# Comparative Analysis of Information Theoretic and Statistical Methods for Line Parameter Estimation

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***Abstract*—Recent studies indicate that the noise characteristics of phasor measurement units (PMUs) can be more accurately described by non-Gaussian distributions. Consequently, estimation techniques based on Gaussian noise assumptions may produce poor results with PMU data. This paper considers the PMU-based line parameter estimation (LPE) problem, and investigates the performance of four state-of-the-art techniques in solving this problem in presence of non-Gaussian measurement noise. The rigorous comparative analysis highlights the merits and demerits of each technique w.r.t. the LPE problem, and identifies conditions under which they are expected to give good results.**

***Index Terms*—Line parameter estimation (LPE), Non-Gaussian measurement noise, Phasor measurement unit (PMU)**

## I. Introduction

In the dynamic landscape of modern power systems, accurate line parameters serve as the linchpin for operational reliability and efficiency. Hence, the ability to estimate them whenever needed using phasor measurement units (PMUs) placed on both ends of the lines was a significant breakthrough for synchrophasor technology [1], [2]. PMUs provide voltage and current phasor measurements, from which line parameters can be found using Ohm's law and Kirchhoff's laws. However, since both voltage and current measurements from PMUs are noisy, the conventional least squares (LS) approach, which only assumes noise in the dependent variable, is unable to give accurate parameter estimates. To confront this challenge, errors-in-variables (EIV) models have emerged as adept tools, capable of providing accurate parameter estimates by accounting for the imprecision present in both dependent and independent variables. The line parameter estimation (LPE) problem has been solved as an EIV problem in [3]–[6].

Traditionally, EIV problems are solved using the total least squares (TLS) method. Similar to LS, TLS is also designed to give optimal estimates when the noises in the dependent and independent variables follow a Gaussian distribution. This is because both LS and TLS rely on second-order statistics and aim to minimize the *sum of squares error* (SSE). However, it has recently been discovered through rigorous statistical analysis that the noise in the PMU measurements can have a non-Gaussian distribution [7], [8]. The non-Gaussianity can stem from diverse operating conditions, aging process of instrument transformers, incorrect time synchronization, errors introduced by phasor estimation algorithms, and/or varying communication channel noises [9]. Consequently, there is a pressing need to delve into advanced estimation techniques for solving EIV problems that are capable of handling non-Gaussian noise in the dependent and independent variables.

This paper starts by analyzing four approaches that have been proposed recently to perform linear estimation[1] for EIV problems in presence of non-Gaussian noise. Minimum total error entropy (MTEE) [10] and maximum total correntropy (MTC) [11] are based on information theoretic learning (ITL) and minimize the "net error" without any additional constraints. Constrained MTC (CMTC), which is also based on ITL, solves the MTC problem with equality constraints [12]. The fourth approach, termed EGLE (EIV for Gaussian mixture model-Lagrange multipliers-Expectation maximization) [13], uses robust statistics to perform noise and parameter estimation simultaneously. Subsequently, the paper applies these state-of-the-art techniques to solve the PMU-based LPE problem under diverse noise conditions. The IEEE 118-bus system is used as the test system for this analysis. The results indicate that (a) with appropriate hyperparameter tuning, MTC/CMTC can give good results; (b) along with extensive hyperparameter tuning, MTEE also needs sufficient time/compute resources to give acceptable results; (c) EGLE gives fast and reasonably accurate line parameter estimates with minimal tuning.

The rest of the paper is structured as follows: Section II provides a brief overview of the MTEE, MTC, CMTC, and EGLE algorithms. Section III explains the PMU-based LPE problem. The performances of the four techniques for the PMU-based LPE problem are described in Section IV. Finally, the conclusion is provided in Section V.

*Notation:* The superscript $\tilde{(.)}$ denotes noisy data, the subscript $(.)_{true}$ denotes the true value, and the superscript $\hat{(.)}$ denotes the estimated value.

