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Understanding Joint Exploration: the Epistemic Positioning Underlying Collaborative Activity in a Secondary Mathematics Classroom

Erika David Parr[1] · Elizabeth B. Dyer[2] · Nessrine Machaka[3] · Christina Krist[3]

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Abstract This study examines how collaborative activity among students and the teacher to investigate disciplinary questions, which we term 'joint exploration', is established and maintained in a secondary mathematics classroom. Although collaborative and active learning is increasingly sought after in mathematics classrooms, studies of instances of joint exploration remain relatively rare. In this study, we use the theoretical perspective of positioning to conceptualize joint exploration as involving the negotiation among participants to position students with epistemic authority and agency. Using a constant comparative method, we use classroom video data of two episodes containing joint exploration and closely analyse the shifts in epistemic positioning within them. We find that shifts in epistemic positioning, especially with respect to students positioning one another with epistemic authority and exercising epistemic agency, help to support continued joint exploration. We also find that the teacher can play an important role in decentring themselves as the epistemic authority. In addition to these findings, this study contributes a distinction in epistemic authority and agency, as we explain how the two concepts are related and involved in establishing and maintaining joint exploration.

Résumé Dans cette étude, on cherche à comprendre comment l'activité collaborative entre les élèves et l'enseignant pour analyser des questions liées à la discipline, ce que nous appelons « exploration conjointe», est établie et maintenue dans une classe de mathématiques du secondaire. Bien que l'apprentissage collaboratif et actif soit de plus en plus recherché dans les classes de mathématiques, les études portant sur les exemples d'exploration conjointe restent relativement rares. Dans cette étude, nous utilisons l'approche théorique du positionnement pour conceptualiser l'exploration conjointe sur la base d'une négociation entre les participants afin de doter les élèves d'une autorité épistémique et d'une capacité d'agir. À l'aide d'une méthode comparative soutenue, nous employons des données

✉ Erika David Parr
parre@rhodes.edu

[1] Department of Mathematics & Statistics, Rhodes College, 2000 N Parkway, 38112 Memphis, TN, USA

[2] College of Education, Health and Human Sciences, The University of Tennessee, Knoxville, TN, USA

[3] College of Education, University of Illinois Urbana-Champaign, Champaign, IL, USA

vidéo montrant deux épisodes d'exploration conjointe en classe et analysons de près les changements de positionnement épistémique observés dans ces épisodes. Nous constatons que les variations de positionnement épistémique, en particulier en ce qui concerne les élèves qui s'attribuent les uns les autres une autorité épistémique et qui exercent également une capacité d'agir épistémique, contribuent à soutenir le maintien de l'exploration conjointe. Nous remarquons également que l'enseignant peut jouer un rôle important en s'éloignant de son rôle d'autorité épistémique. Au-delà de ces résultats, cette étude établit une distinction entre l'autorité épistémique et la capacité d'agir, alors que nous expliquons comment les deux concepts sont liés et impliqués dans la mise en œuvre et le maintien de l'exploration conjointe.

Keywords Epistemic authority · Epistemic agency · Group work · Mathematical exploration · Positioning

Mathematics education reform in the USA has long called for students to collaboratively engage in the broad array of practices used within mathematics (NGA & CCSSO, 2010; NCTM, 1989). To be authentically engaged in the discipline, students should have opportunities to exercise epistemic agency and authority, which emphasize their role in taking on the work of knowledge-building. Epistemic agency extends beyond notions of conceptual agency in mathematics, related to developing solution strategies and meaning of concepts (Cobb et al., 2009), by recognizing the roles students play in making decisions about the process by which ideas are constructed (Damşa et al., 2010; Stroupe, 2014). Specifically, this vision involves students making decisions as part of mathematical inquiry or exploration, including deciding which questions to pursue and how to investigate them.

Authentic engagement in mathematics, through inquiry and exploration, may provide opportunities for students to strengthen their conceptual understanding and build confidence as doers of mathematics. However, school mathematics has traditionally positioned teachers as bearers of a fixed set of disciplinary knowledge and students as recipients of this knowledge, not to be questioned, explored, or theorized about. Indeed, in this hierarchy of the teacher-student relationship, students may be constrained in exercising epistemic agency due to their status in the classroom (Amit & Fried, 2005; Louie, 2020). For students to engage in mathematical exploration requires a fundamental repositioning of the teacher and students to each other and to the disciplinary practices of mathematics itself. As many mathematics classrooms provide little opportunity for students to exercise epistemic agency or hold positions of epistemic authority, when these instances do occur, it is crucial to better understand how teachers and students interact in ways that position students as active participants, particularly during mathematical explorations.

In this paper, we examine *joint exploration* in secondary mathematics classrooms by analysing the epistemic positioning of participants. We aim to answer the following questions: *How are episodes of joint exploration established and maintained in a secondary mathematics classroom? What social and/or epistemic positions do the teacher and students take on, and how do these positions shift throughout the episodes?*

Exploration as a Form of Mathematical Activity

Mathematical exploration is centred around questions, ideas, or problems that are not sufficiently known and thus represent an intellectual need to be fulfilled (Harel, 2001). As such, exploration involves orienting toward inquiry as a fundamental component of the work (Keifert & Stevens, 2019). For students, engaging in exploration involves pursuing knowledge that is new *to them*. In addition to activity orienting toward an intellectual need, exploration also involves determining whether that intellectual need is fulfilled (Keifert & Stevens, 2019). Activities that comprise the exploration process could include (1) stating what is known and not known, (2) offering suggestions of next steps, (3) monitoring and reporting on the status of the activity to the group, and (4) confirming a solution, among others. Thus, mathematical exploration provides a context for students to actively engage in many types of authentic disciplinary work (Engle & Conant, 2002).

We view exploration as a common, essential mathematical practice, not only for mathematicians, but also for learners across levels and content areas. In the early years, researchers have documented the role of children's exploration, framed as play, in building mathematical knowledge (e.g. Vogel, 2013). At the undergraduate level and beyond, exploration is recognized an essential component in the development of mathematical arguments and proofs (Hanna, 2000), sometimes framed in terms of creativity (e.g. Regier & Savic, 2020). Exploration is also touted for its pedagogical value, as inquiry (Artigue & Blomhøj, 2013) by providing students opportunities to build conceptual and coherent mathematical knowledge (Laursen & Rasmussen, 2019).

In the USA, the standards for mathematical practices put forth by the National Council of Teachers of Mathematics allude to student exploration in many places, referencing activities including analysing givens and goals, exploring the truth of their conjectures, and asking questions to advance arguments. In fact, they state 'the very essence of studying mathematics is itself an exercise in *exploring*, conjecturing, examining and testing…' (NCTM, 1989, p. 95). At the secondary level, aspects of exploration have been studied in terms of problem posing (Cai et al., 2015; Headrick et al., 2020) and problem solving (Schindler & Bakker, 2020). Yet, documented instances of broader exploration in the secondary mathematics classroom remain relatively rare.

