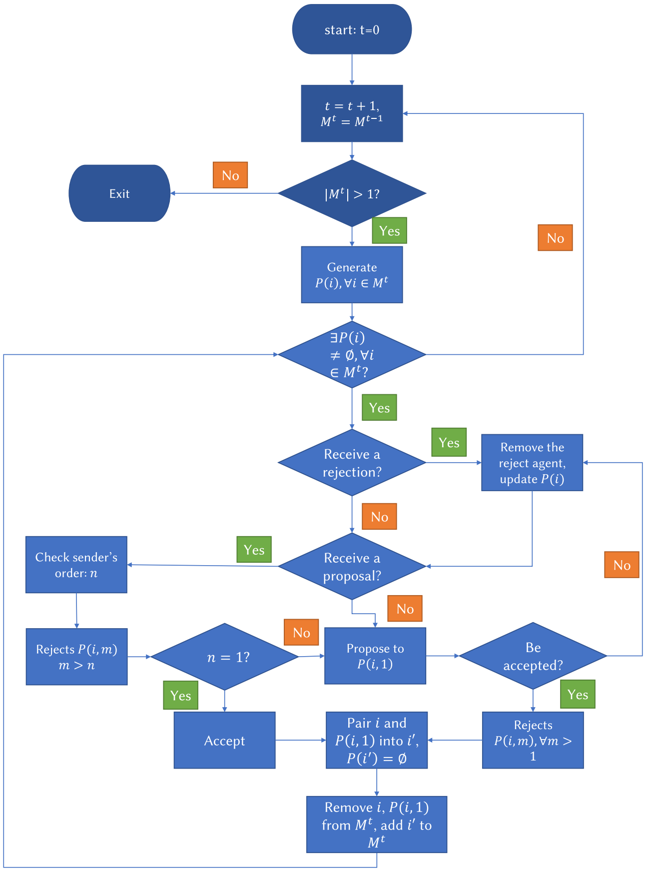


Fig. 11. Flowchart of the deterministic matching procedure.

thereby contradicting with the assumption in Eq. (A.6). Therefore, there must exist a pair (i, j) such that

$$r_{i,j} > \min\{r_{i,p(i)}, r_{j,p(j)}\}.$$

2. Suppose that there exists a pair $(i, j) \in m$ such that $r_{i,p(i)} < r_{i,j}$ and $r_{j,p(j)} < r_{i,j}$, satisfying $r_{i,j} > \min\{r_{i,p(i)}, r_{j,p(j)}\}$. However, it is also impossible because under the stability definition, such a pair (i, j) is a blocking pair for matching m, contradicting the fact that m is a stable matching generated from DMP.
3. As inequalities in (1) and (2) do not hold, every $(i,\ j)$ must satisfy

$$r_{i,j} > \min\{r_{i,p(i)}, r_{j,p(j)}\}, \quad r_{i,j} < \max\{r_{i,p(i)}, r_{j,p(j)}\}.$$

Without loss of generality, suppose that $r_{i,p(i)} > r_{i,j} > r_{j,p(j)}$. Under the stability constraint, if $p(i)$ does not match with i under m', it must match with another agent, denoted as $g^{(1)}p(i)$. Otherwise, if all $p(i)$ stays single under m', $r_{i,p(i)} + r_{j,p(j)} > r_{i,j} + 0$ making $u(m') < u(m)$, a contradiction to the assumption as well. In this way, the pair $(p(i), g^{(1)}p(i))$ is another pair that matches under m' and unmatches under m. The previous analysis can be applied to it as well. Following this logic, one can get a list of agents $g^{(2)}p(i),\ g^{(3)}p(i), \ldots$, such that

$$(g^{(2n)}p(i),\ g^{(2n+1)}p(i)) \in m',\ (g^{(2n+1)}p(i),\ g^{(2n+2)}p(i)) \in m,\ n \geq 0,\ n \in \mathbb{Z},$$

$$r_{g^{(2n+3)}p(i),\, g^{(2n+2)}p(i)} > r_{g^{(2n+2)}p(i),\, g^{(2n+1)}p(i)} > r_{g^{(2n+1)}p(i),\, g^{(2n)}p(i)}, n \geq 0,\ n \in \mathbb{Z}.$$