## II. Solving EIV Problems with Non-Gaussian Noise

A linear estimation problem can be written as

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[1]Note that LPE when done using PMU measurements can be expressed as a linear estimation problem.

$$y_{true} = X_{true}\, w_{true} \tag{1}$$

where, $y_{true} \in \mathbb{R}^{n\times 1}$ is the true value of the dependent variable, $X_{true} \in \mathbb{R}^{n\times p}$ is the true value of the independent variable, $w_{true} \in \mathbb{R}^{p\times 1}$ is the true value of the parameter to be estimated, and $n$ and $p$ denote the number of samples and the number of parameters to be estimated, respectively. In presence of measurement noise, (1) becomes

$$\tilde{y} \approx \tilde{X}\, w \tag{2}$$

where, $\tilde{y}$ and $\tilde{X}$ represent the noisy version of dependent and independent variables, respectively, and $w$ is the parameter to be estimated. The goal is to solve the linear estimation problem by finding the $\hat{w}$ that best fits the observed data. Several techniques, such as TLS, MTEE, MTC, CMTC, and EGLE, can achieve this goal. However, the estimation accuracy varies depending on the noise characteristics and the assumptions on which the techniques are built.

TLS assumes Gaussian noise in both dependent and independent variables, and computes the optimal parameter estimate by solving (3) [14],

$$\hat{w}_{TLS} = [d_{qq}]^{-1}[d_{pq}] \tag{3}$$

where, $d_{pq}$ is the vector of first $p$ elements and $d_{qq}$ is the $(p+1)^{th}$ element, respectively, of the $(p+1)^{th}$ column of the matrix of right singular vectors of the singular value decomposition of $[\tilde{X}\ \tilde{y}]$. However, the performance of TLS deteriorates in presence of non-Gaussian noise in the measurements that comprise the dependent and independent variables. Four techniques that are, by definition, immune to non-Gaussian noises in the EIV context, are described next.

### A. Minimum Total Error Entropy (MTEE)

MTEE involves minimizing the quadratic Renyi's entropy of the total error, $e^{tot}$, to obtain accurate parameter estimates, where $e^{tot}$ is mathematically expressed as

$$e^{tot} = \frac{\tilde{y} - \tilde{X}w}{(\sqrt{\|w^2\| + \epsilon_0^{-2}})}. \tag{4}$$

In (4), $\epsilon_0$ is the square-root of the ratio of the noise intensity of the independent variable to the noise intensity of the dependent variable. If the exact values of noise intensity are not known, but it can be assumed that the noise intensity of the independent variable is comparable to that of the dependent variable[2], then we can set $\epsilon_0 = 1$. The estimator for Renyi's entropy, denoted by $\hat{V}_2(e^{tot})$, can now be expressed as

$$J_{\text{MTEE}} = \hat{V}_2(e^{tot}) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} G_{\sigma\sqrt{2}}(e_j^{tot} - e_i^{tot}) \tag{5}$$

where, $G_\sigma$ represents a Gaussian kernel function with a kernel width of $\sigma$. The MTEE approach aims to iteratively adjust $w$ to maximize $\hat{V}_2(e^{tot})$, and obtain accurate parameter estimates in the process. The necessary condition to find the maximum value of the estimator shown in (5) is given in (6), where $\Delta e_{ij}^{tot} = e_i^{tot} - e_j^{tot}$, $\Delta\tilde{x}_{ij} = \tilde{x}_i - \tilde{x}_j$, where $\tilde{x}_i$ is the $i^{th}$ row of $\tilde{X}$, $\bar{\epsilon} = \epsilon_0^{-2}$, and the superscript $tot$ is suppressed to avoid notational clutter.

$$\begin{aligned} g_{\text{MTEE}} = \frac{\partial \hat{V}_2(e)}{\partial w} &= \frac{1}{\sigma^2 n^2}\cdot \sum_{i=1}^{n}\sum_{j=1}^{n} G_\sigma(\Delta e_{ij}) \\ &\times \left(\frac{\Delta e_{ij}^2 w}{w^T w + \bar{\epsilon}} + \frac{\Delta e_{ij}\Delta\tilde{x}_{ij}^T}{\sqrt{w^T w + \bar{\epsilon}}}\right). \end{aligned} \tag{6}$$

Using (6), the parameter update step can be written in the form of the steepest ascent algorithm as shown below [10],

$$w_{r+1} = w_r + \mu \times g_{\text{MTEE}}\Big|_{w_r} \tag{7}$$

where, $r$ and $\mu$ denote the iteration number and the learning step size of the steepest ascent algorithm, respectively.

