Fluctuations and correlations of transmission eigenchannels within diffusive media

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Abstract: We experimentally and numerically study the fluctuations and correlations of transmission eigenchannel profiles in diffusive media. We find that high-transmission profiles exhibit low realization-to-realization fluctuations and significant correlations exist between low-transmission profiles.

1. Introduction

Coherent wave transport of light, microwaves, or acoustic waves in multiple-scattering media has been a prominent and prolific research topic in recent years. The general goal is to develop the ability to deliver energy deep inside a turbid medium, which requires overcoming the limitations imposed by incoherent diffusion. Coherent wave transport is a deterministic process -in linear systems with static disorder- and therefore, it can be described by a field transmission matrix t which maps the incident flux to the transmitted flux. The eigenvectors of $t^{\dagger}t$ are a complete-basis of input-wavefronts which excite a set of disorder-specific wavefunctions –spanning the system– known as transmission eigenchannels. Each eigenchannel independently propagates through the system with a transmittance given by the corresponding eigenvalue τ. One of the striking theoretical predictions of transmission eigenvalues is their bimodal probability distribution: with maxima at $\tau = 0$ and $\tau = 1$. The eigenchannels corresponding to these values are referred to as closed and open channels.

The fluctuations of, and the correlations between transmission *eigenvalues* have been thoroughly studied and have provided insight into well-known physical phenomena like universal conductance fluctuations and quantum shot noise. The fluctuations of, and correlations between *eigenchannel profiles*, however, remain unstudied. In optics, the recent advent of wavefront shaping has made photonic systems a unique and versatile platform for studying transmission eigenchannels. This has spurred a renewed interest in using transmission eigenchannels for imaging and sensing applications. Open-eigenchannels enhance the transmitted power through and within a diffusive system. The ability to control the energy-density within a disordered system enables manipulating the internal light-matter interactions and nonlinear processes. So far, however, eigenchannel profiles are only well understood after ensemble averaging over many disorder realizations. It is an open question, therefore, if coupling into an open channel *guarantees* a significant enhancement of the energy density inside a single system.

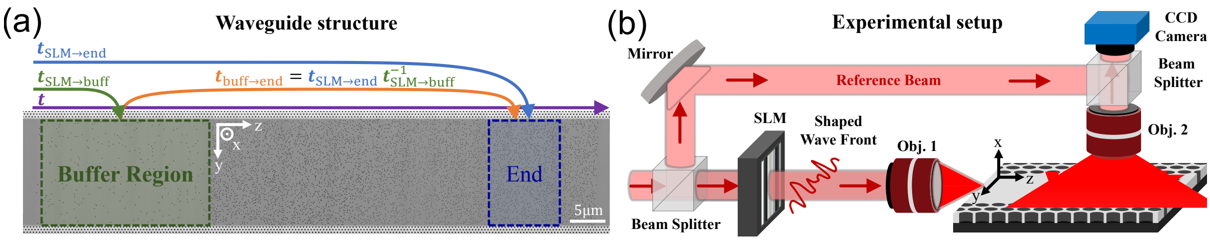

Fig. 1. A scanning electron microscope image of a diffusive waveguide is shown in (a) along with the illustration of field-mapping matrices. In (b) a simplified sketch of the experimental setup depicts the interferometric measurement of the light scattered out of the waveguide while the spatial wavefront of the injected light is shaped by a spatial light modulator (SLM).

Here, we use optical diffusive systems to experimentally investigate the fluctuations and correlations of transmission eigenchannel depth profiles. Using a novel experimental platform which enables us to conduct an interferometric measurement of the light field everywhere inside a disordered waveguide, we measure a proxy for the transmission matrix of an on-chip diffusive waveguide and excite its individual transmission eigenchannels.

Repeating this measurement for different disorder-realizations enables us to conduct the first experimental study of the second-order statistics of transmission eigenchannels. We find that high-transmission eigenchannels exhibit small realization-to-realization fluctuations in their depth profiles. This robustness in open-channel profiles is promising for applications in deep tissue imaging and light delivery.

