Millipath: Bridging Materialist Theory and System Development for Surface Texture Fabrication

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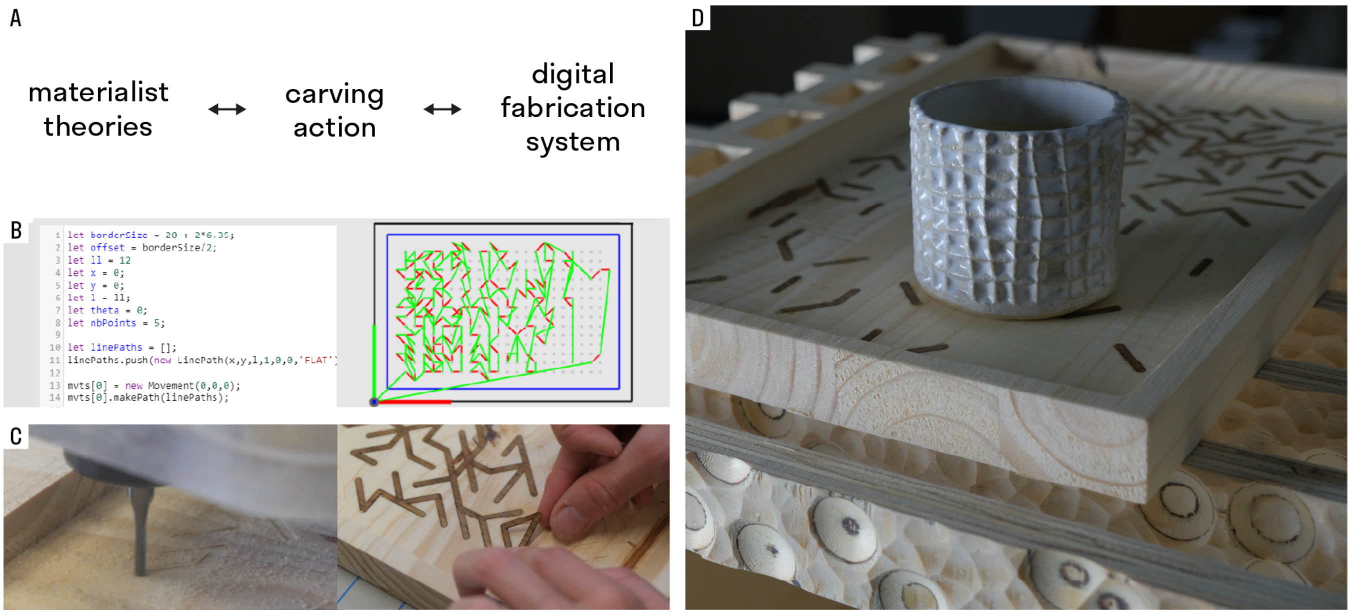

Figure 1: A) This work aims to bridge materialist theories and technical system development for digital fabrication through the use of the bridging concept of action. B) Millipath interface, a system enabling the design of CNC-milled surface texture. C) Computational and manual processes involved in making an inlay tray. D) A scene showing some of the textured artifacts we produced through a research-through-design process, including a tray, a ceramic cup, and a stool.

ABSTRACT

Proponents of digital fabrication argue that future technologies will fundamentally reshape manufacturing; however, we still have a limited understanding of the relationship between contemporary digital fabrication technologies and the values and labor of people who make things. Contemporary materialist theories can offer insights into how interaction modalities with machines and materials influence human production activities. We aim to implement these theoretical principles in technical system development. We focus on *action* as a bridging concept between abstract notions regarding human-machine-material relationships and concrete digital fabrication system features. We use CNC-milled surface texture production on wood as a case study. We follow a research-through-design process to develop Millipath, an action-oriented programming platform enabling the parametric design of machine toolpaths. Through the analysis of autobiographical data from fabricating artifacts, we investigate how digital fabrication systems informed by materialist theories support expressive modes of production and design decisions in response to material behaviors.

CCS CONCEPTS

• **Human-centered computing** → *Interactive systems and tools.*

KEYWORDS

Digital Fabrication, Materialist Theory, Surface Texture, CNC Milling

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1 INTRODUCTION

Digital fabrication technology impacts how humans make physical products and goods, influencing the distribution of resources and labor across societies. Developing durable and sustainable digital fabrication systems, therefore, requires not only technical knowledge but also a better theoretical understanding of the relationship between system functionality, manual and computational skillsets of contemporary manufacturers [28], and the values and labor involved in material production [42]. Despite sharing a focus on human-machine interaction for digital fabrication, researchers making design theory and systems contributions tend to work in parallel tracks, with limited engagement between them [6, 56]. [1] This design-systems divide in HCI reflects the broader gap between design-oriented research, which often remains speculative, and system-oriented research, which frequently overlooks valuable theoretical insights.

While there are benefits to both speculative and applied research outcomes, we argue for the importance of conducting applied digital fabrication systems research driven by theoretical insight. This approach is critical for developing technology that addresses complex problems through an in-depth and interdisciplinary examination of specific domains [6, 56]. Contemporary materialist theory provides a useful framework to examine the fundamental relationships between people and the material world [24]. HCI digital fabrication researchers engaging with materialist theory are especially interested in how decentering the human maker in the design process can support human-machine-material engagements that take into account physical, mechanical, environmental, and social dimensions of production [1, 19]. Given its relevance for digital fabrication research, we believe that this theory can serve as an excellent foundation for addressing the design-systems divide. Therefore, our objective with this work is to explore the methodological and technical opportunities of developing a digital fabrication system informed by materialist theory that can sustain skilled craft practices.

[1]This is in part exemplified by distinct CHI subcommittees for hardware, systems, and design research.

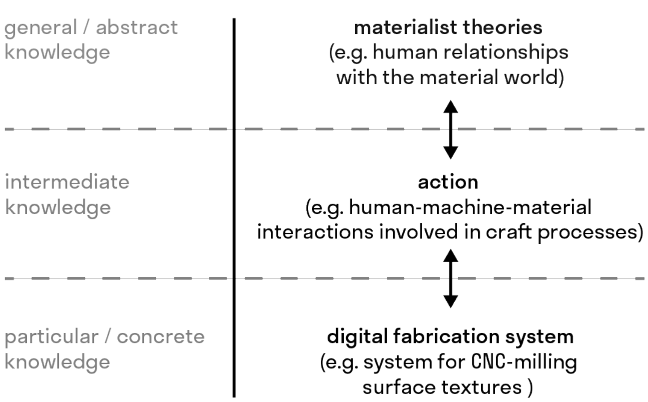

Figure 2: Action as a bridging concept between materialist theories and digital fabrication system. The figure structure is taken from Dalsgaard and Dindler [17].

As a case study for implementing materialist theory in system development, we focus on subtractive manufacturing, with a specific emphasis on woodworking craft. This area of physical production presents a unique opportunity to bridge the design-systems divide, as HCI researchers have investigated it from both design and systems perspectives. Design-oriented researchers have used materialist theory as an analytical tool to examine material practices [14, 20, 23, 50]. Concurrently, systems-oriented researchers have introduced new fabrication modalities into woodworking, including software and hardware that support the automation of complex design and sculpting tasks, such as generating joinery and mitigating sculpting errors through machine guidance [26, 39, 44, 63]. These parallel lines of inquiry demonstrate the potential for integrating theory with practical system development in subtractive fabrication for woodworking.

We draw from Dalsgaard and Dindler's *bridging concepts* to connect these research inquiries. Bridging concepts comprise intermediate-level knowledge "facilitating exchange between theory and practice [17]." We identify the bridging concept of *action*– the site of convergence between a tool and a material during the making process [30, 58]– to guide our development of a digital fabrication system's representations and interactions (Figure 2). Action-oriented modes of production support machine and material expressivity in other manual-digital craft [8]. We focus on the production of surface texture using a CNC-milling machine to develop an action-oriented system characterized by the execution of a sequence of carving actions that reflect manual modes of production [12]. We use a research-through-design process integrating theoretical and engineering stages. We reviewed materialist theories regarding human-material interactions to inform a top-down analysis of a design space of surface-textured artifacts. Through this analysis, we conceptualized three design articulations of the carving action, including (1) the tool movement repeated across (2) a grid within (3) a specified boundary. These articulations informed the development of *Millipath*, a web-based programming platform that supports the creation of parametric surface textures with CNC-milling machines through toolpath specification. We used Millipath to conduct two studies involving the fabrication of texture samples and three artifacts. Finally, by analyzing autobiographical fabrication data, we discussed how working with machine actions fosters deep engagement with specific materials.

We contribute:

(1) A methodology that supports the implementation of materialist theories in system development, using *action* as a bridging concept.
(2) A design space for carved and CNC-milled surface texture highlighting a set of inherent components – movement, grid, and boundary – that reflect the theoretical and practical notion of action in the context of carving.
(3) Millipath, a novel action-oriented system that enables the creation of surface texture on wood and other carvable materials by specifying low-level machine actions.
(4) An analysis of autobiographical data collected during the creation of artifacts with Millipath, emphasizing how using action as a design tool influences human interactions with material and machine.

2 BACKGROUND

Our work builds on prior philosophical, craft, and HCI literature, examining human-machine-material interaction from both theoretical and technical perspectives. We present *action* as a point of convergence to support bridging design theory and systems in digital fabrication research. This section highlights the importance of theoretical, design, and technical knowledge in situating our work between theory and practice.

2.1 Bridging Concept Between Theory and Practice

Building on intermediate forms of knowledge [29, 60], Dalsgaard and Dindler developed the idea of the "bridging concept" as a way to facilitate dialogue between theoretical principles and practical endeavors [17]. They argue that the rapidly evolving human-technological landscape requires regularly updated theories, which, in turn, help clarify the implications of novel systems. Bridging concepts are grounded in three components spanning the theoretical-practical spectrum: (1) a theoretical foundation, (2) practical exemplars, and (3) design articulations– the aesthetic qualities embodied in artifacts [36]. While Dalsgaard and Dindler have focused primarily on existing artifacts, we aim to demonstrate the use of bridging concepts in developing a digital fabrication system for craft. We leverage bridging concepts as a generative theory [6] to design concrete computational functionalities.

