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Novel Formulation for Generalization of Mixed-mode S-parameters for Coupled Differential High-speed Digital Channels

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***Abstract*— As the demand for higher data rates intensifies, achieving accurate S-parameter calculation becomes increasingly critical. The conventional single-ended to mixed-mode S-parameter conversion formulation assumes uncoupled structures, which may not be true for high-speed digital channels. This work introduces a novel, generalized formulation for mixed-mode S-parameters and their corresponding transformation matrices $[M_1]$ and $[M_2]$, enabling comprehensive analysis of multi-pair coupled differential traces. An intra-pair crosstalk analysis of a tightly coupled stripline and microstrip line verifies and highlights the difference between the proposed and old formulations. A loosely coupled case is analyzed as an additional validation of the proposed formulation. Finally, the generalized conversion formulation is used to perform an inter-pair crosstalk analysis which compares the old and the new conversion formulation.**

Keywords—mixed-mode, multi-pair coupled differential trace, transformation matrices, tightly coupled, loosely coupled.

I. Introduction

As the data rate of high-speed digital channels is increasing, signal and power integrity mitigation requires accurate calculation and measurement details. S-parameters are widely used to characterize single-ended high-speed data links. As differential transmission lines are predominantly used in high-speed digital channels to achieve faster data transmission at a lower signal loss, the concept of mixed-mode S-parameters was put forward in [1] to characterize differential systems physically. A single-ended to mixed-mode S-parameter conversion formulation was derived in [2]. The matrix representation of the formulation can be written as $[S_{mm}] = [M] \times [S_s] \times [M]^{-1}$, where $[S_{mm}]$ is the mixed-mode S-parameter, $[M]$ is the transformation matrix, and $[S_s]$ is the single-ended S-parameter. But this formulation is only true for uncoupled transmission lines, thereby making the odd-mode and even-mode impedances equal to the characteristic impedance ($Z_{oe} = Z_{oo} = Z_0$). However, this assumption will not hold for coupled transmission lines. Therefore, the coupling factor between differential transmission lines was introduced in [3] to derive a new

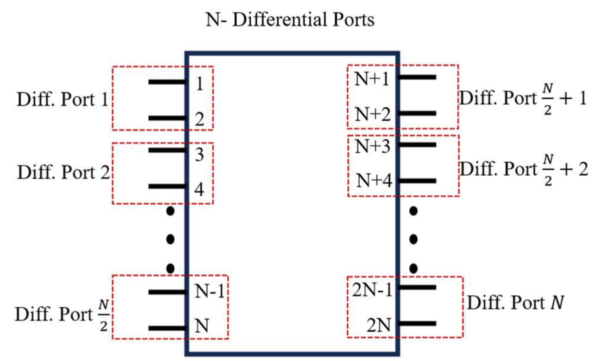

Fig. 1. 'N'-Differential Port Network

formulation. However, the limitation of [3] is that it considers a frequency-independent microstrip model to perform intra-pair analysis.

The absence of a multi-pair crosstalk formalization suitable for both tightly and loosely coupled differential transmission lines necessitates further investigation. In this paper, for the first time, we present a generalized mixed-mode interpretation of crosstalk applicable to any number (N) of differential ports, significantly expanding the scope and impact beyond previous research restricted to specific configurations [1]-[3]. The proposed theory was validated through a comparative study encompassing two transmission line structures crucial to high-speed digital channels: intra-pair stripline and microstrip line. Simulations demonstrated significant discrepancies between the novel and old approaches, highlighting the potential advantages of the proposed method. Further, the inter-pair crosstalk for a multi-pair differential stripline is also analysed with the proposed generalized conversion formulation. The results indicate that the old approach may fail to capture and analyse crosstalk in high-speed channels with sufficient accuracy, potentially compromising the design quality and performance of these systems.

