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Orbitalization properties of random light beams

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Abstract: New properties characterising the pairs of the OAM modes in a scalar random beam such as the degree of orbitalization and the orbitalization ellipse are introduced, in similarity with those of polarization theory. © 2024 The Author(s)

1. Introduction

A large number of Orbital Angular Momentum (OAM) [1] components may be present in an arbitrary scalar light beam. Unlike other light properties such as spectrum or polarization, the OAM components are not local but are given at a fixed radius. The OAM basis consists of the elementary spiral phase functions, $e^{il\phi}$, where l is the OAM index and ϕ is the polar angle. Hence, in general, a multidimensional space is required for quantitative characterization of the individual OAM states. The mode-to-mode OAM correlation properties in random beams are described by the Coherence-OAM matrices [2], [3] of a generally high rank.

We narrow down this general characterization problem to a pair of OAM modes that can be either superposed or sorted out from the total beam and scrutinize the properties of the mixture. We show, in particular, that the single-radius, two-OAM-mode COAM matrix can be uniquely split into completely random and purely harmonic-like parts, which can be then used for determining the degree of statistical similarity between the two OAM modes, and for deriving the elliptical form associated with its harmonic-like portion.

2. Theory

Two-point correlations in a scalar random beam can be characterized by the cross-spectral density function $W(\mathbf{r}_1,\mathbf{r}_2) = \langle E^*(\mathbf{r}_1)E(\mathbf{r}_2)\rangle$ of the electric field E, at a pair of points with position vectors $\mathbf{r}_n = \rho_n\hat{\rho} + \phi_n\hat{\phi}$, $n = 1,2$. Here star denotes conjugation and angular brackets stand for ensemble average. The correlations among different OAM components can then be obtained by filtering out the azimuthal dependence at both points as [2]

$$W_{lm}(\rho_1,\rho_2) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} W(\mathbf{r}_1,\mathbf{r}_2)e^{i(l\phi_1-m\phi_2)}d\phi_1 d\phi_2, \tag{1}$$

for $-\infty < l,m < \infty$. These quantities form the Coherence-OAM (COAM) matrix, viz., $\overleftrightarrow{W}^{OAM}(\rho_1,\rho_2) = [W_{\alpha\beta}(\rho_1,\rho_2)] = [\langle E^*_\alpha(\rho_1)E_\beta(\rho_2)\rangle]$. Here $-\infty < \alpha,\beta < \infty$, in general. We will now segregate a 2×2 sub-matrix formed at two fixed indices, say l and m, and consider it at the coinciding radii, $\rho = \rho_1 = \rho_2$, using notation $S_{lm}(\rho) = W_{lm}(\rho,\rho)$ for its elements: $S^{OAM}_{ij}(\rho)$,$(i,j = l,m)$. It is a non-negative definite, Hermitian matrix, with non-negative diagonal elements. Hence, it yields a unique decomposition $\overleftrightarrow{S}^{OAM}_{l,m}(\rho) = \overleftrightarrow{S}^{unorb}_{l,m}(\rho) + \overleftrightarrow{S}^{orb}_{l,m}(\rho)$. Here "orb" and "unorb" stand for purely orbitalized and completely unorbitalized (unstructured) OAM portions, in similarity with completely polarized and unpolarized portions in a vectorial random beam (for proof see analogous derivation in [4], Sec. 10.9.2). The unstructured portion being a multiple of a 2×2 identity matrix and the purely orbitalized portion being singular, $\mathrm{Det}[\overleftrightarrow{S}^{orb}_{l,m}(\rho)] = 0$, Det standing for matrix determinant, are given as

$$\overleftrightarrow{S}^{unorb}_{l,m}(\rho) = A_{l,m}(\rho)\overleftrightarrow{I_2}, \qquad \overleftrightarrow{S}^{orb}_{l,m}(\rho) = \begin{bmatrix} B_{l,m}(\rho) & D_{l,m}(\rho) \\ D^*_{l,m}(\rho) & C_{l,m}(\rho)\end{bmatrix}, \tag{2}$$

We have retained sub-indices l and m for the elements A, B, C, and D to stress that different matrices are expected for different l and m values. These elements can be related to the matrix $\overleftrightarrow{S}^{OAM}_{l,m}(\rho)$.

For the given pair of states, l and m, let us now consider a ratio of orbitalized portion of the spectral density to the total spectral density of these two states:

$$O_{l,m}(\rho) = \frac{\mathrm{Tr}[\overleftrightarrow{S}^{orb}_{l,m}(\rho)]}{\mathrm{Tr}[\overleftrightarrow{S}^{OAM}_{l,m}(\rho)]} = \sqrt{1 - \frac{4\mathrm{Det}[S^{OAM}_{l,m}(\rho)]}{\mathrm{Tr}^2[S^{OAM}_{l,m}(\rho)]}}. \tag{3}$$