2.2 Materialist and Craft Theories of Action

The idea of *action* spans materialist and craft theories, describing complex relationships between humans and the material world. Karen Barad argued that things acquire meanings as a result of interacting with other things. She described this phenomenon as *intra-action* [3]. Bruno Latour proposed an action-network theory, wherein humans and non-humans evolve in intricately connected networks, eliminating the need for hierarchical distinctions between them [37]. Jane Bennett claimed that all materials, therefore, are dynamic. They do not await human transformation as claimed by Aristotle's prevailing hylomorphic model of creation but are vibrant and responsive [7]. Using these materialist theories to investigate material production, Tim Ingold described the making process as the active negotiations between humans, materials, and environmental forces [30]. In the context of craft, Richard Sennett argued that "making is a time-based rhythm of actions [58]", thus shifting the emphasis of material production from the artifact to the process. For its relevance to both theoretical and practical accounts, we identified the idea of **action** – the convergence of tools and materials at the moment of fabrication [30] – as a potentially powerful bridging concept for digital fabrication research in HCI.

2.3 Machine Actions in Craft-Based HCI

Action as the fundamental unit of making echoes CAM-based design methods in digital fabrication, where an artifact form is specified by machine toolpaths rather than geometry. Artists and designers have demonstrated the benefits of using CAM-based design to create unique and intricate artifacts [9, 16, 35, 40, 45]. HCI researchers have also presented the opportunities of programming machine toolpaths as design strategies to leverage material properties [8, 11], achieve fine control over machine parameters [22, 61], and develop low-volume manufacturing products [28]. Bourgault *et al.* described this approach as *action-oriented*, where each incremental action enacted by a craftsperson or a machine during a fabrication process defines an artifact's final form [8].

We wish to explore the opportunities of action-oriented workflows for subtractive fabrication, focusing on wood carving with CNC-milling machines. Our work takes inspiration from Tran O'Leary *et al.*, who developed a Notebook-based approach for procedural control of CNC-milling machines [65] and the now deprecated ArtCAM software, which allowed craftspeople to use direct manipulation to specify machine toolpaths [2]. These two systems enable low-level manipulation of machine properties and design features, an essential aspect of action-oriented workflows [8].

2.4 Materialist Theories in HCI

Fabrication research in HCI has predominantly used materialist theories as analytical and speculative modes of inquiry to examine human-machine-material interactions. These theories form the basis of what Frauenberger referred to as Entanglement HCI [24], a paradigm describing the relationships between humans and things as inseparable. The concept of entanglement, when applied to fabrication, has informed ethnographic research examining skilled practitioners engaged in working with living materials [14, 20, 23]. Additionally, it has provided guiding principles for developing interactive probes [18, 50, 51]. These works emphasize decentering the human as the exclusive source of meaning and creativity in the making process. Instead, they highlight the importance of paying acute attention to the materials and machines involved, propose new models for collaboration, and promote sustainable practices to address material scarcity through reuse and repair [1, 19]. Although this literature highlights crucial considerations about human-machine-material interactions, the researchers do not propose concrete ways to embed the theory in technical system features. Our work is informed by Devendorf *et al.*'s operationalization of post-human theories in the implementation of a human-embodied 3D printer [18]. Additionally, we aim to leverage these theories in a digital fabrication system that can sustain skilled craft practices.

2.5 Digital Fabrication Systems for Craft in HCI

HCI digital fabrication systems researchers developing tools for craft face the difficult challenge of using simplified computational representations to describe complex material interactions. This endeavor encompasses the development of domain-specific software that extends existing craft [21, 64, 67] and novel hardware that supports craftspeople in complex fabrication tasks [59, 69]. Craft-based HCI has taken a particular interest in woodworking, given the importance of wood in manufacturing processes. HCI system researchers have examined the production of joinery [5, 38, 44, 47, 63, 66], jigs [39], and furniture assembly [26, 41, 57]. The production of surface textures on wood and other surface finish tasks has received limited attention. These tasks are essential in making detailed and refined products, and they provide cultural and social values to artifacts [4]. In this paper, we present a novel system that enables craftspeople to design and fabricate surface texture by specifying low-level machine actions.

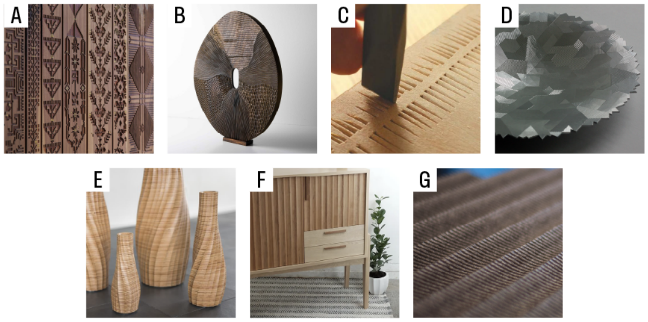

Figure 3: Design space for carved surface texture on wood and metal. A) The Shawl by the Naqsh Collective [48]. B) Fossil by Andrew Daniels [25]. C) Rob Brown's wood decoration techniques with a cold chisel [10]. D) 'ST11', a CNC milled Aluminium platter, copyright Drummond Masterton [46]. E) Aestus vase by Oliver David Krieg [35]. F) Bar Andrés wooden furniture by Maderistica [43]. G) Wavy surface with CNC texture produced with Vespidae on Valchromat [22].

3 DESIGN SPACE

Bridging concepts contain design articulations – "salient parameters that designers can manipulate to change the interaction and experience of an artifact [17]." Identifying the design articulations embodied in a bridging concept facilitates its application. To identify the design articulations of carving action, we collected 30 exemplars of carved artifacts with surface textures produced through manual and digital methods. These exemplars include the work of artists, designers, and researchers whom we have followed for years, alongside works we found through online searches on visual platforms like Pinterest, using prompts such as "carved texture". In Figure 3, we present a selection of seven artifacts from this design space that we believe effectively illustrate the concept of actions in the context of material carving. We analyzed these exemplars visually to find similarities in how textures are applied and organized across the surfaces of these artifacts. We conceptualized the grid, the movement, and the boundary as the constituents of the carving action. We illustrate these components in Figure 4. In 4A, the line texture on the top part of the sculptural work Fossil by Andrew Daniels [25] was created by repeating linear horizontal movements with a rotary tool across a vertical grid within a triangular boundary, while in 4B, the dot texture was produced by downward movements executed across a tilted grid within a similar boundary rotated by 90°. Our design articulations can also describe complex motifs, as shown in Figure 4C. The intricate triangular motif by the Naqsh Collective in the Shawl wooden wall piece [48] is created by a combination of parallel linear movements repeated across a linear grid within a vertical rectangular boundary.

We use the design articulations of grid, movement, and boundary to connect the actions and visual language of carving with the operational language necessary to program a CNC-milling machine.

4 METHODOLOGY

We used a research-through-design approach to develop Millipath, a digital fabrication system informed by materialist theories, and fabricated a series of artifacts [68]. This project took place over ten months, in a research residency at the AUTODESK Technology Center in Boston for the first three months and at our academic research laboratory at UCSB for the seven last. The residency context enabled Sam to extend her knowledge of CNC milling by working with skilled woodworking and machining technicians and residents.

Our system core features are based on the design articulations described in Section 3, which we developed primarily during the first five months of the project. Initially, this development phase involved daily CNC-milling tests on insulation foam to assess the safe operation of the machine with an easily manufacturable material that wouldn't cause damages in case of toolpathing errors. As the system became more predictable and Sam gained confidence, we conducted tests on thin plywood sheets. Once the system's backbone was functional, we began a first study involving the production of 28 texture samples in plywood (Section 6). This first study enabled us to create an initial visual language for CNC-milled surface texture produced with Millipath while continuing to update and add computational features as needed. We conducted a second study involving the design and fabrication of three artifacts: a low stool with surface texture superposition, a set of four ceramic cups molded on CNC-milled plaster slabs, and a tray with walnut inlay (Section 7).

During the five-month process necessary to complete both studies, we compiled written and drawn notes taken during design and fabrication tasks. Sam kept her note application open every time she was programming new features or carving artifacts on the CNC machine. This allowed her to record her observations and reflections in real-time. Finally, we analyzed this autobiographical

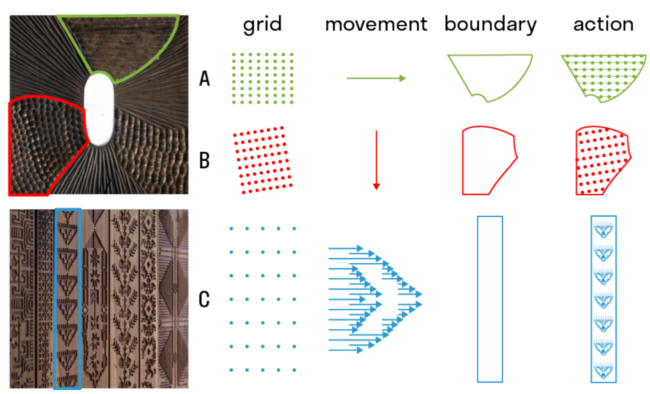

Figure 4: The design articulations of the carving action, including the grid, the movement, and the boundary, characterize the visual organization of surface textures. A) The top right section of Fossil by Andrew Daniels [25] is composed of linear movements executed across a linear grid within a triangle-like boundary, and B) the bottom left region is composed of downward movements executed across a tilted linear grid within a similar but rotated by 90° triangle-like boundary. C) One section of the Shawl wall panel by Naqsh Collective consists of a combination of parallel linear movements repeated across a linear grid within a vertical rectangular boundary.