This paper is divided into five sections. In section II, the newly proposed generalized conversion formulation for an N-differential port network, along with the transformation matrices, is discussed. In section III, an intra-pair comparison of the proposed and existing formulations is conducted using stripline and microstrip configurations to evaluate the impact of the new approach. Also, the proposed formulations are validated by considering loosely coupled stripline and microstrip line pairs. The NEXT and FEXT comparison for a multi-pair differential stripline is showcased in section IV. Finally, section V summarizes the benefits of the proposed formulation for the generalization of mixed-mode S-parameters for coupled differential high-speed channels.

II. Generalization of Coupled Mixed-Mode S-Parameters

A generalized N-differential port network, as shown in Fig. 1, was considered. The differential and common-mode normalized waves, considering a low-loss transmission line, will be [5]

$$a_{dm_j} = \frac{1}{2\sqrt{R_{dm_j}}}\left[V_{dm_j} + I_{dm_j}R_{dm_j}\right] \tag{1}$$

$$b_{dm_j} = \frac{1}{2\sqrt{R_{dm_j}}}\left[V_{dm_j} - I_{dm_j}R_{dm_j}\right] \tag{2}$$

$$a_{cm_j} = \frac{1}{2\sqrt{R_{cm_j}}}\left[V_{cm_j} + I_{cm_j}R_{cm_j}\right] \tag{3}$$

$$b_{cm_j} = \frac{1}{2\sqrt{R_{cm_j}}}\left[V_{cm_j} - I_{cm_j}R_{cm_j}\right] \tag{4}$$

where a_{dm_j}, b_{dm_j}, a_{cm_j}, and b_{cm_j} are the differential-mode and common-mode forward and reverse power waves at the differential port 'j', respectively. V_{dm_j}, I_{dm_j}, R_{dm_j}, V_{cm_j}, I_{cm_j}, and R_{cm_j} are the differential-mode and common-mode voltage, current, and impedance at the j^{th} differential port, respectively. The port number 'j' varies from 1 to N. The differential and common-mode voltage, current, and impedance are given by [1]

$$V_{dm_j} = V_{2i-1} - V_{2i};\ I_{dm_j} = \frac{I_{2i-1} - I_{2i}}{2} \tag{5}$$

$$V_{cm_j} = \frac{V_{2i-1} + V_{2i}}{2};\ I_{cm_j} = I_{2i-1} + I_{2i} \tag{6}$$

$$Z_{dm_j} = \frac{V_{dm_j}}{I_{dm_j}} = 2Z_{oo};\ Z_{cm_j} = \frac{V_{cm_j}}{I_{cm_j}} = \frac{Z_{oe}}{2} \tag{7}$$

where 'i' is the single-ended port number varying from 1 to 2N and 'j' is the differential port number varying from 1 to N. A generalized matrix representation of an N-differential port network will become

$$\begin{bmatrix} b_{d_{m1}} \\ b_{dm_2} \\ \vdots \\ b_{dm_N} \\ b_{cm_1} \\ b_{cm_2} \\ \vdots \\ b_{cm_N} \end{bmatrix} = \begin{bmatrix} S_{dd_{11}} & S_{dd_{12}} & \cdots & S_{dd_{1N}} & S_{dc_{11}} & S_{dc_{12}} & \cdots & S_{dc_{1N}} \\ S_{dd_{21}} & S_{dd_{22}} & \cdots & S_{dd_{2N}} & S_{dc_{21}} & S_{dc_{21}} & \cdots & S_{dc_{2N}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{dd_{N1}} & S_{dd_{N2}} & \cdots & S_{dd_{NN}} & S_{dc_{N1}} & S_{dc_{N2}} & \cdots & S_{dc_{NN}} \\ S_{cd_{11}} & S_{cd_{12}} & \cdots & S_{cd_{1N}} & S_{cc_{11}} & S_{cc_{12}} & \cdots & S_{cc_{1N}} \\ S_{cd_{21}} & S_{cd_{22}} & \cdots & S_{cd_{2N}} & S_{cc_{21}} & S_{cc_{22}} & \cdots & S_{cc_{2N}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{cd_{N1}} & S_{cd_{N2}} & \cdots & S_{cd_{NN}} & S_{cc_{N1}} & S_{cc_{N2}} & \cdots & S_{cc_{NN}} \end{bmatrix} \times \begin{bmatrix} a_{dm_1} \\ a_{dm_2} \\ \vdots \\ a_{dm_N} \\ a_{cm_1} \\ a_{cm_2} \\ \vdots \\ a_{cm_N} \end{bmatrix} \tag{8}$$