### B. Maximum Total Correntropy (MTC)

In the MTC technique, the following correntropy-based cost function is maximized:

$$J_{\text{MTC}} = \mathbb{E}\left[\exp\left(-\frac{e_i^2}{2\sigma_{\text{MTC}}^2\|\bar{w}\|^2}\right)\right]. \tag{8}$$

In (8), $e_i = \tilde{y}_i - \tilde{x}_i w$, $\sigma_{\text{MTC}}$ is the Gaussian kernel width, and $\bar{w} = [\epsilon_0^{-1}\ \ -w]$. The objective of the MTC technique is to minimize the mean of non-linear weighted squared residuals. This can be done using the gradient descent method by computing the partial derivative of the cost function shown in (8), as written below

$$\begin{aligned} g_{\text{MTC}} &= \frac{\partial J_{\text{MTC}}(w)}{\partial w} = \frac{1}{n\sigma_{\text{MTC}}^2}\times \\ &\sum_{i=1}^{n}\left[\exp\left(\frac{-e_i^2}{2\sigma_{\text{MTC}}^2\|\bar{w}\|^2}\right)\times\frac{\left(\|\bar{w}\|^2 e_i\tilde{x}_i^T + e_i^2 w\right)}{\|\bar{w}\|^4}\right]. \end{aligned} \tag{9}$$

At iteration $r$, the parameter update of the gradient-based MTC algorithm is obtained using (9), as shown below [11]:

$$w_{r+1} = w_r + \eta \times g_{\text{MTC}}|_{w_r} \tag{10}$$

where, $\eta = \mu\cdot\sigma_{\text{MTC}}^2$.

### C. Constrained Maximum Total Correntropy (CMTC)

The CMTC technique solves the MTC problem in presence of equality constraints. The mathematical formulation of this linear estimation problem is given below,

$$\begin{aligned} \tilde{y} &\approx \tilde{X}\, w \\ C^T w &= f \end{aligned} \tag{11}$$

in which the second sub-equation is the equality constraint. Through the utilization of the Lagrange multiplier method, the cost function of CMTC is acquired as shown below,

$$J_{\text{CMTC}} = \mathbb{E}\left[G_\sigma\left(\frac{e_i}{\sqrt{\bar{w}^T\bar{w}}}\right)\right] + \lambda^T(C^T w - f) \tag{12}$$

[2]This can happen when noises in both the variables are coming from sensors that satisfy the same standards; e.g., IEC/IEEE 60255-118-1:2018 Standard for PMU measurements [15].

where, $\lambda$ is the Lagrange multiplier, which for the $r^{th}$ iteration, is given by,

$$\lambda_{r+1} = \frac{1}{\eta} \cdot (C^T C)^{-1}(f - C^T w_r - C^T \eta g_{\text{MTC}}(w_r)). \quad (13)$$

By employing the stochastic gradient descent, we derive the updated parameter estimate of CMTC as shown below [12]:

$$w_{r+1} = w_r + \eta \times g_{\text{MTC}}|_{w_r} + \eta C \lambda_{r+1}. \quad (14)$$

### *D. EIV for Gaussian mixture model-Lagrange multipliers-Expectation maximization (EGLE)*

The preceding subsections described three estimation methods that were based on ITL. In contrast, the EGLE algorithm harnesses *robust statistics* to address potential non-Gaussian noise in EIV problems. Another key distinction is that MTEE, MTC, and CMTC were noise-model agnostic, while EGLE explicitly expresses the noise in terms of Gaussian mixture models (GMMs). The mathematical basis for the working of EGLE is that, in the presence of noises modeled as GMMs, minimizing the sum of squares of *standardized* error (SSSE) with respect to the associated Gaussian component in GMM yields optimal parameter estimates (rather than minimizing SSE as done in TLS). The $g^{th}$ Gaussian component of the *standardized* error in the dependent and independent variable (denoted by $e_{g_S}$ in the subscript) can be written in the form:

$$y_{e_{g_S}} = y_{\Sigma_g}^{-\frac{1}{2}} \left(y_{e_g} - y_{\mu_g}\right) \quad (15a)$$

$$X_{e_{g_S}} = X_{\Sigma_g}^{-\frac{1}{2}} \left(X_{e_g} - X_{\mu_g}\right) \quad (15b)$$