2. Methods

We fabricate two-dimensional (2D) disordered waveguide structures, with photonic crystal boundaries, on a silicon-on-insulator wafer with electron beam lithography and plasma etching; which enables us to directly observe the depth profiles of transmission eigenchannels *within* different diffusive systems from the third dimension. In each waveguide 100 nm-diameter holes are randomly etched, providing scattering and a direct probe of the light inside the waveguide: via the light scatters out-of-plane from the holes. At the wavelength of our probe light, $\lambda = 1.55$ µm, the transport mean free path, $\ell_t = 3.2$ µm, is much shorter than the total disordered region length, $L = 50$ µm. Therefore, light passing through each waveguide undergoes multiple scattering and diffusive transport.

To measure the light field inside individual diffusive waveguides, we use the interferometric setup sketched in Fig. 1 (b). Monochromatic light from a wavelength-tunable laser source is split into two beams. One beam is modulated by a spatial light modulator (SLM) and then, via the edge of the wafer, injected into one of the waveguides. The other beam is used as a reference beam, and spatially overlapped on the CCD camera chip with the out-of-plane scattered light from the diffusive waveguide. The CCD camera records the resulting interference pattern, from which the complex field profile across the diffusive waveguide is obtained, and from this a proxy for the field-transmission matrix can be measured, $t_{\mathrm{buff}\to\mathrm{end}}$, as illustrated in Fig. 1 (a). To excite a single eigenchannel, we first perform a singular value decomposition on $t_{\mathrm{buff}\to\mathrm{end}}$, and multiply each singular vector with $t^{-1}_{\mathrm{slm}\to\mathrm{buff}}$ to calculate the requisite SLM phase-modulation patterns for each eigenvector. By displaying one of these patterns on the SLM, we excite a single eigenchannel of the diffusive waveguide, and with sequential measurements we can recover all of the transmission eigenchannel profiles.

3. Results

In Fig. 2 (a) the spatially-averaged depth-profile fluctuations (blue circles and line) of the transmission eigenchannels increase monotonically with the channel index α. The lower index α corresponds to a higher transmission eigenvalue. The high-eigenchannel profiles demonstrate a robustness when compared to either low-transmission eigenchannels or random inputs (green line). The spatially resolved fluctuations for individual eigenchannels are shown in (b). Open channels fluctuate homogeneously along the waveguide depth z, however, closed eigenchannels primarily fluctuate at the front. The covariance of different eigenchannel profiles, shown in (c), indicates their fluctuations are correlated from realization-to-realization. The red circles and dashed line in (a) denote the cumulative covariance, a sum of the covariance of one channel to all other channels. The correlations are weaker for higher-transmission eigenchannels, indicating they are more independent than lower-transmission eigenchannels. Therefore open channels have robust and independent spatial profiles; a promising result for applications in deep tissue imaging and light delivery.

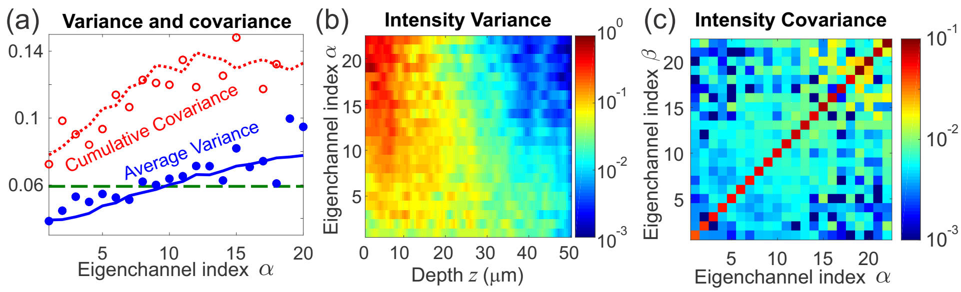

Fig. 2. In (a), the spatially-averaged depth-profile fluctuations of the transmission eigenchannels increase monotonically with the channel index α. The blue dots are calculated from the experimentally-measured depth-resolved intensity fluctuations shown in (b), while the solid blue line represents the numerical simulation result. The green dashed line denotes the fluctuations for random incident wavefronts. In (a) the cumulative covariance (red dots calculated from the experimentally-measured covariance in (c) and red line from simulations) exceeds the variance of any eigenchannel.