

Disclosing a Random Walk

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ABSTRACT

We examine a dynamic disclosure model in which the value of a firm follows a random walk. Every period, with some probability, the manager learns the firm's value and decides whether to disclose it. The manager maximizes the market perception of the firm's value, which is based on disclosed information. In equilibrium, the manager follows a threshold strategy with thresholds below current prices. He sometimes reveals pessimistic information that reduces the market perception of the firm's value. He does so to reduce future market uncertainty, which is valuable even under risk-neutrality.

THE ANALYSIS OF VOLUNTARY DISCLOSURE and its consequences for the functioning of markets intersects with accounting, economics, and finance. Existing literature on strategic disclosure focuses on static or dynamic models, with the asset having a fixed value. This is at odds with a common feature of financial and other markets, namely, that as information arrives, an asset's (expected) value follows a random walk. Such stochastic evolution introduces new strategic considerations. The agent may decide to hide information, hoping to disclose the value in the future only after it has increased. However, if the agent continues to hide information, the market could become more suspicious about the value. Therefore, in deciding whether to disclose information, the agent accounts for the effect of current and future market perceptions. Optimal disclosure decisions depend on the agent's expectations about both the evolution

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DOI: 10.1111/jofi.13290

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of the future value of the firm and the evolution of the market's beliefs. Our aim in this paper is to provide a first step in bridging the gap between static and dynamic voluntary disclosure models.

We study a model of dynamic voluntary information disclosure by a manager of a public firm. The firm's value follows a random walk with a general distribution of increments. In every period, with some probability, the manager holds material information and chooses whether to disclose it. We assume that the market sets the current price at the expected value of the firm conditional on the public history of disclosures. The equilibrium is based on the manager maximizing a weighted average of market prices, and market prices being consistent with the manager's strategy. In particular, the market silence price (i.e., the price when the agent does not disclose) is based on the set of values the manager chooses not to disclose.

We first show that, similar to static models, the equilibrium is based on threshold strategies. At any given time, and given some history, the manager who has information reveals it if and only if it exceeds a certain history-dependent threshold. This feature has some interesting implications. The value process is assumed to be a martingale. Prices also follow a martingale as they reflect expected values conditional on public information. However, since the agent follows a threshold strategy, prices are more positively skewed than the value process.

Second, we characterize the equilibrium disclosure thresholds. Our main result is that they are always lower than the silence price (except for the last period). This implies that with positive probability, the agent discloses information that leads to a lower price than the price that would have prevailed had he decided not to disclose it. This result stands in contrast to a one-period model or myopic behavior. It is important to note that the manager is risk-neutral, and prices are equal to the firm's expected value regardless of the uncertainty in the market. It is perhaps surprising, therefore, that despite being risk-neutral, the manager discloses information that reduces uncertainty about the firm's value at the cost of reducing the current price.

The intuition for why the manager chooses to disclose some values even though by doing so he reduces prices today is that by disclosing today, he reduces future market uncertainty. The key reason for this behavior is the difference in beliefs about future values between the manager and the market: the market forms beliefs based on the public disclosure history. The manager additionally knows the undisclosed information. A decision to withhold information makes the market's beliefs more dispersed than those of the agent. The higher uncertainty implies lower no-disclosure prices in the future, as the market accounts for a fatter left tail. The market is skeptical about the value since there is a chance the agent is hiding information. The more uncertain the value from the market's point of view, the higher the skepticism. Hence, when the firm's value equals the average market belief upon silence, the agent benefits from disclosing.

We next present additional properties of the equilibria. First, "no news is bad news": prices drift down in the periods between disclosures. Second, in a

two-period version of the model, we show that the disclosure threshold in the first period increases in the weight the agent assigns to period 1. Finally, in the two-period model, we show that the disclosure probability in both periods is maximized when the agent cares only about the last period. The last result is based on a generalization of the “minimum principle” (see Cheynel (2009) and Acharya, DeMarzo, and Kremer (2011)), which we refer to as the “suspicious belief principle.” If the market believes that the agent follows a certain disclosure strategy, then it sets silence prices as the expected values conditional on no disclosure. We show that the equilibrium disclosure strategy must satisfy a certain pessimistic-beliefs property (for computation of expected silence prices) for all possible disclosure strategies. This provides a necessary and sufficient condition for a strategy to be an equilibrium strategy. The suspicious belief principle takes a simple form if the agent only cares about the final price. This implies that the last-period price and disclosure threshold are minimized when the agent cares only about the final price.

Our findings relate to a discussion about the firms’ incentives to disclose information. An extensive literature in finance and accounting focuses on the effect of disclosure on the cost of capital (see a recent survey by Bertomeu and Cheynel (2016)). An important question in this literature is whether transparency reduces the cost of capital and thus allows a firm to enjoy more favorable pricing by the market. For example, Easley and O’hara (2004) argue that under risk aversion, disclosing information leads to a lower discount rate and thus higher prices. This can be questioned for two reasons. First, Hughes, Liu, and Liu (2007) and Lambert, Leuz, and Verrecchia (2007) argue that when traders can fully diversify their holdings, asymmetric information affects expected returns only via its impact on premia for systematic risk. Hence, to the extent that firm disclosure is more about specific or idiosyncratic information, this information should not be reflected in the discount rate. Second, Christensen, de la Rosa, and Feltham (2010) point out that disclosure of information does not affect the overall cost of capital if we consider a dynamic model from an *ex ante* perspective, before information is revealed.

Our model is based on risk-neutrality and has an arbitrary number of periods. We nevertheless find that a firm may disclose negative information to reduce future uncertainty. This is because, in our model, pricing is based on pessimistic beliefs. By reducing uncertainty, the firm lowers the market’s sensitivity to this pessimistic view.

We conclude our analysis by considering several variants of the model. First, we argue that our main result extends to several other processes for the evolution of the firm’s value. These processes include a random walk with drift, a random walk with mean reversion, and a geometric random walk. Second, we examine a variant wherein an information event occurs in each period with some probability. This information event, if it occurs, leads to a change in the asset’s value that could be disclosed. Since it is unclear whether such an event has occurred, the manager has discretion as to whether to disclose it or not. If the disclosure is about the asset’s value, the results in our main model continue to hold. However, if the information that the manager can disclose is only

about the change in the asset's value, then the equilibrium is similar to a static model and the disclosure strategy is myopic.

A. Related Literature

The voluntary disclosure literature goes back to Grossman and Hart (1980), Grossman (1981), and Milgrom (1981), who show that if it is common knowledge that the agent is privately informed, then there is full disclosure. Our paper follows Dye (1985) and Jung and Kwon (1988), who show in a one-period model that when investors are uncertain about the information endowment of the agent, there exists a nontrivial equilibrium in which some types withhold information while some disclose it. As mentioned above, despite the vast literature on voluntary disclosure, only a few papers examine multiperiod settings in which information changes over time. Shin (2003, 2006) studies a setting in which a firm may learn a binary signal for each of its independent projects, and each project may fail or succeed. In this binary setting, Shin studies the “sanitization” strategy, under which the agent discloses only good (success) news. The timing of disclosure does not play a role in such a setup. Pae (2005) considers a single-period setting in which the agent can learn up to two normally distributed signals.