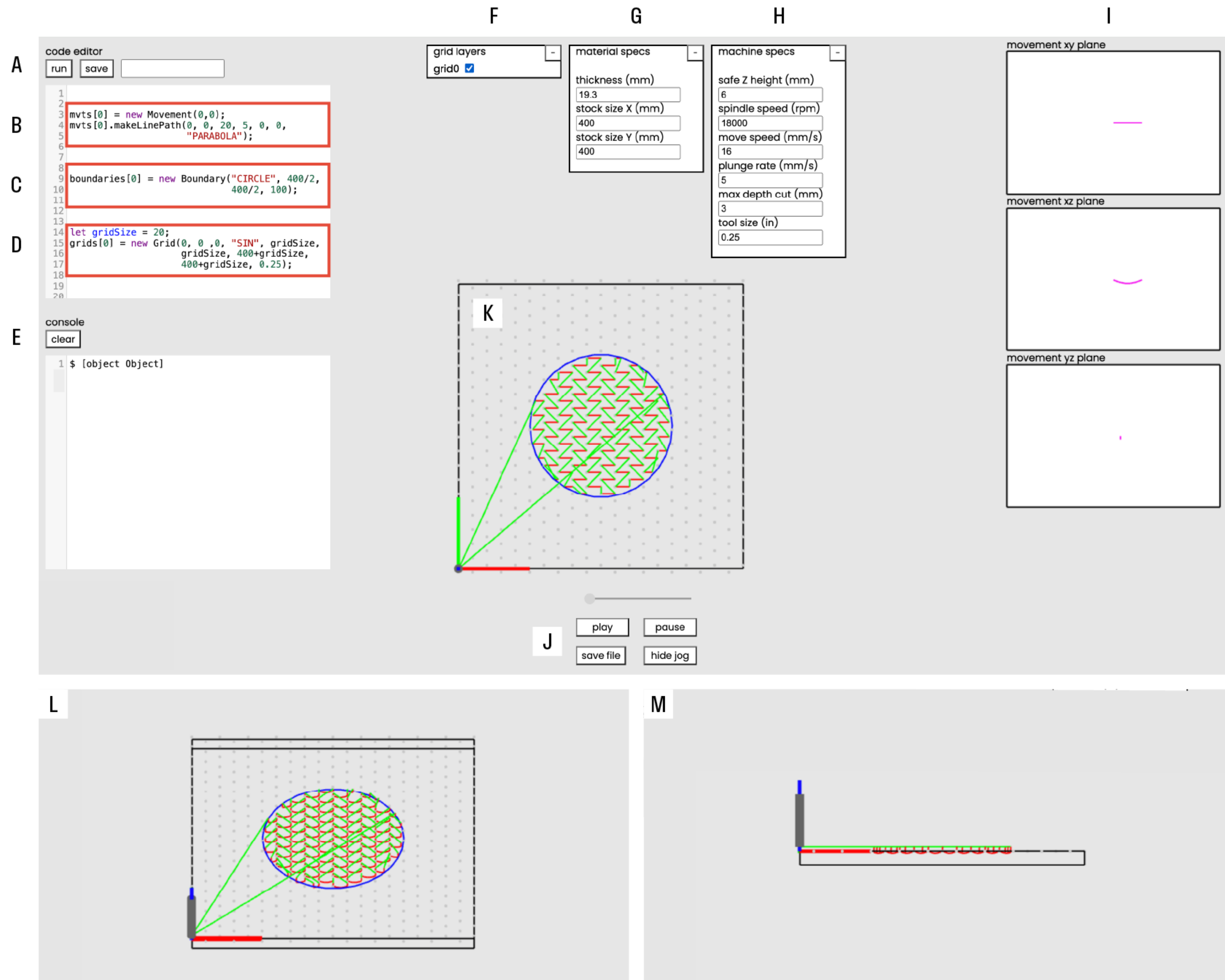

Figure 5: The Millipath web interface. A) Millipath uses a code editor to generate the movement, grid, and boundary components. The run and save buttons are used to execute and save the code. B) a 20mm-long linear movement is created using a *PARABOLA* depth function in this example. C) The boundary is set to a 200mm-diameter circle in the middle of the stock material. D) A sinusoidal grid with 20mm spacing between points is programmed. E) The logged console from the code editor. F) A collapsable box listing the created grids. The listed grids can be visible or invisible by checking their corresponding box. G) A collapsable box containing the material specifications, such as the stock dimensions. H) A collapsable box containing the machine specifications, such as the spindle speed, the move and plunge rates, the maximum depth of cuts, the safe z height for the machine to move, and the tool size. I) The movement is visualized over the XY, XZ, and YZ perpendicular planes. J) A set of buttons to visually play or pause a simulation of the tool movement, to save the generated machine code, and to hide or show the jog moves. A slider to control the orientation of the stock piece. K) A top view of the movements, boundaries, and grids on the specified piece of stock. L) A 45° view of K. M) A side view of K.

data to conceptualize themes [49] around human-machine-material interaction applied to surface texture (Section 8).

5 SYSTEM DESCRIPTION

Through the analysis of our design space (Section 3), we conceptualized three design articulations – movement, grid, and boundary – which informed the development of a programming platform for creating CNC-milled surface texture called Millipath. Millipath is a parametric system that enables the design and fabrication of carving actions by specifying machine toolpaths. Millipath is a web application built with the JavaScript framework p5.js (Figure 5), where a craftsperson can specify machine movements (5B), boundaries (5C) and grids (5D) programmatically, and produce OpenSBP code to execute on a ShopBot CNC router. In Millipath, each grid repeats a specified movement within a designated boundary. A movement is defined by one or more line paths, smaller building blocks characterized by a position, a number of vertices, and a depth function. This section details each of Millipath's components, their associated functions, and the forms of automation supported by the system.

5.1 The Line Path

From our design space, we observed that a movement could be represented by a combination of lines ranging in complexity (see Figure 4). We developed, therefore, the line path object as the building block of every machine movement generated with Millipath. As illustrated in Figure 6A, a line path requires a position `x` and `y`, a length `l`, an orientation `theta`, and the number of vertices `nbPoints`. Additionally, the line path has five optional parameters: a reflection axis `phi` (set to `theta` by default), a depth mode `zMode` (`"FLAT"` by default), a custom depth function, an amplitude `amp` (0 by default) and a period `T` (1 by default). A line path can also be sinusoidal in the XY-plane by modifying the amplitude and period parameters (Figure 6B). The depth mode defines the line path behavior in the XZ-plane, with pre-programmed functions `FLAT`, `LINEAR_UP`, `LINEAR_DOWN`, `SIN`, `PARABOLA`, and `NOISE` (Figure 6C) or a custom function defined as the `customZMode` parameter. Our design space was essential in defining these parameters. The `zMode`, for instance, emerged from observing the variations in depth in the exemplars of Figures 3C and G.

5.2 The Movement

We developed the movement to represent the motion repeated by the CNC-milling machine at each grid point. A movement can be abstract, like in Figure 3E, or figurative, like in Figure 3A. In Millipath, a movement is created with two functions: an instantiation function and a path specification function. Our system provides a set of preprogrammed path specification functions. Table 1 lists a subset of these functions.

The simplest movement can be created with the `makeLinePath` function, which consists of a single line path object. More complex movements combine line paths and require additional parameters. Certain path specification functions such as `makeRectanglePath` allow for multiple depth passes based on the maximum depth of cut value.When multiple passes are enabled, tabs are automatically generated on the last pass of the cut. We implemented the multiple passes functionality to facilitate the creation of exterior boundaries, a feature commonly found in other CNC-milling CAM software. This enabled us to create textures and artifacts concurrently. Finally, to support the generation of more complex movements, path specification functions are customizable by using a line path array as input to the `makePath` function.

5.3 The Grid

To organize machine movements in space, we developed the grid object. In Millipath, a grid is always associated with a movement and a boundary. As shown in Figure 7, a grid is defined by a position

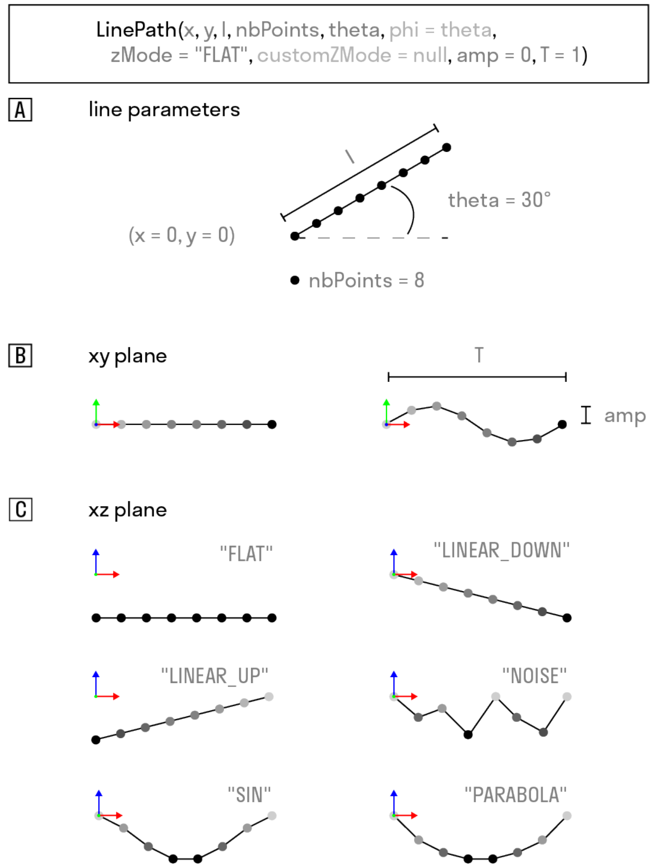

Figure 6: The line path is the building block of every movement in Millipath. A) A line path is defined by a position `x` and `y`, a length `l`, a number of vertices `nbPoints`, and an orientation `theta`. B) The line path can take a sinusoidal shape in the XY-plane by specifying an amplitude `amp` and a period `T`. C) The line path can be mapped to six preprogrammed depth functions. It also accepts custom depth functions as the `customMode` parameter.

`(x,y)`, a mode, spacings between the grid's points in x and y, and sizes in x and y. Millipath provides four different grid modes: linear, sinusoidal, Perlin noise, and random noise. We found that these parameters and grid modes could describe the spatial organization of the texture shown in our design space exemplars.

To support movement transformation across the grid, we implemented transformation matrices that map each matrix element to each grid point. These transformations include rotation, reflection, scale, translation, and visibility. These transformations are applied to the associated movement at each grid point. For instance, if the element on row 2 and column 3 of the `visibility` matrix is 0, the movement will not be executed at the corresponding grid point. We found that these transformations could support creating geometrical and organic-looking patterns. Finally, we implemented an optional `linkState` parameter for the grid to enable or disable the retraction of the machine between grid points. A linked `linkState` is useful for facing operations, e.i. leveling the stock material surface

Table 1: Some of the Preprogrammed Movement Path Specification Functions Available in Millipath

Function	Inputs	Description
makePath	<linePath>	Create a movement from a custom array of LinePath objects.
makeLinePath	x, y, l, nbPoints, θ, ϕ, *zMode*, *customZMode*	Create a line of length l with *nbPoints* points. The first point is at (x, y), rotated around $(0, 0)$ by θ radians, and reflected across ϕ axis. The line follows a "zMode" or custom depth function.
makeArcPath	r, arcAngle, nbSides, θ *zMode*, *customZMode*, *multiDepth*	Create an arc of radius r and arc angle arcAngle made of nbSides sides. The arc is centered and rotated around $(0, 0)$ by θ radians, with each sidefollowing a "zMode" or custom depth function.
makeRectanglePath	rx, ry, nbPoints, θ, *zMode*, *customZMode*, *multiDepth*	Create a rectangle of half sides rx and ry with each side made of *nbPoints* points. The rectangle follows a "zMode" or custom depth function with possibly multiple passes.
makeRectanglePathWithTab	rx, ry, nbPoints, θ *zMode*, *customZMode*	Create a rectangle of half sides rx and ry with each side made of *nbPoints* points. The rectangle follows a "zMode" or custom depth function with one tab per side.

