Hydrodynamic Slip in Nanoconfined Flows: A Review of Experimental, Computational, and Theoretical Progress

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Abstract

Nanofluidics has made significant impacts and advancements in various fields, including ultrafiltration, water desalination, biomedical applications, and energy conversion. These advancements are driven by the distinct behavior of fluids at the nanoscale, where fluid behaviors are affected or altered due to interactions with solid surfaces. A key challenge in nanofluidics is understanding hydrodynamic slip, a phenomenon where liquid flows past solid boundaries with a non-zero velocity, deviating from the classical no-slip boundary condition. This review consolidates experimental, computational, and theoretical efforts to elucidate the mechanisms behind hydrodynamic slip in nanoconfined flows. We evaluate essential experimental methodologies, including surface force apparatus, atomic force microscopy, and micro-particle image velocimetry, which have been instrumental in characterizing slip at the nanoscale. The review also discusses the contributions of molecular dynamics simulations, including both non-equilibrium (NEMD) and equilibrium (EMD) approaches, in modeling interfacial phenomena and slip behavior. Additionally, it explores the influence of factors such as surface wettability, shear rate, and confinement on slip, emphasizing the interaction between liquid structuring and solid-liquid interactions. Advancements made so far have uncovered more complexities in nanoconfined flows which have not been considered in the past, inviting more investigation to fully understand and control fluid behavior at the molecular level.

Keywords: Nanofluidics, slip length, hydrodynamic slip, interfaces, molecular dynamics.

1. Introduction

Investigations on nanoconfined fluids date to the first half of the last century. Following early investigations of fluid transport in nanoconfinements,[1] the study of fluid flow in the nanoscale emerged as a new field of science called nanofluidics, a term coined in the 1990s.[2,3] Furthermore, the integration of nanofabrication with nanofluidic technology offers potential breakthroughs in the manipulation and control of fluids at the molecular level, which could revolutionize our approach to a variety of scientific and industrial challenges. This surge in interest is driven by the capability of nanofluidic devices to operate with high efficiency and sensitivity due to their minute scale and the unique properties of fluids confined to such small dimensions.

The application of nanofluidic devices could impact promising technology, including biomedical applications, ultrafiltration and desalination processes, and ionic transport for energy conversion. In the biomedical field, nanofluidics may reduce the amount of genomic material and time for analysis of DNA.[4–6] Nanochannel devices have therapeutic applications on high-precision drug delivery systems;[7–10] while, nanoengineered fluidic devices enable low-cost cell analysis and disease diagnostics.[11,12] Similarly, the integration of nanoconduits in lab-on-a-chip systems allows for single-cell analysis, increasing the reliability of portable point-of-care medical diagnostic systems.[13,14] Beyond biomedical applications, nanoconfined fluid flows offer precise manipulation of ionic concentrations and enhance electrochemical-mechanical energy conversion in batteries.[15] Such technologies also facilitate groundbreaking electrical energy production from salinity gradients.[16,17] Furthermore, the active control of ion charge concentration in nanoflows is applied to the development of fluid-based devices analogous to micro-electronics, such as nanofluidic transistors[18–20] and diodes.[21–24] Another main research venue on nanofluidics applications takes advantage of the filtration capabilities of nanotubes and nanopores.[25] Capillary devices with atomic-scale precision are used in ultrafiltration,[26,27] while nanometer-scale porous membranes are promising alternatives for seawater desalination.[28–32] In addition to the highly specialized applications discussed so far, the behavior of thin, confined fluids is essential in numerous industrial operations, mainly involving lubrication[33], nanoencapsulation[34], and nanofabrication.[35] Electrospinning technologies leverage the interaction

61 between electrostatic forces and working fluids at the spinneret nozzle to create innovative nanofabrication

62 methods to produce hollow, core-shell, and multichannel nanofibers, which are crucial for high-performance

63 materials used in catalysis, drug delivery, and energy storage.[35]

64   Along with the enhanced capabilities of nanofluidic devices comes a major increase in flow friction, which

65 is an inevitable challenge. Fluid flow through minuscule confinements experiences a vast increase in flow

66 resistance, per the classical hydrodynamics theory. The significance of friction in nanoconfined flows is attributed

67 to the interfacial interactions as the surface-to-volume ratio increases at the nanoscale.[36] However, due to the

68 dominant effects of interfacial properties such as wettability and surface roughness, fluid flow in nanochannels

69 can be tailored to overcome the hydraulic limitations imposed by high confinement levels. For instance,

70 experimental investigations of flow in carbon nanotubes (CNT) have reported flow enhancements up to five times

71 higher than expected from the conventional continuum theory.[37–39] CNTs are atomically smooth surfaces with

72 hydrophobic properties, a combination that hypothetically offers a reduced resistance to flow.

73   Nanoconfined flows have been described by classical fluid dynamics combined with atomistic modeling to

74 account for interfacial interactions in nanochannels with diameters down to 1.4 nm.[40] The applicability of the

75 continuum approach facilitates the creation of a theoretical framework to model fluid flow. However, the boundary

76 condition compatible with the continuum approach remains a challenge, i.e., accounting for and quantifying

77 hydrodynamic slip.[41–43] The earliest efforts to simulate flow through nanochannels resulted in flow velocities

78 between 2 to 10 orders of magnitude higher than experimental measurements,[44] indicating an overestimation of

79 the hydrodynamic slip. Sophisticated simulations, considering complex wettability interactions[41] and long

80 averaged simulated times[44] ($\sim$ 100 μs) have been conducted in recent years, obtaining flow regimes comparable

81 with the flow velocities reported in the experimental literature. However, a complete understanding of the

82 mechanisms and significant parameters in nanoconfined flows is still needed. [41]

83   In this literature review, the basics of hydrodynamics in nanoconfined flows are introduced in Section 2.

84 A review of the experimental efforts aimed at explaining the different variables affecting hydrodynamic slip is

85 presented in Section 3; the most relevant experimental techniques and the tendency towards higher resolution

86 measurements are highlighted as well as the controversy regarding the effects of wettability and liquid structuring

87   metrics on the hydrodynamic slip. Subsequently, a more comprehensive review of the modeling efforts for

88   investigating the complex phenomena of nanoconfined flows is reported in Section 4. The different non-

89   equilibrium and equilibrium methods for modeling fluid friction in nanochannels are revised and compared. Lastly,

90   in Section 5 a more fundamental review of the underlying physics behind slip as well as the effects of wettability,

91   shear rate, and confinement is presented. Figure 1 illustrates a knowledge map of this review's contents.
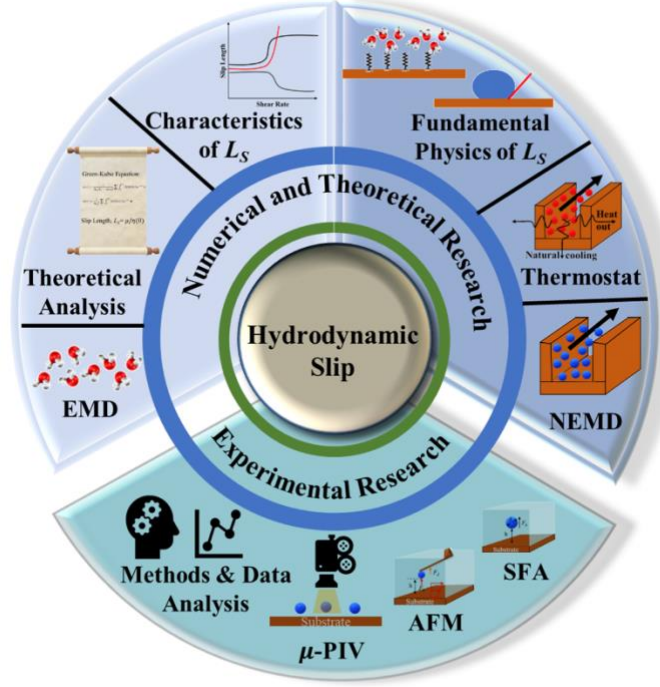

92

93   Figure 1. Knowledge map of our literature review.

94   **2. Friction and hydrodynamic slippage in nanoconfined liquids**

95   In macroscale fluid dynamics, the no-slip boundary condition has allowed experimental results to align

96   with numerical and analytical models across various applications. Despite its widespread use, the no-slip boundary

97   condition is phenomenological and not derived from fundamental physical principles.[45] In Navier's early work, [46]

98   an alternative boundary condition that permits slippage has been suggested:

$$u_s = L_s \frac{\partial u}{\partial z}\big|_{z=0} \tag{1}$$

99

100   where u_S and L_S represent the slip velocity and slip length, respectively. Here, z is the coordinate normal to the

101   interface where the velocity gradient is assessed, and L_S is defined as the distance at which the linearly extrapolated

4

102 velocity reaches zero. $L_S = 0$ denotes the no-slip condition. However, this slip boundary condition, as described in

103 Eq. (1), is also empirical and lacks a solid theoretical foundation.

104   Conversely, the governing equations of continuum fluid mechanics have a solid theoretical foundation. In the

105 continuum assumption, Newton's second law is applied to infinitesimal volume elements large enough to preserve

106 the bulk values of the thermophysical and transport properties of the fluid. After applying the Newtonian

107 corollaries to the relation between stresses and deformations, the famous Navier-Stokes (NS) equations arise.

108 However, these equations break down when the flow system is reduced to dimensions comparable to the molecular

109 size due to the uncertainty of the continuum assumption. [40]

110   In the hydrodynamic regime where the Navier-Stokes (NS) equations are applicable, there is a distinct

111 separation in length and time scales between bulk properties and surface effects.[3] In bulk systems with particles in

112 the order of Avogadro's number, the molecular degrees of freedom can be described with only a few variables

113 such as pressure, velocity field, temperature, etc., while the complexity of the transport phenomena is lumped into

114 transport coefficients. Bocquet and Charlaix [3] calculated the applicability range of the NS by determining the

115 lower-scale limit for the concept of shear viscosity η, which is represented by the Green-Kubo relation:

$$\eta = \frac{1}{V k_B T} \int_0^\infty \langle \sigma_{xy}(t)\sigma_{xy}(0) \rangle_{eq} dt \qquad (2)$$

116

117 where V, k_B, T, $\langle \sigma_{xy}(t)\sigma_{xy}(0) \rangle_{eq}$ are the system volume, Boltzmann constant, absolute temperature, and

118 autocorrelation function of the off-diagonal component of the stress tensor, respectively. The validity of this

119 equation assumes that the timescale of the stress-stress correlation function τ_σ is smaller than any hydrodynamic

120 timescale. For instance, the relaxation time of momentum, $\tau_q = (vq^2)^{-1}$, where v is the kinematic viscosity and q is

121 a wave vector; thus, $vq^2 \tau_\sigma < 1$ fixes the limit for timescales at confinements w larger than a viscous length scale,

122 namely $w > \sqrt{v\tau_\sigma}$. For water $\tau_\sigma \sim 10^{-12}$ s and $v = 10^{-6}$ m²/s at 20 ºC, which yield a limit of $w \approx 1$ nm, proving the

123 robustness of the NS equations. Thus, for water, confinement levels in the nanometer scale can be modeled using

124 the classical governing equations of fluid mechanics. It is noteworthy that Thomas and McGaughey [40] confirmed

125 the bulk-like behavior of water flowing through CNT of diameters ~1.4 nm.

3. Experimental investigations of hydrodynamic slip

Nanoconfined flowing liquids can be described by the equations of bulk hydrodynamics down to scales of approximately 1 nm (approximately three water molecular diameters). However, characterizing the boundary conditions for these flows remains challenging. The no-slip boundary condition has been successfully applied to macroscopic systems, where it remains phenomenologically valid. However, at smaller scales dominated by surface interactions, deviations from the no-slip boundary condition have been observed, meaning its validity is questionable.