Einhorn and Ziv (2008) study a setting in which in each period the manager may obtain a single signal about the period's cash flows. At the end of each period, the realized cash flows are publicly revealed, which eliminates the dynamic considerations that are at the heart of the present paper. Acharya, DeMarzo, and Kremer (2011) examine a dynamic model in which a manager learns one piece of information at some random time, and his decision to disclose it is affected by the release of some external news. Bertomeu, Beyer, and Dye (2011) study a reputation model in which the manager may learn a single private signal in each of the two periods. The manager can be “forthcoming” and disclose any information he learns, or he can be “strategic.” At the end of each period, the firm's signal/cash flow for the period becomes public, and the market updates beliefs about the value of the firm and the type of the agent. In the work of Frenkel (2020), a dynamic model of over-the-counter asset sales is analyzed. This model involves the seller receiving compensation that depends on the stock's performance, and the transaction itself serves as a means of conveying information regarding the firm's value. Importantly, the option to wait for a better signal that is behind our main result is not present in any of these papers.

Guttman, Kremer, and Skrzypacz (2014) examine a two-period model in which there are potentially two pieces of information. The main result is that in the second period, the market values the same signal more if it is disclosed in the second period than in the first period. This is different from the result we obtain for the case with stale information disclosure. This difference stems from the nature of the evidence we consider. In our model, the evidence pertains to the current value, and so it has a time stamp. When it is disclosed late, the market knows that it was disclosed with delay. Moreover, the information

sets in the different periods can be described as filtration. Information in the first period is not only less informative than information in the second period, but also a garbling of the value in the second period. Aghamolla and An (2021) study a two-period model in which the value is normally distributed. In their model, it is common knowledge that the manager knows the value of the firm. Disclosure, however, is costly. When deciding whether to disclose, the manager optimizes the benefits of disclosure against the cost of disclosure in each period. In that model the authors obtain a result that is similar to our excessive disclosure result: the manager may disclose information in the first period even though it reduces his price. We allow arbitrary distributions and an arbitrary number of periods. Our model is not based on costly disclosure but rather on the market being uncertain about the information the agent has.

The remainder of the paper is organized as follows. Section I provides a detailed explanation of the model and explore its underlying assumptions. In Section II, we delve into a preliminary analysis, commencing with the static model as presented by Dye (1985). Section III is dedicated to presenting our primary finding concerning excess disclosure. The comparative statics analysis is covered in Section IV. In Section V, we introduce and elucidate our novel concept, the “suspicious belief principle.” Finally, Section VI offers concluding remarks.

I. The Model

We consider a model of dynamic strategic disclosure with a single agent who can be viewed as a manager of a public firm. Time is discrete, $t \in \{1, 2, \dots, T\}$. The starting value of the firm, V_0 , is known. The firm’s value evolves as a random walk,

$$V_t = V_0 + \sum_{\tau=1}^t \Delta V_{\tau},$$

with increments $\Delta V_{\tau} \equiv V_{\tau} - V_{\tau-1}$, which are zero-mean independent identically distributed (i.i.d) random variables, with cumulative distribution function F , strictly positive density function f , and finite variance.

In every period t , with probability $\pi \in (0, 1)$ the agent learns the current value of the firm and can credibly/verifiably disclose it.

A strategy of the agent is a disclosure rule that specifies which values of V_t to disclose. Denote by $H_t = \{d_1 \dots d_t\}$ the (public) history of disclosures at time t , where $d_{\tau} = V_{\tau}$ if the agent discloses the value of the firm, and $d_{\tau} = \emptyset$ if he does not. The agent can reveal the value of the firm only when he has verifiable information. The agent’s time- t disclosure strategy is denoted by $\sigma_t(H_{t-1}, V_t) \in [0, 1]$, which is the probability of disclosure of the current value if he can disclose it.

The market sets prices to be equal to the expected value of V_t based on rational expectations and conditional on the history of the agent’s disclosures. We denote the market prices by $P_t(H_t)$.

The agent maximizes a weighted sum of prices,

$$\sum_{t=1}^T w_t \cdot P_t(H_t)$$

for some known weights $w_t \geq 0$ and $w_T > 0$.

This general specification of weights allows us to capture a standard discounted utility model, the case in which managerial compensation is more sensitive to stock prices on specific dates, and the case in which the agent cares only about the price at the terminal date T .

DEFINITION 1: An equilibrium of this model is a disclosure strategy of the agent and market prices, $\{\sigma_t(H_{t-1}, V_t), P_t(H_t)\}$, such that:

1. (Sequential Rationality) After every history, the agent maximizes his expected payoff given market prices.
2. (Sequential Consistency of Prices) The price at time t equals the expected value of V_t conditional on the public history and the agent's disclosure strategy.

Since we present the equilibrium using a verbal description rather than a mathematical expression, we make a few comments. When the agent maximizes his expected payoff given prices, he considers how disclosure affects his current and future prices.

In calculating prices, there are two cases: after the agent discloses V_t and after no disclosure (silence). When the agent discloses information, that is, for any H_t such that $d_t = V_t$, we have $P_t(H_t) = V_t$ because we assume that disclosed information is credible.

When the agent does not disclose, that is, for any H_t such that $d_t = \emptyset$, let $\tau \in \{0, \dots, t-1\}$ be the last period in which $d_\tau \neq \emptyset$ (or zero if the agent had never disclosed). For those histories, prices are equal to

$$P_t(H_t) = V_\tau + \mathbb{E} \left[\sum_{s=\tau+1}^t \Delta V_s \mid \{\sigma_s(H_{s-1}, V_s), d_s = \emptyset\}_{s=\tau+1}^{s=t} \right].$$

Note that the agent's equilibrium strategy $\{\sigma_t(\cdot, \cdot)\}$ since the last disclosure is used to calculate the expected value conditional on no disclosure.

The equilibrium conditions apply to histories on and off the equilibrium path. On the equilibrium path, prices follow Bayes' rule. The market observes off-path events only when the agent discloses a value to which σ assigns zero probability. In the period of such unexpected disclosure, the price is equal to the revealed value. Thereafter, the continuation strategy and prices form an equilibrium of the model as if the starting value were $V_0 = V_\tau$ and the model had horizon $\tilde{T} = T - \tau$.¹

¹ It would also be possible to write the model as a game between the agent and a competitive set of investors. In that case, we could use as the equilibrium notion perfect Bayesian equilibrium with the requirement that beliefs satisfy proper subgame consistency (see Kreps (2023)).

We show in the next section that the equilibrium disclosure strategies are threshold rules. It is therefore convenient to define the pricing function $\hat{P}_t(H_t, x_1, \dots, x_t)$ as the expectation of V_t conditional on the history and conditional on the agent using some arbitrary disclosure thresholds $x_1, \dots, x_t$. For equilibrium thresholds, we have $\hat{P}_t(H_t, x_1, \dots, x_t) = P_t(H_t)$, but for some of our arguments it is helpful to define prices for arbitrary market beliefs. We denote the equilibrium threshold strategies by $x_t^*(H_{t-1})$.

A. Discussion of Assumptions

We assumed that the increments ΔV_t have identical distributions and that the probability that the agent can disclose information is constant. These assumptions are made to simplify the exposition. Our qualitative results are robust to allowing F and π to vary over time. Our assumption that increments have zero mean and are independent implies that the value process follows a martingale. This is a common feature of finance models that captures the fact that in efficient markets, prices are equal to the expected value conditional on available information. One way to interpret this model is that there is a fixed terminal value initially unknown to the agent and the market, and the agent gradually learns what that value is. Prices then follow a martingale, as the expected value conditional on an increasing set of information is a martingale process. We discuss other stochastic processes of the evolution of V_t in Section V.B.