To convert the single-ended S-parameters into mixed-mode S-parameters, the differential and common-mode power waves should be represented in terms of the single-ended power waves. Since the odd-mode and even-mode impedances of a coupled transmission line are unequal, i.e., $Z_{oe} \neq Z_{oo}$, the coupling factors k_{oo} and k_{oe} need to be introduced [3]. There are a couple of different methods to calculate the mode impedance. One way is to use the closed-form formulations already defined in previous literature [5]. Another way is to use 2D field solvers to generate the odd- and even-mode impedances. The definition of coupling coefficient 'C' in microwave directional coupler is given by [6]

$$\text{C} = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \tag{9}$$

The coupling factors k_{oo} and k_{oe} can be calculated from the coupling coefficient,

$$k_{oo} = \sqrt{\frac{1-C}{1+C}},\ k_{oe} = \sqrt{\frac{1+C}{1-C}} \tag{10}$$

Therefore, the modal impedances for the coupled transmission lines can be calculated from the following equations.

$$Z_{oo} = k_{oo} \times Z_0 = \sqrt{\frac{1-C}{1+C}} \times Z_0 \tag{11}$$

$$Z_{oe} = k_{oe} Z_0 = \sqrt{\frac{1+C}{1-C}} \times Z_0 \tag{12}$$

The generalized differential and common–mode power waves for coupled differential pairs at a differential port 'j' in terms of the corresponding single-ended power waves can be derived by substituting equations (5)-(7) and (11)-(12) into (1)-(4).

$$a_{dm_j} = \frac{(1+k_{oo})(a_{2i-1} - a_{2i}) + (1-k_{oo})(b_{2i-1} - b_{2i})}{2\sqrt{2k_{oo}}} \tag{13}$$

$$b_{dm_j} = \frac{(1+k_{oo})(b_{2i-1} - b_{2i}) + (1-k_{oo})(a_{2i-1} - a_{2i})}{2\sqrt{2k_{oo}}} \tag{14}$$

$$a_{cm_j} = \frac{(1+k_{oe})(a_{2i-1} + a_{2i}) + (1-k_{oe})(b_{2i-1} + b_{2i})}{2\sqrt{2k_{oo}}} \tag{15}$$

$$b_{cm_j} = \frac{(1+k_{oe})(b_{2i-1} + b_{2i}) + (1-k_{oe})(a_{2i-1} + a_{2i})}{2\sqrt{2k_{oo}}} \tag{16}$$

where 'i' is the single-ended port number and 'j' is the differential port number. Substituting (13)-(16) into the matrix (8) will give a generalized formula for mixed-mode S-parameters that consider transmission line coupling.

$$[S_{mm}]_{i\times i} = ([M_1]_{i\times i} \times [S_s]_{i\times i} + [M_2]_{i\times i}) \times ([M_1]_{i\times i} + [M_2]_{i\times i} \times [S_s])^{-1} \tag{17}$$

where 'i' is the single-ended port number ranging from 1 to '2N'. Here, the matrices $[M_1]$ and $[M_2]$ can be derived into their generalized representation for an N-differential port network, as shown in equations (18) and (19). From the generalized equations (17), (18), and (19), we will be able to perform a multi-pair analysis of coupled transmission lines.