Accurately measuring interfacial properties in nanofluidics is challenging due to the dynamic interactions at liquid-solid interfaces and spatial resolution limitations.[3] Recently, these experimental challenges have been addressed significantly. For instance, the surface force apparatus (SFA) and atomic force microscopy (AFM) can directly access the hydrodynamic forces with high sensitivity and in a controlled environment. Additionally, μ-PIV enables visualization of the velocity field in real time, which is critical for slip measurements. This Section will discuss these primary methodologies—SFA, AFM, and μ-PIV—and their recent developments in measuring L_S in nanoconfinements. Moreover, Section 3.4 will explore emerging techniques for measuring L_S using suspended microchannel resonators (SMR), Dynamic Quartz Crystal Microbalance (QCM-D), and hybrid graphene/silica nanochannel techniques. Along with L_S metrologies, surface characterization techniques, such as sum frequency generation (SFG) spectroscopy and X-ray reflectometry (XRR), will also be presented in Section 3.5. These techniques are useful in uncovering the origins of hydrodynamic slip behavior, particularly concerning the chemical interactions of interfacial water molecules.
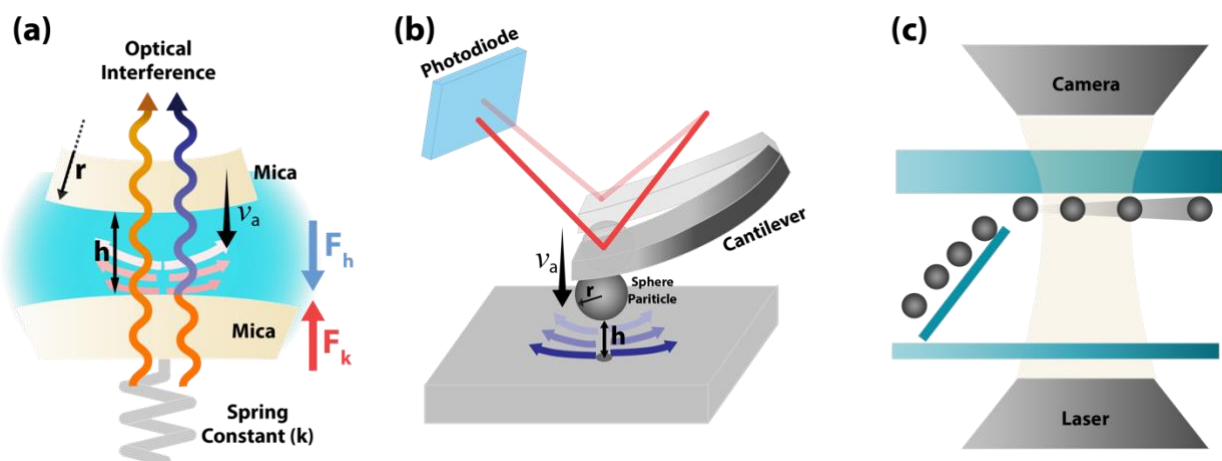
146   Figure 2. Schematic of the operational principles of (a) SFA, (b) AFM, and (c) μ-PIV. In (a) and (b), the radius of

147   the sphere (r), separation distance (h), and approach velocity (v_a) are indicated as the parameters of the

148   hydrodynamic drag force (F_h) and the restorative force of the cantilever (F_k), which are balanced for hydrodynamic

149   slip measurement.

150

151   **3.1 Surface force apparatus (SFA)**

152   The SFA quantifies the viscous force, F_h, between two surfaces submerged in a liquid with viscosity η, based

153   on their separation distance h, and balanced by the cantilever restorative force, F_k,[47] as shown in Figure 2(a). This

154   viscous force is detected using piezoelectric materials, and the gap between the surfaces is determined through

155   interferometry. Employing SFA in conjunction with multiple beam interferometry enables measurements with

156   sub-nanometer accuracy. The expression for F_h is provided in Eq. (3) where v_a represents the relative approach

157   velocity of the surfaces, r is the radius of the sphere immersed in the liquid, and f^* is a correction factor that

158   compensates for hydrodynamic slip. When $f^* = 1$, Eq. (3) corresponds to the Navier-Stokes (NS) solution in the

159   lubrication approximation. For surfaces of comparable characteristics, Vinogradova [48] derived the solution for f^*

160   as presented in Eq. (4).

$$F_h = \frac{6\pi\eta r^2 v_a}{h} f^* \tag{3}$$

$$f^* = \frac{h}{3L_s}\left[\left(1 + \frac{h}{6L_s}\right)\ln\left(1 + \frac{6L_s}{h}\right) - 1\right] \tag{4}$$

161

162   Chan and Horn[49] investigated the drainage of three non-polar organic Newtonian liquid films between

163   molecularly smooth mica surfaces using the SFA. Their results were in good agreement with the Reynolds theory

164   of lubrication (no-slip boundary condition) for film thicknesses above 50 nm; however, for thinner films, an

165   apparent enhancement of the viscosity was observed. Chan and Horn[49] indicated that as the film thickness

166   decreases, the surface effects are more significant and produce a "solid-like" ordering in liquid layers near the

167   wall, causing an increase in the liquid resistance to shear. A modification to the formerly static SFA apparatus is

168   reported in Luengo et al.[50] where a shear attachment was added to the original design, now allowing for dynamic

169   rheological analyses. A series of transition regimes in the rheological behavior of polymer melts as well as a

170   reduction of viscosity with film thickness were found using the modified SFA.

171   Baudry et al.[51] used a similar version of the dynamic SFA to investigate the slippage of glycerol in contact

172   with wetting and non-wetting surfaces. On a wetting cobalt surface, the no-slip boundary condition was found to

173   hold; alternatively, when cobalt was coated with a thiol, the surface became non-wetting, and $L_S \approx 65$ glycerol

174   molecular diameters were measured. The liquid confined between wetting surfaces showed a constant viscosity as

175   the confinement was varied; in contrast, for non-wetting cobalt-thiol, a reduction in the liquid viscosity was

176   observed as the separation between surfaces decreased.

177   High shear rates have been observed to trigger bubble nucleation, leading to increased slip. Zhu and Granick[52]

178   observed extensive L_S with water and tetradecane on hydrophobic surfaces, noting that L_S escalated unboundedly

179   as shear rates increased in their experiments, reaching up to the micrometer range. Furthermore, Cottinn-Bizonne

180   et al.[53] investigated the hydrodynamic boundary conditions of water and dodecane on both hydrophobic and

181   hydrophilic surfaces, employing a dynamic SFA under shear rates up to 5×10^3 s^{-1}. They found that the viscosity

182   of water remained consistent with its bulk value in confinements as narrow as 10 nm. On hydrophilic Pyrex

183   surfaces, neither water nor dodecane exhibited hydrodynamic slip; however, on hydrophobic surfaces, both fluids

184   showed slip with L_S reaching approximately 20 nm.

185   The findings by Cottinn-Bizonne et al.[53] present notable contradictions to those of Zhu and Granick.[52]

186   Cottinn-Bizonne et al.[53] noted that the oscillation amplitude of pressure in their SFA experiments remained below

187   the vapor pressure of the examined liquids, precluding cavitation, which might have been possible under the

188   conditions used by Zhu et al.[52] Furthermore, they hypothesized that discrepancies could arise from the

189   contamination of surfaces by hydrophobic materials. Continuing this investigation, Cottinn-Bizonne et al.[54] also

190   explored potential experimental inaccuracies affecting their results. They highlighted that even small

191   miscalculations in measuring the separation distance between surfaces could significantly impact the calculated

192   L_S. Their further studies using water and water mixtures aimed to discern the impact of viscosity on different

193   wettability surfaces, finding that surfaces with higher hydrophobicity exhibited greater L_S, though not exceeding

194   20 nm.

195   **3.2 Atomic force microscopy (AFM)**

196   AFM, as described in Figure 2(b), also employs the same approaches and equations Eq. (3)-(4) as the SFA.

197   However, AFM has smaller probed areas, which are determined by the size of the spherical bead attached to the

198 AFM probe.[3] Craig et al.[55] utilized an AFM setup to assess the drainage force in aqueous sucrose solutions between

199 a spherical tip and a flat surface. They employed analytical models to determine variables such as L_S from the

200 measured force-distance curves. They observed that the L_S varied with the fluid's viscosity and the rate at which

201 the AFM cantilever approached (shear rate). At lower velocities, there was no slip, indicative of a "free system"

202 behavior, whereas at higher velocities where $L_S \leq 20$ nm. A notable limitation of AFM in measuring viscous

203 forces in nanoconfined liquids is its sensitivity to deflections in the AFM cantilever caused by viscous drag.[56] To

204 address this, Vinogradova et al.[56] developed multiple models aimed at curbing or even eliminating the increase in

205 viscous drag as the speed of the AFM cantilever escalated. Further, Vinagroda et al.[57] engineered an AFM probe

206 that reduced drag. Employing a data reduction method previously outlined by Vinogradova et al.[56], they

207 successfully identified the no-slip boundary condition and measured an L_S of 10 nm on hydrophobic surfaces.

208   Using AFM, Bonaccurso et al.[58] assessed the hydrodynamic force between hydrophilic mica and glass in the

209 presence of aqueous solutions, measuring L_S of $8 - 9$ nm on these surfaces, irrespective of the AFM probe approach

210 speed. The experiments were conducted at high shear rates (10^4 s^{-1}), which accounted for the observed

211 hydrodynamic slippage on the hydrophilic surfaces. Honig and Ducker[59] explored the impact of rapidly

212 approaching AFM probes on measuring viscous forces within sucrose solutions varying in viscosity. They

213 observed no significant increase of L_S exceeding zero, even at shear rates up to 2.5×10^5 s^{-1}, contrasting with the

214 results from Bonaccurso et al.[58] when examining similar systems. However, their findings were in line with those

215 of Vinagroda et al. [57] The discrepancies are thought to stem from the different methods used to measure the gap

216 between the surface and the probe. Traditional AFM experiments calculate this distance using the combined

217 displacement of the piezoelectric scanner movement and the cantilever deflection, while Honig and Ducker [59]

218 derived the separation distance through the intensity of scattered evanescent waves. Bushan et al. [60] investigated

219 L_S for different surface conditions using the AFM tapping mode. The authors reported L_S values of 43 nm and 232

220 nm for hydrophobic and superhydrophobic surfaces, respectively. Maali et al.[61] enhanced the design of

221 commercially available AFM cantilevers by adapting them for improved acoustic excitation in liquid environments.

222 They integrated an anti-reflective coated glass slide into the cantilever holder to reduce unwanted oscillation peaks.

223 Subsequently, Maali et al.[62] used this improved version of the AFM to measure the L_S of water on graphitic-carbon

224 surfaces at a value of 8 nm.

225   In AFM experiments, instrumental uncertainties such as offsets and improper calibrations result in significant

226   errors in determining the slip length.[54,63] Such errors originate from the necessity of independently providing

227   accurate values for the liquid viscosity, sphere radius, and spring constant when calculating L_S, as inaccuracies in

228   these parameters can propagate and lead to incorrect L_S estimations.[54] For example, Bonaccurso et al.[58] reported

229   L_S of approximately 8 nm for a hydrophilic substrate (mica–water interface), while Maali et al.[64] and Zhang et al.[65]

230   reported values close to zero. To overcome these discrepancies, a new data analysis model was proposed, assuming

231   that the diameter of the microsphere is much larger than the slip length.[54,66,67]

$$\frac{v}{F_h} = \frac{h + L_s}{6\pi\eta R^2} \tag{5}$$

232

233   where h is the separation between the microsphere and the surface of the substrate, v is the approach speed of the

234   sphere to the substrate, F_h is the hydrodynamic force acting on the tip, η is the viscosity of the liquid, and R is the

235   diameter of the microsphere. Because v/F_h is proportional to $h + L_s$, the slip length is determined by its intercept

236   on the x-axis without knowing the viscosity of the liquid and the size of the microsphere. Haruya Ishida et al.[68]

237   proposed an analytical alternative to Eq. (5) adding the assumption that the L_S of the substrate is identical to that

238   of the microsphere on the AFM tip. With their new analytical approach, they proposed that the L_S of a mica

239   substrate with water is close to zero.

240

241   **3.3 Micro-particle image velocimetry (µ-PIV)**

242   The µ-PIV method, as illustrated in Figure 2(c), involves tracking particle movement within a liquid flow

243   constrained to microscale dimensions. This technique focuses image velocimetry analysis on areas close to the

244   surface to sample the velocity profile and observe interfacial phenomena. Although theoretically applicable to

245   nanoscale conduits, µ-PIV faces significant challenges in accurately tracking particles that are only a fraction of

246   the size of the nanochannels. Additionally, as the resolution of µ-PIV improves, the measurements become noisier

247   due to increased Brownian motion affecting smaller tracer particles.[3] Consequently, µ-PIV is typically employed

248   in microchannels, with efforts concentrated on enhancing resolution near the surface to detect the presence of

249   hydrodynamic slip. Tretheway and Meinhart[45] utilized standard µ-PIV to investigate slip in hydrophilic and

250   hydrophobic channels with a cross-section area $A_{cross-section}= 30\times300$ μm². They used fluorescent polystyrene

251   spheres with a diameter of 300 nm as tracers in the sampled region measuring 25×100 μm. They reported L_S =1

252   μm for hydrophobic surfaces, while no-slip (L_S = 0) for hydrophilic conditions. These findings exceeded theoretical

253   predictions but aligned with the expected effects of wettability.