where, for the corresponding variable, the subscripts $e_g$ denote the measurement noise, and $\mu_g$ and $\Sigma_g$ denote the mean and covariance, of the $g^{th}$ Gaussian component, respectively. The overall minimization problem for optimal noise and parameter estimation by EGLE is mathematically expressed as

$$\begin{aligned} J_{\text{EGLE}} = \arg\min_{w} \; & \sum_{g=1}^{m} \frac{1}{2}(y_{e_{g_S}})^T (y_{e_{g_S}}) \\ & + \sum_{g=1}^{m} \frac{1}{2}(\text{vec}(X_{e_{g_S}}))^T (\text{vec}(X_{e_{g_S}})) \quad (16) \\ \text{s.t. } & [\tilde{y}_g - y_{e_g}] = (w^T \otimes I_{n_g})(\text{vec}(\tilde{X}_g) - \text{vec}(X_{e_g})) \\ & \forall g \in [1, \ldots, m]. \end{aligned}$$

where, vec denotes the vectorization operation, $\otimes$ is the Kronecker product, $I_{n_g}$ is the identity matrix of size $n_g$, where $n_g$ is the number of samples belonging to the $g^{th}$ Gaussian component, and $m$ is the number of Gaussian components. Expressing (16) as an unconstrained optimization problem using Lagrange multipliers ($\alpha_g$), and substituting $y_{e_{g_S}}$ and $X_{e_{g_S}}$ from (15), the optimal parameter estimate is obtained by solving the following system of non-linear equations:

$$f(w) = \sum_{g=1}^{m} (\tilde{X}_g - X_{e_g})^T \alpha_g = 0 \quad (17)$$

where,

$$\alpha_g = (\Gamma_{\Sigma_g})^{-1}(\tilde{y}_g - \tilde{X}_g w - \Gamma_{\mu_g}) \quad (18a)$$

$$\Gamma_{\mu_g} = y_{\mu_g} - \sum_{j=1}^{p} X_{\mu_g} w_j \quad (18b)$$

$$\Gamma_{\Sigma_g} = y_{\Sigma_g} + \sum_{j=1}^{p} X_{\Sigma_g} w_j^2. \quad (18c)$$

The above-mentioned system of non-linear equations can be solved using the standard Newton's method as shown below:

$$w^{(r+1)} = w^{(r)} - \left[\text{Jac}(f(w^{(r)}))\right]^{-1} f(w^{(r)}) \quad (19)$$

where, $\text{Jac}(f(w^{(r)}))$ is the Jacobian of $f(w^{(r)})$ at $r^{th}$ iteration. When equality constraints are present, the system of non-linear equations will be solved using the Newton's method with equality constraints. The components of the noise estimates of $y_e$ and $X_e$ are obtained by solving (20), where $x_{e_{g_j}}$ and $x_{\mu_{g_j}}$ denote the $j^{th}$ column of $X_{e_g}$ and $X_{\mu_g}$, respectively, and $w_j$ is the $j^{th}$ parameter of $w$.

$$\hat{y}_{e_g} = y_{\Sigma_g} \alpha_g + y_{\mu_g} \quad (20a)$$

$$\hat{x_{e_{g_j}}} = -w_j X_{\Sigma_g} \alpha_g + x_{\mu_{g_j}}. \quad (20b)$$

The updated GMM parameters are obtained by using expectation maximization on these noise estimates. Subsequently, the obtained GMM parameters along with the noisy measurements are set as inputs to the parameter estimation step explained in (17)-(19). The noise estimation (using (20)) and the parameter estimation (using (17)-(19)) are carried out iteratively until an optimal solution is obtained for a particular value of $m$. This procedure is then repeated for all $m$ in the range $[1, m_{\text{max}}]$. Finally, the Bayesian information criterion is utilized to select the optimal $m$ ($m^*$), and the corresponding parameter estimate is chosen as the output of EGLE [13].

This completes the description of the four state-of-the-art linear estimation methods that are equipped to handle non-Gaussian noises in EIV problems. The subsequent sections of this paper delve into a detailed comparison of these methods across diverse scenarios for the LPE problem. We start by providing a brief discussion of the significance of LPE in the power system and its underlying mathematical basis.

## III. Need for Periodic LPE using PMUs

Typically, power utilities have a set of line parameter values for each line in their database. However, the line parameters vary over time due to temperature, humidity, and aging [16], [17]. Hence, it is important to conduct LPE periodically to update the values in the utility database, as it will then benefit *all* downstream applications that rely on these values.