A more substantial assumption is that the agent can only reveal timely information, and if he does not reveal V_t at t , he cannot reveal it later.² Under Section 13 OR 15(d) of the Securities Exchange Act of 1934, firms are required to disclose current material information promptly. However, as Dye (1985) writes in his seminal paper, “investors are often unsure whether the manager has any such information... This is an important qualification to the disclosure principle and can lead managers to suppress some information which they possess.” Late disclosure would be evidence that the manager has concealed information in violation of the Securities and Exchange Commission rule.³

We also assumed that the agent learns V_t only when he can disclose it. Our analysis and results would remain unchanged if, instead, the agent observed V_t even in periods when he could not disclose it. This is because the agent can

² It is important to note that when considering a two-period scenario, if the agent is allowed to disclose outdated information later, it may lead to myopic behavior in the first period. This is because the agent may choose to hold back on revealing any information that suggests values below the silence price. As a result, our primary finding that managers tend to disclose negative information that reduces the market's perception of the company's value relies on this assumption.

³ A related scenario is when the firm has some information that can be validated in real time but not ex post. For example, an outside investor could approach the manager with a credible price offer (for the firm or a part of it). When the offer is made, the agent must decide whether to treat it as material information. He is compelled to reveal it to shareholders immediately if he does. If he does not, the offer expires and it may not be possible to reveal it credibly. Moreover, revealing it later could land the manager in trouble.

take action only when he has verifiable information, and once that information arrives, the current realization of V_t is a sufficient statistic for the firm's future value.

Finally, we assume that when the agent obtains information, then this information fully reveals the value of the firm V_t . We could instead interpret V_t as the best (possibly noisy) estimate available to the agent at the time of disclosure. For example, if instead of observing at t the value V_t the agent observed the value V_s for some $s < t$, then as long as the time that he observes the signal can be verified and the agent is constrained to reveal V_s only in the period he observes it, our model with timely disclosure would apply. In other words, what matters in the model for information to be "timely" is not whether the information is about current or past values of V_t , but rather when the agent received that information: if he can disclose the information only in the period in which he received it, our model applies.

II. Preliminary Analysis

A. Disclosure in a One-Period Model

We begin the analysis by considering a static model (which is useful for understanding disclosure in period T). We review some results that are due to Dye (1985), Jung and Kwon (1988), and Acharya, DeMarzo, and Kremer (2011). These results form a benchmark against which we will compare disclosure policies in the dynamic model.

In the static model, the asset value is given by V_1 and is distributed according to some distribution F . With probability π , the agent learns the value and can decide whether or not to disclose it. In equilibrium, the agent follows a threshold strategy. The agent chooses not to disclose if and only if the value is below a certain threshold. The reason is that his payoff is a fixed price in case of no disclosure and increases in his type if he discloses. As a result, the incentive to disclose is increasing in type. The threshold equals the price the agent would obtain upon no disclosure, $P(\emptyset)$. Given that the price equals the market's expected value conditional on no disclosure, we have

$$P(\emptyset) = \frac{(1 - \pi)E[V_1] + \pi * \Pr(V_1 < P(\emptyset))E[V_1|V_1 < P(\emptyset)]}{(1 - \pi) + \pi * \Pr(V_1 < P(\emptyset))}.$$

One can express the equilibrium threshold, x^* , as a fixed point. Consider the expected value of V_1 conditional on no disclosure and the disclosure threshold given by

$$\hat{P}(\emptyset, x) \equiv \frac{(1 - \pi)E[V_1] + \pi * \Pr(V_1 < x)E[V_1|V_1 < x]}{(1 - \pi) + \pi * \Pr(V_1 < x)}.$$

Then x^* is a solution to $\hat{P}(\emptyset, x) = x$. This model was first introduced by Dye (1985). Jung and Kwon (1988) show the existence and uniqueness of such a fixed point (and hence equilibrium). Acharya, DeMarzo, and Kremer (2011) (and independently Cheynel (2009)), show an alternative characterization—

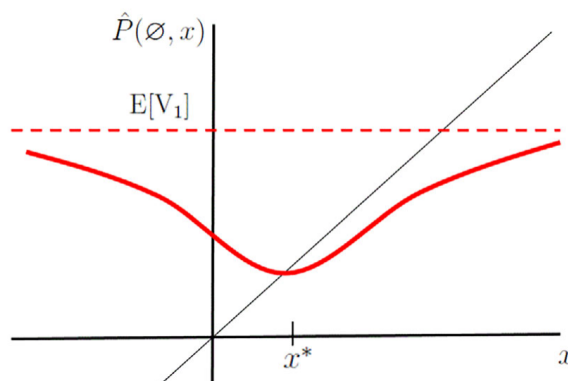


Figure 1. The no-disclosure price in a one-period model as a function of the threshold x . The unique equilibrium threshold is x^* and, as the graph shows, it equals the minimum no-disclosure price. (Color figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/terms-and-conditions))

the “minimum principle”—to which we will refer later:

$$P(\emptyset) = \min_x \hat{P}(\emptyset, x). \quad (1)$$

The intuition for why the fixed point is at the minimum silence price is that at that point the marginal type (the threshold) and the average type (the silence price) are equal. If we change the threshold from $x^* \equiv P(\emptyset)$ to $x^* + \Delta$, then we add to the pool of types that do not disclose those types in $[x^*, x^* + \Delta]$ that are better than average. If we change the threshold to $x^* - \Delta$, then we would remove from the pool of types that do not disclose those types in $[x^* - \Delta, x^*]$ that are worse than average. Either direction would increase the silence price, which equals the average value in this set.

Note that an immediate implication of this condition is the uniqueness of the equilibrium (because if the minimum is also a fixed point, then $\hat{P}(\emptyset, x)$ has only one minimum; see Figure 1 for an illustration). Moreover, for any x such that $x < \hat{P}(\emptyset, x)$, we have $x < x^* < \hat{P}(\emptyset, x)$, and for any x such that $x > \hat{P}(\emptyset, x)$, we have $\hat{P}(\emptyset, x) > x^*$ (see Figure 1).

Finally, consider the expected payoff for the agent. With probability π he is informed, and his payoff is $\max\{P(\emptyset), V_1\}$, because he discloses if and only if doing so increases his price. With probability $1 - \pi$, the agent has no discretion, and his payoff is $P(\emptyset)$. The law of iterated expectations implies that the agent’s average payoff equals the firm’s average value based on the distribution of F , that is,

$$E[V_1] = (1 - \pi)P(\emptyset) + \pi \int_{V_1} \max\{P(\emptyset), V_1\} dF.$$

B. Basic Properties of Equilibria in the Dynamic Model

We now turn to the dynamic game and provide preliminary results about the existence and structure of the equilibrium. In the following sections, we use these results to characterize equilibrium disclosure.

To analyze the agent's equilibrium strategy, we need to compare his payoffs conditional on disclosing or remaining silent. Fix an equilibrium, time $t < T$, and an arbitrary history H_{t-1} . Define $h(v)$ to be the expected continuation payoff of type $V_t = v$ if he does not disclose today.⁴ Since every type has a positive probability of not being able to disclose today, $h(v)$ is consistent with equilibrium play for every v . If the current silence price is $P_t(H_{t-1}, \emptyset)$ and the agent does not disclose, his payoff is

$$w_t P_t(H_{t-1}, \emptyset) + h(v).$$

In contrast to the static model, the continuation payoff $h(v)$ is not constant because the agent's current value is correlated with future prices.