$$M_1 = \begin{bmatrix} \frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & 0 & 0 & \cdots & 0_{1,2N-1} & 0_{1,2N} \\ 0 & 0 & \frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & \cdots & 0 & 0_{2,2N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1+k_{oo}}{2\sqrt{2k_{oo}}}_{N,2N-1} & -\frac{1+k_{oo}}{2\sqrt{2k_{oo}}}_{N,2N} \\ \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & 0 & 0 & \cdots & 0_{N+1,2N-1} & 0_{N+1,2N} \\ 0 & 0 & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & \cdots & 0_{N+2,2N-1} & 0_{N+2,2N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}}_{2N,2N-1} & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}}_{2N,2N} \end{bmatrix} \tag{18}$$

$$M_2 = \begin{bmatrix} \frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & 0 & 0 & \cdots & 0_{1,2N-1} & 0_{1,2N} \\ 0 & 0 & \frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & \cdots & 0 & 0_{2,2N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1-k_{oo}}{2\sqrt{2k_{oo}}}_{N,2N-1} & -\frac{1-k_{oo}}{2\sqrt{2k_{oo}}}_{N,2N} \\ \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & 0 & 0 & \cdots & 0_{N+1,2N-1} & 0_{N+1,2N} \\ 0 & 0 & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & \cdots & 0_{N+2,2N-1} & 0_{N+2,2N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}}_{2N,2N-1} & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}}_{2N,2N} \end{bmatrix} \quad (19)$$

III. INTRA-PAIR ANALYSIS

A stripline and a microstrip line, as shown in Fig. 2, are considered for the analysis. The figure shows the port numbering, with the number inside the bracket representing the port on the opposite side. The cross-sectional geometry, material property, and modal impedance values are mentioned in Table I. For the stripline pair, the odd-mode and even-mode impedances are calculated from the closed-form formulation shown in (20), given by [5]

$$\left.\begin{aligned} z_{c,i} &= \frac{30\pi(b-t)}{\sqrt{\varepsilon_r}\left(W + \frac{bC_f}{2\pi}A_i\right)} \\ A_e &= 1 + \frac{\ln[1+\tanh(\theta)]}{\ln 2} \\ A_0 &= 1 + \frac{\ln\left[1+\frac{1}{\tanh(\theta)}\right]}{\ln 2} \\ \theta &= \frac{\pi S}{2b} \\ c_f\left(\frac{t}{b}\right) &= 2\ln\left(\frac{2b-t}{b-t}\right) - \frac{t}{b}\ln\left[\frac{t(2b-t)}{(b-t)^2}\right] \end{aligned}\right\} \quad (20)$$

Compared to the stripline, the closed-form formulations for the odd-mode and even-mode impedances of the microstrip line are complicated. Therefore, a 2D-solver, Ansys Q2D, was used to obtain the mode impedances of the microstrip line pair. The single-ended S-parameter of the stripline and microstrip line was generated from the 3D simulation performed in Ansys HFSS. Two cases were considered-

1. **Tightly coupled case:** The intra-pair spacing (S) was assumed to be 3 mil.
2. **Loosely coupled case:** The intra-pair spacing (S) was assumed to be 40 mils.

The tightly coupled case was considered to realize the discrepancy between the new and old formulations. Fig. 3 shows the results from the stripline pair. The differential return loss ($S_{DD_{11}}$) and insertion loss ($S_{DD_{21}}$) are plotted in Fig. 3(a) and 3(b). Similarly, Fig. 4(a) and 4(b) show the differential return loss and insertion loss of the microstrip line.

The following can be summarized from the tightly coupled case:

- For striplines, the new formulation gives lower differential return loss compared to the old formulation.
- For microstrip lines, the new formulation gives higher differential return loss compared to the old formulation.