The use of PMU measurements for LPE is extremely popular because of three reasons. First, PMUs directly produce voltage and current phasors (magnitude and angle). Hence, they can be used for LPE without any apriori estimation; they also make the LPE problem linear. Second, the output rate of PMUs is much higher than the speed with which line parameters change. Therefore, PMU-based LPE is a "static" estimation problem that can be solved off-line; judiciously

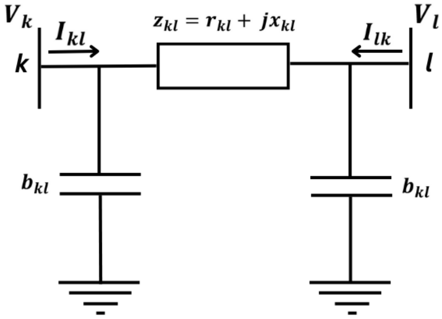


Fig. 1. $\pi$-Model for performing LPE

choosing PMU measurement samples also makes the LPE problem linearly independent (rank-sufficient). Third, as PMU measurements are time-stamped, PMU-based LPE is free from polling and time-skew errors.

The LPE problem is mathematically formulated for a medium-length transmission line whose $\pi$-model is shown in Fig. 1. The "from end" and "to end" of the line are denoted by $k$ and $l$, respectively. The objective of the LPE problem is to update the line parameter values in the utility database using noisy voltage ($\tilde{V}$) and current ($\tilde{I}$) measurements available from PMUs placed at both ends of the line. The series resistance ($r_{kl} \in \mathbb{R}$), series reactance ($x_{kl} \in \mathbb{R}$), and shunt susceptance ($b_{kl} \in \mathbb{R}$) are the line parameters that must be estimated. Applying Kirchhoff's laws at both ends of the line, the following relationship between true voltage phasors, true current phasors, and line parameters is obtained,

$$\begin{aligned} I_{kl_{true}} &= b_{kl} V_{k_{true}} + (V_{k_{true}} - V_{l_{true}})/z_{kl} \\ I_{lk_{true}} &= b_{kl} V_{k_{true}} - (V_{k_{true}} - V_{l_{true}})/z_{kl} \end{aligned} \tag{21}$$

where, $z_{kl} = r_{kl} + jx_{kl}$. By representing the complex currents, voltages, and line parameters of (21) in their Cartesian form, replacing the true values by their noisy counterparts, and vertically stacking the resulting expressions at different time instances, we obtain the linear regression form shown below,

$$\begin{bmatrix} \tilde{I}_{k_r}(1) \\ \tilde{I}_{k_i}(1) \\ \tilde{I}_{l_r}(1) \\ \tilde{I}_{l_i}(1) \\ \cdot \\ \cdot \\ \cdot \\ \tilde{I}_{k_r}(t) \\ \tilde{I}_{k_i}(t) \\ \tilde{I}_{l_r}(t) \\ \tilde{I}_{l_i}(t) \end{bmatrix} \approx \begin{bmatrix} \tilde{V}_{k_r}(1) & \tilde{V}_{k_i}(1) & \tilde{V}_{l_r}(1) & \tilde{V}_{l_i}(1) \\ \tilde{V}_{k_i}(1) & -\tilde{V}_{k_r}(1) & \tilde{V}_{l_i}(1) & -\tilde{V}_{l_r}(1) \\ \tilde{V}_{l_r}(1) & \tilde{V}_{l_i}(1) & \tilde{V}_{k_r}(1) & \tilde{V}_{k_i}(1) \\ \tilde{V}_{l_i}(1) & -\tilde{V}_{l_r}(1) & \tilde{V}_{k_i}(1) & -\tilde{V}_{k_r}(1) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tilde{V}_{k_r}(t) & \tilde{V}_{k_i}(t) & \tilde{V}_{l_r}(t) & \tilde{V}_{l_i}(t) \\ \tilde{V}_{k_i}(t) & -\tilde{V}_{k_r}(t) & \tilde{V}_{l_i}(t) & -\tilde{V}_{l_r}(t) \\ \tilde{V}_{l_r}(t) & \tilde{V}_{l_i}(t) & \tilde{V}_{k_r}(t) & \tilde{V}_{k_i}(t) \\ \tilde{V}_{l_i}(t) & -\tilde{V}_{l_r}(t) & \tilde{V}_{k_i}(t) & -\tilde{V}_{k_r}(t) \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} \tag{22}$$