254   Lumma et al.[69] enhanced the precision of velocity profiling within a 100 μm wide microchannel by using the

255   fluorescence correlation spectroscopy (FCS) method to cross-correlate the fluorescence signals of clearly

256   identified tracer particles with a diameter of 40 nm. This approach distinguishes between flow and diffusion

257   effects, revealing L_S ranging from 0.2 to 1 μm. They recognized that their measurements of L_S might be larger,

258   influenced by interactions between the liquid and surface and the repulsive forces among the tracer colloids. In

259   another investigation using conventional μ-PIV, Ou and Rothstein[70] measured L_S of 7.5 μm in micro-ridges with

260   ultra-hydrophobic surfaces and a shear-free configuration. Their results also matched independent experiments

261   using a pressure drop calculation of slip. Joseph and Tabeling[71] refined the μ-PIV method significantly, achieving

262   near-simulation accuracy. They used fluorescent beads ranging from 100 to 200 nm in diameter within a

263   microchannel measuring 10×100 μm. To precisely ascertain the wall position, they tracked the location of tracer

264   particles that adhered to the walls, reporting L_S < 100 nm, with uncertainties comparable to the measured values.

265

266   **3.4 Other techniques for direct slip measurement**

267   In addition to the hydrodynamic slip characterization techniques described in Sections 3.1 – 3.3, there are

268   methods to directly measure L_S. Collis et al.[72] first proposed L_S measurements on individual gold nanoparticles

269   immersed in water. They used suspended microchannel resonators (SMRs) introducing gold nanoparticles into

270   "U-shaped" channels embedded in a cantilever. When a nanoparticle passes through the channel, it increases the

271   inertial mass of the sensor. In the experiments, the flow at the particle surface is closely related to the

272   hydrodynamic boundary condition, and the slip at the particle surface is the result of the discrepancy between

273   measured and actual mass. Thus, L_S can be calculated by fitting the varying excitation frequencies of each

274   vibrational mode versus mass discrepancies. The main advantage of using SMRs is the measurement without

275   confinement conditions which can modify the nature of the slip. Unfortunately, further study of the hydrodynamic

276 slip using SMRs has not been reported, but the authors suggested the measurement capability of particle wettability,

277 particle crystal structure, particle surface functionalization, particle surface charge, system temperature, liquid

278 viscosity, and polarity.

279 Xie et al.[73] developed a hybrid graphene/silica nanochannel for L_S measurements. The ratio of mass flow

280 resistance between the silica and graphene sections was calculated using the meniscus movement in the graphene

281 section and the corresponding capillary flow constants. The variation of mass flow resistance between graphene

282 and the hybrid nanochannels is likely due to variations in slippage between the different sections of the hybrid

283 nanochannel. This approach enables an indirect measurement of L_S in the graphene section of the nanochannel. In

284 their result, the measured L_S =16 nm is smaller than what has been estimated in the MD simulation of graphene

285 capillaries with pristine multilayered graphene (L_S =60 nm).[74] They hypothesize that the observed variation of the

286 graphene slip length is due to the functional groups and charges on the graphene surface during the CVD

287 process.[75,76] This implies that there is growing interest in the relationship between hydrodynamic slip and

288 interfacial chemistry.

289 Dynamic quartz crystal microbalance (QCM-D) is another emerging technique to study hydrodynamic

290 slip. QCM-D measures changes in the resonant frequency of the crystal under oscillation, both before and after

291 the deposition of mass onto the substrate; Zhdanov and Kasemo [77] reported the simulation-based observation of

292 hydrodynamic slip on the surface of a QCM-D sensor. According to theoretical calculations, at low oscillation

293 amplitudes, the amplitude of the substrate matched that of the central bead deposited on it, indicating a no-slip

294 boundary condition. However, as the substrate oscillation amplitude increased, the oscillation amplitudes of the

295 substrate and bead deviated from the theoretical prediction. Zhdanov and Kasemo[77] hypothesized that this

296 mismatch is due to a transition from sticking to slipping at the interface between the substrate and the bead. Their

297 additional finding that a lower transition amplitude occurred with a Ca2+-containing buffer (which changed the

298 bead-support interaction) supported this hypothesis, inferring that the frequency shift and energy dissipation in

299 QCM-D may increase or decrease depending on the slippage between the substrate and the beads (or fluids). This

300 approach could provide valuable insights into the influence of interfacial interactions on hydrodynamic slip,

301 though further fundamental studies are needed to fully develop it.

302   **3.5 Interfacial liquid characterization**

303   In the previous Section, we introduced experimental techniques for the characterization of hydrodynamic slip

304   at solid-liquid interfaces. The fundamental mechanisms of slip at the interface are not fully understood, but theories

305   and hypotheses suggest a significant influence of interfacial liquid properties, structure, and ordering on the nature

306   of momentum transfer at solid-liquid interfaces. In this Section, analytical tools to probe the water/liquid interface

307   at the molecular level will be introduced. It should be noted that while the techniques presented here are not capable

308   of directly measuring hydrodynamic slip, they can provide important properties linked to its origin and

309   fundamental principles.

310

311   **3.5.1 Sum frequency generation (SFG) vibrational spectroscopy**

312   SFG vibrational spectroscopy is a non-linear optical process where two photons are combined at a surface

313   and a new photon with its energy equal to the sum of two input photons is generated. This process requires

314   noncentrosymmetry. In the bulk liquid phase, all molecules are randomly moving, and such randomness is

315   equivalent to centrosymmetry because the positive and negative directions on any axis are equivalent. Thus, bulk

316   liquid cannot generate an SFG signal. In contrast, at solid-liquid interfaces, the randomness is broken, creating

317   noncentrosymmetry; thus, SFG can detect interfacial molecular species without interference from the bulk phase

318   molecules of the same species.

319   SFG has been extensively employed to investigate the interaction between water molecules and solid surfaces

320   by analyzing the behavior of OH stretching signals. One study on fused quartz revealed that all free OH groups at

321   the silica surface are hydrogen-bonded to water molecules in contact with liquid water, which induces

322   noncentrosymmetric ordering of water molecules near the surface. The distribution of disordered and ordered

323   water structures in the interfacial region varies depending on pH.[78] The dipole direction of water at the liquid/solid

324   interface flips by 180º when the pH of the aqueous solution crosses the isoelectric point of the surface.[79] This is

325   significant as previous computational studies have demonstrated that the structuring of liquids at the solid-liquid

326   interface plays a pivotal role in capturing the hydrodynamic slip behavior across different solid-liquid interfaces

327   (see Section 5.2 for further information). Hence, the experimental characterization of liquid structure and

13

328   orientation at the interface is crucial for bridging the knowledge gap between measurements and calculations of

329   slip.

330   When a solid surface interacts with a polar solvent such as water, it can be charged. This surface charge can

331   arise through two mechanisms: either by the dissolution of surface groups into the contacting liquid and/or by the

332   adsorption of ions from the solution. This process leads to the formation of electrical double layers (EDL), which

333   can significantly influence solid-liquid friction. Using the SFG technique, Wei et al.[80] reported the relationship

334   between pH, electrolyte concentration, and hydrogen bonding interactions at the interface. They observed a

335   significant reduction of the SFG intensities of the OH stretching region ($3000 \sim 3800$ cm^{-1}) as increasing electrolyte

336   concentration, and this drop was primarily attributed to the decrease of the net orientation of water molecules

337   orienting toward the solid surface in the EDL. Notably, numerical and theoretical models show that surface charges

338   and salt concentration directly influence the hydrodynamic slip behavior. The reduction of L_S with surface charge

339   was consistently reported in previous studies.[81–85] Geng et al.[83] highlighted that at high surface charge density, the

340   L_S behavior is dominated by ionic interactions rather than solid-liquid binding forces. Rezaei et al.[85] conducted

341   MD simulations to study the electro-osmotic flow of an aqueous NaCl solution on a charged silicon surface and

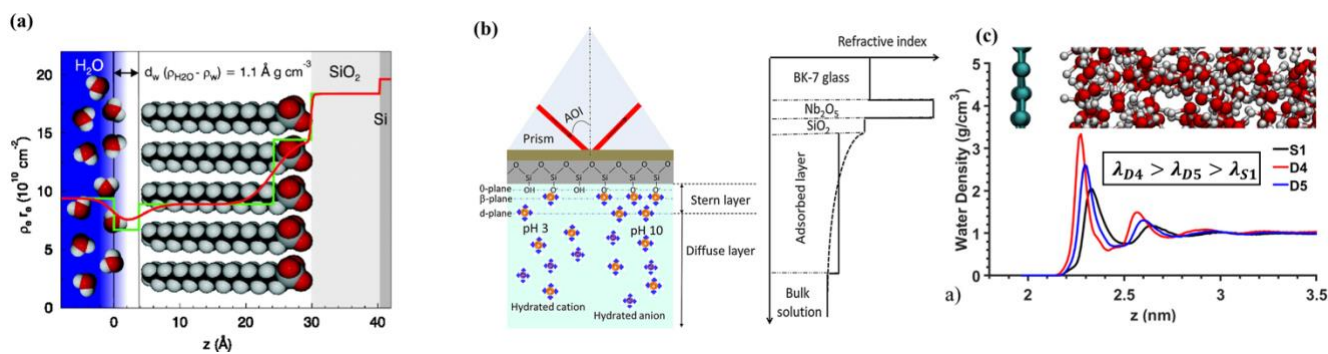342   observed a similar relationship between L_S and surface charge density.

343   Recently, Wang et al.[86] successfully detected SFG signals at graphene/water interfaces. Previously, isolating

344   the graphene/water interaction was challenging due to interference from substrate/graphene signals because of the

345   transparent nature of graphene. They employed a method of suspending graphene on the water surface, creating

346   air/graphene/water interfaces under dry conditions to avoid signal interference from the air side. They observed

347   that the OH peak appears at nearly the same frequency and amplitude as at the air/water interface, suggesting that

348   graphene has only a weak effect on the organization of interfacial water. This result and approach open new

349   opportunities to study the chemical interactions between graphene and water, which is of great interest for

350   applications in microfluidic devices.

351   In addition to using spectroscopy metrologies for the characterization of solid-liquid interfaces, atomistic

352   scale simulations are pertinent for investigating local transport properties, such as viscosity (η), the EDL thickness,

353   and diffusion coefficients, particularly in nanoconfined electrolytes under varying electric fields and surface

354   charge conditions.[87–89] For example, Masuduzzaman et al.[87] explored the effect of electric fields on nanoconfined

14

355   aqueous electrolytes, showing that the EDL thickness increased while the local η decreased due to the bulk motion

356   of counter-ions along the current flow direction. Further expanding this work, they also studied the impact of

357   surface charge, revealing that the high local η in the first and second hydration layers results from strong

358   electrostatic interactions and enhanced hydrogen bonding, which leads to a more ordered water structure.[88] This

359   molecular ordering restricts mobility, increasing resistance to flow and thus elevating local viscosity compared to

360   the bulk fluid ($\eta_{first\ layer} > \eta_{second\ layer} > \eta_{bulk}$). In addition, Masuduzzaman et al.[89] investigated the effect of

361   molecular interface position on EDL thickness and hydrodynamic properties. Their findings showed that

362   employing the hydration layer as a boundary rather than the solid substrate's first atomic layer position resulted in

363   better convergence toward the continuum assumptions. Furthermore, Ma et al.[90] conducted a comprehensive study

364   on the relationship between water ordering and friction at the interfaces of water-TiO_2 and water-silicone surfaces

365   using SFG and AFM. The wettability of TiO_2 and silicone substrates was systematically varied under different

366   heating or plasma-treated conditions to assess its effect on L_S. Their results revealed a stark contrast: while

367   hydrophobic TiO_2 substrates exhibited low friction, hydrophobic silicone surfaces demonstrated significantly

368   higher friction. To investigate this potential discrepancy, the structuring of interfacial water molecules was

369   analyzed through SFG. The spectra indicated that the interfacial water exhibited both loosely hydrogen-bonded

370   "water-like" (with a peak at 3300–3600 cm^{-1}) and strongly hydrogen-bonded "ice-like" (with a peak at 3100–3300

371   cm^{-1}) structures, with the inhomogeneity in water structuring contributing to higher friction at the solid-liquid

372   interface. Conversely, more uniformly ordered water structures (ice-like structures) in the first monolayer were

373   reported to reduce friction on both TiO_2 (hydrophobic) and silicone (hydrophilic) surfaces. This suggests that the

374   structuring of interfacial water, rather than wettability alone, plays a critical role in governing frictional behavior.