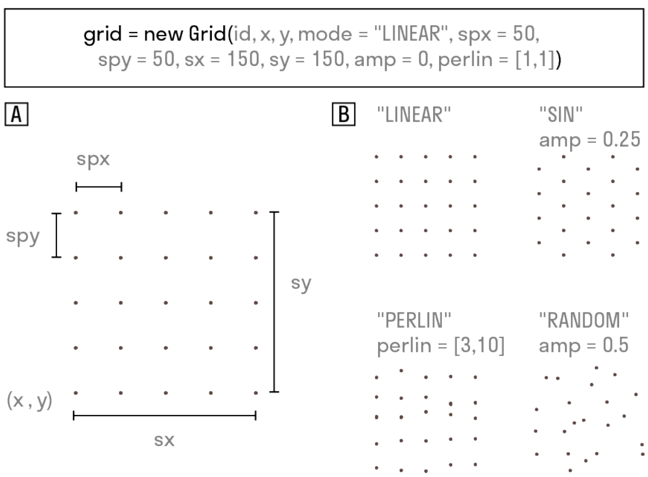

Figure 7: Millipath's grid object. A) The spatial parameters of the grid include its bottom left corner position (x,y), the spacing between its points in x, spx, and in y, spy, and its size in x, sx, and in y, sy. B) The system implements four different grid modes: linear, sinusoidal, Perlin, and random mode. The last three modes use the optional amp parameters to scale their amplitude, and the Perlin mode uses a 2D perlin size to scale the noise level.

by removing the top layers and pocketing operations, e.i. creating a bounded hole at a set depth.

5.4 Boundary

To define the application perimeter of a grid, we developed the boundary object. A boundary is instantiated with a `mode`, a position `(x,y)`, and size specifications `rX` and optionally `rY`. The available modes include: `CIRCLE`, `RECTANGLE`, `TRIANGLE` and `POLYGON`. Internally, a boundary evaluates a signed distance function to determine if a movement's line path vertex is within or outside the boundary [55]. This structure facilitates the generation of more complex perimeters by combining and subtracting boundaries through Boolean operations [13, 34]. The boundary object also accepts custom signed distance functions at instantiation. Millipath provides simple polygonal boundary utility functions to simplify the creation of these functions. By default, a boundary is an outside perimeter (Figure 8A), meaning that the movements within its perimeter are executed. Setting the optional parameter `outsideBoundary` to false reverses this behavior (Figure 8B). We developed these different methods for combining simple boundary functions to enable more accurate texture placement on particular sections of an artifact.

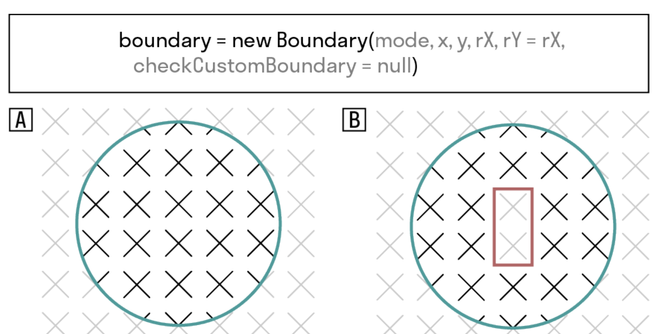

Figure 8: Boundary where X's represent machine movements. A) The movements are only executed within the teal circular boundary. B) The red rectangle acts as an inverse boundary, where movements outside are executed and inside are not.

5.5 The Code Generator

The code generator script combines the components and generates the OpenSBP code executable by a ShopBot CNC-milling machine. It associates each grid with a movement and a boundary object and applies the grid's transformation modifiers (rotation, reflection, scale, translation, and visibility) to each movement. It checks the position of each movement's line path vertex and removes the vertices outside the boundary. When a movement is fragmented, the code generator adds a retraction motion to a safe z height before moving to the next section of the movement.

6 TEXTURE SAMPLES STUDIES

To gain insights into the expressive potential of our movement-grid-boundary model of the carving action, we applied Millipath to the design and fabrication of five collections of texture samples. The samples were designed around five visual components that recurred systematically throughout our design space: dot, line, motif, pattern superposition, and triangle. We present the first four components in this section and the last in appendix A. Making these texture samples allowed us to systematically explore Millipath's multi-dimensional design space and adapt system features to enable specific outcomes. Each sample records an interaction between code, machine, and material, thereby providing "concrete embodiments of theory and technical opportunities [68]." Drawing inspiration from the role played by swatches in textile practices and research [33, 53], we envisioned these texture samples as design patterns that could be repurposed, extended, and remixed in the creation of artifacts. Each sample is a 120 by 120 mm square positioned on a 1 by 1 foot plywood sheet. We analyze the design and fabrication process for making these texture samples in more detail in Sections 8 and 9.

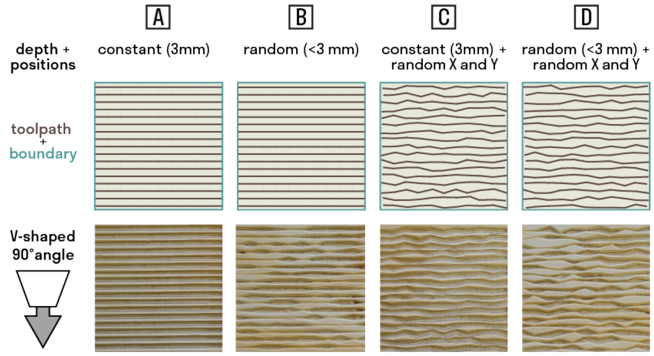

Figure 10: Line study through variations of movement's vertices position using a V-shaped 90° end mill. A) Straight vertical movements. B) Vertical movements with random vertices depth ranging from 0 mm to 3 mm. C) Vertical movements with random vertices X and Y positions between 0 mm and 2 mm from the initial position. D) Vertical movements with B and C modifications.

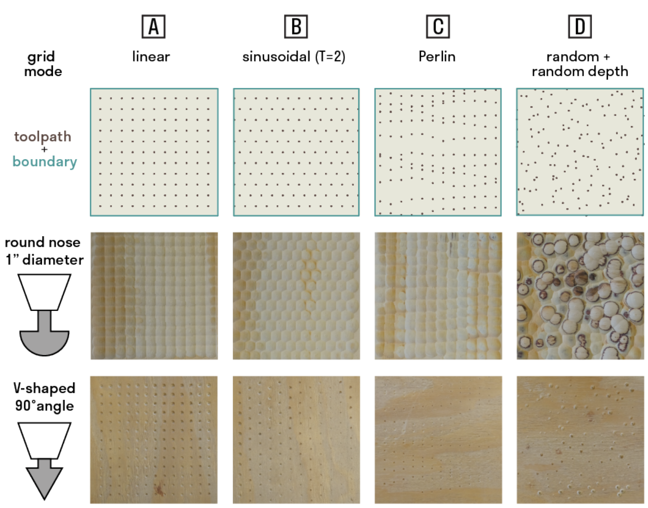

Figure 9: Dot study with grid mode variations. The same toolpaths are used with two different end mills, a round nose of 1" diameter, and a V-shaped with a 120° angle. The results are shown on plywood on row 2 and row 3, respectively. A) Arc movements following a linear grid within a rectangular boundary. B) Arc movements following a sinusoidal grid with period $T = 2$ within a rectangular boundary. C) Arc movements following a Perlin noise grid within a rectangular boundary. D) Arc movements following a random grid within a rectangular boundary.

6.1 Study on Dots

Inspired by how Andrew Daniels executes carving actions with a power rotary tool across a region of his sculpture to produce dot-like marks (Figure 4B), we wished to use Millipath to create dot-like etchings. With these first texture samples, we used variations of downward vertical movements with two different end mills. This enabled us to observe how incremental changes in design features and machine properties could affect the final textures.

As illustrated in Figure 9, we used a 1"-diameter round nose and a V-shaped 90° end mills to execute the same carving actions. We programmed four different grid modes, e.i. linear, sinusoidal, Perlin noise, and random, and introduced random depths in the last sample (9D). These resulting samples show how organizing the carving actions around different grid configurations and using one end mill over another can support a range of aesthetic surface qualities. They also demonstrate how CNC-milling carving actions, while highly simplified compared to manual woodworking, can reproduce visual aspects of manually created carving actions.

6.2 Study on Lines

As depicted in Figures 3B, C, E, F, and G, linear surface textures are commonly used in woodworking. With these second texture samples, we explored variations on lines through modifications of the movement vertices.

We executed linear vertical movement across these samples using a V-shaped 90° end mill. We began our exploration by generating straight horizontal linear movements at a constant depth of 3 mm (Figure 10A). We then added random depth values ranging from 0 mm to 3 mm deep (Figure 10B) and random X and Y position values ranging from 0 mm to 2 mm to the movement's vertices

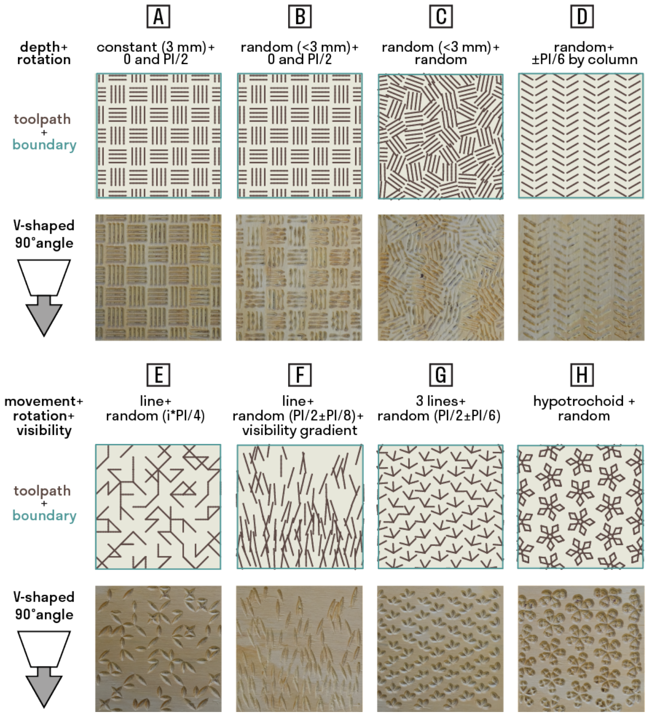

Figure 11: Motif study using a V-shaped 90° end mill. A) Four parallel line movements organized on a linear grid with alternating $PI/2$ rotation. B) Same as A with random depth. C) Same a B with random movement rotation. D) Chevron-like movement with random depth. E) Linear movements with parabolic depth and random integer of $PI/4$ rotation organized on a linear grid. F) Linear movements with random rotation around $PI/2$ organized on a random grid with a gradient in visibility. G) Three converging line movements organized on a sinusoidal grid. H) Hypotrochoid movements with random rotation on a sinusoidal grid .