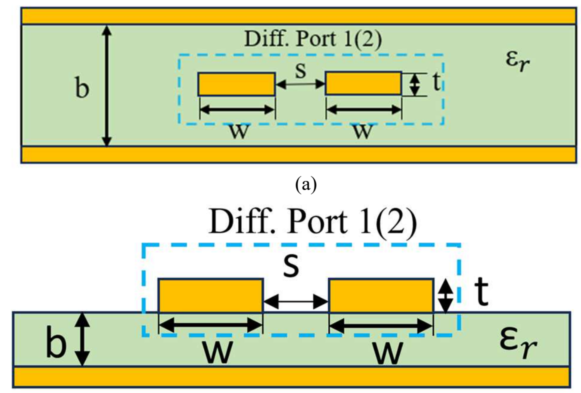

Fig. 2. Cross-section: (a) Intra-pair stripline (b) Intra-pair microstrip line

TABLE I
STRIPLINE/MICROSTRIP- PAIR PARAMETERS

Dielectric Constant (ε_r)	3.67
Dielectric Loss-tangent	0.01
Intra-Pair Spacing (S)	3 mils
Conductor Thickness (t)	0.6 mils
Conductor Length (L)	1000 mils (1- inch)
Conductivity	5.67e7 S/m
Stripline Substrate Thickness (b)	9.8 mils
Microstrip line Substrate Thickness (b)	2.7 mils
Line Width (W)	4 mils
Stripline Odd-Mode Impedance (Zoo)	42.6485 Ω (Closed-Form Formulation)
Stripline Even-Mode Impedance (Zoe)	58.3141 Ω (Closed-Form Formulation)
Microstrip line Odd-Mode Impedance (Zoo)	49.5 Ω (Ansys Q2D)
Microstrip line Even-Mode Impedance (Zoe)	67.2 Ω (Ansys Q2D)

The loosely coupled case was considered to validate the new formulation. An intra-pair spacing of 40 mils (10×W, W-trace width) was considered for both the stripline and microstrip line. Under the loosely coupled case, the assumption that the odd-mode and even-mode impedance are equal will hold ($Z_{oe} = Z_{oo}$). As a result, the coupling factors become unity ($k_{oo}=k_{oe}=1$). This will reduce the transformation matrix $[M_2]$ to zero ($[M_2] = 0$). Therefore,