375   To support these experimental findings, molecular dynamics simulations were conducted, showing that more

376   ordered water structures reduced friction by decreasing hydrogen bonding and attractive interactions between the

377   first monolayer and the bulk water molecules. The reduction in hydrogen bonds and energy barriers facilitated a

378   smoother interlayer movement, significantly lowering friction. The integration of both experimental and numerical

379   approaches provides deeper insights into the complex interplay between water structuring, friction, and

380   hydrodynamic slip behavior.

381   **3.5.2 X-ray reflectometry and ellipsometry**

382   In Section 3.5.1, SFG spectroscopy was described as a surface-sensitive technique used to investigate the

383   molecular water structure within the EDL regime, which can influence hydrodynamic slip behavior at solid-liquid

384   interfaces. Studying the density profile of liquid at solid-liquid interfaces is crucial to understand the hydrodynamic

385   slip behavior. Furthermore, density profile is used to compute density depletion length, in the previous MD

386   investigations, which effectively captures hydrodynamic slip behavior; see Section 5.2 for a detailed discussion.

387   However, measuring the density profile of water molecules near the interface is challenging due to the need for

388   high surface sensitivity at the appropriate scale. In this Section, X-ray reflectometry (XRR) and ellipsometry are

389   introduced as unique optical techniques capable of measuring the density of water molecules near the surface.

390

391   Figure 3. (a) Raw (Red line) or step functioned (green line) density profile of bulk

392   water/depletion/OTS/SiO$_2$/Si layers using XRR (Copyright 2006 by The National Academy of Sciences of the

393   USA),[91] (b) schematic illustration of the experiment configuration of water/SiO$_2$/Nb$_2$O$_5$/BK-7 prism and refractive

394   index profile used in the analysis of ellipsometry measurement (adapted from Wang et al.[92] with permission), (c)

395   density profiles obtained from MD simulations for different solid-liquid interactions. Adapted from Paniagua et

396   al.[93] with permission. Copyright 2020, American Chemical Society.

397

398   XRR is used to obtain quantitative information on the electron density profile by observing changes in

399   reflectivity (in-plane) of X-ray radiation at grazing incidence angles. In a multi-layer structure, reflected X-ray

400   beams at each interface generate constructive or destructive interference patterns that can be compared to

401   theoretical calculations based on the Fresnel equation. Using XRR, Mezger et al.[91] determined the density profile

402   of water/OTS(octadecyl-trichlorosilane)/SiO$_2$/Si layers, as shown in Figure 3(a). They highlighted the depletion

403   of water density at the interface between water and hydrophobic OTS layers, which was not detected by other

404   experimental techniques. XRR was also used to investigate the surface of ionic liquid (1-methyl-3-

405   octadecylimidazolium tris(perfluoroethyl) trifluorophosphate, $[C_{18}mim]^+$ $[FAP]^-$).[94] The coexistence of positively

406   and negatively charged parts of the ionic liquid allows for structuring at the air-liquid surface. However, due to its

407   structural complexity, the degree of structural ordering by positively or negatively charged particles periodically

408   oscillates with a depth-decaying, showing the convergence of density as away from the surface. This result

409   demonstrates the capability of XRR to measure the density profile of ionic liquid multilayers at the surface, as

410   well as the limitations of general measurement methods for slip length (described in Sections 3.1-3.3), which

411   typically involve fitting a single force-distance curve to calculate slip length.

412   Ellipsometry is also capable of detecting the density profiles of liquids near solid surfaces. It measures

413   changes in polarization of the reflected light and relates them to the refractive index change as a function of

414   distance. The refractive index profile can be related to the density profile via the Clausium-Mossotti relation. Wang

415   et al.[92] measured the refractive index profile of a water/SiO_2/Nb_2O_5/BK-7 prism under acidic and basic conditions.

416   At pH 3, the surface is almost uncharged due to the isoelectric point of SiO_2, making the EDL layer negligible

417   (referred to as the absorbed layer in the paper). However, at pH 10, an additional layer representing the EDL is

418   observed due to the more negatively charged silica surface by the deprotonation of the silanol groups, as shown in

419   Figure 3(b).

420   Although both XRR and ellipsometry lack chemical specificity, these techniques provide important structural

421   properties (density and thickness) of liquid molecules near the surface, which are critical for determining

422   hydrodynamic slip behavior. The experimental results from these techniques can be used to validate computational

423   hydrodynamics simulations such as density profiles predicted from MD simulations. An example is shown in

424   Figure 3(c).[88]

425   **3.6 Summary**

426   In this Section, widely used experimental methodologies including SFA, AFM, and μ-PIV were described

427   for the direct measurement of L_S. SFA and AFM quantify the viscous drag force, which is balanced by the

428   restorative force imposed on the surfaces compressing a liquid. μ-PIV can directly measure the surface velocity of

429   liquid near a solid wall by tracking particle movement. Additionally, other approaches such as using microchannel

430 resonators, hybrid nanochannels, and QCM-D with varying flow resistances were presented. These techniques are

431 capable of measuring L_S at the solid-liquid interface, but they also have limitations in providing detailed

432 information on the fundamental mechanisms of hydrodynamic slip. Specifically, these dynamic measurement

433 methods cannot provide any information on the liquid-surface interactions. Such information can be obtained with

434 SFG, XRR, and ellipsometry. But, since these techniques work for the static condition, the link between the

435 dynamic slip and the static interfacial structure remains elusive. The working principle and limitations of all these

436 experimental techniques are summarized in Table I.

437 Table I. Summary of experimental techniques used to analyze the hydrodynamic slip behavior at the
438 liquid-solid interface and the surface chemistry effect on liquid structure.

Technique	Description	Limitation
Surface force apparatus (SFA)	• Measures the repulsive force as two surfaces approach one another • Directly measures of hydrodynamic slip and slip length based on the rigid theories	• Interpretation is model-based • Requires highly controlled and smooth surfaces; limited to the chemistry control on mica • Limited to relatively small separations
Atomic force microscopy (AFM)	• Measures the repulsive force between a cantilever tip and a surface • Sensitive to very small-scale interaction tip • Directly probe hydrodynamic slip and slip length on the surface	• Interpretation is model-based • Sensitive to probe type and geometry • Difficult to reproducibly control the chemistry of the probe surface
Micro-particle image velocimetry (μPIV)	• Tracks particle movement and velocity within a liquid flow confined to a microscale dimension channel.	• Lack of surface chemistry control • Limited resolution • Requires the use of appropriate tracer
Microchannel resonators (SMR), Hybrid nanochannel	• Measures mass changes of flows through a vibrating microchannel caused by the interaction at the surface boundaries (such as channel wall or nanoparticles)	• Lack of surface chemistry • Requires complex fabrication and precise design • Complex interpretation

Dynamic QCM (QCM-D)	• Measures the frequency shift and energy dissipation of a vibrating quartz crystal as a liquid interacts with its surface • Sensitive to very small changes in interfacial properties	• Deconvoluting from the bulk fluid effect • Insufficient studies on hydrodynamic slip
Sum frequency generation (SFG)	• Nonlinear optical technique by combining two laser beams to produce an SFG signal • Can provide molecular-level information about the alignment and behavior of liquid molecules at the solid-liquid interface	• Does not provide a direct measurement of dynamic slip • Limited to surfaces and interfaces that are optically accessible
X-ray reflectometry (XRR)	• Measures the reflectivity of X-rays from a surface with high resolution and sensitive to atomic-scale structural changes • Can provide information about the density profile of a liquid near the solid interface	• Does not provide a direct measurement of dynamic slip • Limited to well-defined, homogeneous surfaces
Ellipsometry	• Measures changes in polarization of the reflected light providing the density profiles of liquids near solid surfaces	• Does not provide a direct measurement of dynamic slip • Requires precise knowledge of the optical properties of the interface

439

Combining dynamic and static characterization methods as complementary probes is paramount to understanding the underlying mechanisms of hydrodynamic slip; however, such a combination of multiple experimental techniques for the same interfacial system has not been done systematically yet. In contrast, studies bridging the complex interplay between water structuring, friction, and hydrodynamic slip behaviors, have been done computationally, which is reviewed in Section 4.

4. Molecular dynamics (MD) simulations for the analysis of hydrodynamics of nanoconfined liquids
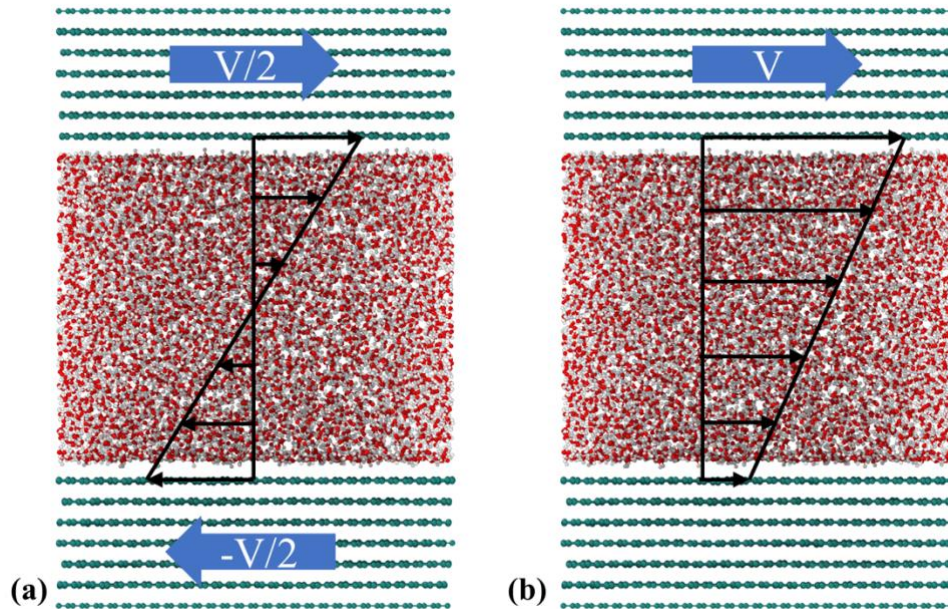
4.1 Non-equilibrium molecular dynamics (NEMD)

NEMD simulations have been applied to determine the friction coefficient and L_S for different solid-liquid interfaces. A key aspect of NEMD simulations is the application of a gradient across the molecular or atomic configuration to observe the system's linear response. The variables of interest are monitored over time, and once

450   a quasi-steady state is achieved, transport or interfacial properties can be determined by fitting the data to a

451   continuum-based model, typically involving a gradient. While the NEMD method is conceptually straightforward

452   and closely mimics real experimental setups, it is significantly influenced by the size of the simulation box and

453   the large gradients needed to extract information constrained by the limited time scales accessible in MD

454   simulations. Additionally, defining the location of the solid-liquid interface is paramount for performing the

455   velocity extrapolation needed to compute L_S in the NEMD model. Karim et al.[95] reported that L_S and η calculations

456   based on a solid-liquid interface defined at the first hydration layer closely matched experimental data and

457   predictions from the modified Hagen–Poiseuille equation.
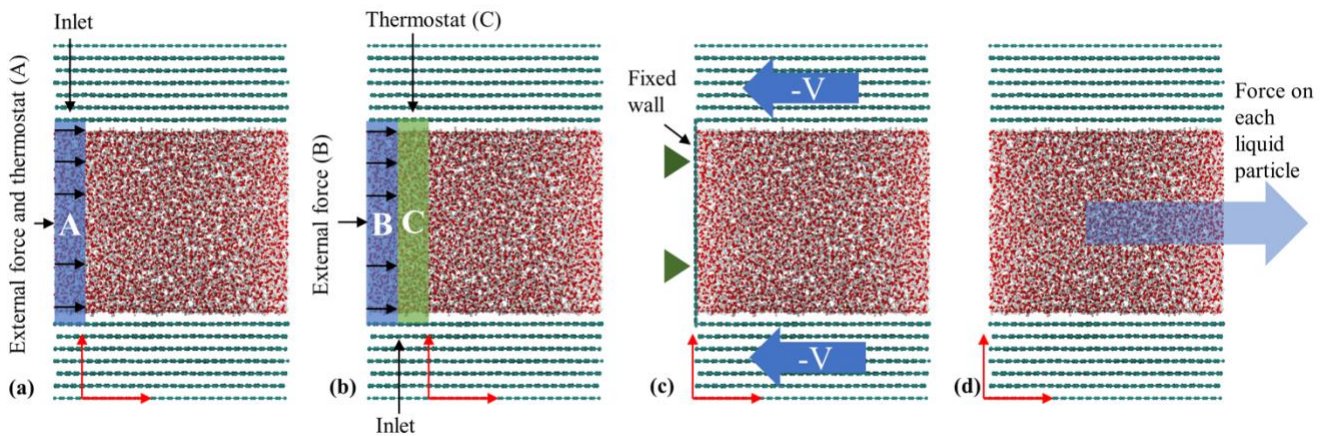
458   For the analysis of nanochannels flow, NEMD simulations that resemble Couette and Poiseuille flow

459   configurations have been used. NEDM simulations consider flow between parallel plates, due to the restrictions

460   imposed by the utilization of periodic boundary conditions in MD simulations. Generally, the simulations involve

461   placing liquid molecules between solid walls and applying an external force to induce unidirectional motion of the

462   liquid particles. To create Couette flow, one wall is moved at a constant speed while the other remains stationary,[96–

463   101] or both walls are moved in opposite directions,[102–106] see Figure 4. Couette flow is generally preferred for

464   slippage and liquid-solid friction investigations due to its simplicity of implementation and the constant shear rate

465   observed over the whole liquid domain.