(Figure 10C). We combined both modifications on the last texture sample (Figure 10D). From straight to organic-looking lines, these samples show how subtle adjustments in the machine movement's properties can significantly alter the surface texture's aesthetic.

6.3 Study on Motifs

As shown in Figure 3A and 6B, figurative and abstract motifs are used as decorative elements. Producing these patterns requires repeating complex movements with precise positioning to create an ordered visual outcome. We fabricated eight motif texture samples inspired by woven textile patterns and floral-figurative elements.

In the first three texture samples, we used a four-line movement (Figure 11A) and modified its depth (Figure 11B) and rotation (Figure 11C) across a linear grid. In Figure 11D, we implemented a chevron-like movement with random depth (Figure 11D).

The movements used in the last four samples follow a parabolic depth function. Using a V-shaped 90° end mill, we created lemon-shaped etchings with integer multipliers of $PI/4$ rad rotation (Figure 11E), grass-like patterns with a visibility gradient in Y (Figure 11F), flower-like motifs with three connected line paths (Figure 11G), and a hypotrochoid movement (Figure 11H). These texture samples demonstrate how the carving tool's geometrical properties can support aesthetic qualities not accounted for in the toolpath representation.

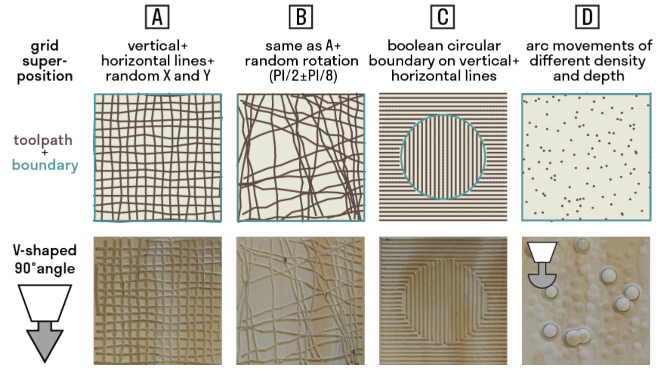

Figure 12: Study on Superposition. A) The superposition of vertical and horizontal movements generates square-like positive forms. B) The superposition of vertical and horizontal movements with random rotations creates uneven parallelograms and triangles. C) A custom circular boundary shows vertical movement within it and horizontal ones outside it. D) In this grid superposition, dense and shallow circular movements contrast with sparse and deep circular movements.

6.4 Study on Superposition

The last texture samples showcase grid superposition and the use of custom boundaries to create more complex textures. As shown in Figures 3A and F, overlaying grids and movements result in uncarved positive forms at the intersection of carved regions. In these samples, we explored the aesthetic relations emerging from overlaying dot and line movements.

We overlaid two grids consisting of vertical and horizontal movements at a constant depth of 3 mm (Figure 12A), showing $\pm PI/8$ rad variations in rotation (Figure 12B) and grid differentiation through custom complementary circular boundaries (Figure 12C). In Figure 12D, we overlaid two grids in random mode composed of dense and shallow circular movements in contrast with sparse and deep circular movements. Since the plywood has varying colored layers, topographic circles appear in the larger circular dots, creating

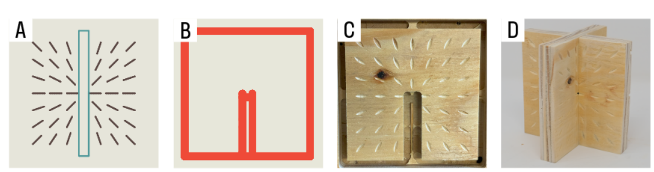

Figure 13: Decorated Dog Bone Joint. A) Linear movements with parabolic depth oriented towards the middle of the part for the surface texture. B) A rectangle movement and a dog-bone movement for the contour. C) One part of the joint is fabricated with tabs in plywood. D) Two joined parts.

additional motifs. These samples show how using grids as design objects in combination with custom boundaries results in more intricate region-specific surface textures.

6.5 Comparing Millipath with a Typical CAD-CAM Workflow

By making 28 texture samples, we observed some benefits of using an action-oriented model for surface texture production over the traditional CAD-CAM workflow. First, our system enabled us to leverage machine properties – position, depth of cut, toolpath stepover through movement density, etc. – as design features. Craftspeople working with digital fabrication use this approach to create unique surface finishes [28]. With Millipath, the combined effect of movement path and end mill shape describe the visual outcomes. This approach to design supports unconventional CAM operations and opens up new ways to work with subtractive fabrication by offering a formal representation of the machine's actions. Second, Millipath enables the parametric specification of machine toolpaths, departing from traditional digital fabrication workflows where the toolpaths are directed by the geometry of the fabricated artifact. This supports the rapid generation of design variations from precise parameter adjustments or the creation of different surface textures centered around a specific visual component. While Millipath enables machine control at a low level, it also automates certain tasks like other CAM software to reduce code repetitions. For instance,

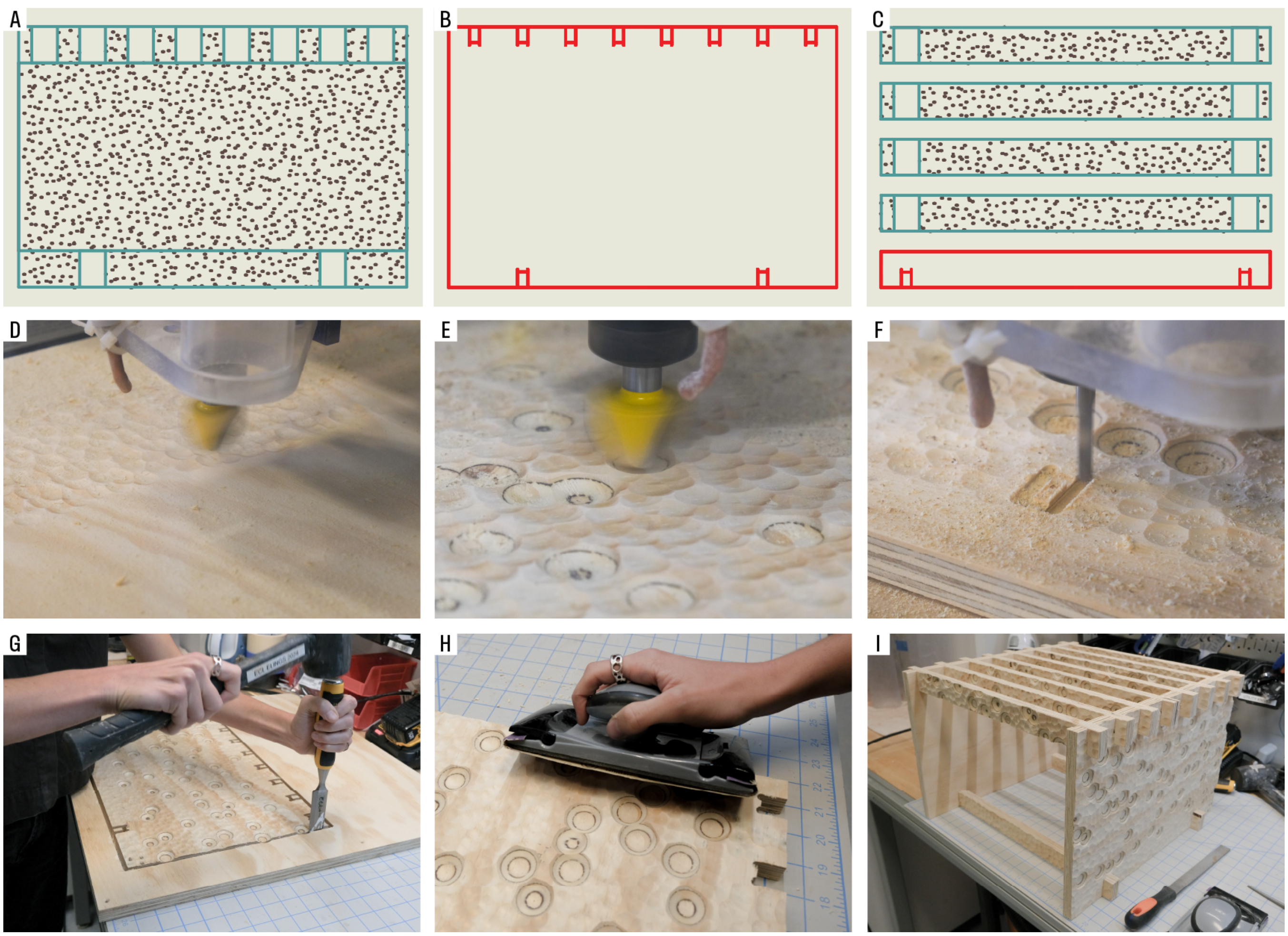

Figure 14: Low textured stool. A) Surface texture toolpath for the sides of the stool with inverse boundaries to prevent texture on the joints. B) The contour toolpath for the side of the stool has eight joints for the top slats and two for the feet. C) Four surface texture variations and the contour toolpath for the slats. D) The first texture layer on the side of the stool is made of short 1.2 to 2 mm-deep lines following a dense random grid. A region without texture on the bottom left corner of the image shows the effect of an inverse boundary. E) The second texture layer on the side of the stool is made of 3.5 to 4 mm-deep arc movements following a loose random grid. F) Cutting the first layer of a dogbone joint on the side of the stool. G) Removing the tabs with a chisel and hammer. H) Sanding the tabs and splinters. I) Assembling the pieces to complete the stool.

it supports the automatic generation of lift and job movements between carving actions, multi-pass cuts, and tabs to secure through cuts.

7 ARTIFACTS STUDY

To evaluate the viability and opportunities of CNC-milled surface textures in craft production, we used Millipath to create three artifacts using different materials and decorative techniques. During five months, we iterated on the design and fabrication of a plywood stool, four textured ceramic cups, and an inlaid wooden tray using the actions and visual language developed in our first study.