With this notation, we can state how continuation payoffs depend on the agent's type and his decision as to whether to disclose it or not at t .

LEMMA 1 (Continuation Payoffs): *For any equilibrium, $t < T$, history, and type $V_t = v$:*

(i) *If the agent discloses, his expected payoff (starting at t) is*

$$v \sum_{s=t}^T w_s.$$

(ii) *If the agent does not disclose, his expected payoff is*

$$w_t P_t(H_{t-1}, \emptyset) + h(v),$$

where $h(v)$ is increasing and convex, with slope strictly less than $\sum_{s=t+1}^T w_s$.

(iii) *Let the last disclosed value be V_τ . Let ψ be a distribution of values V_t with mean $E_\psi[V_t] = v$ and the same support as $V_\tau + \sum_{s=\tau}^t \Delta V_s$.*

Then, the expected payoff of type $V_t = v$ satisfies

$$h(v) < E_\psi[h(V)].$$

We prove these results in the [Appendix](#) but provide the economic intuition here. The first part of the lemma follows from the values process being a martingale and the market being correct on average in equilibrium. If the agent discloses $V_t = v$, there is no asymmetric information between the agent and the market at this point. Therefore, both the agent and the market expect the average price in any future period to be equal to the current value.

To understand the properties of $h(v)$, consider a two-period version of our model. If type v does not disclose in period 1, his continuation payoff is

$$h(v) = w_2((1 - \pi)P_2(\emptyset, \emptyset) + \pi E[\max(P_2(\emptyset, \emptyset), V_2)|V_1 = v]).$$

⁴ This payoff depends on H_{t-1} , but for clarity we suppress this dependence in the notation.

This expression uses the fact that in the second period, the optimal strategy is the same as in the static model, so the agent discloses if and only if $V_2 > P_2(\emptyset, \emptyset)$. The derivative of $h(v)$ is:

$$h'(v) = w_2 \pi \Pr(V_2 > P_2(\emptyset, \emptyset) | V_1 = v).$$

The intuition is that the agent with a higher V_1 benefits from it in period 2 only if he can disclose and the realized value is higher than $P_2(\emptyset, \emptyset)$. This derivative is positive and strictly less than w_2 , as claimed in Lemma 1. Finally, since higher types V_1 are more likely to have V_2 above the disclosure threshold in period 2, the derivative of $h(v)$ is increasing.

The economic intuition for the convexity of $h(v)$ (and a proof for a general T) follows from future disclosure being an option. If the agent had to disclose in all future periods, or if disclosure probability could not depend on the realized value, $h(v)$ would be linear.

Suppose the agent could secretly (without changing the market's beliefs) replace his value v with any mean-preserving random draw. After such replacement, the agent could follow the strategy of disclosing for the same value increments as does type v . That strategy would give the agent the same average payoff as that of type v , $h(v)$. Since the agent could choose an even better strategy, $h(v)$ is convex. This argument does not rely on the details of future silence prices, so it holds for any T , not just in the two-period model. The last part of the lemma considers mean-preserving spreads of v with full support to ensure the richness of possible future types, so they do not all follow the same strategy. Such mean-preserving spreads yield a strictly higher payoff than $h(v)$.

With the help of Lemma 1, we establish the following lemma.

LEMMA 2 (Existence and thresholds):

- (i) *In any equilibrium, the agent follows a threshold strategy.*
- (ii) *An equilibrium exists.*

The term “threshold strategy” in (i) means that for any given history H_{t-1} , if a type $V_t = v$ discloses with positive probability, then all higher types $V_t > v$ find it strictly optimal to disclose. Thus, a threshold x_t (that depends on the history H_{t-1}) exists such that types above it disclose and those below it do not.

PROOF (of part (i), threshold strategies): For the proof of part (i), fix an equilibrium and a history H_{t-1} . By Lemma 1, the agent with type $V_t = v$ prefers to disclose if and only if (ignoring indifference for the threshold type)

$$v \sum_{s \geq t} w_s > w_t P_t(H_{t-1}, \emptyset) + h(v). \quad (2)$$

The derivative of the left-hand side (LHS) with respect to v is $\sum_{s \geq t} w_s$, while the derivative of the right-hand side (RHS) is strictly less than $\sum_{s \geq t} w_s$, so for at most one type the two sides are equal. This is the equilibrium threshold in that period. $\square$

We prove part (ii) (existence) in the [Appendix](#) via relatively standard arguments that use the Theorem of the Maximum and a fixed point theorem.

Note that we do not claim that the equilibrium is unique. While in all our numerical examples we find that the equilibrium is unique, we cannot prove it in general. The difficulty is that the disclosure threshold in period t affects all future silence prices. Without additional assumptions, we cannot assure that the fixed point is unique. We discuss uniqueness further in the [Internet Appendix](#)⁵ and include some sufficient conditions that imply uniqueness.

III. Excess Disclosure

We now present our main result: there is more disclosure in the dynamic model than a myopic model would predict. We show two results. First, the dynamic-model disclosure thresholds are below the static-model ones. Second, the agent discloses some information even though it decreases his current price to below the silence price.

Given a history and an equilibrium, denote by x_t^{myopic} the disclosure threshold of the static model. Recall that x_t^* and $P_t(\emptyset)$ are the equilibrium threshold and silence price in period t , respectively. We then have the following result.

THEOREM 1 (Excess Disclosure): *For every $t < T$, history H_{t-1} , and equilibrium,*

$$x_t^* < x_t^{myopic} < P_t(\emptyset).$$

The intuition for this result is that since disclosure is an option, the market makes adverse inferences when the agent does not disclose. Those inferences are worse when the market starts with more dispersed beliefs: a more dispersed distribution means a worse left tail of those that hide. For example, in the static model, the silence price decreases as we increase the dispersion of values in the second-order stochastic dominance sense.

Compare two situations: one in which the market knows that the agent has value $V_t = v$ and another in which the market has dispersed beliefs about V_t , with mean v . The second scenario will make the market more skeptical when the agent is silent, which will lead the agent to want to disclose even if it costs him a bit of a payoff today, implying that the disclosure threshold has to be below the silence price.

PROOF: Fix an arbitrary history and equilibrium. Let the period t silence price be $P_t(\emptyset)$. Consider type $V_t = x$ that equals that price, $x = P_t(\emptyset)$. We claim that it is strictly optimal for this type to disclose.

First, the current period payoff is the same whether he discloses it or not. Hence, the optimal decision depends on the ranking of expected continuation

⁵ The [Internet Appendix](#) may be found in the online version of this article.

payoffs. From Lemma 1 this depends on the sign of

$$x \sum_{s=t+1}^T w_s - h(x). \quad (3)$$

Second, since the silence price is $P_t(\emptyset)$, then from the market's perspective values in every future period will be on average $P_t(\emptyset)$. Moreover, the expected price in every future period is also $P_t(\emptyset)$. The reason is that the market has correct expectations on average, and values are a martingale. This implies that the average continuation payoff of all types that do not disclose at time t is equal to

$$\mathbb{E}[h(V_t)|V_t \in ND_t] = P_t(\emptyset) \sum_{s=t+1}^T w_s,$$

where ND_t is the set of types that do not disclose at t .