Collaborative Activity in Support of Exploration in Math and Science Classrooms

In this paper, we focus particularly on *joint* exploration, highlighting the instances in which students participate *collaboratively*. Collaboration as a site of learning in mathematics and science classrooms has been given extensive attention in recent years (e.g. Amit & Fried, 2005; Cohen & Lotan, 2014; Dunleavy, 2018; Esmonde & Langer-Osuna, 2013; Kotsopoulos 2014; Wood, 2013). Prior research on collaborative activity has shown that students working together on the same task in small groups (group work) can promote learning effectively (e.g. Johnson & Johnson, 1999; Smith & Confrey, 1991; Webb, 1982). For group work to be effective, students must communicate with each other and the task in specific ways. Students who successfully collaborate in group work (1) listen and make sense of other group members' ideas, (2) negotiate and decide upon goals or a plan of action to take, (3) discuss and resolve differing ideas or perspectives, and (4) recognize connections, similarities, and differences among multiple representations, strategies, or perspectives of group members (e.g. Koichu et al., 2021; Kontorovich et al., 2012; Rummel & Spada, 2005; Volet et al., 2009).

Collaborative activity often supports the goals of exploration. Working with collaborators, rather than individually, may increase the diversity of ideas shared (Abdu & Schwarz, 2020), which benefits exploration, even if some ideas do not end up being pursued (Koichu et al., 2021). Despite the potential for collaborative activity to be an effective means of promoting exploration, students may not always take up productive ideas to advance the group's goals (e.g. Barron, 2003; Koichu et al., 2021). Status may influence whose ideas are taken up (Langer-Osuna, 2016; Adams-Wiggins et al., 2020), and some students may be silenced in groups, limiting their ideas (Kotsopoulos, 2014). As Wood (2016) found, students' perceived status in terms of mathematical knowledge may influence whether they choose to engage in exploration activities, or whether they fall back to 'ritual activity', including copying answers from the perceived authority in the group to meet a teacher's expectations.

Our focus on joint exploration in this study helps to uncover the complex interactional and social dynamics involved in establishing and maintaining opportunities for exploration, given that it is not normative for mathematics learning to incorporate this activity. Thus, we claim that joint exploration, which involves collaborative activity, likely necessitates re-negotiating the positions and participatory structures of traditional classroom contexts that distribute the social and intellectual authority and agency to students (Ko & Krist, 2019; Ha & Kim, 2021). To investigate these interactions related to epistemic agency and authority, we apply the theoretical lens of positioning.

Positioning Theory as a Theoretical Lens

Positioning theory considers both social and intellectual roles and authority in analysing interaction. It highlights the interactional nature of activity, which is afforded and constrained by normative possibilities of the authority and responsibilities associated with different roles (Davies & Harré, 1990). From this perspective, participants take on roles or positions, which afford them specific ways of acting and recognition among participants. These positions are flexible and continuously re-negotiated during interaction (Esmonde, 2009). Shifts in positions during interaction tend to indicate important moments of activity because they typically involve participants negotiating and coordinating roles. We use the theoretical construct of positions in ways that align with others' use of positional framing (e.g. Ha & Kim, 2021; Shim & Kim, 2018; van de Sande & Greeno, 2012).

Positioning theory has been used to analyse discourse in mathematics classrooms and highlight the role of identity and power in micro-interactions among students and the teacher (Herbel-Eisenmann et al., 2017). For example, research on epistemic positioning in mathematics classrooms has identified two main positions that students and the teacher commonly take up during mathematical activity: (1) a *knower* or *source* who provides mathematical information and (2) an *actor* who performs an action doing mathematics (González & DeJarnette, 2015; Lo & Ruef, 2020; van de Sande & Greeno, 2012). Within these positions, participants can take on primary or secondary roles depending on whether they provide or request knowledge or the activity to be completed.

We also leverage the constructs of authority, agency, and status to account for the various positions students and teachers take on in classrooms, and how these positions may shift. By *authority*, we refer to the social power to give information or request actions in a way that is unquestioned by others. Specifically, *epistemic authority* refers to the authority to generate and validate knowledge (Engle et al., 2014), which normatively resides with the teacher (Forman & Ford, 2014). In our view, the traditional teacher-student relationship comprises an inherent hierarchical structuring within the sociocultural context of a classroom. In this hierarchy, the teacher is in a position of authority, viewed by the students as the conduit of disciplinary knowledge creating an inherent asymmetry. Further, the teacher decides what happens in the classroom in terms of what tasks are to be worked on, and gives directions to control the activity in the classroom (Mercer & Dawes, 2008). Applying the positions of *knower* and *actor* to this hierarchy, the teacher is, by default, in the position of a primary knower, who provides or confirms information, and a secondary actor, who directs the students' activity (González & DeJarnette, 2015; Lo & Ruef, 2020). However, even these default positions are continuously negotiated among the teacher and students, and may give way to shifts from these default positions (e.g. Lo & Ruef, 2020).

Related to authority, *agency* refers to the capacity of an individual to behave in a way that is self-directed, which is both afforded and constrained by broader social dynamics and altered by individual behaviour (e.g. Fu & Clarke, 2020). Specifically, when students exercise *epistemic agency* in the classroom, they direct their own knowledge building (Ko & Krist, 2019; Damşa et al., 2010; Stroupe, 2014), including posing problems, suggesting novel solutions, and testing uncertain claims. Exercising epistemic agency involves a command over certain disciplinary practices and viewing oneself as capable of engaging in the work of the discipline (Basu & Calabrese Barton, 2009). When a student exercises epistemic agency, they share in the epistemic authority traditionally held by the teacher (Miller et al., 2018). Yet, exercising epistemic agency extends beyond assuming authority; a student may assume epistemic authority through confidence in his own correct solution to a problem, whereas a student who exercise epistemic agency decides when and how to find a solution, or even which problem to solve. We see epistemic agency as related to the notion of intellectual autonomy used by Yackel and Cobb (1996), which supports students' capacity to participate in a community of inquirers. Their definition of intellectual autonomy stems from Piaget's (1948/1973) work and includes the ability to think for oneself, and decide between what is true and untrue and what constitutes a mathematical contribution. We view a difference in emphasis in the two terms: intellectual autonomy emphasizes the independence of the individual in their intellectual reasoning, whereas epistemic agency centres the activity of the individual and an individual's power to guide that activity in their environment to seek and validate knowledge.

When a student exercises epistemic agency, such as in cases of joint exploration, the position of primary knower may temporarily shift to the student, meaning a student's *status* in the classroom, or position relative to the teacher, may increase. Such a shift would be a temporary reversal in the typical arrangement in which the teacher is the primary knower. Yet, if such a reversal occurs, it does so within the broader context of the normative social backdrop of the classroom. Thus, a temporary reversal in status between teacher and student(s) may occur while the overall teacher-student hierarchy remains. Although who is teacher and who is student cannot fundamentally change, who may have or share in epistemic authority or exercise epistemic agency may be negotiated. In this paper, we describe how these positions, through such interactional negotiations, can be altered or upended from the default arrangement during instances of joint exploration.