7.1 Surface Texture on Plywood Stool

Our first artifact is a low stool with two textured side panels and ten slats. The surface texture of each component is based on the superposition of two grids created in sample 12D. In this section, we first present the creation of a dogbone joint, an integral component of the stool's structure. We then describe the creation of the complete stool. We discuss the requirement for deliberate texture positioning in planning for part assembly.

7.1.1 Dog Bone Joinery. Creating a dogbone joint was a fundamental step in making our textured stool. While Millipath was developed for surface texture, we wanted to investigate its potential for creating contours and joinery. To make a joint, we designed a surface texture consisting of linear movements with parabolic depth oriented toward the center of the part (Figure 13A). We then created the movements for the done bone joint and the rectangular contour of the part (Figure 13B). We fabricated the texture with a 60° V-shaped end mill and cut the contours with a 1/4" flat end mill (Figure 13C). Figure 13D shows the resulting two-parts joinery.

7.1.2 Carved Stool. Following the creation of a functional joint, we started to design our textured stool. We created a series of 1:6 stool models in the Rhino software and decided on a design consisting of two side panels and ten slats for the seat and the feet of the stool. For surface texture, we updated sample 12D to fit the dimension of each stool's part. We added inverse boundaries where the parts were meant to connect to avoid having textures around the joints (Figures 14A and C). We generated four different texture patterns for the slats to get visual variations (Figure 14C). Each variation results from executing the code once, generating new random grid point positions every time. We fabricated the surface texture first (Figures 14D and E) and then the contours, including the joints (Figures 14B, C, and F). Once all the parts were fabricated, we removed the tabs (Figure 14G) and sanded the surfaces (Figure 14H). We assembled the parts to get the finished stool (Figure 14I).

This stool exemplar shows how Millipath not only facilitates the creation of surface textures but, through its grid-movement-boundary model, also enables the production of functional joinery and furniture. Incorporating custom boundary objects allows the intentional placement of textures, which is particularly beneficial for circumventing regions requiring proper contact between two parts, as in the case of a joint.

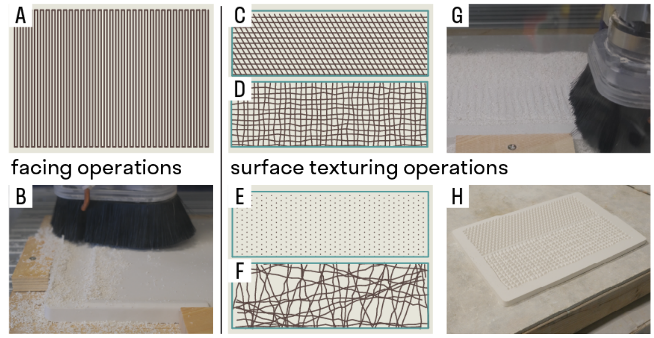

Figure 15: Creating texture on plaster slabs: A) Toolpath with half-toolsize stepover. B) Facing with a 1/4" flat end mill. C) Overlay of 120° and horizontal movements. D) Overlay of vertical and horizontal movements with random point positions. E) Small arc movements on a sinusoidal grid. F) Movements with $\pm\pi/6$ angles. G) CNC-milling the textures with a 90° V-shaped end mill. H) Final plaster molds.

7.2 A Set of Textured Ceramic Cups

This second exemplar shows our process of creating textured plaster molds for imprinting patterns on slabs of clay. Using slab-building techniques, we used these textured slabs to form ceramic cups. We made four cups with different surface textures. We discuss using subtractive fabrication to create additive surface textures through a manual-computational workflow [64].

Making the plaster molds required a facing and a texturing stage, with distinct Millipath-generated code for each (Figure 15). We generated a continuous movement that alternated directions at constant intervals equal to half the tool diameter, e.i, 1/4" (Figures 15A and B). We then generated four different texture designs (Figures 15C, D, E, and F) that we executed with a 60° V-shaped end mill (Figure 15G) to obtain our final textured plaster molds (Figure 15H).

We followed the workflow illustrated in Figure 16 to make each cup. We rolled a slab of wet clay between two 1/8" slats of plywood to get a uniform height (Figure 16A). We then pressed it on the textured plaster mold and cut the imprinted slab to our desired dimensions (Figures 16B to E). We formed the cup and attached a circular base (Figures 16F to I). We waited three days for the clay wrapped in plastic to become leather-hard before finishing the base on a pottery wheel. We proceeded with the traditional firing and glazing stages of ceramic fabrication to get the final artifacts (Figure 16J).

These cup exemplars show how using CNC-milled surface texture as mold emphasizes the aesthetic qualities of the carving end mill. The textures extend beyond their immediate use and become tools themselves. The cup-making process demonstrates how CNC-milling actions can integrate complex manual ceramic decoration workflows to create functional artifacts.

7.3 Inlay Tray

This last exemplar shows the creation of a wooden tray with walnut inlays. Inlays are decorative elements of varying colors inserted in shallow recesses on an artifact's surface [62]. Inlays require the production of uniform depth for the recessed areas and precise cuts and fit for the inlay pieces. Our inlay tray demonstrates the integration of Millipath in this existing woodworking practice.

To create a leveled surface at the base of the tray for the subsequent fabrication stages, we designed and executed a 300 by 200 mm continuous movement and fabricated a 12.7 mm-deep pocket (Figures 17A and D). We then created the inlay pattern with linear movements on a linear grid with randomly generated integers of

Figure 16: The creation of a ceramic cup molded on textured plaster. A) Rolling a slab of clay using guides to create an even height. B) Placing the clay slab on the textured plaster. C) Rolling the clay slab on the texture. D) Resulting textured clay slab. E) Measuring and cutting the clay slab to the desired dimensions. We cut a 2.5" x 11" rectangle to form the wall of the cups. F) Forming the wall of the cup. G) Letting the slab firmed up. H) Scoring and joining the two sides of the clay slab. I) Adding a base to the cup. J) Final cups after glazing.

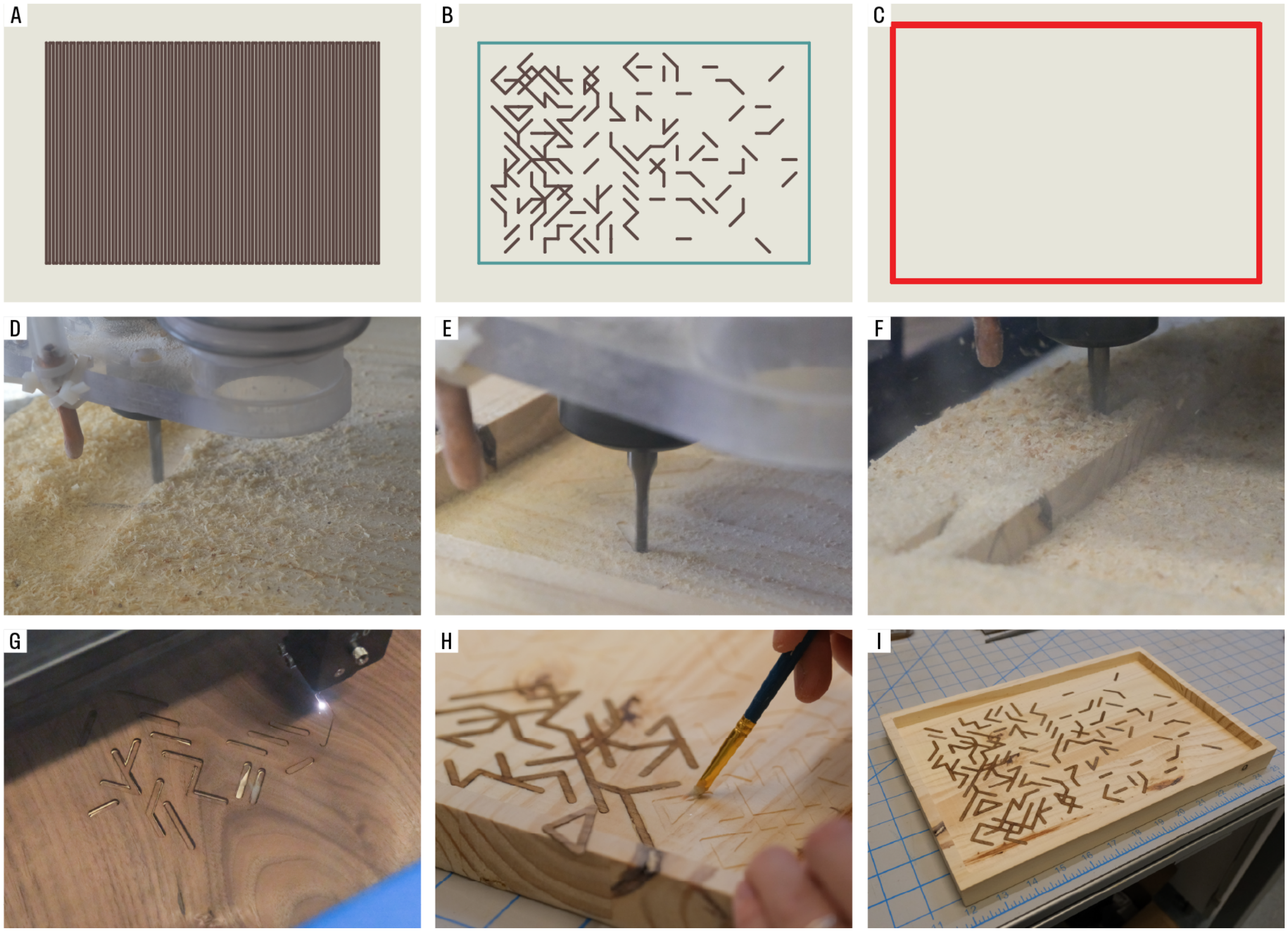

Figure 17: Tray with walnut inlay. A-C) The movements for the inner pocket, the inlay, and the contour of the tray generated with Millipath. D) Milling the inner pocket. E) 0.52 mm-deep linear cuts on a piece of plywood. F) Cutting the contour of the tray. G) Laser cutting the inlays in a walnut veneer. H) Installing the veneer in the cuts with wood glue. I) Final inlaid tray.

$\pi/4$ rotations and a decreasing gradient in visibility along the X-axis (Figure 17B). The movements were executed with a 1/8" flat end mill at a constant depth of 0.52 mm to accommodate the veneer's thickness (Figure 17E). The width of the end mill dictated the width of the inlays. We cut the tray contour (Figures 17C and F) and produced the walnut veneer pieces on a laser cutter from the movement SVG file generated by Millipath (Figure 17G). We glued the veneer in the recesses (Figure 17H) to obtain our final artifact (Figure 17I).