where, $Y_1 = y_{kl_r}$, $Y_2 = -(b_{kl} + y_{kl_i})$, $Y_3 = -y_{kl_r}$, $Y_4 = y_{kl_i}$, and $y_{kl}$ is the inverse of $z_{kl}$. Note that (22) is equivalent to (2) in the sense that the dependent variables are the current measurements and the independent variables are the voltage measurements, both of which are obtained from PMUs. Furthermore, since $Y_1 + Y_3 = 0$, (22) is also compatible with (11). Lastly, since line parameters lie within $\pm 30\%$ of their database values [18], the values in the power utility database are good starting conditions for the iterative techniques described in Section II.

Once (22) is solved using an appropriate technique, the line parameters can be recovered using the following equation:

$$\begin{aligned} r_{kl} &= \frac{2(Y_1 - Y_3)}{(Y_1 - Y_3)^2 + (2Y_4)^2} \\ x_{kl} &= \frac{-4Y_4}{(Y_1 - Y_3)^2 + (2Y_4)^2} \\ b_{kl} &= -(Y_2 + Y_4). \end{aligned} \tag{23}$$

This concludes the mathematical basis of the LPE problem. In the next section, we evaluate how the four methods described in Section II compare against each other for this problem under diverse measurement noise conditions.

## IV. Results

The IEEE 118-bus system is used to evaluate the performance of MTEE, MTC, CMTC, and EGLE in solving the LPE problem. As power utilities typically place PMUs on the highest voltage (HV) buses first [19], it is assumed that the HV lines of this system have PMUs at both ends. There are ten such lines in the 118-bus system, and so PMU-based LPE is done for them. The data for this study is generated in MATPOWER by simulating the morning load pickup in which the load (and generation) rises steadily [20]. The power flow solutions at different loading conditions provide the true voltage and current phasors in p.u. The noisy data is created by adding appropriate noise to the true values. Finally, the noisy voltage and current measurements along with an initial guess of the line parameters are set as inputs to the four techniques.

For analyzing the performance of the different estimation techniques, absolute relative error (ARE) is chosen as the performance metric. It is defined as

$$w_{\text{ARE}} = \left| \frac{\hat{w} - w_{true}}{w_{true}} \right| \tag{24}$$

where, $\hat{w}$ denotes the estimated value of the parameter vector.

### A. Evaluation in presence of GMM noise

For the first study, a non-Gaussian noise in the form of a two-component GMM is used to create the noisy data. The characteristics of the GMM (in p.u.) are as follows: $\mu$ = [0, 0.01], $\sigma$ = [0.002, 0.002], $\Lambda$ = [0.3, 0.7], where $\mu$, $\sigma$, and $\Lambda$ represents the mean, standard deviation, and weight vectors of the GMM, respectively. The estimation error in terms of %ARE for the four methods is compared in Figs. 2-4.

Figs. 2 and 3 compare the estimation error for resistance and reactance estimates, respectively. It can be observed that all four methods provide very good estimates under non-Gaussian measurement noise for most branches. One possible reason for the last two branches ($L_{65-68}$ and $L_{68-81}$) having a higher error could be the relatively poorer conditioning of the EIV problem form of these two lines in comparison to the other lines. The estimation accuracy of the shunt susceptance is compared in Fig. 4. It can be observed from the figures