In particular, we hypothesize that the presence of the teacher in a group interaction could both support and constrain students to be actively involved in joint exploration. We hypothesize that when a teacher seeks to support students in engaging in exploration, the inherent asymmetry of the teacher-student hierarchy needs to be challenged by positioning students in roles that are associated with intellectual and social authority and agency. Therefore, in this paper, we present two episodes in which the teacher differentially influences moments of joint exploration by sharing or releasing authority.

Methods

Data and Episode Selection

We analysed two comparative episodes of joint exploration drawn from a large classroom video dataset collected by the second author (Dyer, 2016) investigating secondary mathematics teachers' responsiveness to student thinking. In this study, we use classroom video and audio data from one Integrated Mathematics III class at a selective enrolment public school in a large urban school district in the Midwestern United States. The majority of students in the class were in grade 11, with some in grades 10 and 12. The course used the Interactive Mathematics Program curriculum (Fendel et al., 2011, 2012), which covers advanced algebra and trigonometry, including units on trigonometric, exponential, and logarithmic functions. This classroom was selected for analysis because the teacher, Mrs. Perry, devoted large portions of class time for students to work in small groups solving problems collaboratively. In fact, students' desks were grouped in fours, facing each other, around the room to support collaboration. Additionally, previous research has documented that Mrs. Perry's teaching practice is responsive to student thinking (Dyer & Sherin, 2016). We believed that both of these factors, the structure of the class and Mrs. Perry's responsiveness, would make it more likely for joint exploration to occur with the teacher present.

Data from Mrs. Perry's classroom included 10 videotaped 100-min lessons, filmed approximately every week for the final 3 months of school, which corresponded to about a quarter of her lessons during that time period due to block scheduling. Video was collected from three different angles and a separate audio was captured for each group of students. We selected two lessons, one from each of the first 2 months, from units of instruction on (1) trigonometric functions and (2) exponential and logarithmic functions. These lessons were selected for further examination as they included significant portions of the lesson devoted to groupwork, increasing the potential of uncovering instances of joint exploration.

Two episodes of joint exploration were selected, one from each of the two lessons by watching video and listening to audio of each group and identifying potential instances of joint exploration as a sensitizing concept (Blumer, 1954) in which the teacher was present with the group. First, episodes in which the teacher was interacting with a group for longer than 1 min were identified. In total, nine episodes from the first lesson and 11 from the second were analysed for possible joint exploration. We note that these episodes may have included the teacher briefly shifting her attention from the group, provided the teacher returned to the group. Then, audio and video were analysed for instances of joint exploration. We defined instances of *joint exploration* as the collaborative activity of investigating disciplinary questions and ideas among students and/or teachers through interaction. Thus, our analytic criteria specified that instances must include (a) at least two participants contributing substantively, either intellectually or socially, to the group's sensemaking activity, and (b) participants seeking to construct new knowledge (to them) related to a content idea or question to fulfil an intellectual need.

We selected two contrasting cases based on the different types of participation from the teacher in each of these episodes, as we hypothesized the teachers' participation would have significant influence on how episodes of joint exploration are established and maintained. In the first episode, Mrs. Perry serves as a guide to the students as they seek to answer a question she designed, while in the second, she is engaged in answering a spontaneous question posed by a student, alongside the students in the group.

Episode Analysis

We used a constant comparative method (Strauss & Corbin, 1990) from the tradition of grounded theory that involved constructing rich descriptions of the epistemic positioning for each episode. We bounded episodes by starting with the teacher approaching the student(s) and ending when she left to interact with another group. We created transcripts of each episode, which we used in tandem with the video and audio records in all subsequent analyses. Two authors independently wrote descriptive accounts of the positioning according to the two dimensions of epistemic authority and epistemic agency at the beginning, during, and at the conclusion of joint exploration. Specifically, we centred our descriptions around the questions shown in Table 1, which served as an analytic framework.

Table 1 contains questions about epistemic positioning, in relation to epistemic authority and epistemic agency. In our analysis, we saw three distinct stages of joint exploration, relative to these two dimensions: initiation, process, and conclusion. In the initiation stage, group members decide what problem to explore and who or what to seek for knowledge in the exploration. At this first stage, relevant aspects of epistemic positioning include who is making decisions about what to explore and who is being sought for knowledge. In the process stage, group members devise and execute a plan for exploration. Within the process stage, members may take up, reject, or ignore others' ideas, and express varying levels of certainty. The conclusion stage involves confirming that a solution has been found. At this final stage, relevant aspects of epistemic positioning include who decides the exploration is completed and who confirms that knowledge that was previously unknown or uncertain is now known. The questions associated with each stage of joint exploration are the ones likely to be most relevant in that stage, but need not be limited exclusively to that stage of exploration.

Table 1 Our analytic framework: two dimensions of epistemic positioning analysed in episodes of joint exploration

		Dimension analysed	
		Epistemic authority	Epistemic agency
Stage of joint exploration	Initiation	Who is seeking knowledge? Who (or what) is being sought for knowledge?	Who decides what knowledge to seek?
	Process	What is an individual's level of certainty about the knowledge? Who contributes ideas? Whose ideas are solicited and/or taken up?	Who decides how that knowledge is sought? Who takes up the search for knowledge? Who decides which ideas to take up?
	Conclusion	Who confirms the knowledge as certain?	Who decides a search has been satisfactory?

We used our descriptive accounts of positioning to identify portions of the episodes in which the positioning of a participant shifted. We considered a shift in epistemic positioning to be anytime there was a change in an answer to one of the questions listed in Table 1, or a change from the roles of the traditional teacher-student hierarchy. These shifts included the following seven types:

1. An individual being newly sought as a source of knowledge (initiation; shift in authority)
2. An individual expressing a change from a state of certainty to uncertainty (or vice versa) about a piece of information (process; shift in authority)
3. A change in who contributes ideas or is solicited for ideas (process; shift in authority)
4. A change in who confirmed information as valid (conclusion; shift in authority)
5. A change in who decides what knowledge to seek (initiation; shift in agency)
6. A change in who decides how the knowledge is sought (process; shift in agency)
7. A change in who decides when a search is satisfactory (conclusion; shift in agency)

The authors of the analytic descriptions discussed and refined their descriptions with all authors and compared their analysis throughout the stages of the two selected episodes.

We note that the *identities* of students (and the teacher), including their race, ethnicity, gender, and socio-economic status, are fundamental components to the broader dynamics of their social interaction in mathematics classrooms (e.g. Ehrenfeld & Heyd-Metzuyanim, 2019), especially as related to authority and agency in learning environments (Langer-Osuna 2017; Lerman, 2012). In this study, we have chosen to focus our analysis of these episodes locally in terms of how shifts in positioning contribute to instances of joint exploration to conduct a more focused investigation, and do not analyse the role of the identities of students.

Results

We present two contrasting episodes of joint exploration and report shifts in epistemic positioning that we identified in each episode. These two episodes differ in the teacher's participation in the joint exploration, the extent to which the teacher's epistemic authority is shared or released and the extent to which students exercise epistemic agency.