Third, if an agent with value x could replace his value with a random draw from the distribution used by the market to form beliefs after no disclosure, he would be indifferent between disclosing and not. But he is not a random draw, so he does not enjoy the same option value of future disclosure. By the last part of Lemma 1, since the distribution of types that do not disclose at t is a mean-preserving spread of x with full support, we can bound $h(x)$ from above

$$h(x) < \mathbb{E}[h(V_t)|V_t \in ND_t] = x \sum_{s=t+1}^T w_s.$$

Combining this bound with (3) implies that type $V_t = x$ strictly prefers to disclose in equilibrium, so that

$$x_t^* < P_t(\emptyset).$$

The last part of the theorem is that the myopic threshold is between those two numbers x_t^* and $P_t(\emptyset)$. This follows immediately from our analysis of the static game. The silence price is above the disclosure threshold only for thresholds below the myopic threshold. Moreover, for all thresholds that are different from the myopic threshold, prices are higher than that myopic threshold/price. $\square$

IV. Comparative Statics

We present two simple comparative static results. First, we argue that the silence prices decline over periods of no disclosure. Define $P_t(\emptyset)$ to be the silence price if the agent does not disclose in periods $\tau = 1, \dots, t$.⁶

⁶ Analogously, if the agent discloses at τ , the result is that silence prices decline from V_τ over the next period of no disclosure.

PROPOSITION 1: $P_t(\emptyset)$ is decreasing in t .

PROOF: The claim follows from $P_t(\emptyset)$ being the average V_t conditional on no disclosure. It equals the unconditional mean of values at $t + 1$ for an agent who remained silent up to time t . At $t + 1$, the agent follows a threshold strategy, so the average price conditional on disclosure is higher than the unconditional mean $P_t(\emptyset)$. Hence, the silence price at $t + 1$ satisfies $P_{t+1}(\emptyset) < P_t(\emptyset)$. $\square$

For the second comparative static, we examine the effect of weights that the agent assigns to the different periods in a two-period case ($T = 2$). To this end, we assume that the model's parameters are such that the equilibrium is unique.⁷

PROPOSITION 2 (Disclosure in a two-period model): *In the two-period model:*

- (i) *The first-period disclosure threshold is increasing in w_1 .*
- (ii) *The second-period disclosure threshold is minimized when $w_1 = 0$.*

Proposition 2 implies that the amount of information disclosed is maximized when the agent cares only about the final price. We provide the proof for (ii) in the next section when we introduce the suspicious belief principle. Specifically, the claim is shown in Corollary 1. While we provide the formal proof of (i) in the Appendix, the economic intuition for (i) is clear. The larger the weight w_1 , the less important the continuation payoff and hence the equilibrium disclosure strategy gets closer to being myopic.

Part (ii) follows from the disclosure strategy in the second period being myopic (since this is the last period) and Corollary 1 in Section V.A. The corollary states that when the agent cares only about the last period, the equilibrium disclosure strategy minimizes the silence price at T .

Figure 2 illustrates these results using a numerical example. We normalize $w_2 = 1 - w_1$. The left panel shows as a function of w_1 the first-period silence price (top curve) and the first-period threshold (bottom curve). The two are equal when the agent is myopic, $w_1 = 1$, while the gap between them increases as w_1 decreases. The first-period threshold is minimized when the agent cares only about the last period, $w_1 = 0$. The right panel shows the second-period silence price and threshold if the agent did not disclose in the first period. They are minimized when $w_1 = 0$ and are increasing in w_1 .

V. Discussion and Extensions

A. The Suspicious Belief Principle

A key property in static disclosure models is the minimum principle. According to this principle, in equilibrium, whenever the agent does not disclose information the market assumes the worst possible scenario. For instance, when

⁷ A similar result can be obtained for the case of multiple equilibria, but comparative statics become clearer when the equilibrium is unique.

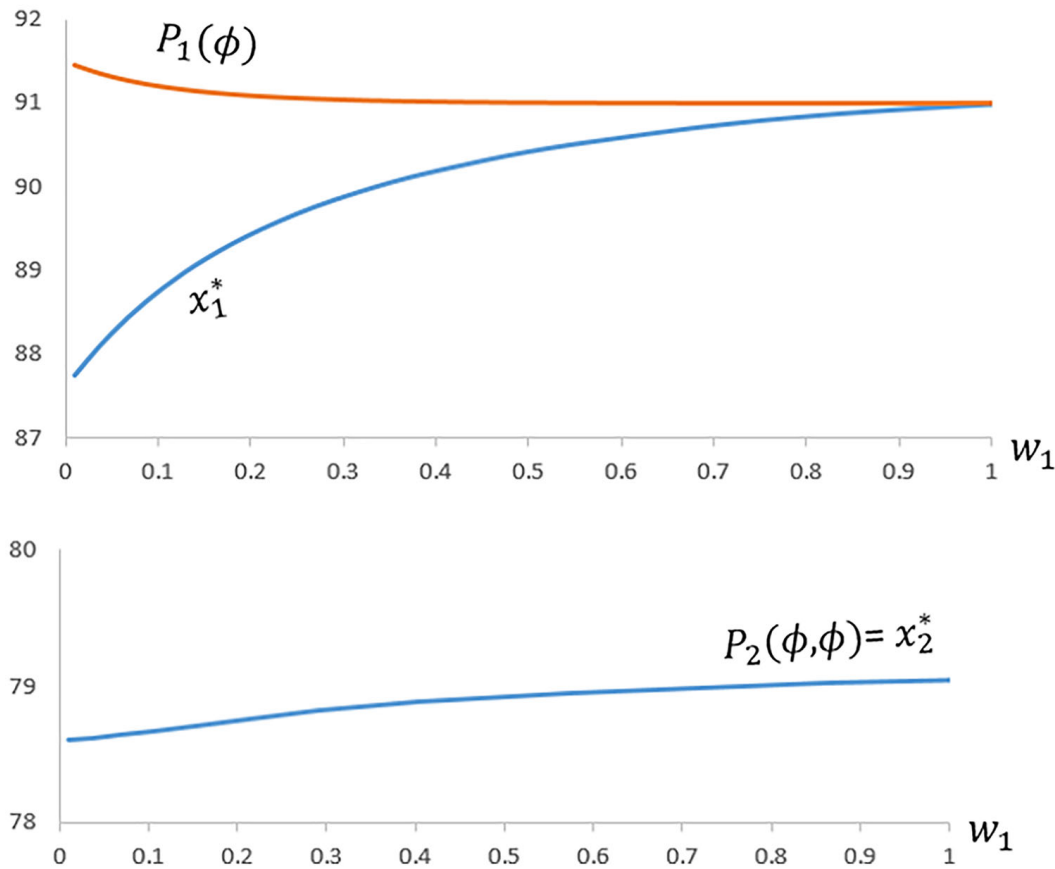


Figure 2. Equilibrium prices and thresholds in a two-period model as functions of w_1 . Each of the two random steps has a Normal distribution with zero mean and a standard deviation of 10. In addition, $V_0 = 100$ and $\pi = 0.9$. In this example, the equilibrium is unique for any value of w_1 . (Color figure can be viewed at wileyonlinelibrary.com)

it is common knowledge that the agent is informed, as in Grossman (1981) and Milgrom (1981), the equilibrium silence price is the lowest possible value. Moreover, when it is not common knowledge that the agent is informed, the silence price is the lowest expected value that any disclosure policy can support; see (1) and Figure 1.

In contrast, in our dynamic model the disclosure policy does not minimize the first-period no-disclosure price (Theorem 1). But does the spirit of the minimum principle survive in any way in our game?