466

467   Figure 4. NEMD methods for generating Couette flow, (a) moving walls in opposite directions and (b) moving

468   one wall while keeping the other wall fixed.

469   Poiseuille flow simulations are used for more general purposes, such as studying the size effect on

470   hydrodynamic slip[107] and the characterization of flow regimes in nanochannels.[108,109] Unlike Couette flow, the

471   Poiseuille flow in NEMD can be modeled in several ways. One common approach involves applying an external

472   force to liquid particles within a defined region, often referred to as the inlet,[110,111] and allowing the system to

473   reach equilibrium to achieve a parabolic velocity profile, as shown in Figure 5(a). An optimization of this method

474   proposed by Ge et al.[112], considers the addition of an external force to the fluid particles in region B, and then the

475   velocity is rescaled in region C to generate a constant inlet temperature every time the fluid moves between

476   periodic boundaries from outlet to inlet, see Figure 5(b). Another method to create a pressure-driven flow,

477   introduced by Zhang et al.[113], involves confining fluid particles with three walls; one stationary at $x = 0$ and two

478   moving walls advancing at a constant speed perpendicular to the stationary one, illustrated in Figure 5(c). This

479   approach successfully produces a parabolic velocity profile and a linear correlation between mean velocity and

480   pressure drop. Lastly, Poiseuille-like flow can be generated by applying a constant force on every liquid atom in

481   the desired direction of flow,[107–109,114,115] see Figure 5(d). Although this method does not resemble a pressure-

482   driven flow but a body-force-driven flow, the outcome regarding the velocity profile and shear rates are similar.[116]
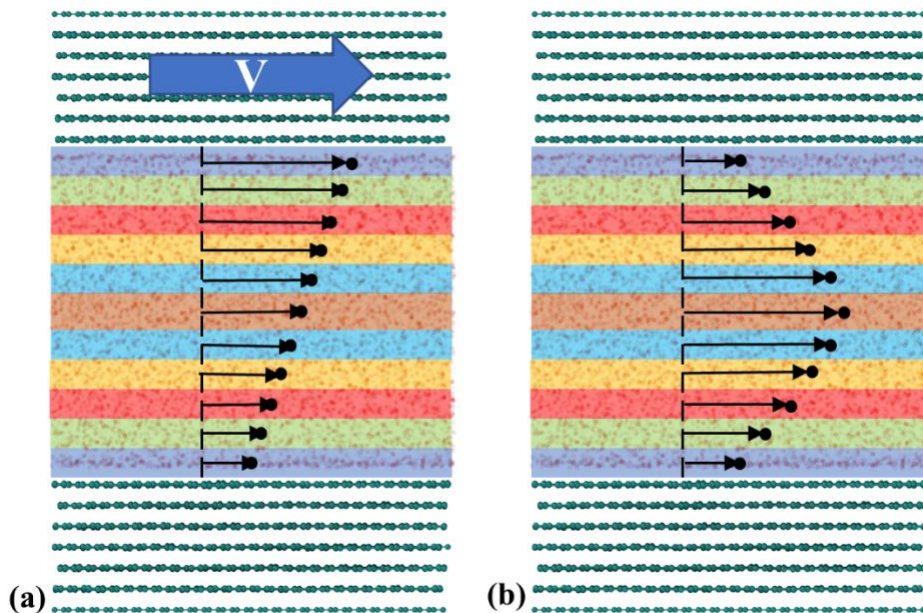

483

484   Figure 5. NEMD methods for generating a Poiseuille flow: (a) inlet-driven flow where an external force and

485   thermostat are applied in the same finite region (based on the methodology used in Ref.[110,111]), (b) optimized inlet-

486   force-driven flow (based on the methodology used in Ge et al.[112]), (c) pressure-driven flow using three walls (based

487   on the methodology used in Zhang et al.[113]), and (d) driving force on each liquid particle (based on the methodology

488   used in Ref. [107–109,114,115]).

489   To calculate L_S, a velocity profile is usually needed to calculate from NEMD data. To analyze the velocity

490   profile, the confined liquid region is divided into segments, each recording the streamwise velocity of individual

491   atoms, see Figure 6. These velocities are then averaged across particles for each timestep and subsequently over

492   multiple timesteps to accurately delineate the velocity profile. The resulting data is typically modeled using either

493   a linear or parabolic fit. The slip velocity is determined by extrapolating the fluid velocity to the wall or any other

494   characteristic length (usually one molecular diameter from the wall). Finally, the L_S is calculated as

$$L_S = \frac{\Delta u_s}{\partial u/\partial z|_{z_0}} \tag{6}$$

495

496   where Δu_s is slip velocity at the interface and $\partial u/\partial z$ presents velocity calculated at the position z_0 where z

497   corresponds to the direction perpendicular to the wall. If the observed velocity profile is plug-like, one can track

498   the force and the slip velocity over several time steps and calculate the friction coefficient from $\lambda = F_A/u_s$, where

499   F_A is the force acting on the solid walls per unit area and u_s is the slip velocity.[117] L_S is calculated then as $L_S = $

500   η/λ, here η represents the shear viscosity.

501

Figure 6. NEMD methodology to generate velocity profiles using discrete bins for (a) Couette flow (linear profile), and (b) Poiseuille flow (parabolic profile).

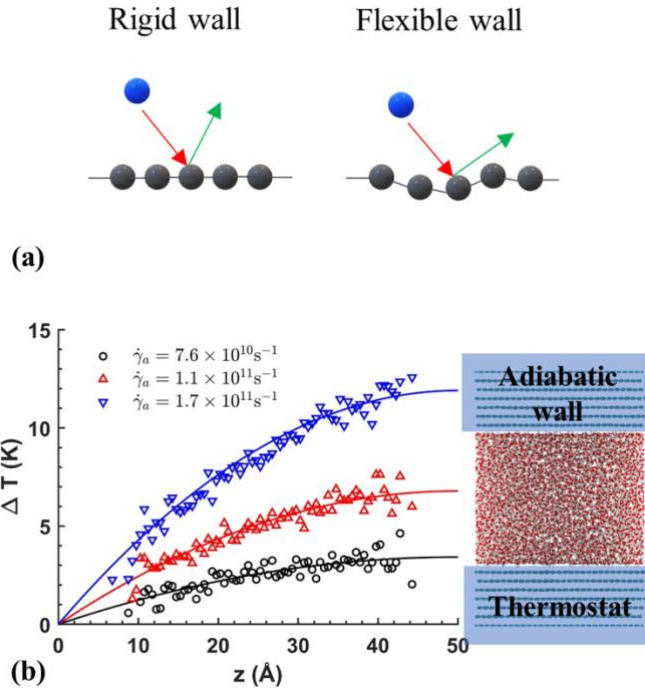NEMD simulations of flow in nanochannels are intuitive for implementation and analysis. Unfortunately, the high shear rates necessary to dampen the numerical noise, are significantly larger than the experimentally achieved ones. Additionally, the work done by the external forces produces viscous heating in the liquid particles, which is a collateral effect. To avoid this problem, many authors have applied the following methods: thermostating the liquid atoms while keeping the solid atoms frozen;[96–98,113,118,119] thermostating all particles;[96,98,99,102] thermostating the fluid particles only in the perpendicular direction of flow so that the equations in the direction of motion are not affected (solid atoms are either mobile or inert);[101,103,104,116,120–124] and thermostating only the solid atoms so they can act as natural heat sinks to remove viscous heating.[107–110,114,115,125–129] Notably, most of the earlier NEMD nanochannel flow simulations employed the liquid thermostating method or all-particles-thermostating method, due to the limited computational resources available at that time. Moreover, most of these investigations were concerned with the calculation of L_S in uniform temperature flows. Alternatively, early works where the solid thermostating method was applied were concerned with the relationship between heat dissipation and hydrodynamic slip. Thus, the only manner of accounting for both phenomena was to allow viscous heating in the fluid.

Discussion and justification of the thermostating approaches are reported in the literature. Martini et al.[106] carried out multiple simulations to investigate the hydrodynamic behavior of a nanoconfined fluid under different shear rates. The main finding of this investigation was that when the liquid atoms are subjected to thermostating and the solid wall atoms remain frozen, L_S increases exponentially with the shear rate; conversely, if the solid atoms are thermostated and allowed to vibrate the L_S growth is bounded to a constant value at high shear rates. Ho et al.[99] indicated that either thermostating the liquid or the solid was consistent with experimental conditions of high and moderate heat dissipation, respectively. They also observed that the use of different thermostats on the fluid did not affect the velocity profiles.

In a more in-depth investigation, Bernardi et al.[130] indicated that thermostating could alter the physical behavior of a system, if not applied correctly to a system where inhomogeneities exist, such as in a nanoconfined

528 fluid flowing. Two scenarios were considered using a 2-D Couette flow: thermostating the fluid particles while

529 keeping inert the solid atoms and thermostating the solid atoms while allowing the liquid to heat up. The first

530 finding is that the properties of the wall affect the density profiles. If the solid walls are allowed to vibrate, the

531 liquid can slightly push the walls and make the channel effectively bigger. Moreover, thermostated fluids presented

532 unrealistic shear distribution curves, which eventually could lead to incorrect results in studies where thermal

533 properties are included. The effect of the wall atom dynamics was like that observed by Martini et al.[106], showing

534 that at low shear rates, the vibrating walls allowed larger slip than frozen walls. The wall dynamics effect was

535 explained by looking at the elasticity of particle collisions, using stationary walls and walls consisting of particles

536 tethered by elastic springs. Depending on the collision angle, a frozen wall particle would limit the direction of

537 motion of a liquid particle after a collision, whereas an elastic wall allows more easily a particle to move in the

538 direction of flow after a collision (see Figure 7(a) for the schematic of rigid and flexible wall types).

539   Yong and Zhang[131] performed a series of MD simulations with three different thermostat setups: (i) applying

540 the thermostat only to liquid atoms, (ii) applying it to both solid and liquid atoms, and (iii) applying it solely to

541 solid atoms. They investigated the mechanical and thermal properties of the system for each configuration. Their

542 findings showed that using a thermostat on the solid walls resulted in a parabolic temperature profile, which

543 aligned with the solution of the energy equation. However, deviations from theoretical expectations occurred when

544 isothermal conditions were applied to the liquid atoms at high shear rates. Similarly, Shuvo et al.[129] applied

545 thermostating to the walls only to mimic natural cooling, reporting a parabolic temperature distribution in the

546 nanochannel consistent with the energy equation, as illustrated in Figure 7(b). It was concluded that letting the

547 system cool down through the walls as would naturally occur is preferred over direct liquid thermostating.
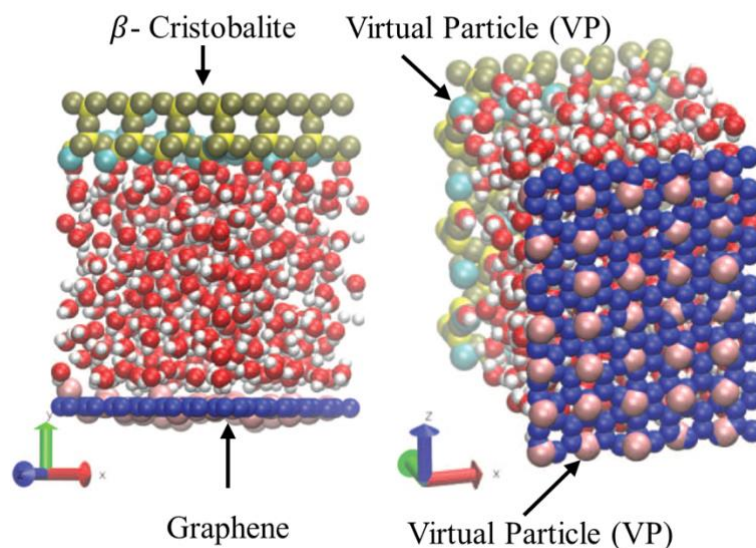
548

Figure 7. (a) Schematic of rigid and flexible wall NEMD model, redrawn from Bernardi et al.[130], (b) temperature profile obtained by Shuvo et al.[129] (adapted with permission).