Episode I: Solve a Cosine Equation Using a Graph

The first episode involves a group of three students, Ellie, Nick, and Theo, and the teacher, Mrs. Perry, jointly exploring a task that she provided to the class. This task asked students to solve $-15 = 20\cos(30x)$ for the portion of the function shown in the provided graph (Fig. 1a), where x is measured in degrees. Using the inverse cosine function to solve the equation yields one solution ($x \approx 4.62$). Students could then use the graph and the period of the cosine function (12) to find the remaining three solutions ($x \approx 16.62, -4.62, 7.38$) by locating the x-values of the points of intersection of the function and the line $y = -15$ (Fig. 1b). We note the mathematical richness of this task, allowing students to explore and use the periodicity and symmetry of the graph of the cosine function to find additional solutions, rather than applying a rote procedure of adding to or subtracting from the period.

This episode involves joint exploration, as the students and Mrs. Perry work collaboratively to find the remaining solutions. Each participant contributes by posing questions about the task, offering solution strategies, or directing the next steps of the activity to advance the search for remaining solutions. We present three main shifts in epistemic positioning that served to establish and maintain joint exploration

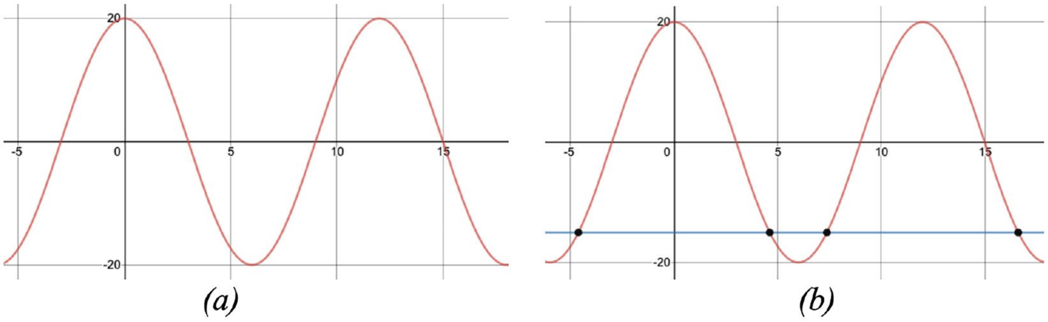

Fig. 1 Solve a cosine equation graph provided (**a**) and its solutions (**b**)

below. This episode comes from an 11-min interaction between Mrs. Perry and this group, with the interaction reported occurring over 3.5 min.

Initiation—Ellie Positions Nick as Having Epistemic Authority

The exchange below initiates the episode of joint exploration. The first shift involves a change in who is sought for knowledge (type 1). This shift occurs when Ellie looks to Mrs. Perry, who is standing up to briefly leave, as a source of knowledge to confirm her solution. As Mrs. Perry leaves to attend to another student, Ellie then turns to Nick to confirm her answer of 4.61.

> Ellie:Yes, I don't know if I'm right, is this right? (*moves paper towards Mrs. Perry who stands up to briefly leave group, then turns to Nick*) What did you get Nick?
> Nick: I got 4.62, 16.62
> Ellie: Yesss (*exclaims, holds both fist in the air*). Wait, I just got 4.61. (*Mrs. Perry returns*)
> Nick: You're supposed to find all this, just one more. All you have to do is add the period…
> Ellie: But how do you make that into like an equation so I can solve it?
> Nick:(*Stands up, leaning over Ellie's work across from him*) You don't necessarily, … put it into the equation. What you can do… knowing that this is going to be repeating over and over again … so say you want here… you would add this to 12.

At first, Ellie seeks confirmation for her answer on the task by looking to the teacher, Mrs. Perry. When Mrs. Perry leaves, Ellie instead turns to Nick to confirm her answer, indicating a shift in Nick's status relative to Ellie. Ellie cheers as Nick gives an equivalent answer to hers, indicating that she considers his solution to be confirmation of hers, but then questions him when he lists a second answer. Ellie continues to ask Nick to explain how he found the second solution, further considering him a valid source of information. Nick offers an explanation involving adding the period, 12, to the first solution, which he justifies by explaining that the graph is 'repeating'. In this exchange, not only does Ellie position Nick as having epistemic authority, but also Nick fulfils this role by responding to Ellie with his own solutions, explaining how to find the additional solution he found, notably, with Mrs. Perry present. Nick even stands up as the interaction progresses, a physical display of a higher status, coinciding with his continued knowledge sharing.

Process—Mrs. Perry Shares Epistemic Authority

The second shift that occurs in this episode of joint exploration involves Mrs. Perry sharing epistemic authority with the students in the group by repositioning herself as one who gives directives without giving mathematical information, thus implicitly positioning the students with epistemic authority. In doing so, she helps to sustain the joint exploration in this exchange. This change in positioning is a shift in who contributes ideas (type 3). When Mrs. Perry re-enters the conversation, she continues to generally redirect the students to the graph, rather than providing a clear next step.

Nick: So, you just, I want to say you add to the period, but I know I'm wrong because that would get to here.
Mrs. Perry: Well, where's the one, what x does that say? (*pointing to Nick's work*)
Nick: 4.61, I got 4.62 because I rounded.
Mrs. Perry: Can you like find that, mark that on the x-axis where that is?
Nick:That would be about, let's assume, here (*writing on paper*) …
Mrs. Perry: Maybe you should mark that on there, like you were about to, because maybe that will help you to think about, okay, how can I?
Nick:Because I know you can add to the period to get this
Mrs. Perry: Okay so that will get you that one [the second solution, 16.62], so you could at least get that one by adding the period and we just have to figure out how to get the other two.

Mrs. Perry's moves to reposition herself include clarifying which x-value the students are referring to and directing them to mark that value on the x-axis, a hint to use the graph provided more fully. Mrs. Perry could have given the students the solution strategy to find the remaining two solutions, or more pointed guidance to look specifically at the symmetry of the graph over the y-axis. Instead, Mrs. Perry affirms Nick's proposed solution of adding the period to the first solution to find a second solution. Then, she summarizes what is left to find, the remaining two solutions, rather than providing more specific epistemic guidance. These moves position Nick and the group of students with authority to continue the exploration, independent of Mrs. Perry as the epistemic authority. In this process, Mrs. Perry creates opportunities for the students to exercise agency in the path to the remaining solutions.

Process and Conclusion—Theo and Nick Exercise Agency and Position Each Other with Epistemic Authority

The third set of shifts in positioning occurs when Theo enters the conversation and begins to work with Nick to find the other solutions. In doing so, Nick and Theo shift to position each other as valid sources of information (type 3). As Nick poses the question of how to find the remaining solutions, in addition to 4.62 and 16.62, Theo offers a solution strategy that Nick takes up. By offering a strategy and taking it up, Theo and Nick exercise agency in the exploration process (type 6). These shifts in positioning, specifically in agency and authority, also serve to maintain the joint exploration.