Notice first that no one belief minimizes all no-disclosure prices in our dynamic setting. As a result, if we wanted to find a market belief that minimizes the sum of no-disclosure prices weighted by w_t and the probability the agent reaches t without disclosing, the minimizer would depend on the disclosure strategy (via the probabilities).

As a result, instead of the minimum principle, a more general property holds in our game. We call it the suspicious belief principle. This new principle

simplifies to the minimum principle if the agent assigns positive w_t to only one period.

To define the suspicious belief principle, let $P_s^{\hat{\sigma}}(\emptyset)$ be the no-disclosure price at time s , given that there was no disclosure up to time s and the market believes that the agent follows $\hat{\sigma}$. Next, suppose that the agent follows σ , but the market believes that he follows $\hat{\sigma}$. For these two strategies, define the weighted sum of expected no-disclosure prices as

$$\phi(\sigma, \hat{\sigma}) = \mathbb{E} \left[\sum_{s=1}^{\tau(\sigma)-1} w_s \cdot P_s^{\hat{\sigma}}(\emptyset) \right], \quad (4)$$

where $\tau(\sigma)$ is a random variable that is equal to the first disclosure period given σ and the expectation is with respect to that random variable.⁸

Note that $\hat{\sigma}$ affects $\phi(\sigma, \hat{\sigma})$ only via the no-disclosure prices and σ affects it only via the stopping times. As we discussed above, changing $\hat{\sigma}$ can increase the no-disclosure prices in some periods and reduce them in other periods. For example, the strategy that minimizes $P_1^{\hat{\sigma}}(\emptyset)$ (the static model strategy) does not minimize $P_2^{\hat{\sigma}}(\emptyset)$.

With this notation, $\phi(\sigma, \sigma)$ is the sum of expected no-disclosure prices if the market believes correctly that the agent follows σ . By contrast, $\phi(\sigma, \hat{\sigma})$ is the sum for the same agent strategy if the market believes incorrectly $\hat{\sigma}$. The following proposition states that σ^* is an equilibrium strategy if and only if for any potential strategy σ , the equilibrium belief σ^* is worse (in terms of the sum of no-disclosure prices) than if the market believed σ .

PROPOSITION 3: *Consider the beginning of the game or any history following disclosure. The strategy σ^* is an equilibrium strategy if and only if for any strategy σ ,*

$$\phi(\sigma, \sigma^*) \leq \phi(\sigma, \sigma).$$

Our proof shows that if this condition is violated for some σ , then this σ would be a profitable deviation. The intuition starts with noting that if the market observed the deviation to σ , the average price in every period would be V_0 , as in equilibrium (since the market updates correctly). However, since the market does not observe deviations, if $\phi(\sigma, \sigma^*) > \phi(\sigma, \sigma)$, then the expected deviation payoff would be strictly higher.

One special case of our dynamic setting is when the agent cares only about the last-period price. In this case, there is one belief that minimizes the value upon no disclosure for all strategies as in the static model, and therefore the minimizing strategy is the unique equilibrium strategy. In other words, in the special case in which the agent cares only about the final price, the condition of Proposition 3 implies the familiar “minimum principle.”

⁸ To simplify notation, we define $\phi(\sigma, \hat{\sigma})$ only from the perspective of $t = 1$. However, recall that our model “resets” after every disclosure: if the agent discloses $V_t = v$, then the continuation equilibrium is also an equilibrium of a game with horizon $T - t$ and starting value $V_0 = v$. For periods following disclosure, the function in (4) is recomputed by replacing $s = 1$ with $s = t$.

COROLLARY 1: If $\forall s \leq T - 1$, $w_s = 0$, and $w_T = 1$, then

$$\sigma^* = \arg \min_{\sigma} P_T^{\sigma}(\emptyset). \quad (5)$$

This corollary has an interesting implication regarding the information revealed about the final value V_T . Consider an arbitrary weight profile $\{w_t\}$ and compare it to the case in which the agent cares only about the final price. The corollary above implies that more information is being revealed regarding the final value in the latter case. The agent discloses more values when he cares only about the final price.

There is one more sense in which the equilibrium beliefs are pessimistic. Consider a different auxiliary game: the market first chooses beliefs (about the agent's strategy), and the agent then chooses his disclosure strategy. The agent maximizes his payoffs given the market prices, and the market chooses beliefs *to minimize the agent's expected payoff*. It turns out that the equilibria of the two models coincide—a disclosure strategy is an equilibrium of our model if and only if it is also an equilibrium of this auxiliary game where the market tries to minimize the agent's payoff! In a one-period game, this implies minimization of the no-disclosure price. In a multiperiod game, it reflects the suspicious belief principle.

B. Beyond a Random Walk

So far we have assumed that the firm value follows a random walk with i.i.d increments. This model captures a scenario with an initially unknown fixed terminal value and a process in which the agent gradually learns what that value is.

However, our main results extend to other processes. Three that are of particular interest are:

1. Random walk with a drift:

$$V_t = V_0 + \sum_{\tau=1}^t \Delta V_{\tau},$$

with increments $\Delta V_{\tau} \equiv V_{\tau} - V_{\tau-1}$, which are i.i.d random variables with a positive mean $E(\Delta V_{\tau}) > 0$.

2. Geometric random walk:

$$V_t = V_0 \times \prod_{\tau=1}^t \Delta V_{\tau},$$

with multiplicative increments $\Delta V_{\tau} \equiv V_{\tau}/V_{\tau-1}$, which are i.i.d random variables.

3. Random walk with mean reversion:

$$V_t = \alpha V_{t-1} + \Delta V_t$$

for some $\alpha < 1$ and i.i.d. zero-mean increments ΔV_t .

We first claim that in all three cases, equilibria are found in threshold strategies. A proof similar to that for Lemma 1 implies that for any equilibrium, $t < T$, history, and type $V_t = v$, (i) if the agent discloses, his expected payoff (starting at t) is $v \cdot M + c$, where M and c are constants independent of v , and (ii) if the agent does not disclose, his expected payoff is

$$w_t P_t(H_{t-1}, \emptyset) + h(v),$$

where $h(v)$ is increasing and convex, with slope strictly less than M .

What changes across the three cases are the values of M and c :

1. In the case of the random walk with or without drift,

$$M = \sum_{s=t}^T w_s$$

and the constant is $c = \sum_{s=t+1}^T w_s \mathbb{E}[\Delta V_s]$.

2. In the case of the geometric random walk, $c = 0$ and

$$M = w_t + \sum_{s=t+1}^T w_s \cdot \mathbb{E} \left[\prod_{\tau=t+1}^s \Delta V_\tau \right].$$

3. In the case of the random walk with mean reversion, $c = 0$ and

$$M = \sum_{s=t}^T w_s \alpha^{s-t}.$$

Since the slope of the disclosure payoffs with respect to the disclosed value is M and the slope of the payoffs after no disclosure is strictly less than M , there is a unique threshold type that is indifferent between disclosing and not.

Moreover, in all these cases the logic of the proof of Theorem 1 applies. The type equal to the silence price today strictly prefers to disclose, so in equilibrium the disclosure threshold is below the myopic threshold and the silence price is above the myopic threshold.

The same reasoning can be applied to other processes that satisfy the property that future values are increasing linear functions of today's value. That said, our results do not hold for arbitrary processes for values, in particular for stochastic processes where the increments are not independent. Once we allow the distribution of ΔV_t to depend on the value of V_t , equilibria in threshold strategies may not exist. For a simple illustration, consider a two-period model with V_1 that has two values, -1 , 1 . Suppose that if $V_1 = -1$, then the increment has no variance, $V_2 = V_1$. In contrast, if $V_1 = 1$, then $V_2 = 1 + \Delta V_2$, for ΔV_2 with zero mean and large variance. In this example, the silence price after disclosing $V_1 = 1$ may be lower than after disclosing $V_1 = -1$. If so, then for sufficiently low w_1 , the agent would reveal $V_1 = -1$ and hide $V_1 = 1$, which is not a threshold strategy.