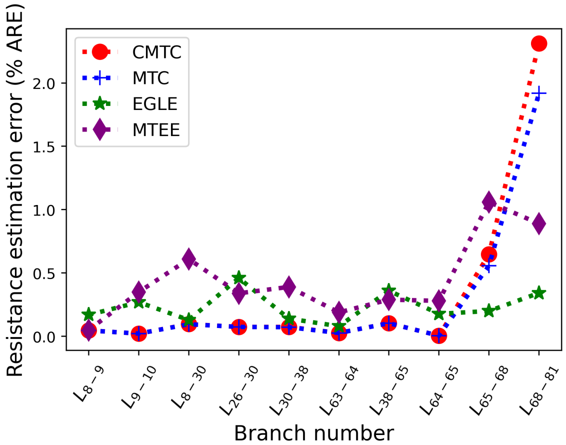


Fig. 2. Resistance ARE comparison

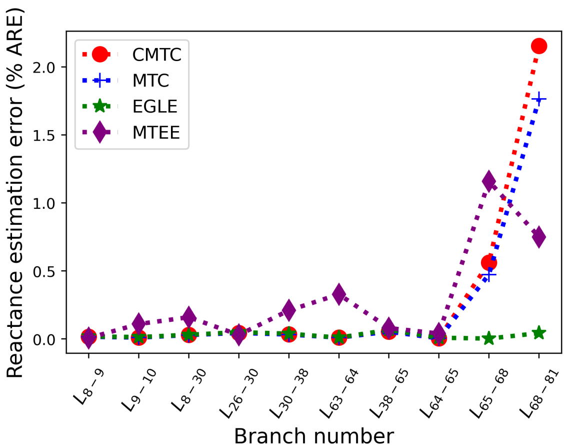


Fig. 3. Reactance ARE comparison

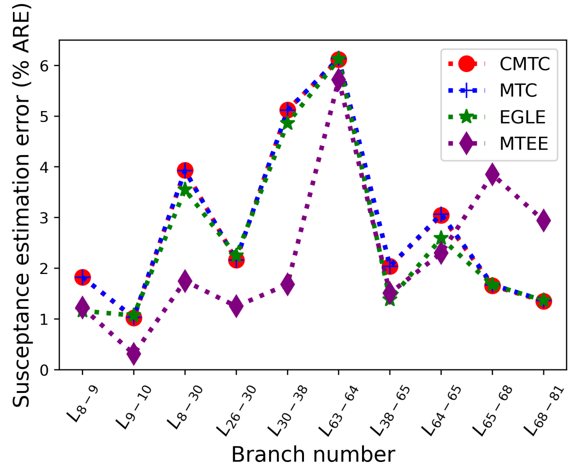


Fig. 4. Susceptance ARE comparison

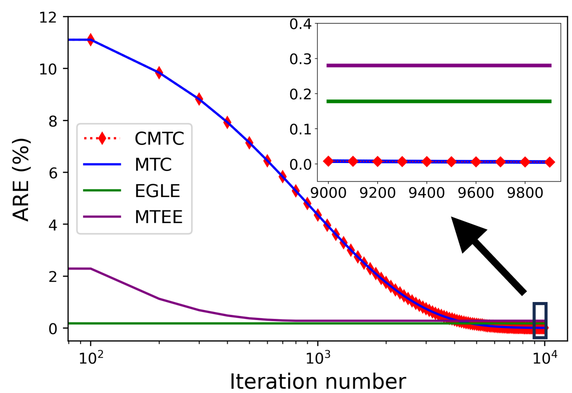


Fig. 5. ARE variation with iterations for $L_{64-65}$

that MTEE generally has better accuracy for the susceptance estimates, whereas MTC, CMTC, and EGLE have lower errors for the resistance and reactance estimates. The relatively high error values for the susceptance estimates can be explained as follows: even though $Y_2$ and $Y_4$ are individually estimated with high accuracy, when they are combined to calculate susceptance using (23), the errors could get added up to result in a high net %ARE for the susceptance estimates. This problem can be tackled by creating alternate forms of (22), and will be explored in the future. To analyze convergence behavior of the four methods, %ARE of resistance estimates w.r.t. number of iterations are plotted for $L_{64-65}$ in Fig. 5. EGLE is found to converge within very few iterations ($\approx$20 iterations), MTEE takes around $10^3$ iterations, while MTC and CMTC keep improving until about $10^4$ iterations. Similar convergence behavior was observed for reactance and susceptance estimates as well.

### *B. Evaluation in presence of other noise distributions*

In this subsection, we evaluate performance of the four methods under Laplacian and Gaussian noise conditions. The Laplacian noise has a mean of 0 and a scale of 0.005. The Gaussian noise has zero mean with a standard deviation of 0.005. The results obtained for $L_{64-65}$ along with the time taken to do so, are displayed in Tables I and II, respectively, in which TV refers to the *true value*. All four methods produce reasonably good estimates for the Laplacian noise case; this confirms that they can effectively handle different types of non-Gaussian noises in PMU measurements. For the Gaussian noise case, the four techniques were also compared with TLS. It was observed from the two tables that EGLE had the best performance while also being the quickest among the four techniques. Particularly, MTEE took *approximately four orders of magnitude* longer time than the other methods. This was due to the double summation that must be computed over all the samples (see (6)), implying that MTEE can only be used when there is sufficient time and compute resources.