Nick: How would you solve for that one, though?
Theo: You would add 12
Nick: When you're adding 12, you're just going through an entire period, 6 you're going through half a period
Theo: Yes, which wouldn't work because it's a cosine...Oh, could you add the three? Possibly?
Nick: A fourth of the period?
Theo: Yeah.
Nick: Let's find out! (*typing into calculator*) No, that's not what I wanted...Nope!... There we go, that's another one.

When Nick asks, 'how would you solve for that one, though?' Theo responds with an answer, although it is restating the strategy Nick was employing, adding one period (12) to the first solution. This continues the joint exploration between Nick and Theo as they search for a way to find the remaining two solutions. Theo then proposes a strategy of adding 3, which he softens with a tone of inflection and the addition of 'possibly'. Theo's proposed strategy, though uncertain (and ultimately incorrect), indicates an important shift in his agency and authority—he has moved from restating a previous strategy to providing a new strategy of his own accord. Nick reinforces Theo's status of epistemic authority when he takes up this strategy. Nick also displays epistemic authority, by reframing the strategy relative to the graph as 'a fourth of the period', and exercises agency by deciding to test the strategy using a calculator. Nick reports back as he continues to use the calculator and eventually seems to have success. Nick and Theo together take up the epistemic authority shared with them and exercise their agency to find the remaining solutions. In the process, Nick and Theo appear to consider each other as matched in status, relative to their epistemic authority. Theo's offer of a possible strategy and Nick's uptake of this strategy maintain the exploration, which ends when Nick finds another solution.

Episode II: 'Is There a Natural Logarithm that is Equivalent to e?'

The second episode of joint exploration comes from a different day of class and involves a student, James, posing a problem of his own creation related to the concept of logarithms. James calls for Mrs. Perry's attention while she is at his group's desks and asks her whether there is a number whose natural logarithm is equivalent to e (there is, the number e^e, approximately 15.15, has a natural logarithm which equals e). This question does not appear to be directly from the homework assignment they were going over at the time. For context, Mrs. Perry interacted with the group for about 4 min and did not leave the group in that time. This episode of joint exploration lasts about 1 min and 40 s in its entirety.

We consider this episode to be an instance of joint exploration among Mrs. Perry, James, and a third student, Sergey, seated beside James. Each participant contributes intellectually (with information) or socially (with directives) to answer the question posed by James. Further, the participants are all uncertain of their proposed answers at first and seek to fulfil an intellectual need. We present four main shifts in epistemic positioning that served to establish and maintain joint exploration below.

Initiation—James' Spontaneous Problem Posing as an Act of Epistemic Agency

The question that James poses at the beginning of the episode, which establishes the episode of joint exploration, is an act of exercising epistemic agency, as it is a shift in who decides what knowledge to seek (type 5). James frames his question as 'messing around' as he asks it and looks to Mrs. Perry for an answer.

> James: Is there, I've just been messing around a little bit. Is there a natural log that is equivalent to e? (*looking at calculator, then looks up at Mrs. Perry, resting head on hand*) Like 15 point something?

The question James poses is one derived from his own exploration, not from a homework or classwork question. Because this question is student-sourced, and not from the tasks Mrs. Perry asked them to be working on, James posing this question is a proposed alteration of the structure of the lesson and what is 'supposed to be happening' during this portion of class. James appears to recognize that posing this question could make him appear to be 'off-task', and frames the exploration that led him to this question as 'messing around', perhaps to soften such negotiation.

Despite his dismissive qualification, James' question itself indicates a deeper conceptual pondering about the natural logarithm. James appears to have a decimal approximation for the answer to his question, '15 point something', but may not yet have the exact number (which is e^e). From our perspective, James' question is a significant one, which can be used to build conceptual understanding. By posing this question, James exercises epistemic agency as his question is one of self-directed activity. His willingness to pose the question and risk appearing off-task to seek an answer indicates the intellectual merit he places on the question and, by extension, his own mathematical reasoning.

Process—Mrs. Perry's Initial Release of Epistemic Authority

Mrs. Perry's initial reaction to James' question involves a change in epistemic positioning that serves to support the joint exploration. This shift involved a release of epistemic authority by Mrs. Perry, as she changes from being perceived as having knowledge to expressing uncertainty (type 2).

> James: Is there, I've just been messing around a little bit. Is there a natural log that is equivalent to *e*? (*looking at calculator, then looks up at Mrs. Perry, resting head on hand*) Like 15 point something?
> Mrs. Perry: (*slowly*) Is there a natural log that is equivalent to *e*? (*pauses, steps back,*) You mean like, you take the natural log of something and you get *e*? (*looking at James, who nods*) Is that what you mean? (*pauses, puts hand up to mouth and then brings it down*) Ahhh so... (*leans slightly back briefly writing in the air*)

The interaction begins with James positioning Mrs. Perry as an epistemic authority, asking her this question and looking to her for an answer. Notably, James *does* have an accurate estimate to answer his question when he poses it. Yet, he is not certain and looks to Mrs. Perry as a source of knowledge to confirm his hypothesis of the existence of a number whose natural logarithm is equivalent to *e*. This is followed by Mrs. Perry restating James' question, pausing, stepping back, and leaning backward, away from James and Sergey, seated beside James. Mrs. Perry's words and movement seem to suggest that the answer is unknown to her, which stands in contrast to her as the epistemic authority. By pausing to consider the question, Mrs. Perry creates an opportunity for James and Sergey to take up the epistemic authority within the conversation and exercise agency to explore it. Mrs. Perry continues to release authority as the episode unfolds.

Process—James' Repositioning of Sergey and Himself as Epistemic Authorities and Exercise Agency

As Mrs. Perry shifts away from being an epistemic authority, James and Sergey position each other as epistemic authorities. This is a shift in whose ideas are taken up (type 3). In response to James' question, Sergey joins the conversation and affirms the existence of such a value, continuing the exploration. As the conversation unfolds, James decides to use a calculator to support his search, which is a change in who decides how knowledge is sought (type 6).

> Sergey: I mean, yeah, cuz it'd be a power.
> James: Of *e*, right? (Mrs. Perry: yeah)
> Sergey: Would it just be 1? (*looking at James*)
> James: It would be *e* to the power of *e* (*looking at Mrs. Perry*)
> Sergey: So the log, log base *e*
> James: Oh, log of *e* to the *e* (*pauses*) is *e*? (*looking at Sergey, laughs*) Wait a second, is that right? Lemme check. (*picks up calculator*)
> Mrs. Perry: Well wait, write it down, write it down. I can't think right. I have to see it. (*bending down to table, leaning over student work*) So,
> James: I'm wondering, so I think we just figured out that (*writing*) log base *e*. … It would be ln of *e* to the *e*. Okay. So, what's wait. Does that, does that work?
> Mrs. Perry: That seems right. Log (*looking at work*)
> James: Let's try that so (*typing into calculator*) *e* to the *e* to the 1

Mrs. Perry: Waaait no (*long pause, bends head all the way forward*)

Sergey supports James in exploring his question. He first affirms that such a value would exist, explaining that 'it would be a power [of *e*]'. Sergey initially offers 1 as a solution, which James quickly disregards. Sergey is undeterred, however, in contributing information. Sergey reminds James that he is interested in the natural logarithm (base *e*), rather than the common logarithm, and reorients James to this in several instances, which James acknowledges and takes up. Notably, James shifts his gaze from Mrs. Perry to Sergey, indicating a shift from positioning Mrs. Perry as an authority to Sergey as supporting his exploration and as a valid contributor of knowledge. James begins to talk through his hypothesis that 'log of *e* to the *e* is *e*' and decides to use his calculator to confirm his proposed solution, repositioning himself as an epistemic authority, rather than rely on Mrs. Perry for confirmation.