Although the most realistic manner of performing NEMD simulations of nanofluidics is by thermostating the walls and allowing the liquid to expel the excess viscous heating through the walls, significant computational demand is inherent to this approach. If the solid walls are thermostated, it means that the equations of motion must be solved for the solid particles too. Hence, less expensive computations have been performed for sole hydrodynamic effects by keeping the solid walls inert, i.e., with no internal degrees of freedom. The drawbacks of such a conventional method were fundamentally discussed in Bernardi et al.[130] and recently De Luca et al.[132] proposed a novel methodology for conducting physically realistic simulations without solving the dynamics of the wall. Such a new thermostating method consists of employing "ghost" particles/virtual particles with no interaction other than with the liquid and being tethered to their lattice site via an elastic spring model, see Figure 8. The wall particles are inert, while ghost particles are allowed to interact with the fluid atoms for thermostating. Although this method was proven to be efficient and did not significantly alter the dynamics of the fluid, several trial runs

562   must be conducted before finding the most appropriate configuration (number and position) of ghost atoms for a

563   particular system.

564

565   Figure 8. Computational domain of water confined between graphene sheets (dark blue sphere) and β-cristobalite

566   (yellow sphere) wall with virtual particles (pink and light blue spheres). Adapted from De Luca et al.[133] with

567   permission. Copyright 2014, American Chemical Society.

568   **4.2 Equilibrium Molecular Dynamics (EMD)**

569   EMD simulation is employed to achieve the friction factor at the interface. These methods do not resemble

570   experimental setups as in NEMD but have the advantage of not requiring shear rates larger than those found in

571   real experiments in nanoconfined fluids. Likewise, these methods are more reliable than NEMD simulations for

572   systems with large values of slip, such as water flowing in carbon nanochannels[127], where the velocity profiles are

573   difficult to resolve. Additionally, EMD methods are limited to the low shear rate regime when a comparison with

574   NEMD is required. The calculation of the friction factor using EMD is based on the fluctuation-dissipation theory,

575   which leads to Green-Kubo-like expressions. The fluctuation-dissipation of particle forces, velocities, and their

576   cross-correlations have been used to determine the friction factor employing four principal methods that are

577   discussed herein.

578   Bocquet and Barrat[118] proposed avoiding the arbitrariness of choosing the hydrodynamic boundary conditions

579   by performing first-principles calculations. A "phenomenological" model of momentum transport based on the

580   Navier-Stokes equations was formulated using a statistical mechanics approach, selecting the L_S and the

581   hydrodynamic distance to the wall (z_{wall}) as fitting parameters. Analytical expressions for the momentum density

582   correlation function were obtained for perfectly flat and rough walls. Molecular dynamics simulations were

583   performed to get the "exact" values of the momentum density correlation function. Equilibrium simulations of

584   nano-confined atomic liquids interacting through purely repulsive potentials were conducted at constant

585   temperature and varying the size of the channels. Through parameter fitting, Bocquet and Barrat[118] calculated L_s

586   and z_{wall}, and found that the analytical models matched the simulation results for repulsive walls with and without

587   corrugation. However, confinements imposed by attractive walls were not correctly described by the

588   phenomenological model due to the presence of slip-locking. Lastly, they conducted NEMD simulations of

589   Couette flow to prove the effectiveness of the "phenomenological" model in predicting the velocity profile. The

590   analytical model fitted through EMD simulations accurately matched the NEMD velocity profiles.

591   Given the success of their equilibrium calculations, Bocquet and Barrat[118] formulated a method to calculate

592   L_s and z_{wall} as equilibrium properties using a Green-Kubo-like approach. They employed linear response theory

593   and the Mori-Zwanzig formalism separately to derive equilibrium coefficients based on the time-dependent

594   correlation functions of the fluid. A perturbation Hamiltonian $H[\dot{\gamma}, z_0] = \dot{\gamma} \sum_{i=1}^{N}(z - z_0)P_{i,x}$ was chosen to

595   generate a Poiseuille flow in the x-direction with a fictitious shear field $\dot{\gamma}$, where $p_{i,x}$ is the momentum of particle

596   i in the x-direction, and z_0 is the position at which the velocity profile vanishes while $v_x(z) = \dot{\gamma}(z - z_0)$ is a first-

597   order approximation of the tangential velocity. Applying linear response theory, and having non-equilibrium

598   friction force $\langle F_x \rangle(t)$ as the response and H as the perturbation field, the following expression was developed:

$$\langle F_x \rangle(t) = \frac{\dot{\gamma}}{k_B T} \int_0^t ds \langle F_x(t-s)[\sigma_{xz}(0) - z_0 F_x(0)] \rangle$$

$$= \frac{\dot{\gamma}}{k_B T} \int_0^t ds \langle F_x(t-s)\sigma_{xz}(0) \rangle - \frac{\dot{\gamma} z_0}{k_B T} \int_0^t ds \langle F_x(t-s)F_x(0) \rangle \tag{7}$$

$$\langle F_x \rangle(t) = \frac{1}{k_B T} \int_0^t ds \langle F_x(s)F_x(0) \rangle [\dot{\gamma}(z_{wall} - z_0)] = -A\lambda\dot{\gamma}(z_0 - z_{wall})$$

27

599   where F_x represents the total force exerted in the x-direction on the solid wall by the liquid atoms; k_B, T, and A

600   denote the Boltzmann constant, temperature, and the area of the wall, respectively; and the wall friction coefficient

601   (λ) is:

$$\lambda = \frac{1}{Ak_BT} \int_0^\infty ds \langle F_x(s)F_x(0) \rangle \tag{8}$$

602

603   from which the slip length is obtained as $L_s = \eta/\lambda$.

$$z_{wall} = \frac{\int_0^\infty ds \langle F_x(s)\sigma_{xz}(0) \rangle}{\int_0^\infty ds \langle F_x(s)F_x(0) \rangle} \ and \ \sigma_{xz} = \sum_{i=1}^{N} [\frac{P_{x,i}P_{z,i}}{m} + (F_{x,i}^{fluid} + F_{x,i}^{wall})z_i] \tag{9}$$

604

605   According to Navier's friction model[46] $F_x = -\lambda A v_x(\Delta z)$, where F_x is the total tangential force exerted by the

606   liquid on the wall and $v_x(\Delta z)$ is the tangential velocity at an equilibrium position Δz away from the wall; clearly,

607   Bocquet and Barrat [118] assumed that $\dot{\gamma}(z_o - z_{wall}) = v_x(\Delta z)$ to find an expression for the friction coefficient.

608   Petravic and Harrowell[134] indicated the incorrectness of such an assumption given that $\dot{\gamma}$ is an artificial constant

609   field used to induce a perturbation into the system and no physical correlation exists with F_x which is the actual

610   friction force on the wall.

611   Petravic and Harrowell[134] addressed the equilibrium perturbation issue using Doll's equations of motion.

612   They induced a disturbance within the system by generating a relative velocity, referred to as Δv_{wall}, between the

613   confining walls, while also considering the constraints of a heterogeneous system in a boundary-driven flow

614   context. The system's linear response was then evaluated in the context of a small Δv_{wall}, leading to the

615   determination of a new friction coefficient.

$$\mu_i = \lim_{\Delta v_{wall} \to 0} \frac{\langle F_{xi}(t) \rangle / A}{\Delta v_{wall}} = \frac{1}{Ak_BT} \int_0^t ds \langle F_{xi}(s)F_{xi}(0) \rangle \tag{10}$$

616

617   where μ_i is determined when $t \to \infty$ (statistically equilibrium stage). At this stage, $\mu_1(t) = \mu_2(t) = \mu$ (i=1, 2 reference

618   two confining walls). Eq. (9) bears similarity to Eq. (7), with the distinction that Δv_{wall} is utilized in place of slip

619   velocity to relate the friction force to velocity. Consequently, Eq. (9) accounts for the entire thickness of the

620 confined fluid and addresses the size-dependent friction coefficient observations noted in Petravic et al.[134] Finally,
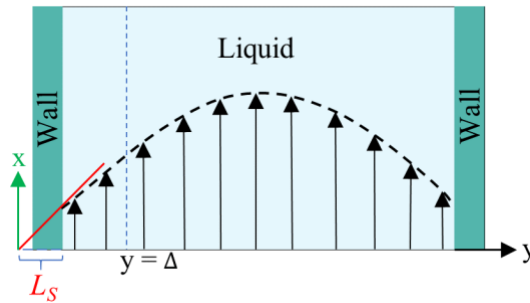
621 the L_S can be obtained as follows:

$$\frac{F_x/A}{\dot{\gamma}_{bulk}} = \frac{F_x/A}{\Delta v_{wall}/(D + L_{s1} + L_{s2})} = \eta \; ; \quad \mu = \frac{\eta}{D + L_{s1} + L_{s2}} \tag{11}$$

622

623 where D represents the distance between the solid walls. Eq. (10) can be simplified when considering identical

624 walls.

625 Hansen et al.[125] emphasized that Eq. (9) accounts for the friction of the whole system, including both the walls

626 and the liquid, and highlighted the importance of separating the region affected by the wall from the bulk fluid to

627 accurately determine the true wall friction. They expanded on Navier's foundational ideas, addressing the issue of

628 wall friction by focusing on a thin layer of liquid close to the wall, see Figure 9. The analysis considers the velocity

629 profile of a liquid confined between two walls, separated by a distance L_y, with the liquid layer defined at $y = \Delta$,

630 and is evaluated using Newton's second law: The velocity profile of a liquid flowing between two walls separated

631 by a distance L_y, as depicted in Figure 9. The liquid slab delimited at $y = \Delta$ is analyzed using Newton's second

632 law:

$$m\frac{du_{slab}}{dt} = F_x'(t) + F_x''(t) + mF_e \tag{12}$$

633

634 where the mass of the liquid m, while u_{slab} denotes the slab's center-of-mass velocity. The friction force resulting

635 from the interactions between the wall and the slab is referred to as F_x', whereas F_x'' is the friction force due to

636 the wall-slab interactions. Lastly, F_e is an external force applied per unit mass.