C. Disclosure When Value Changes

Consider now a variant of the model in which the value changes in some periods only, and when it does, the manager has verifiable information. In particular, in every period, with probability π an information event occurs. If so, the value, V_t , increases by ΔV_t , and the manager has information he can disclose at that time (as before $\{\Delta V_t\}$ are i.i.d random variables). With probability $1 - \pi$, the value does not change so that $V_t = V_{t-1}$ and the manager cannot prove that the value has not changed. One can think about two different cases for such a game. In the first, the manager can disclose the cumulative value V_t . In the other, he can disclose only the current increment ΔV_t .

When the manager discloses V_t , note that the model is not immediately a special case of our main model. We have assumed so far that the ability to disclose is independent of the value. Nevertheless, one can verify that our results from Section III continue to hold with essentially the same proofs because this model maintains that the probability the agent can disclose in the future is independent of the current value. The proof for why there is excess disclosure is also valid for this case. The agent follows a threshold strategy where the thresholds are lower than the myopic ones.

When instead the manager discloses the increments ΔV_t , the agent becomes myopic. In period t , the price equals the time $t - 1$ plus the expected value of the increment conditional on the time t disclosure, $P_t = P_{t-1} + E[\Delta V_t | d_t]$. It follows that P_{t-1} does not affect the disclosure decision. Hence, the disclosure decision is the same as the static model discussed above, where the agent decides whether to disclose ΔV_t .

This comparison shows that our model yields different results depending on the nature of the information the agent can disclose. If it is just current increments, disclosure does not depend on past decisions. If it is cumulative value, previous disclosure decisions do affect current disclosure. It matters whether the market knew V_{t-1} , so $P_{t-1} = V_{t-1}$, or whether P_{t-1} was based on partial information (even if the price last period was the same).

VI. Conclusions

We study a dynamic model of voluntary information disclosure in a setup where the firm's true value follows a random walk. The manager occasionally learns verifiable information and chooses what to disclose to maximize a weighted sum of stock prices.

We derive two main results. First, the equilibrium disclosure threshold is strictly lower than the silence price. The manager in equilibrium sometimes discloses information even though doing so reduces the current stock price. This result, which we refer to as "excess disclosure," runs counter to most existing models. The intuition is that the manager has an incentive to reduce the market's future uncertainty (despite all agents being risk-neutral). Second, we show the extent to which the minimum principle from static models extends to our dynamic model. The minimum principle states that when the market

sets the silence price, it assigns the most pessimistic beliefs about the agent's strategy. We discover that this property does not hold in the dynamic model and is replaced by a more general suspicious belief principle.

Initial submission: February 3, 2022; Accepted: January 13, 2023
Editors: Stefan Nagel, Philip Bond, Amit Seru, and Wei Xiong

Appendix

PROOF OF LEMMA 1 (Continuation Payoffs): The argument for part (i) is in the text.

For part (ii), to see that $h(v)$ is increasing, consider any two types $v' > v$. Fix the optimal strategy of type v . Type v' can follow that strategy by disclosing for the same value increments as those for which v discloses. If v' does so, he gets the same prices until the first disclosure, a strictly higher price upon the first disclosure (by $(v' - v)$), and a strictly higher expected continuation payoff by the first part of the lemma. Since type v' can do even better than mimicking type v , we must have $h(v') \geq h(v)$.

Next, we can bound the slope as $h(v') - h(v) < (v' - v) \sum_{s>t} w_s$ using similar reasoning. Now fix the optimal strategy of the higher type v' and let the lower type mimic it, by disclosing for the same value increments as v' . If v does so, in the event of no disclosure the lower type gets the same price as the higher type. In the event of disclosure, the lower type gets only $(v' - v)$ less (and this gap is the same for the continuation payoffs). Therefore, the slope is bounded above by $\sum_{s=t}^T w_s$.⁹

Finally, the convexity of $h(v)$ follows from disclosure being an option. Take any type $V_t = v$ and his optimal disclosure strategy. Consider any distribution of types with mean v . If all types in the support of that distribution follow the same strategy as v (in the sense of disclosing for the same value increments), their average payoff would also be $h(v)$. To see this, note that when v does not disclose, they get the same price as v . When v discloses, they would disclose as well and on average get the same price and the same expected continuation payoff. Types other than v can do even better than mimicking the strategy of v , so the average equilibrium payoff of those types is at least as high as $h(v)$. Since this is true for every distribution of types with mean v , $h(v)$ is convex.

For part (iii), note that a weak inequality follows from the convexity of $h(v)$. Strict inequality follows because $h(v)$ is not linear over the whole support of V_t that is consistent with the history. Suppose by contradiction that $h(v)$ is linear in v . By the reasoning in the proof of part (ii), that can be true only if for all types (up to measure zero) the optimal strategy depends only on the realized increments. But that contradicts our characterization of the equilibrium in the last period, namely, that it is optimal for the agent to disclose if and only if V_T

⁹ Here we assume that $w_T > 0$. This assumption is sufficient for the strict inequality since there is a strictly positive probability that the agent will not be able to disclose in periods t to T . In that event, both types get the same payoff.

is above the silence price. That strategy depends on both the increments and the starting value.¹⁰ $\square$

PROOF OF LEMMA 2: We prove part (i) in the text.

Part (ii) (existence): Start with an auxiliary game. For each $\tau \in \{0, \dots, T-1\}$ define the game as follows. First, the game starts at $t = \tau + 1$ and we normalize $V_\tau = 0$. Second, in this game, V_t has the increments distributed in the same way as in the original game. The agent observes each V_t with the same probability as in the original game and decides whether to disclose in every period. If he does not disclose at t , he obtains a payoff flow $w_t P_s(H_t^\emptyset)$, where $H_t^\emptyset$ is the history of no disclosure between $\tau + 1$ and t . If he discloses at t the game ends and he gets a final payoff of $V_t \sum_{s \geq t} w_s$. An equilibrium of the auxiliary game for a fixed τ is a sequence of disclosure thresholds, x_t^* , and silence prices, P_t , for all $t \in \{\tau + 1, \dots, T\}$ such that the thresholds are optimal given the prices and the prices are consistent with the thresholds and Bayes' rule.

If we can find the equilibria of this auxiliary game for every τ , then we can construct an equilibrium for our original model. The intuition is that in our model the economic situation “resets” every time the agent discloses his type. To see this, note that if in the original game $V_\tau = v$ is disclosed, and if we make the continuation prices (until the next disclosure) and disclosure thresholds equal to the prices and thresholds from the auxiliary game plus v , then all incentives continue to be satisfied, and prices continue to be consistent with equilibrium strategies.