In this exchange, Mrs. Perry also continues to release epistemic authority. When she does direct James to 'write it down', she seems to do so to catch up on the students' thinking, explaining she 'can't think right' and needs 'to see it'. Her final comment of 'waaait no' as she bends her head forward appears to be a final release of epistemic authority, as the students are not deterred by her 'bowing out' of the exploration, but continue to confirm James' proposed solution. Rather than participate directly in the joint exploration, Mrs. Perry essentially removes herself from the discourse, allowing James and Sergey to conclude the exploration themselves.

Conclusion—James' and Sergey's Status Begins to Even Out

As James and Sergey continue to take up the epistemic authority in the interaction, their intellectual status relative to each other appears to become matched. Their newfound status and authority are a shift in who traditionally confirms information as valid in a classroom, and a shift from the beginning of the episode just two minutes earlier in who is sought for knowledge. This shift can be seen as they reach an answer that they are both satisfied with (types 4 and 7).

James: *e* to the *e* to the 1. Yeah, fifteen point, yeah, there it is. And then log of that is *e*
Sergey: No (James: Just kidding) *natural* log of that (*looking at James' work*)
James: (*mumbles*) ln. Yeah, my bad.
Sergey: Yeah (*looking at work*)
James: Yeah

Prior to this exchange, Sergey appeared to join the conversation with lower status relative to James. For instance, when he offered 1 as a solution, James disregarded it without comment, rather than take it up. In this later exchange above, James acknowledges Sergey's contribution of clarifying that they were in fact referring to the *natural* log (or log base e) rather than the (common) logarithm. This time, James readily takes up Sergey's correction and acknowledges his mistake with 'yeah, my bad'. Their mutual affirmations of 'yeah' at the end of this episode indicate that they are confident in their solution to the initial question. Furthermore, these affirmations acknowledge each other's confirmation as well, indicating an even intellectual status. At the conclusion of the exploration, Sergey and James arrive at a solution they were both satisfied with, without any confirmation from Mrs. Perry, who had already begun to move on to working with the other two students of the group.

Comparison of Two Episodes

Both episodes of joint exploration share common features in how they are established and the shifts in positioning that maintained them. In both episodes, exploration is initiated by a student asking Mrs. Perry a question, positioning her as an epistemic authority. Mrs. Perry's responses in both instances, either briefly leaving, or restating the question, are a shift away from her being positioned as the epistemic authority and coincide with the start of joint exploration. In both episodes, joint exploration is maintained by two shifts in positioning: (1) Mrs. Perry, the teacher, redistributing authority by repositioning the students as capable of seeking the answer; and (2) the students (Nick and Theo in Episode I and James and Sergey in Episode II) positioning themselves and each other as epistemic authorities in the situation, and exercising epistemic agency, as they both exchanged ideas and took up the ideas offered.

The extent to which Mrs. Perry releases epistemic authority to the groups, however, differs between episodes. In episode 1, Mrs. Perry supports the students in working through the assigned task by directing Nick to mark his solution at a particular place on the graph and giving the status of the group's work. In doing so, Mrs. Perry shared some of the epistemic authority with the students, yet still seemed in command of the material as she guided the students toward the next step, implying she held the solutions to the task. However, in the second episode, Mrs. Perry appeared to release her epistemic authority almost entirely after James asked a question to which she does not immediately know the answer. The moment when Mrs. Perry says, 'waaait no' as she leans her head down, effectively bowing out of the conversation, appears to be when she fully releases her epistemic authority, as James and Sergey are not deterred by her refutation. Rather, they continue toward their solution in spite of her lack of direction or affirmation.

In both episodes, joint exploration ended when a student became the one to determine whether knowledge was correct, without confirmation from the teacher. In both episodes, students were able to resolve their own uncertainties in the presence of the teacher, without relying on her as the source of knowledge. Additionally, as students worked to resolve their own uncertainty, they utilized calculators as tools to verify their proposed solutions. While the students had to determine what to input into the calculator, and how to interpret the output, the calculator was positioned by students as an external authority for validating knowledge.

Discussion and Conclusion

These episodes are encouraging as they indicate that joint exploration can indeed occur in secondary mathematics classrooms, even with the teacher present. In fact, our findings suggest that the teacher's actions may help to establish and maintain joint exploration. In both episodes, joint exploration was initiated by students with the teacher present and acting in ways that implicitly decentred herself as the epistemic authority and primary director of activity. It is not clear that this was the teacher's intended purpose in leaving the group briefly or expressing hesitation in response to James' question. This suggests that the teacher's role in fostering joint exploration can be subtle.

The findings also show the teacher positioning students in more explicit ways while maintaining joint exploration. For example, the teacher summarized what solutions the students knew, framed the group's next steps, and restated a student's question with more clarity. These actions do not position students to provide information they already know about the problem, which would reflect the 'knower' or 'source' position in the literature (González & DeJarnette, 2015; Lo & Ruef, 2020; van de Sande & Greeno, 2012). Instead, these findings suggest that an additional position, beyond the positions of knower and actor, might better characterize the roles subtly implied by the teacher, which are taken up by students during the episodes of joint exploration. Future research could investigate whether coding schemes for positions of participants in groupwork (e.g., González & DeJarnette, 2015; Lo & Ruef, 2020) can be extended beyond the positions of knowers and actors, perhaps to include the role of 'explorers'.

Future research may also investigate factors associated with which students become positioned with epistemic authority and agency, such as the role of identity, in terms of race, ethnicity, gender, and socio-economic status. As others have shown (e.g. Langer-Osuna, 2016), students' identities greatly influence who becomes positioned with power in collaborative situations. Future studies of repeated instances of joint exploration over time may indicate how students develop identities as agents of their own mathematical understanding (Boaler & Greeno, 2000). We offer our analytical framework of epistemic agency and authority as a starting to point for such studies.