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Here Tr stands for a trace of a matrix. This quantity is the OAM counterpart of the degree of polarization defined for vectorial optical beams, being the ratio of the spectral density of a polarized portion of the beam to that of the total beam. We term quantity $O_{l,m}(\rho)$ the *degree of orbitalization* (DO). Further, matrix $\overleftrightarrow{S}^{orb}_{l,m}(\rho)$ is singular and, hence, factorizes:

$$\overleftrightarrow{S}^{orb}_{l,m}(\rho) = \begin{bmatrix} E_l^*(\rho)E_l(\rho) & E_l^*(\rho)E_m(\rho) \\ E_m^*(\rho)E_l(\rho) & E_m^*(\rho)E_m(\rho) \end{bmatrix}, \tag{4}$$

where $E_l(\rho)$ and $E_m(\rho)$ are the spatial counterparts of the equivalent monochromatic realizations of the OAM components: $\mathscr{E}_l(\rho,t) = E_l(\rho)e^{i\omega t}$, $\mathscr{E}_m(\rho,t) = E_m(\rho)e^{i\omega t}$. They can be related to the elements of matrix $\overleftrightarrow{S}^{orb}_{l,m}(\rho)$ as $E_l(\rho) = \sqrt{B_{l,m}(\rho)}e^{i\delta_l(\rho)}$, $E_m(\rho) = \sqrt{C_{l,m}(\rho)}e^{i\delta_m(\rho)}$. Hence the real-valued realizations of modes l and m become: $\mathscr{E}_l^{(r)}(\rho,t) = \sqrt{B_{l,m}(\rho)}\cos[\delta_l(\rho)+\omega t]$, $\mathscr{E}_m^{(r)}(\rho,t) = \sqrt{C_{l,m}(\rho)}\cos[\delta_m(\rho)+\omega t]$. Further, since $D_{l,m}(\rho) = E_l^*(\rho)E_m(\rho)$, the phase difference between the two OAM modes relates to $D_{l,m}(\rho)$ as $\delta_{l,m}(\rho) = \delta_m(\rho) - \delta_l(\rho) = \arg[D_{l,m}(\rho)]$. Here arg stands for an argument of a complex number. Therefore, the removal of the time-dependence portion from field realization leads to an elliptic form:

$$C_{l,m}(\rho)\left[\mathscr{E}_l^{(r)}(\rho)\right]^2 + B_{l,m}(\rho)\left[\mathscr{E}_m^{(r)}(\rho)\right]^2 - 2\mathrm{Re}[D_{l,m}(\rho)]\mathscr{E}_l^{(r)}(\rho)\mathscr{E}_m^{(r)}(\rho) = \mathrm{Im}[D_{l,m}(\rho)], \tag{5}$$

where Re(Im) are real(imaginary) parts. This derivation is similar to that for polarization ellipse [5]. We term this curve the *orbitalization ellipse* (OE).

3. Numerical examples

Figure 1 (A), (B) shows the DO for three pairs of the OAM indices, while (C), (D) represents the OE for $l = 1$ and $m = 2$ of the incoherently superimposed pair of the I_j-Bessel correlated beams [6] varying with: (A), (C) propagation distance z at $\rho = 1$mm; (B), (D) radial distance ρ at the source $z = 0$. Figure 1 (A) shows that for some pairs of OAM modes (i.e., $l,m = 1,2$ or $0,1$), the DO can reach zero value. This occurs if $S_{ll} = S_{mm}$. For $z \to \infty$ the DO tends to 1. Similar behavior can also be observed in Fig. 1 (B), however at $\rho \to \infty$, the DO saturates at a value less than unity. In Fig. 1(C), the OE is plotted at fixed $\rho = 1$mm. It starts from a linear state along the $\mathscr{E}_l^{(r)}$ axis, shrinks to a point, and then expands in a linear state along the $\mathscr{E}_m^{(r)}$ axis. A comparable pattern for the OE can also be observed Fig. 1 (D) plotted at source. However, the OE starts and ends as a point, due to the zero axial intensity pertinent to the OAM-carrying beams and vanishing intensity beyond the beam edge.

Note that the space spanned by basis functions $e^{il\phi}$ and $e^{im\phi}$ is not the physical $x-y$ plane customary for polarization analysis. However, since the COAM matrix is directly measurable [3], the OE can be readily deduced and presented in the $(\mathscr{E}_l^{(r)}, \mathscr{E}_m^{(r)})$ coordinates. If needed, the OE can be mapped to the real Cartesian plane by the devices of polarization optics inserted behind the rings in the measurement procedure of [3].

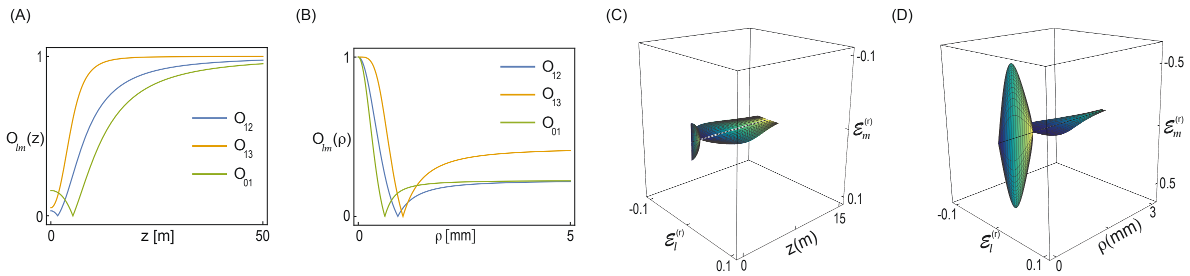

Fig. 1. DO and OE for a pair of I_j-Bessel correlated beams.

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