### *C. Discussion*

The analyses conducted in the previous subsections confirmed that MTEE, MTC, CMTC, and EGLE can be used to perform LPE in presence of any type of noise in the PMU measurements. In this subsection, we identify conditions under which these techniques are expected to give good results. This will be extremely beneficial to power system operators as it will help them select the appropriate method for their problem.

TABLE I
ESTIMATES IN P.U. UNDER LAPLACIAN NOISE FOR $L_{64-65}$

| | TV | MTC | CMTC | EGLE | MTEE |
|---|---|---|---|---|---|
| r | 0.00269 | 0.00267 | 0.00267 | 0.00269 | 0.00268 |
| x | 0.0302 | 0.0300 | 0.0301 | 0.0302 | 0.0301 |
| b | 0.3800 | 0.3797 | 0.3798 | 0.3797 | 0.3796 |
| Time (s) | | 4.07 | 4.24 | 1.68 | 7.3e+4 |

TABLE II
ESTIMATES IN P.U. UNDER GAUSSIAN NOISE FOR $L_{64-65}$

| | TV | TLS | MTC | CMTC | EGLE | MTEE |
|---|---|---|---|---|---|---|
| r | 2.69e-3 | 2.69e-3 | 2.71e-3 | 2.71e-3 | 2.69e-3 | 2.70e-3 |
| x | 30.2e-3 | 30.2e-3 | 30.4e-3 | 30.4e-3 | 30.2e-3 | 30.2e-3 |
| b | 0.3800 | 0.3801 | 0.3801 | 0.3801 | 0.3800 | 0.3801 |
| Time (s) | | 0.35 | 4.14 | 4.18 | 1.68 | 7.3e+4 |

*Remark 1: Tuning.* One challenge faced while implementing the ITL techniques (MTEE, MTC, and CMTC) is the need to tune the hyperparameters, such as step size and kernel width. Specifically, for all three ITL techniques, the step size was found to play a crucial role in achieving both accuracy as well as speed. Hence, it had to be independently tuned for every line. In this regard, the EGLE algorithm stands out as it does not need to be separately tuned for the different lines.

*Remark 2: Applicability.* EGLE had a lower efficacy when the initial guess was far away from the actual values of the line parameters. However, given that the variability of the values is constrained to be within established limits ($\pm 30\%$ [18]), this is not a major concern for the LPE problem. For applications that do not have such constraints on the parameters, modifications to the EGLE technique may be necessary. Conversely, MTEE, MTC, and CMTC exhibited less sensitivity to initial conditions. Hence, a potential approach could be to first solve the estimation problem using MTC (with zero initial conditions) and then use the MTC results as an input to EGLE.

*Remark 3: Operating guidelines.* In MTEE, the quadratic Renyi's entropy captures the uncertainty in the error distribution, which can lead to better generalization in scenarios where the underlying distribution is complex. MTC's use of the correntropy function enhances its robustness against outliers. Specifically, the function's reduced sensitivity to substantial deviations compared to squared error plays a pivotal role in enhancing the speed of MTC. This could not be demonstrated in the identified application because LPE is typically done offline using pre-cleaned (outlier-free) data. CMTC retains the merits of MTC, with its additional equality constraint further boosting the accuracy under appropriate scenarios. EGLE, which models noise as a GMM followed by parameter estimation, is both anticipated and demonstrated to proficiently manage diverse noise distributions with minimal user effort.

## V. CONCLUSION

Four techniques are investigated in this paper to solve the PMU-based LPE problem. This problem is a linear EIV problem with the possibility that the noises in the dependent and independent variables have non-Gaussian distributions. Each method was found to be competent for this task. The accuracy of MTEE was a function of rigorous hyperparameter tuning as well as availability of sufficient time and compute resources. MTC/CMTC are commendable alternatives to the (computational burden of the) MTEE technique; however, they have comparatively lower estimation accuracy. The EGLE method yields fast and accurate results without the need for any hyperparameter tuning. Its effective performance hinges on intelligent initialization, which requires domain knowledge.

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