Finally, we return to the notion of the teacher-student hierarchy described earlier. In the two episodes we analysed, we contend that the repositioning that occurred served to flatten the hierarchy between teacher and student. This repositioning involved the teacher sharing or releasing authority, and the students exercising epistemic agency. In the first episode, as Mrs. Perry shared her epistemic authority with Nick and the other students, she elevated their intellectual status relative to herself and the discipline of mathematics. In episode 2, we observed a more extreme flattening of the hierarchy among Mrs. Perry and James and Sergey. Mrs. Perry released her epistemic authority, essentially admitting she did not have the answer that James was seeking with Sergey. Following this release of authority, James and Sergey exercised epistemic agency to validate their hypothesized solution to James' question, arriving at a satisfactory conclusion without confirmation from Mrs. Perry. We do not view Mrs. Perry's actions in this moment as negative. Rather, we highlight that the repositioning moves that she employed, either intentionally or not, served to support students in jointly exploring a significant question. We consider both episodes to be examples of how joint exploration may unfold in a secondary mathematics classroom, suggesting the positive impact of repositioning students with epistemic authority and agency. In order for students to exercise epistemic agency, then, we contend that a teacher must share or release her authority to the students. We also theorize that a teacher sharing authority with students does not guarantee that students will act in ways that are self-directed; such repositioning likely must be reinforced consistently in order for students, with years of schooling experience, to become agents of their own disciplinary learning.

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Declarations

Conflict of Interest The authors declare no competing interests

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References

Abdu, R., & Schwarz, B. (2020). Split up, but stay together: Collaboration and cooperation in mathematical problem solving. *Instructional Science*, *48*, 313–336. https://doi.org/10.1007/s11251-020-09512-7

Adams-Wiggins, K. R., Myers, M. N., & Dancis, J. S. (2020). Negotiating status hierarchies in middle school inquiry science: implications for marginal non-participation. *Instructional Science*, *48,* 427-451. https://doi.org/10.1007/s11251-020-09514-5

Amit, M., & Fried M. N. (2005). Authority and authority relations in mathematics education: A view from an 8th grade classroom. *Educational Studies in Mathematics, 58*, 145-168. https://doi.org/10.1007/s10649-005-3618-2

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education, 45*, 797-810. https://doi.org/10.1007/s11858-013-0506-6

Barron, B. (2003). When smart groups fail. *Journal of the Learning Sciences*, *12*(3), 307–359. https://doi.org/10.1207/S15327809JLS1203_1

Basu, S.J., & Calabrese Barton, A. (2009). Critical physics agency: further unraveling the intersections of subject matter knowledge, learning, and taking action. *Cultural Studies of Science Education*, *4*, 387–392. https://doi.org/10.1007/s11422-008-9155-4

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45–82). Ablex.

Blumer, H. (1954). What is wrong with social theory? *American Sociological Review*, *19*(1), 3–10. https://doi.org/10.2307/2088165

Cai J., Hwang S., Jiang C., & Silber S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In F. F. Singer, N. Ellerton & J. Cai (Eds.), *Mathematical Problem Posing. Research in Mathematics Education* (pp. 3–34). Springer. https://doi.org/10.1007/978-1-4614-6258-3_1

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40–68. http://www.jstor.org/stable/40539320

Cohen, E. G., & Lotan, R. A. (2014). *Designing groupwork: Strategies for the heterogeneous classroom*. Teachers College Press.

Damşa, C. I., Kirschner, P. A., Andriessen, J. E., Erkens, G., & Sins, P. H. (2010). Shared epistemic agency: An empirical study of an emergent construct. *The Journal of the Learning Sciences*, *19*(2), 143–186. https://doi.org/10.1080/10508401003708381

Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour*, *20*(1), 43–63. https://doi.org/10.1111/j.1468-5914.1990.tb00174.x

Dunleavy, T. (2018). High school algebra students busting the myth about mathematical smartness: Counterstories to the dominant narrative "get it quick and get it right." *Education Sciences, 8*(2). https://doi.org/10.3390/educsci8020058

Dyer, E. B. (2016). *Learning through teaching: An exploration of teachers' use of everyday classroom experiences as feedback to develop responsive teaching in mathematics* [Dissertation, Northwestern University]. http://gradworks.umi.com/10/16/10160667.html

Dyer, E. B., & Sherin, M. G. (2016). *Instructional reasoning about interpretations of student thinking that supports responsive teaching in secondary mathematics. ZDM, 48*(1–2), 69–82. https://doi.org/10.1007/s11858-015-0740-1

Ehrenfeld, N. & Heyd-Metzuyanim, E. (2019). Intellective identities in the construction of a hybrid discourse: The case of an ultra-orthodox Jewish mathematics classroom. *International Journal of Science and Mathematics Education*, *17*, 739-757. https://doi.org/10.1007/s10763-018-9885-z

Engle, R. A., & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners classroom. *Cognition and Instruction*, *20*(4), 399–483. https://doi.org/10.1207/S1532690XCI2004_1

Engle, R. A., Langer-Osuna, J. M., & de Royston, M. M. (2014). Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument. *Journal of the Learning Sciences*, *23*(2), 245-268. https://doi.org/10.1080/10508406.2014.883979

Esmonde, I. (2009). Ideas and identities: Supporting equity in cooperative mathematics learning. *Review of Educational Research*, *79*(2), 1008-1043. https://doi.org/10.3102/0034654309332562

Esmonde, I., & Langer-Osuna, J. (2013). Power in numbers: Student participation in mathematical discussions in heterogeneous spaces. *Journal for Research in Mathematics Education, 44*(1), 288–315. https://doi.org/10.5951/jresematheduc.44.1.0288

Fendel, D., Resek, D., Alper, L., & Fraser, S. (2011). *Interactive Mathematics Program: Integrated High School Mathematics, Year 3* (2nd edition). Key Curriculum Press.

Fendel, D., Resek, D., Alper, L., & Fraser, S. (2012). *Interactive Mathematics Program: Integrated High School Mathematics, Year 4* (2nd edition). Key Curriculum Press.

Forman, E. A., Ford, M. J. (2014). Authority and accountability in light of disciplinary practices in science. *International Journal of Educational Research*, 64, 199-210. https://doi.org/10.1016/j.ijer.2013.07.009

Fu, G., & Clarke, A. (2020). Moving beyond the agency-structure dialectic in pre-collegiate science education: Positionality, engagement, and emergence. *Studies in Science Education*, *55*(2), 215-256. https://doi.org/10.1080/03057267.2020.1735756

González, G., & DeJarnette, A. F. (2015). Teachers' and students' negotiation moves when teachers scaffold group work. *Cognition and Instruction*, *33*(1), 1–45. https://doi.org/10.1080/07370008.2014.987058

Ha, H., & Kim, H. B. (2021). Framing oneself and one another as collaborative contributors in small group argumentation in a science classroom. *International Journal of Science and Mathematics Education 19,* 517–537. https://doi.org/10.1007/s10763-020-10071-z

Hanna, G. (2000). Proof, explanation, and exploration: An overview. *Educational Studies in Mathematics, 44,* 5-23. https://doi.org/10.1023/A:1012737223465

Harel, G. (2001). The development of mathematical induction as a proof scheme: A model for DNR-based instruction. In S. Campbell & R. Zazkis (Eds.), *Learning and Teaching Number Theory*. In C. Maher (Ed.), *Journal of Mathematical Behavior* (pp. 185–212). Ablex.