637

638 Figure 9. Sketch of the system used in the friction analysis in Hansen et al.[125] Figure 9 is redrawn from Hansen et

639 al.[125]

640   The friction force F_x' was described by the following expression:

$$F_x'(t) = - \int_0^t \zeta(t-\tau)\Delta u(\tau)d\tau + F_r'(t) \tag{13}$$

641
642   where ζ is a friction kernel, $\Delta u = u_{slab} - u_{wall}$, and F_r' represents a random force that has a zero mean and is

643   uncorrelated with u_{slab}. In steady-state conditions, the friction forces are given as:

$$\langle F_x' \rangle = -\zeta \langle \Delta u \rangle \tag{14}$$

$$\left\langle F_x' \right\rangle = A\eta \langle \dot{\gamma} \rangle = A\eta \frac{\partial u}{\partial y} \big|_{y=\Delta} \tag{15}$$

644   where $A = L_x L_z$ is the cross-section area of the system and ζ_0 is the zero-frequency friction coefficient.[125] Hansen

645   et al.[125] calculated L_S using Eq. (11 - 14), in combination with the Couette and Poiseuille flow solutions obtained

646   with integral boundary conditions (finite liquid regions of width Δ). L_S calculated from such solutions matched the

647   expected value of $L_S = \eta/\xi_0$ where $\xi_0 = \zeta_0/A$ in the limit when $\Delta \rightarrow 0$. The analytical solutions were compared with

648   EMD simulations from which the friction coefficient was obtained from the Laplace transform of the velocity-

649   velocity and force-velocity autocorrelation functions (ACFs) as:

$$\tilde{C}_{uF_x}(s) = - \sum_{i=1}^n \frac{B_i \, \tilde{C}_{uu}(s)}{s + k_i} \tag{16}$$

650

651   where the ACFs are defined as

$$C_{uF_x}(t) = \langle u_{slab}(0)F_x'(t) \rangle \text{ and } C_{uu}(t) = \langle u_{slab}(0)u_{slab}(t) \rangle \tag{17}$$

652
653   And

$$F_x'(t) = \sum_{\substack{i \in slab \\ j \in wall}} F_{ij}(t) \text{ and } u_{slab}(t) = \frac{1}{m} \sum_{i \in slab} m_i v_{i,x}(t) \tag{18}$$

654

655   The Laplace transform of the friction kernel was obtained using a Maxwellian memory function for convenience

656   as indicated in Hansen et al.[125],

$$\zeta(t) = \sum_{i=1}^n B_i e^{-\kappa_i t} \tag{19}$$

657

658 Thus, the zero-frequency friction coefficient can be found by fitting B_i and κ_i in Eq. (18) using data from EMD

659 simulations as:

$$\zeta_0 = \int_0^\infty \zeta(t)dt = \sum_{i=1}^n \frac{B_i}{\kappa_i} \tag{20}$$

660

661 Hansen et al.[125] discovered that the value of ζ_0 depended on Δ, requiring multiple trials to determine the

662 appropriate Δ value. Narrow slabs failed to capture the entire wall-slab interactions, while larger slabs may

663 introduce unnecessary bulk particles. They noted that the friction coefficient was influenced by the channel width,

664 particularly for channels with $L_y \leq 7\sigma$ in their molecular configuration A comparison was conducted between the

665 L_S predicted using their EMD approach and NEMD calculations using Couette and Poiseuille flows. A remarkable

666 agreement was found for flows with small shear rates.

667 Bocquet and Barrat[135] responded to the criticisms in the definition of their model in Bocquet et al.[118],

668 suggesting that the sensitivity of their interfacial friction coefficient stemmed from the specific approach taken in

669 handling system size and time limits extending to infinity, as derived from MD simulations. To reinforce the

670 generality of their previous model, they introduced a new formulation for the Green-Kubo relationship for the

671 friction coefficient λ, offering a more robust and fundamental approach grounded in the general Langevin equation.

672 This formulation was applicable to both planar and cylindrically confined fluids. In their study, Bocquet and Barrat

673 [135] defined a confined liquid system in which solid walls of a large mass M are allowed to move in the tangential

674 direction only. In the presence of solid-liquid friction, the fluctuations in the wall velocity $U(t)$ are given by

$$M\frac{dU}{dt} = -\lambda A v_s(t) + \delta F(t) \tag{21}$$

675

676 where the slip velocity is $v_s(t) = U(t) - v_f(t)$ and $v_f(t)$ is the fluid velocity, A is the wetted area, and $\delta F(t)$ is a lateral

677 fluctuating force. In the linear response regime, the slip velocity is related to the wall velocity

$$v_s(t) = \int_{-\infty}^\infty dt'\psi(t-t')U(t') \tag{22}$$

678   where ψ is a friction memory kernel with origins in the hydrodynamic shear modes in the fluid. After substituting

679   Eq. (21) into (20), the Langevin equation was Laplace-transformed and the force correlation function

680      $\langle F_w(t)F_w(0)\rangle$ was found

$$\phi(t) = \langle F_W(t)F_W(0)\rangle = -M^2 \frac{d^2}{dt^2}\langle U(t)U(0)\rangle \tag{23}$$

681

682   Eq. (22) is best handled in the Laplace space from which the friction coefficient was found when $\tilde{\varphi}(s)$ is evaluated

683   in the $s \to 0$ limit, yielding

$$\lambda = \frac{1}{Ak_BT}\int\limits_0^\infty dt\langle F_W(t)F_W(0)\rangle \tag{24}$$

684

685   which is the same expression previously found in Bocquet et al. [118] but this time using more general arguments

686   and without the approximations involved in the first derivation.

687   Huang and Szlufarska[123] noted a significant concern in the discourse regarding friction coefficients derived

688   from equilibrium calculations. They argued that the friction coefficient should be considered a local parameter

689   rather than a bulk property. For instance, while solid-liquid friction is present in liquids moving through a carbon

690   nanotube, achieving the thermodynamic limit in such a system, as proposed by Bocquet and Barrat.[118,135], is not

691   feasible. Moreover, when dealing with heterogeneous surfaces or fluid mixtures in contact with a solid boundary,

692   using a bulk property equation like Eq. (9) fails to capture the localized variations at the interface where friction

693   takes place. Bocquet et al.[135] applied the general Langevin equation together with a set of sum rules to the

694   fluctuating velocity of a wall of large mass. In a new formulation, Huang and Szlufarska[123] applied a mechanical

695   perturbation Hamiltonian to individual liquid particles at the solid-liquid interface, $H = -xfe^{i\omega t}$ where x is the

696   displacement of particles parallel to the solid walls and $fe^{i\omega t}$ is an external drag force with frequency ω and time t.

697   Being u_i the drift velocity of an interfacial particle moving parallel to the solid wall, and Fourier transforming the

698   linear correlation function, the particle mobility ϕ_i was obtained

$$\langle u_i\rangle_\omega(t) = \frac{fe^{-i\omega t}}{k_BT}\int\limits_0^\infty \langle u_i(0)u_i(t)\rangle e^{i\omega t}dt \tag{25}$$

699

$$\phi_i(\omega) = \frac{1}{k_B T} \int_0^\infty \langle u_i(0) u_i(t) \rangle e^{i\omega t} dt \tag{26}$$

Linear response theory was applied a second time using F_i as the force exerted by the wall on a single interfacial particle i yielding

$$\langle F_i \rangle_\omega(t) = \frac{f e^{-i\omega t}}{k_B T} \int_0^\infty \langle u_i(0) F_i(t) \rangle e^{i\omega t} dt \tag{27}$$

Now, by definition of the friction coefficient

$$\overline{\eta_\iota} = -\frac{\langle F_i \rangle_\omega(t)}{\langle u_i \rangle_\omega(t)} = \frac{\int_0^\infty \langle u_i(0) F_i(t) \rangle e^{i\omega t} dt}{\int_0^\infty \langle u_i(0) u_i(t) \rangle e^{i\omega t} dt} \tag{28}$$

where the total friction coefficient can be obtained by summing all the contributing particles and normalizing it by the area of the interface

$$\lambda = \frac{1}{A k_B T \mu_i(\omega)} \sum_i \int_0^\infty \langle u_i(0) F_i(t) \rangle e^{i\omega t} dt \tag{29}$$

where the short-range nature of F_i allows us to evaluate Eq. (28) across a substantial number of liquid particles, without compromising the interfacial aspects of the calculations. Huang and Szlufarska[123] observed difficulty in achieving a well-converged value of ϕ_i due to the particles not staying sufficient time near the wall and due to the sensitivity to the spatial definition of the interfacial region. This problem was solved by obtaining the right-hand side terms of Eq. (28) through a Langevin formalism where single particles are analyzed. Using linear response theory and after many mathematical manipulations, they obtained

$$\bar{\eta}(\omega) = \frac{1}{A k_B T[1 - \alpha(\omega)]} \sum_i \int_0^\infty \langle F_i(0) F_i(t) \rangle e^{i\omega t} dt \tag{30}$$

$$\alpha(\omega) = \frac{1}{A k_B T} \sum_i \int_0^\infty \langle F_i(0) u_i(t) \rangle e^{i\omega t} dt \tag{31}$$

716   where the static friction factor $\bar{\eta}(0)$ can be used to obtain $L_s = \frac{\eta}{\bar{\eta}(0)}$. Table II summarizes the EMD models for

717   the calculation of L_S with key highlights.

718

719   Table II. Overview of theoretical analysis for computing friction factor via EMD

Source	Equations	Key Highlights
Bocquet and Barrat[118]	$\lambda = \frac{1}{Ak_BT}\int\limits_0^\infty ds\langle F_x(s)F_x(0)\rangle$	▪ Utilized both linear response theory and the Mori-Zwanzig formalism independently to derive equilibrium coefficients. ▪ Introduced an artificial shear rate into the fluid using a Hamiltonian to simulate Poiseuille flow. ▪ Analyzed the ACF of the friction factor focusing on the solid atoms. ▪ Determined that the ACF should reach zero when $t\to\infty$, and calculated λ at this point
Petravic and Harrowell[134]	$\mu_i = \lim\limits_{\Delta v_{wall}\to 0}\frac{\frac{\langle F_{xi}(t)\rangle}{A}}{\Delta v_{wall}}$ $\mu_i = \frac{1}{Ak_BT}\int\limits_0^t ds\langle F_{xi}(s)F_{xi}(0)\rangle$	▪ Discovered that the ACF's integral does not reduce to zero over time but instead stabilizes at a constant value, implying a smooth decay of the force ACF. ▪ Employed a method similar to Bocquet and Barrat[118] for calculating the friction factor, though they disagreed on the interpretation of the findings. ▪ Identified that the friction coefficient varies with system size, indicating it is not an intrinsic interfacial property.
Hansen et al.[125]	$\tilde{C}_{uF_x}(s) = -\sum\limits_{i=1}^n \frac{B_i\,\tilde{C}_{uu}(s)}{s+k_i}$	▪ Proposed isolating the region near the wall from the bulk to accurately determine wall friction. ▪ Developed a dynamic analysis of a thin liquid slab adjacent to the wall, correlating the friction force with the slab velocity using a memory function. ▪ Conducted the ACF analysis focusing on the interfacial liquid atoms.
Bocquet and Barrat[135]	$\lambda = \frac{1}{Ak_BT}\int\limits_0^\infty dt\langle F_w(t)F_w(0)\rangle$	▪ Refined their earlier model to address criticisms regarding the generality of their Green-Kubo formulation. ▪ Introduced a non-Markovian general Langevin framework to investigate perturbations in wall velocity and slip behavior.

| Huang and Szlufarska[123] | $\bar{\eta}(\omega)$ $$= \frac{1}{A k_B T [1 - \alpha(\omega)]} \sum_i \int_{\infty}^{\infty} \langle F_i(0) F_i(t) \rangle e^{i\omega t} dt$$ $$\alpha(\omega) = \frac{1}{A k_B T} \sum_i \int_0^{\infty} \langle F_i(0) u_i(t) \rangle e^{i\omega t} dt$$ | ▪ Applied linear response theory to a system of liquid particles subjected to perturbations by a Hamiltonian.
 ▪ Assumed that particles interact independently, with interfacial interactions considered additive.
 ▪ Conducted ACF analysis on the liquid atoms.
 ▪ The model indicated that several Langevin equations are needed to explore both wall velocity and slip behavior, indicating a linear relationship between these variables. |

720

Bocquet and Barrat[118] identified the deficiencies of integrating the force ACF in Eq. (7) in the limit when the lag time goes to infinity. As an alternative, it was proposed to evaluate the integral only up to the point where the ACF reached its first zero. A year earlier, Español and Zuñiga[136] highlighted the issues with evaluating the friction factor integral from zero to arbitrary limits as indicated in Bocquet et al.[118] The solid-liquid friction phenomenon was studied using Hamilton's equation with projection operators on a Brownian particle of infinite mass interacting with other particles. Complimentary EMD simulations were carried out by analyzing a fixed liquid particle (infinite mass particle) interacting with several other liquid particles to prove a correlation between the decay of the momentum ACF and the friction factor. The force ACF decreased fast and smoothly but the integral of such was rather noisy with a tendency to decay after long simulation times. No plateau of Eq. (7) was found for long simulation times, as suggested when the thermodynamic limit was reached, but approximations of the friction factor could be extracted from shorter-time behaviors.

Español and Zuñiga [136] concluded that EMD calculations using Green-Kubo-like models are hindered by the order in which the thermodynamic limit (infinite number of particles) and the infinite time limit are taken, during the model formulation, since they do not commute. However, as the simulation systems get larger, it is expected that the friction coefficient calculations approach the obtained results in the limits discussed. Notably, several authors reported no issues with the evaluation of Eq. (7).[36,116,117,128,134] In these investigations, significantly smooth time-dependent friction factors are reported with a plateau at which point the steady state friction factor is evaluated. Furthermore, consistency between EMD calculations using Eq. (7) and NEMD has been reported.[116,117,128] Liang and Keblinski[128] obtained the friction factor of argon flowing between graphene surfaces and observed a steady friction factor plateau over a window of 12 ps analyzing data over 10 ns.