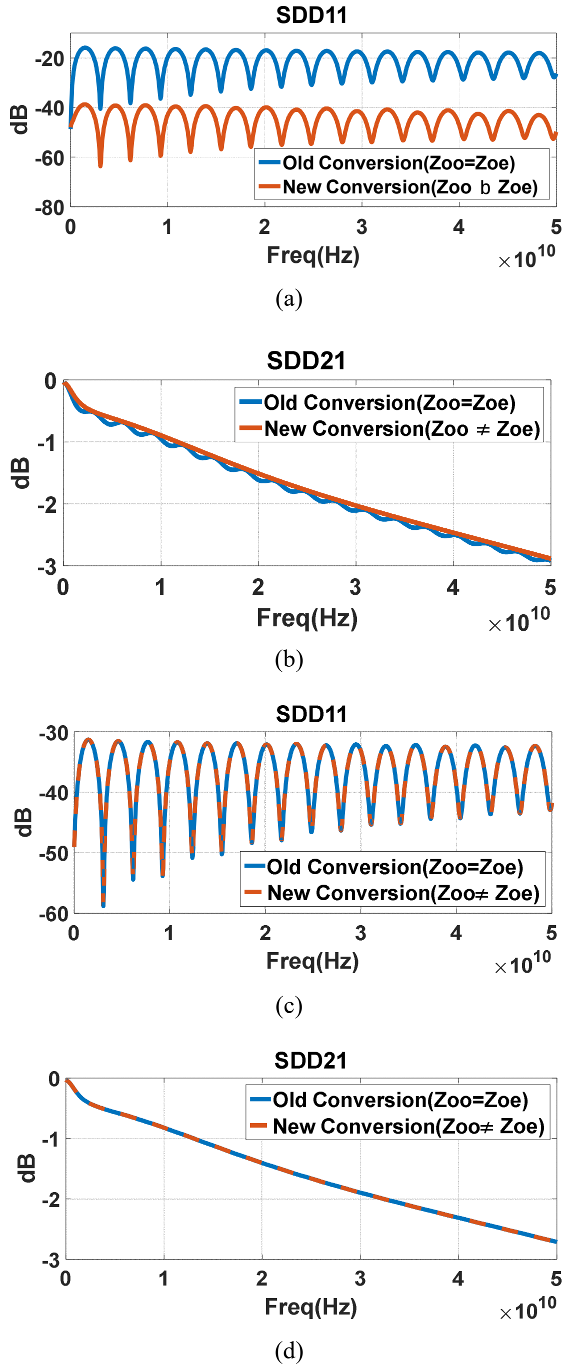

Fig. 3. Coupled Stripline: (a) Differential Return Loss (tightly coupled), (b) Differential Insertion Loss (tightly coupled), (c) Differential Return Loss (loosely coupled), (d) Differential Insertion Loss (loosely coupled)

the new conversion formulation in equation (17) will be reduced to the old conventional formulation.

$$[S_{mm}] = [M_1] \times [S_s] \times [M_1]^{-1} \tag{21a}$$

$$\text{where } [M_1] = [M] = \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{21b}$$

From the differential return loss and insertion loss plots of the stripline in Fig. 3(c) and 3(d), the old and new conversion results overlap. This validates the explanation given above. The same is true with the microstrip line results in figures 4(c) and 4(d).

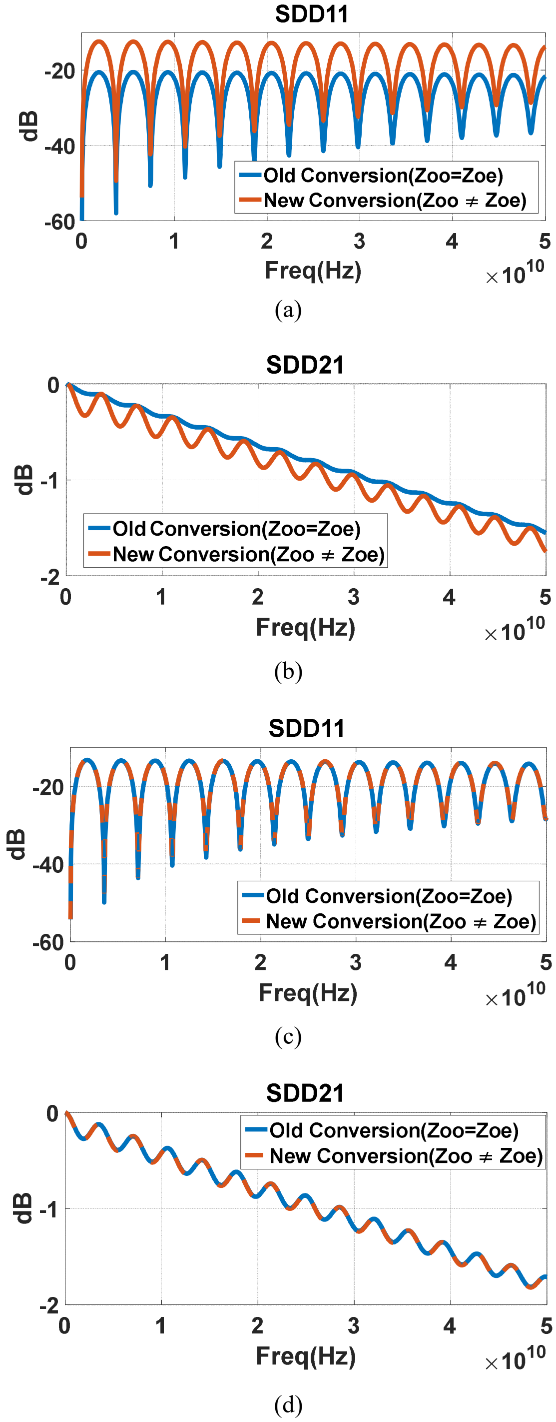

Fig. 4. Coupled Microstrip line: (a) Differential Return Loss (tightly coupled), (b) Differential Insertion Loss (tightly coupled), (c) Differential Return Loss (loosely coupled), (d) Differential Insertion Loss (loosely coupled)