Headrick, L., Wiezel, A., Tarr, G., Zhang, X., Cullicott, C. E., Middleton, J. A., & Jansen, A. (2020). Engagement and affect patterns in high school mathematics classrooms that exhibit spontaneous problem posing: an exploratory framework and study. *Educational Studies in Mathematics, 105*(3), 435–456. https://doi.org/10.1007/s10649-020-09996-7

Herbel-Eisenmann, B. A., Meaney, T., Pierson Bishop, J., & Heyd-Metzuyanim, E. (2017). Highlighting heritages and building tasks: A critical analysis of mathematics classroom discourse literature. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 722–765). National Council of Teachers of Mathematics.

Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory Into Practice, 38*(2), 67–73. https://doi.org/10.1080/00405849909543834

Keifert, D., & Stevens, R. (2019). Inquiry as a members' phenomenon: Young children as competent inquirers. *Journal of the Learning Sciences*, *28*(2), 240–278. https://doi.org/10.1080/10508406.2018.1528448

Ko, M.-L. M., & Krist, C. (2019). Opening up curricula to redistribute epistemic agency: A framework for supporting science teaching. *Science Education, 103*(4), 979–1010. https://doi.org/10.1002/sce.21511

Koichu, B., Parasha, R., & Tabach, M. (2021). Who-Is-Right tasks as a means for supporting collective looking-back practices. *ZDM Mathematics Education.* https://doi.org/10.1007/s11858-021-01264-z

Kontorovich, I., Koichua, B., Leikinb, R., & Berman, A. (2012). An exploratory framework for handling the complexity of mathematical problem posing in small groups. *The Journal of Mathematical Behavior*, *31*(1), 149–161. https://doi.org/10.1016/j.jmathb.2011.11.002

Kotsopoulos, D. (2014). The case of Mitchell's cube: Interactive and reflexive positioning during collaborative learning in mathematics. *Mind, Culture, and Activity, 21*, 34-52. https://doi.org/10.1080/10749039.2013.790905

Langer-Osuna, J. M. (2016). The social construction of authority among peers and Its implications for collaborative mathematics problem solving. *Mathematical Thinking and Learning*, *18*(2), 107–124. https://doi.org/10.1080/10986065.2016.1148529

Langer-Osuna, J. M. (2017). Authority, identity, and collaborative mathematics. *Journal of Research in Mathematics Education, 48*(3), 237–247. https://doi.org/10.5951/jresematheduc.48.3.0237

Laursen, S.L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education, 5*(1), 129–146. https://doi.org/10.1007/s40753-019-00085-6

Lerman, S. (2012). Agency and identity: Mathematics teachers' stories of overcoming disadvantage. In T.-Y. Tso (Ed.), *Proceedings of the 36th conference of the International Group for the Psychology of Mathematics Education* (pp. 99–106). Taipei: PME.

Lo, M. & Ruef, J. (2020). Student or teacher? A look at how students facilitate public sensemaking during collaborative groupwork. *Journal of Urban Mathematics Education*, *13*(1), 15–33. https://doi.org/10.21423/jume-v13i1a372

Louie, N. (2020). Agency discourse and the reproduction of hierarchy in mathematics instruction. *Cognition and Instruction, 38*(1), 1-26. https://doi.org/10.1080/07370008.2019.1677664

Mercer, N., & Dawes, L. (2008). The value of exploratory talk. In N. Mercer & S. Hodgkinson (Eds.) *Exploring Talk in School: Inspired by the work of Douglas Barnes*, (pp. 55–71). SAGE Publications Ltd. https://doi.org/10.4135/9781446279526.n4

Miller, E., Manz, E., Russ, R., Stroupe, D., & Berland, L. (2018). Addressing the epistemic elephant in the room: Epistemic agency and the next generation science standards. *Journal of Research in Science Teaching*, *55*(7), 1053-1075. https://doi.org/10.1002/tea.21459

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Authors.

Piaget, J. (1948/1973). *To understand is to invent: The future of education.* Grossman Publishers.

Regier, P., & Savic, M. (2020). How teaching to foster mathematical creativity may impact student self-efficacy for proving. *Journal of Mathematical Behavior, 57*, Article 100720. https://doi.org/10.1016/j.jmathb.2019.100720

Rummel, N., & Spada, H. (2005). Learning to collaborate: An instructional approach to promoting collaborative problem-solving in computer-mediated settings. *The Journal of the Learning Sciences, 14*(2), 201–241. https://doi.org/10.1207/s15327809jls1402_2

Schindler, M., & Bakker, A. (2020). Affective field during collaborative problem posing and problem solving: A case study. *Educational Studies in Mathematics, 105,* 303-324. https://doi.org/10.1007/s10649-020-09973-0

Shim, S. Y., & Kim, H. B. (2018). Framing negotiation: Dynamics of epistemological and positional framing in small groups during scientific modeling. *Science Education, 102*, 128–152. https://doi.org/10.1002/sce.21306

Smith, E., & Confrey, J. (1991, April). *Understanding collaborative learning: Small group work on contextual problems using a multi-representational software tool.* A paper presented at the Annual Meeting of the American Educational Research Association. Chicago.

Strauss, A., & Corbin, J. M. (1990). *Basics of qualitative research*. Sage Publications, Inc.

Stroupe, D. (2014). Examining classroom science practice communities: How teachers and students negotiate epistemic agency and learn science-as-practice. *Science Education*, *98*(3), 487–516. https://doi.org/10.1002/sce.21112
van de Sande, C. C., & Greeno, J. G. (2012). Achieving alignment of perspectival framings in problem-solving discourse. *Journal of the Learning Sciences, 21*(1), 1-44. https://doi.org/10.1080/10508406.2011.639000
Vogel, R. (2013). Mathematical situations of play and exploration. *Educational Studies in Mathematics, 84*(2)*,* 209-225. https://doi.org/10.1007/s10649-013-9504-4
Volet, S., Summers, M., & Thurman, J. (2009). High-level co-regulation in collaborative learning: How does it emerge and how is it sustained? *Learning and Instruction, 19*(2), 128 143. https://doi.org/10.1016/j.learninstruc.2008.03.001
Webb, N. (1982). Group composition, group interaction, and achievement in cooperative small groups. *Journal of Educational Psychology,74*(4), 475-484. https://doi.org/10.1037/0022-0663.74.4.475
Wood, M. B. (2013). Mathematical micro-identities: Moment-to-moment positioning and learning in a fourth-grade classroom. *Journal for Research in Mathematics Education, 44*(5), 775. https://doi.org/10.5951/jresematheduc.44.5.0775
Wood, M. B. (2016). Rituals and right answers: Barriers and supports to autonomous activity. *Educational Studies in Mathematics 91*, 327-348. https://doi.org/10.1007/s10649-015-9653-8
Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal of Research in Mathematics Education, 27*(4), 458-477. https://doi.org/10.5951/jresematheduc.27.4.0458