This example shows how Millipath enables the layering of operations wherein each operation is contingent on the previous one. This includes pocketing, texturing, and cutting. In addition to providing the code representation of the toolpath, Millipath generates a vectorial toolpath representation. This versatile representation can be used with other digital fabrication machines, offering a broad range of possibilities for diverse decorative applications.

8 FINDINGS

This section describes our analysis of autobiographical data collected by Sam during our two studies. This dataset includes detailed accounts of the machine's actions, observations regarding material behaviors, and considerations on the aesthetic qualities of the produced surface textures. We describe three themes we conceptualized from this analysis: 1) the design and fabrication modularity provided by an action-oriented system, 2) the surface finish quality determined by design, machine, and material interactions, and 3) the necessity of the fabrication process for developing surface textures.

8.1 Implications of Modularity in an Action-Oriented System

The Millipath's movement-grid-boundary model enabled us to work modularly with the CNC-milling machine. Enabling changes in the machine action at different levels of spatial organization and

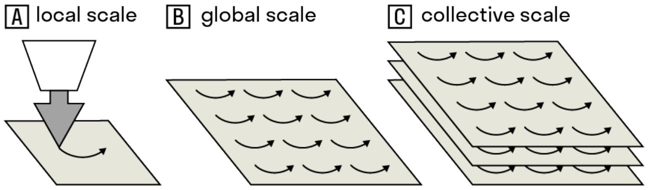

Figure 18: Our movement-grid-boundary model provided space to work modularly across different design scales. A) The movement supported changes at the local scale. B) The grid and boundary enabled changes at the global scale of a surface texture. C) The superposition of distinct surface textures supported changes at a collective scale.

breaking down the design and machine processes into modular tasks was especially suitable for creating repetitive movements and layered surface textures.

Our action-oriented approach to design supported changes at different design scales. Modifications to the movement changed how the end mill entered a material locally (Figure 18A). For instance, to get a subtle but distinctive dot-like texture in samples 9A and B, we tested multiple movement depths until we settled for the visual quality afforded by a 1.5 mm depth. Modifications at the grid level supported global changes, particularly in pattern density and alignment (Figure 18B). For the stool texture, we tried different grid spacing values for the deeper hole texture to create contrast with the shallower holes. We also discovered that surface textures generated as code files could be used as design components through superposition (Figure 18C). For example, we produced sample 12D by layering two instances of sample 9B with variations in density and depth. Working at different design scales enabled the meticulous exploration of diverse decorative options and the iterative fine-tuning of selected textures.

Working at the level of action also facilitated the representation of functional features. For instance, describing the dogbone joint as a series of movements eliminated the need for modeling geometry. As illustrated in Figure 19A, concave cuts on a CNC-milling machine generate round corners with a curvature equal to the tool's radius. When fabricating interlocking joinery, these round corners prevent the parts from fitting tightly into one another. Traditionally, this issue is addressed by modeling circular openings of the tool's diameter at these inner corners in CAD software (Figure 19B). Using Millipath's action model, we define the joint as a combination of linear movements that extend into the part by an end mill's radius on the side of the joint (Figure 19C). This approach to joint design provides an action-oriented solution to an action-oriented problem that results from the interaction between toolpath design and machine properties.

The integration of the CNC machine as part of this modular design workflow also supported the fine-tuning of surface textures. Initially, the flexible qualities of the ShopBot were not apparent to Sam, who assumed there was no space for rapid design modifications during fabrication. The ShopBot interface, however, provides options to pause, resume, and execute specific lines of code and exposes machine parameters that significantly affect the aesthetic of a cut. In creating our texture samples, control over these parameters was essential for fine-tuning machine movements, accommodating irregularities in material surfaces, and positioning surface textures precisely on an artifact's surface. In our first study, we frequently adjusted the Z zero position, skipped code fragments, or changed the end mill mid-cut to compare the visual effects of different depths and end mill geometries. To create the ten stool slats, we incremented the Y zeroes after each cut, allowing us to use the same plywood sheet for multiple parts efficiently. Repeating actions directly by manipulating machine parameters instead of generating and fabricating all slats using a single code file enabled us to adapt to the space available on the stock material. While we didn't implement these ShobBot features, these examples underscore the potential of flexible machine positioning and code execution to augment action-oriented workflows.

8.2 Design, Machine, and Material Interactions for Surface Finish

Human supervision is imperative in CNC milling, as even minor errors can significantly damage the material and machine. Therefore, over fifty hours of monitoring, we gained insights into the relationship between design features, material properties, and machine actions in shaping the aesthetic quality of surface textures. While some observations may seem trivial for experienced machinists or woodworkers, they highlight the significance of an action-oriented workflow in facilitating material exploration.

Movement properties and end mill geometry impacted the finish of surface textures differently on different materials. As depicted in Figures 20A and B, movements executed with non-flat end mills left foam filaments and wood splinters inside the cuts. Although performing a finishing pass by re-executing the same toolpath helped diminish these irregularities, it didn't eliminate them. Additionally, non-flat end mills tended to leave marks on the plywood surface, evident in the indentations caused by 0.1 mm-diameter arc movements in sample 12D (Figure 20C). However, the same carving actions on MDF didn't create marks, a phenomenon we attribute to the material's homogeneity (Figure 20D). We used short linear movements instead of arcs to address this issue in the stool dot-like texture. While fabricating sample 12C, we observed noticeable visual and audible differences when cutting plywood parallel and perpendicular to its fibers. Figure 20E shows precise horizontal cuts achieved through linear movements aligned with the fibers next to indented cuts resulting from vertical movements perpendicular to the fibers. To make the stool and the tray, we strategically oriented the longer cuts parallel to the wood fibers to achieve a better finish.

Figure 19: Describing dogbone joints through actions instead of geometries: A) Concave cuts on a CNC create round corners, affecting the fit in part assembly. B) Dogbone joinery ensures a tight fit and is typically modeled as a rectangle with two end mill-diameter half-circles. C) In Millipath, a doggone joint is defined by movements extending by an end mill's radius along joint sides.

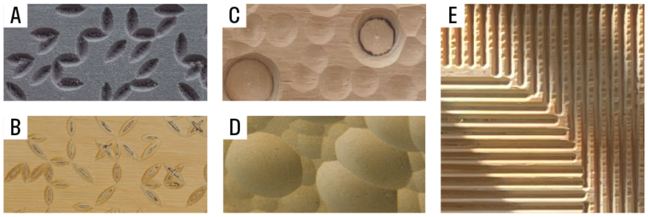

Figure 20: Material and machine-dependent irregularities and uniformity. A) Foam filaments and B) plywood splinters persist in cuts made by a non-flat end mill. C) Downward movements leave end mill marks in plywood, but D) no marks in MDF. E) Perpendicular movements to fibers indent wood; parallel movements do not.

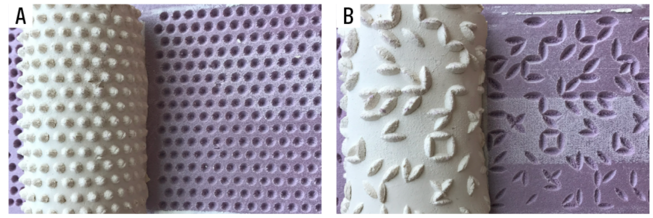

Figure 21: Initial tests using Millipath for textured mold for clay emphasize the end-mill geometry used in carving. A) Clay doesn't fill the dot-like textured mold, producing roundly shaped spikes. B) Clay has room to expand in the linear etchings, creating sharper features.

The irregularities in stock material, particularly in plywood, also impacted the quality of the surface textures. Despite our best efforts to level the materials on the machine bed before fabrication, achieving flatness was sometimes impossible without affecting the material integrity because of how some plywood warps. This led to surface textures displaying unequal depth (Figures 11G and H), which was especially visible with non-flat end mills where depth defines the cut's width or depth-sensitive applications like the inlay tray. While these irregularities were undesirable in most cases, we observed that handmade-looking textures could accommodate more irregularities without considerably affecting their aesthetic quality. Intending to push our system beyond machine-made geometrical patterns, we sought to produce surface textures inspired by Figures 3B, C, and F. In this context, slight execution anomalies could be beneficial. This led to the development of the "RANDOM" grid mode and the X, Y, and Z translation matrices. This feature enabled the creation of linear movements with random depth and positions, such as in samples 10B, C, and D, and samples 11B, C, and D.

These observations on surface finish emphasized two principles in using CNC-milling for creating textures: 1) *the production of fine surface texture details requires fine control over machine properties and deep understanding of material properties* and 2) *small changes in material and machine properties result in significant aesthetic changes in surface textures.*

8.3 Fabrication as an Essential Stage of Texture Design

Fabricating prototypes of surface texture was essential to our artifact production process. While our first study began as a testing ground for computational feature refinement, it provided a visual language that significantly informed the iterative design process of our artifacts. Physical iterations helped us assess both the functionality and aesthetic of our surface textures.

The design of the ceramic cups was informed by molding clay on different textured surfaces. We began by doing initial tests with air-dry clay on our CNC-carved foam and observed how textures created with a 60° V-shaped end mill gained dimensionality when transferred to clay (Figure 21). Instead of casting shadows inside the cuts, light bounced off the edges of the raised texture, emphasizing the end mill geometry used in the carving process. However, the spike texture had rounded corners (Figure 21A) compared to the sharp ridges of the linear texture (Figure 21B). This prompted us to use channel-like movement for our ceramic cups' design to leave room for clay to expand when imprinted.

Fabricating and testing the interaction between aesthetic and functional features was critical in creating the stool. Initially, a surface texture was uniformly applied to all stool parts, including the joint areas. The deeper holes in the texture, however, compromised the integrity of the joints and led to empty spaces between the parts (Figure 22A). We fixed this issue by adding inverse boundary objects around each joint to avoid machine movements in these regions. The interaction between texture and joint highlighted the potential for intentional visual relationships between them. In the decorative joint example, we embraced the presence of the central hole to create a subtle eye-like motif (Figure 22B). We also leveraged unplanned fabricated features as desirable aesthetic qualities. For instance, random-depth downward movements produced in sample 9D revealed darker plywood layers, generating additional circular motifs (Figure 22C). As depicted in Figure 22D, we used this feature to enhance the contrast between two overlaid textures, ultimately selected as the stool texture.

These examples show how the fabrication process reveals the interplay between surface textures, functional features, and environmental factors like light.