IV. INTER-PAIR ANALYSIS

The generalized formulation (17) shall be used to perform an inter-pair analysis on transmission line structures. Here, as an example, a 2-differential pair stripline, designed for a

TABLE II
INTER-PAIR STRIPLINE PARAMETERS

Dielectric Constant (ε_r)	3.1
Dielectric Loss-tangent	0.01
Intra-Pair Spacing (S)	5 mils
Inter-pair Spacing (S')	5 mils
Conductor Thickness (t)	1.2 mils
Conductor Length (L)	1000 mils (1- inch)
Conductivity	5.67e7 S/m
Stripline Substrate Thickness (b)	12 mils
Line Width (W)	5 mils
Stripline Odd-Mode Impedance (Zoo)	45.91 Ω
Stripline Even-Mode Impedance (Zoe)	57.19 Ω

differential impedance of Zd = 92-ohm is used to compare the NEXT and FEXT between the old and new conversion. The geometry parameters, material properties, and calculated impedances are tabulated in Table 2.

V. CONCLUSION

In this work, we proposed a novel and generalizable formulation for converting single-ended to mixed-mode S-parameters. This framework leverages an N-differential port network, representing a significant advancement over existing methods that often lack scalability. Also, the generalization of transformation matrices, $[M_1]$ *and* $[M_2]$ makes it possible to perform multi-pair differential trace analysis. A three-dimensional electromagnetic simulation was performed on stripline and microstrip line intra-pairs to compare the differential return loss and differential insertion loss between the old and new conversion formulations. Based on the results shown in Fig 3 and Fig 4, there is a visible discrepancy between new and old approaches. The validated new closed-form formulation (17) makes it suitable for accurate analysis of both tightly and loosely coupled differential traces. The multi-pair crosstalk comparison performed on a differential stripline also shows the discrepancy in the results. The newly established conversion formulation has overcome the mode impedance disparity ($Z_{oe} \neq Z_{oo}$) thereby allowing high-speed channel design to be more accurate. Finally, the extension to N-differential port generalization makes the theory applicable to any pattern (vertical or horizontal), any structures (traces, via, etc.), and any number of coupling structures.

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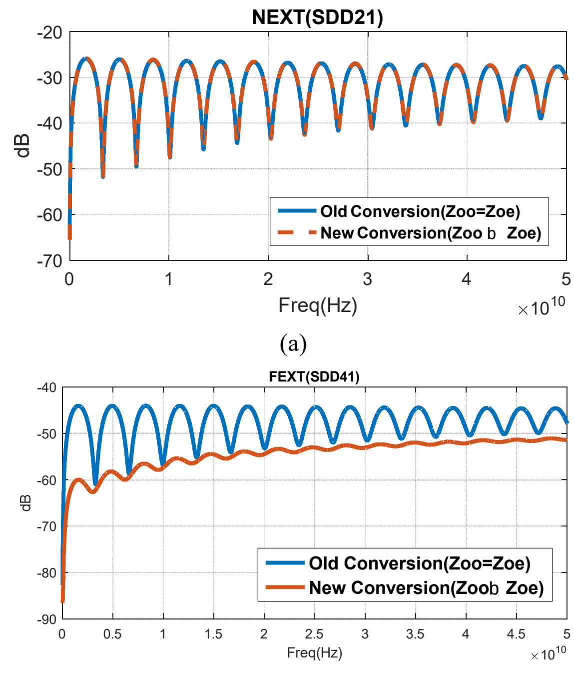

Fig. 6. Inter-pair Stripline: (a) Differential NEXT, (b) Differential FEXT

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