Now fix any τ and consider an arbitrary vector of disclosure probabilities in every period $t \in \{\tau + 1, \dots, T\}$ conditional on not disclosing before (and conditional on having verifiable information in that period). These probabilities pin down uniquely the disclosure thresholds in all periods. Next, compute for that vector of disclosure probabilities the implied thresholds, and in turn, the implied silence prices. These prices are continuous in the vector of probabilities. For any arbitrary vector of prices $\tilde{P}_s$ for $s > \tau$, consider the best-response problem of the agent in the auxiliary game given those prices. The objective function of the agent is continuous in both the prices and the probabilities. It follows from the Theorem of the Maximum that the best-response correspondence (from prices to optimal probabilities of disclosure) is upper semicontinuous. Moreover, we claim (below) that the best response is unique, and so the best response is a continuous function from the vector of prices to the vector of disclosure probabilities (implied by the optimal deterministic thresholds).

Putting these operations together (from the vector of disclosure probabilities to silence prices using Bayes' rule, and then from silence prices back to probabilities of disclosure using the best response), we define a continuous mapping

¹⁰ For some distributions, it is possible that for very large or small values of v , $h(v)$ is locally linear (and hence not strictly convex). This can arise if the support of future increments is sufficiently small that given an extreme realization of $V_t = v$, the agent's optimal continuation strategy would not depend on the realized increments. For example, it could occur if V_{T-1} is sufficiently low that for all possible realizations of ΔV_T the agent prefers not to disclose at T . In that case, h would be constant in the neighborhood of that v .

from conjectured vector of disclosure probabilities back to the vector of disclosure probabilities. By Brouwer's fixed point theorem, this mapping has a fixed point. That fixed point is an equilibrium of the auxiliary game.

To see that the best-response disclosure policy is unique for any vector of silence prices $\tilde{P}_s$, note the following. First, the disclosure threshold at T is equal to $\tilde{P}_T$. Second, for any other period t , the disclosure threshold is independent of disclosure thresholds in previous periods and can be found by solving:

$$x_t \sum_{s \geq t} w_s = w_t \tilde{P}_t + h(x_t | \tilde{P}), \quad (\text{A.1})$$

where $h(v | \tilde{P})$ is the expected optimal continuation payoff of type v if he does not disclose today, given the future silence prices.

The derivative of the left-hand side (LHS) of (A.1) is $\sum_{s \geq t} w_s$ while the derivative of the right-hand side (RHS) is strictly less (by the same reasoning as in the proof of Lemma 1 that we used to bound the derivative of h from above).¹¹ Hence, for all x_t the LHS is larger (so the best response is to disclose with probability one), the RHS is larger (so the best response is to disclose with probability zero) or there is a unique interior x_t that satisfies (A.1). $\square$

PROOF OF PROPOSITION 2: (i) Suppose that $x_1^*(w_1)$ is the threshold for disclosure in the first period given weight w_1 . It is based on the indifference condition

$$w_1 * (\hat{P}_1(\emptyset, x_1^*) - x_1^*) = w_2 * (x_1^* - E[\hat{P}_2(\emptyset, d_2, x_1^*, x_2^*) | V_1 = x_1^*]).$$

The LHS is the $t = 1$ gain from no disclosure by the threshold type; we show in Section III that it is positive. The RHS is the time $t = 2$ expected loss from no disclosure at $t = 1$ (the expectation is with respect to the optimal disclosure d_2). Now consider $w_1^2 < w_1^1$. If we keep the threshold for disclosure at $x_1^*(w_1^1)$, as we decrease w_1 from w_1^1 to w_1^2 , the LHS becomes smaller, implying that the agent would strictly prefer to disclose. Therefore, the equilibrium threshold in period 1 has to change with w_1 . If instead we take the threshold x_1 to be very low, then the LHS will continue to be positive and the RHS will become negative. The intermediate value theorem (and our assumption that the equilibrium is unique) implies that $x_1^*(w_1^2) < x_1^*(w_1^1)$. $\square$

PROOF OF PROPOSITION 3: First, recall that for σ^* to be an equilibrium strategy, it must be the case that after disclosure of $V_t = v$, the continuation strategy is also an equilibrium in the game that starts with value $V_0 = v$ and has horizon $T - t$. So, without loss of generality, we prove the statements for arbitrary T . Given the equilibrium structure we focus on how σ affects the first disclosure time, $\tau(\sigma)$.

Necessity: Suppose σ^* is an equilibrium strategy.

¹¹ The reasoning is as follows. Fix the optimal continuation strategy of type v' and consider a lower type $v < v'$. That type can mimic v' by disclosing for the same value increments. Conditional on disclosure, their payoffs differ by $(v' - v)$ times the sum of remaining weights. Conditional on no disclosure, their payoffs are the same.

For any strategy σ , if the agent follows it and the market believes that he does so, the expected payoff is the same because prices satisfy Bayes' rule,

$$\forall \sigma, \quad V_0 \sum_{s=1}^T w_s = \phi(\sigma, \sigma) + \mathbb{E} \left[V_{\tau(\sigma)} \cdot \sum_{s=\tau(\sigma)}^T w_s \right], \quad (\text{A.2})$$

where $V_{\tau(\sigma)}$ is the value disclosed at $\tau(\sigma)$ (these are both random variables).

Since this is true for every strategy, it is true also for the equilibrium strategy,

$$V_0 \sum_{s=1}^T w_s = \phi(\sigma^*, \sigma^*) + \mathbb{E} \left[V_{\tau(\sigma^*)} \cdot \sum_{s=\tau(\sigma^*)}^T w_s \right]. \quad (\text{A.3})$$

Now consider a deviation to some strategy σ_1 until the first disclosure and then σ^* is followed. For σ^* to be an equilibrium, this deviation cannot be profitable. This deviation yields the expected payoff

$$\phi(\sigma_1, \sigma^*) + \mathbb{E} \left[V_{\tau(\sigma_1)} \cdot \sum_{s=\tau(\sigma_1)}^T w_s \right]. \quad (\text{A.4})$$

Hence, for the deviation not to be profitable we must have that (A.4) is weakly smaller than the RHS of (A.2) (which has the same payoff as the equilibrium payoff in (A.3)). This implies $\phi(\sigma_1, \sigma^*) \leq \phi(\sigma_1, \sigma_1)$, as claimed.

Sufficiency: Suppose that for some σ^* the condition holds at time $t = 1$ and at every history following disclosure (where $\phi(\sigma, \hat{\sigma})$ is redefined to sum over times following the disclosure). Suppose by contradiction that σ^* is not an equilibrium, so there exists a profitable deviation $\hat{\sigma}$. Moreover, there must exist at least one period such that it is profitable to deviate from σ^* to $\hat{\sigma}$ up to $\tau(\hat{\sigma}) - 1$ (the time of the first disclosure given the deviation strategy) and thereafter play σ^* . Otherwise, by induction, deviating to $\hat{\sigma}$ would not be profitable. Without loss of generality, suppose that this deviation is profitable at $t = 1$.

For $\hat{\sigma}$ to be profitable, we must have

$$V_0 \sum_{s=1}^T w_s < \phi(\hat{\sigma}, \sigma^*) + \mathbb{E} \left[V_{\tau(\hat{\sigma})} \cdot \sum_{s=\tau(\hat{\sigma})}^T w_s \right]. \quad (\text{A.5})$$

Applying (A.2) to $\hat{\sigma}$ we also get

$$V_0 \sum_{s=1}^T w_s = \phi(\hat{\sigma}, \hat{\sigma}) + \mathbb{E} \left[V_{\tau(\hat{\sigma})} \cdot \sum_{s=\tau(\hat{\sigma})}^T w_s \right]. \quad (\text{A.6})$$

Putting these conditions together yields $\phi(\hat{\sigma}, \sigma^*) > \phi(\hat{\sigma}, \hat{\sigma})$, a contradiction. $\square$

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.