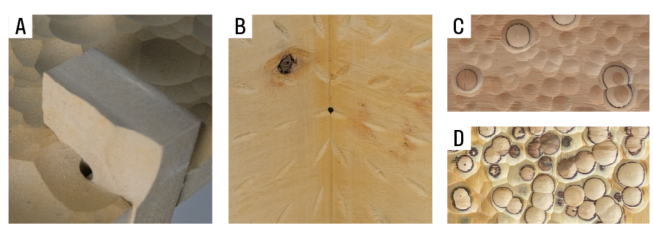

Figure 22: Interaction of surface texture with environment: A) Mispositioned texture creates holes, compromising joinery. B) Joint and texture interaction can form intentional motifs, like this eye-like motif. C) Random depth movements expose plywood's multicolored layers. D) Plywood's colors can highlight layered textures.

9 DISCUSSION

Our analysis of autobiographical data and fabricated artifacts highlighted the implications of using action to inform our digital fabrication system for surface texture production. In this section, we propose general technical principles for developing and working with action-oriented systems and discuss philosophical tensions that arise from bridging materialist theory and system development.

9.1 Principles of Action-Oriented Workflows for Systems Development

Digital fabrication models like digital crafts-machine-ship [1] and action-oriented workflows [8] challenge how we interact with machines and materials in the making of things. These models can help orient the implementation of system functionalities by defining machine abstractions that promote deeper engagement with machines and materials. Drawing from these models, we highlight three principles we conceptualized from our research-through-design process. These principles relate to system modularity, material properties understanding, and physical production.

First, *modularity in action representations and machine operations can support ad-hoc design decisions*. Applying modifications across design scales with Millipath digitally and positioning of the CNC machine physically supported rapid iteration. This iterative process enabled a near real-time interaction with the material when fabricating surface textures. However, when creating functional features like joints and contours, we experienced the need for pre-planned toolpaths. This was because implementing functional features required higher-level action representations with stronger ties to physical constraints, such as dimensions, multi-pass cuts constrained by the maximum depth of cut, and proper fit between parts. To account for these constraints, we automated the generation of multiple passes and tabs in our movement abstractions. These forms of automation enhanced our movement-grid-boundary model, enabling flexible design of both decorative and functional components.

Second, *working with low-fidelity action representations in the digital space can foster physical understanding in the material space.* Machine toolpaths, as represented in Millipath interface, don't account for the end mill shape and the CNC-milling machine positioning. We specify these machine parameters at the time of fabrication. Moving away from simulation, typically used for CAM operations with CNC-milling machines, and choosing to control these features physically required understanding specific machine-material interactions. This understanding grew over time, echoing a small part of a much longer process that constitutes a professional craftsperson's lifelong skill development [30, 54]. While our machining and woodworking expertise remains limited, making design and operational decisions in the physical space enhanced our ability to anticipate material behaviors in response to machine actions.

Finally, *we need to materialize machine actions to assess their expressive qualities.* The critical role of our first study became obvious as we began relying on the texture samples as a starting point for our artifact design. Unlike static representations of geometries achievable through 3D modeling or drawing, machine actions exist in time and must be executed–in whole or in part– to be understood. Carved textures gain significance in relation to light and touch; a digital representation cannot fully capture this contextual experience. Our texture samples served, therefore, as physical notes documenting the expressive potential of our system. Through iteration and comparison, they enabled us to identify density, scale, and alignment of machine actions as the foundation of our aesthetic explorations. Recognizing these overarching spatial elements helped us connect action properties to surface texture material qualities.

While we applied action in the specific context of CNC-milled surface texture production, we see broader application of these principles in digital fabrication systems research. The low-level machine control that is intrinsic to action-oriented workflows supports precise collaboration between humans and machines when working with materials. This interaction, in turn, has the potential to influence the quality and value of the products we fabricate and use. We are excited about the possibilities of applying our action-oriented model to more complex CNC-milling tasks, such as creating three-dimensional artifacts. We encourage future researchers investigating this computational practice to consider working with low-level and modular action representations that incorporate machine functionalities and skilled manual operations.

9.2 Materialist Theories and Post-Human Principles in Digital Fabrication Research

We conclude this paper by reflecting on the implications of post-human principles highlighted by HCI fabrication research engaging with materialist theories (Section 2.4). These principles include 1) decentering the human in the making process, 2) attending to the material and the machine, and 3) fostering non-hierarchical modes of production through collaboration between agents [18, 20, 23, 24, 31, 50, 51]. These principles informed our approach to building Millipath and fabricating our artifacts; however, the results of our design experience challenge the notion that humans can indeed be decentered in a digital-making process. We discuss tensions associated with human intentions, design requirements, and skill development.

While we aimed to adhere to these post-human principles in our work, it felt like a misrepresentation to claim non-hierarchical collaboration with the non-human agents – wood, plaster, the computer, the CNC-milling machine, etc. – in the process. We argue there is a difference between developing digital fabrication tools that support rather than abstract material qualities and developing technologies and processes that enact genuine collaboration between humans, machines, and materials. In human terms, collaboration suggests a joint effort toward a shared goal with mutual benefits for all involved parties. While material and machine qualities shaped our decisions, human intentions primarily directed our research-through-design process. The requirements of our research method, e.g., the physical production of artifacts and the writing of a paper discussing results, centered our intentions over the machine and material agencies. We believe, however, that being accountable for the work being done and clarifying our objectives with respect to non-human agents can help establish authentic post-human engagement. We, for instance, aimed to develop technology that could support the creation of beautiful artifacts that emphasize material properties using digital fabrication machine operations. Our experience indicates that truly non-hierarchical modes of knowledge production in digital fabrication research are challenging, if not impossible, to achieve within human-defined contexts. We hope future research using post-human principles delves deeper into discussing this reality.

By definition, design is an intervention [52], an attempt to "transform the world from its current state to a preferred state" [68]. While, at first, we felt that this active role in designing material-machine interactions conflicted with a post-human attitude, we observed that as we shaped the material, it shaped us back. Working at the level of action helped us understand material behaviors, which, in turn, informed our design decisions and improved our understanding of machine opportunities. Working with a CNC-milling machine displaying a simplified motion range, in contrast to the human hand, also supported a systematic approach to understanding the forces involved in producing surface textures. Over the course of ten months, this multi-directional dialogue between the machine, the material, and ourselves deepened as we progressively gained confidence in harnessing material properties through repeating machine actions. This repeated interaction led to the development of our machining and woodworking skills.

We argue, therefore, that developing fabrication expertise is an intrinsically post-human practice. This practice, however, requires time and dedication. Professional craftspeople and machinists develop skills through years of engagement with tools and materials. Their craftsmanship depends on their capacity to flexibly adapt their fabrication techniques in response to the vibrant nature of materials [20, 54], a form of dynamic predictability. While digital fabrication research engaging with post-human principles has encouraged speculative design as a mode of interaction with material properties, we believe this is insufficient to truly listen to the machine and material. We also believe that some forms of hierarchies can act as valuable catalysts for engagement in digital crafts-machine-ship. For instance, using foam and MDF as prototyping materials helped alleviate our apprehensions about working with the CNC-milling machine. These materials are easily machinable and behave predictably under milling conditions. Using these materials as temporary prototyping sites was liberating at times, as it allowed us to transition our focus from the material to the system and machine at play.

Finally, our process challenged our conceptions about making – the act of transforming matter. For philosopher Elizabeth Grosz, controlled chaos is at the root of creation, "the capacity to elaborate an innovative and unpredictable response to stimuli, to react or, rather, simply to act, to enfold matter into itself, to transform matter and life in unpredictable ways [27]." This definition is exciting as it applies to human and non-human agencies. While the study of physical making lacks a formal name, we firmly believe it should hold a status akin to ontology and epistemology, given its fundamental connection to human and non-human existence.

10 CONCLUSION

Through the research-through-design process of producing CNC-milled surface textures, we showed how developing a digital fabrication system rooted in a materialist theoretical framework can support deep engagement with machines and materials. We applied *action* as a bridging concept to address the design-systems divide we observe in HCI and engage materialist theories in the sub-field of digital fabrication systems research. Our work provides a foundation for future research paradigms where design, software, and hardware contributions are applied and valued in equal measure and where interdisciplinary approaches are used to develop durable technology.

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A STUDY ON TRIANGLES FOR V-SHAPED END MILL

Through material exploration, we realized the opportunities of using V-shaped end mills to create geometrical ornamentation, including diamond, chevron, and cross shapes. These geometrical motifs are commonly used in the decorative practices of chip carving [15] and kumiko design [32]. With these samples, we wished to explore the relationship between the movement's depth and the angle of the end mill to create motifs with sharp corners.

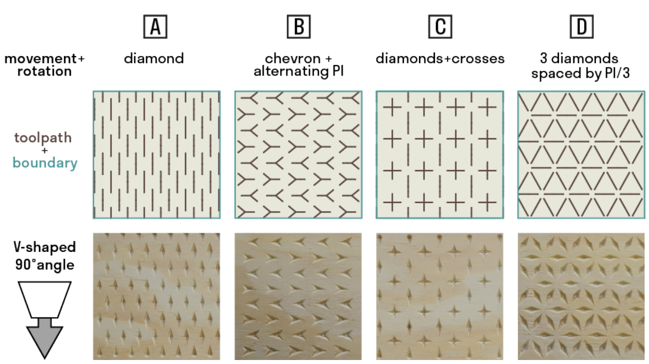

Figure 23: Study on triangles for V-shaped end mill. A) Diamond motifs using down and up linear movements across a sinusoidal grid. B) Chevron-shaped motifs using three linear movements based on the angle of the end mill across a linear grid with alternating rotation of PI rad. C) Two overlaid grids consisting of diamond and cross motifs, respectively. D) Movement made of three $PI/3$-spaced linear paths across a sinusoidal grid, creating a repeating star-like pattern.

In Figure 23A, a linear movement moving down and up linearly results in a diamond motif. A half movement at the top and bottom boundaries produces a drop-like motif. In Figure 23B, the movement composed of three converging line paths creates a chevron motif. Straight lines on all edges of the chevron motifs depend on the angles between the line paths, the angle of the end mill, and the movement's desired depth. The movement path specification function $makeChevronForVBit(x, y, l, nbPoints, theta, bitAngle)$ automates this computation. In Figure 23C, we offset two grids, one with a diamond movement and the other with a cross movement. The last sample repeats a three-branch $PI/3$-spaced movement across a sinusoidal grid, with each branch producing a diamond motif. This spatial organization results in a star-like pattern. These samples demonstrate the combined use of mathematical relationships between movement depth, movement angle, and end mill angle in creating geometrical motifs.