As Fig. 12 shows, the matching under m' is colored in red, and that under m is colored in black.
Since the number of agents in each step is finite, the list is finite as well. Assume that the last element in the list $g^{(2n+1)}p(i), n \in \mathbb{Z}_+$. Moreover, stability indicates that

$$P(g^{(2n)}p(i), 1) = g^{(2n+1)}p(i), \quad P(g^{(2n+1)}p(i), 1) = g^{(2n)}p(i).$$

In other words, $g^{(2n)}p(i)$ and $g^{(2n+1)}p(i)$ are the first preferences of each other. If it is not the case, one can always lengthen the list based on previous analysis. However, as m is generated by DMP and according to Lemma 4.3, we also have

$$P(g^{(2n-1)}p(i), 1) = g^{(2n)}p(i), \quad P(g^{(2n)}p(i), 1) = g^{(2n-1)}p(i).$$

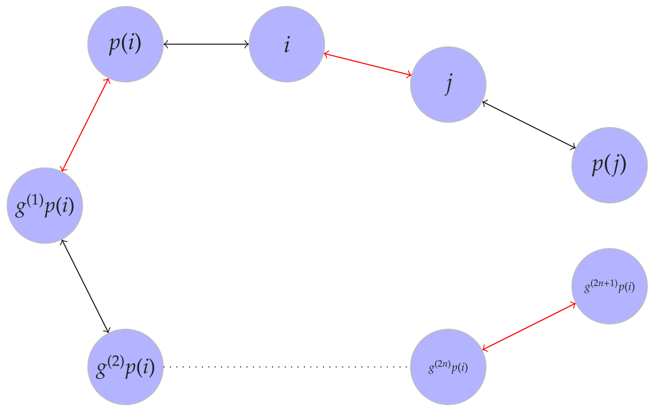


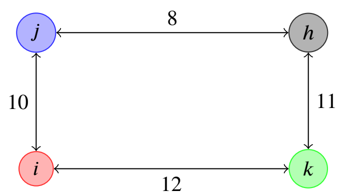
Fig. 12. List of agents under matched pairs in m and m'.

Together with the previous conditions, agent $g^{(2n)}p(i)$ has two first preferences, which is obviously a contradiction when the preference is strictly ordered. On the other hand, if the last element in the list is $g^{(2n)}p(i)$, $n \in \mathbb{Z}_+$,

$$\sum_{k=1}^{n} r_{g^{2n}p(i),g^{2n-1}p(i)} + r_{p(i),i} + r_{j,p(j)} > \sum_{k=2}^{n} r_{g^{2n-1}p(i),g^{2n-2}p(i)} + r_{g^{1}p(i),p(i)} + r_{i,j},$$

meaning that the difference parts between m and m' generated by (i, j) has a greater value in m than in m'. If all the difference between m and m' are represented in this list, $u(m) > u(m')$ accordingly. If not, combining all the results from disjoint lists, one can still achieve a result of $u(m) > u(m')$.

In sum, a stable and more efficient matching m' does not exist. m achieves the maximum system utility among all stable matchings. A toy example illustrates this result as follows.



Four agents, i, j, k, h, can be matched into two pairs. The corresponding total utility for each pair is $r_{ij} = 10$, $r_{jh} = 8$, $r_{hk} = 11$, $r_{ki} = 12$. According to Algorithm 1, we have (i, k), $(j, h) \in m$. Another matching m' includes the pairs (i, j) and (h, k). Though

$$u(m') = 21 > 20 = u(m),$$

meaning that matching m' achieves a higher utility than m, it is unstable because (i, k) is a blocking pair. □

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