741   Tocci et al.[36] used *ab initio* MD and force field MD simulations to obtain the friction factor for graphene and

742   hexagonal boron nitride in contact with water. By using both sources of atomic trajectories evaluated from 50 ps

743   to 10 ns in a 1 ps time window, they obtained smooth friction coefficient integrals. Falk et al.[117] evaluated the

744   friction factor from force ACFs evaluated over 0.4 ns with a time window of 2 ps. It was indicated that at long

745   timescales (typically nanoseconds), the integral vanishes due to the finite size of the system, but at intermediate

746   times a plateau of the integral can be observed. Additionally, they did not observe confinement dependence on the

747   friction factor. Contrariwise, Wei et al.[116] used Eq. (7) to determine the L_S in water confined between graphitic-

748   carbon walls and observed the confinement effect on the friction coefficient after the viscosity was adjusted to the

749   confinement level. The investigation by Harrowell and Petravic[134] focused on giving a better interpretation of Eq.

750   (7) and throughout their analysis, smooth time-dependent friction coefficients were observed. On the other hand,

751   Huang and Szlufarska[123] observed rather fluctuating time-dependent friction coefficients.

752   Arguments supporting and disproving the properties of the original friction coefficient expression derived by

753   Bocquet and Barrat[118,135] can be found all over the literature. The vanishing behavior of the integral in Eq. (7) has

754   been observed by some but not by others. Likewise, the confinement effect has been reported by some authors,

755   but others did not capture that in their analysis. New analytical approaches and reinterpretations of the initial model

756   have been proposed, but they have not been widely investigated. For example, only Kannam et al.[126,127] used the

757   method, proposed by Hansen et al.[125], to study the friction between liquids and graphite surfaces obtaining

758   consistent results with NEMD simulations in the low shear rate limit. There is a notable debate surrounding the

759   EMD analysis of hydrodynamics in nanoconfined liquids, and more comprehensive studies are necessary to reach

760   definitive conclusions. The methodologies for analysis and simulation are not thoroughly detailed in existing

761   literature, and the inconsistencies observed across various studies may stem from errors in postprocessing or data
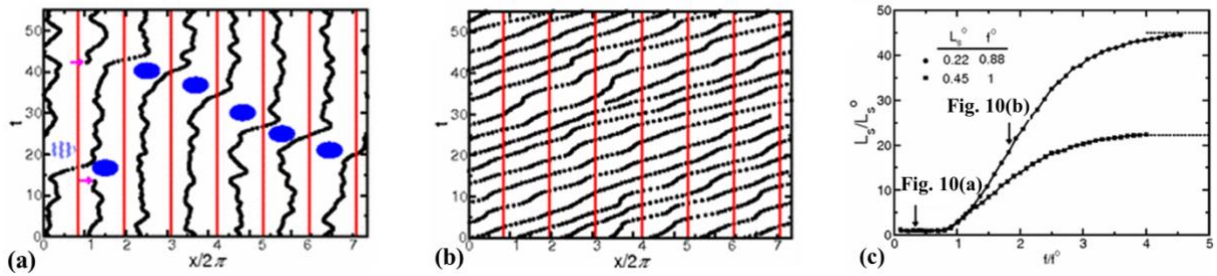
762   sampling during EMD simulations.

763

764   **5 The hydrodynamics of nanoconfined flows**

765   **5.1 Hydrodynamic slip mechanisms and molecular origins**

766   Thompson and Robbins[98,137] made significant contributions to the understanding of the stick-slip mechanisms

767   of liquids moving past solid surfaces, examining this phenomenon from a thermodynamic standpoint rather than

768   considering it as a hydrodynamic instability. Through NEMD simulations of Couette flow, they recorded the

769   friction force, wall displacement, and structure factor over time as the wall velocity varied. Their findings revealed

770   that increasing the binding strength between solid and liquid atoms led to solid-to-liquid transitions among the

771   liquid particles at the interface. In some extreme cases, crystallization of interfacial liquid particles occurred, which

772   was then disrupted by the high shear stresses present in the Couette flow. This interplay between solid-liquid

773   binding and shear-induced disruption resulted in periodic phase transitions, thereby framing hydrodynamic slip

774   within a thermodynamic context.

775   Lichter et al.[138] proposed that liquid molecules spend sufficient time near the wall to warrant a dynamic

776   treatment of their molecular motion, based on the observed ordering of liquid particles in the direction

777   perpendicular to the wall and the mass flux towards the solid. They developed a stochastic differential-difference

778   equation for particles in the first adsorption layer near the wall, allowing for mass exchange between the bulk and

779   interfacial particles. This approach was termed the variable-density Frenkel-Kontorova model (vdFK). The vdFK

780   model was qualitatively successful in predicting the relationship between shear rate and L_S observed in NEMD

781   simulations. Moreover, the model identified two distinct slip mechanisms observed in NEMD simulations: (1) slip

782   caused by localized defect propagation, where particle exchange occurs between interfacial vacancies and the

783   bulk, and (2) simultaneous slip of large liquid regions. At low shear rates, localized defects emerge within the

784   liquid layer, with adjacent molecules quickly filling the resultant vacancies, as depicted in Figure 10(a). This defect

785   propagation is notably slow under low-shear conditions. In contrast, at high shear rates, the shear forces are

786   sufficient to induce concurrent slip across large domains of the liquid layer, see Figure 10(b). Figure 10(c) presents

787   the response of L_S to the applied force as modeled by the vdFK model, demonstrating that at low levels of force,

788   the L_S remains relatively constant. This stability is due to the sparse nature of molecular defects, which propagate

789   slowly through the liquid layer without significantly affecting the overall slip behavior. These defects do not cover

790   a substantial area, thus minimally impacting the bulk liquid behavior. However, as the force increases, a sharp

791   transition occurs due to the intensification of local defects. Ultimately, the system reaches a new plateau at higher

792 applied forces, indicating that the liquid layer moves uniformly over the solid surface. Further increases in force

793 do not significantly impact the L_S, suggesting a saturation of mobility mechanisms at the interface.
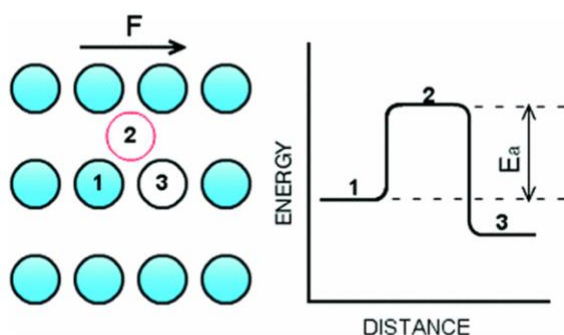

794

795 Figure 10. Trajectories of liquid atoms predicted by the vdFK model, (a) at a low shear rate (defect propagation

796 stage), and (b) at a high shear rate (translation of the entire first liquid layer or concurrent slip). The black solid

797 dots represent the positions of liquid atoms at the interface. For an eye guide, the blue dots indicate the generating

798 vacancies and filling the resultant vacancies. The vertical red solid lines illustrate the positions of seven solid

799 atoms at the interface, (c) L_S as a function of applied force calculated from the vdFK model. L_S and force graph

800 were normalized to collapse the low-forcing data points onto a single curve for different ground states which

801 denotes the ratio of liquid-liquid and solid-solid spacing. The arrows in panel (c) indicate the L_S for the

802 corresponding molecular trajectories of the panel (a) and (b). Figures 10 (a)-(c) were adapted from Lichter et al.

803 [138] with permission.

804   Martini et al. [139] reported defect slip (like in the vdFK model) using low shear rate NEMD simulations. In

805 this regime, liquid particles adjacent to the solid surface hop between equilibrium sites within a potential field

806 generated by the solid, following Arrhenius dynamics. At higher shear rates, they observed global slip, where the

807 entire layer of liquid particles moves collectively. At the smallest wall velocity, atom movement was almost

808 indiscernible, with minimal movement either upstream or downstream. As the wall velocity increased to

809 intermediate levels (5 ms^{-1} and 50 ms^{-1}), atoms displayed periods of stillness interspersed with sudden downstream

810 shifts. At 50 ms^{-1}, the behavior began to show collective trends where groups of atoms might slip simultaneously,

811 indicating the onset of more coordinated movement. At the highest wall velocity simulated, a distinct global slip

812 was observed where all atoms within the first liquid layer move uniformly downstream, presenting parallel

813 trajectories that indicate a cohesive and uniform motion across the solid wall. These observations align with the

38

814   vdFK model. Additionally, Martini et al. [139] found a critical wall velocity for their system—a specific wall velocity

815   that demarcates the transition from defect-driven slip to this observed global slip.

816   When molecular vacancies at the interface are widely spaced during liquid slip, the movement of a single

817   atom from one equilibrium position to another happens independently. This independence allows for studying the

818   dynamics of individual atoms and the application of transition-state theory.[139] Babu and Sathian[120] utilized Eyring's

819   theory of reaction rates (transition-state theory), which models viscous flow as a chemical reaction where the

820   primary process involves molecular hopping between equilibrium positions, see Figure 11. In this context, liquid

821   molecules must surpass an energy barrier created by neighboring molecules to reach a new equilibrium position.

822   A comprehensive analytical model comprising six equations was developed, with shear viscosity and the friction

823   coefficient being the primary outputs. NEMD simulations of water confined between graphene sheets and carbon

824   nanotubes (CNTs) were conducted to directly compute the friction coefficient and estimate the activation energy

825   of the liquid molecules—an essential input for the analytical model. The model's validity is contingent upon

826   maintaining a low shear rate to ensure defect slip, as noted by Martini et al.[139] The friction coefficient predictions

827   from the analytical model generally aligned with the numerical simulations, although there were instances of

828   underestimation and overestimation for the various confinement levels studied. The authors indicated that they

829   used different driving forces, in a Poiseuille flow configuration, and varied the channel dimensions. If this process

830   is not performed carefully, very different shear rates would be produced if the driving force is kept constant while

831   the channel size varies. Additionally, Babu and Sathian[120] found size size-dependent friction coefficients for flow

832   between graphene sheets but Falk et al.[117] reported otherwise.

833

834   Figure 11. Schematic of atom transition from one equilibrium position to another following Eyring's theory of

835   reaction rates. Adapted from Babu and Sathian[120] with permission.

836   Understanding friction at the solid-liquid interface on the nanoscale remains a significant challenge. For

837   instance, Tocci et al.[140] investigated the friction of water at the interfaces of graphene and hexagonal boron nitride

838   (h-BN) using *ab initio* molecular dynamics (AIMD) simulations. Notably, graphene and h-BN generated a similar

839   interfacial water structure. Furthermore, the AIMD calculations revealed nearly identical contact angles for water

840   droplets on graphene and h-BN sheets.[141] Despite structural and wettability similarities, the calculated friction

841   coefficient on h-BN was approximately three times higher than on graphene. This significant difference was

842   attributed to the greater corrugation of the energy landscape on h-BN, determined by the differences in the

843   electronic structure of the two 2-D materials. To further investigate this phenomenon, Secchi et al.[142] conducted

844   experiments on water transport inside carbon nanotubes (CNTs) and boron nitride nanotubes (BNNTs). Their

845   study revealed a significant radius-dependent slippage in CNTs, where water flow through the nanotubes exhibits

846   nearly frictionless interfaces, leading to exceptionally high flow rates. In contrast, BNNTs showed almost no

847   slippage, despite their similar crystallography to CNTs. This difference highlights the influence of subtle atomic-

848   scale interactions at the solid-liquid interface, suggesting a connection between hydrodynamic behavior and the

849   electronic properties of the confining material.

850   Recently, Kavokine et al.[143] developed a quantum theory of the solid-liquid interface and introduced a

851   new concept of quantum friction caused by the coupling of charge fluctuations in water to electronic excitations

852   in the solid surface. In this theory, the authors argued that hydrodynamic friction arises not only from the static

853   roughness of a solid surface (classical friction) but also from the interaction between water fluctuations and solid

854   electronic excitations (quantum friction). Thus, this concept could be understood from the electronic contribution

855   to the solid-liquid friction behavior. They also investigated water interactions with graphitic materials, where (i)

856   graphene, a 2-D material, exhibited very low energy excitations at very small momenta ($q \leq 0.02$ Å^{-1}); suggesting

857   that the electronic excitations in graphene are less likely to interact with water molecules over large distances,

858   contributing negligible quantum friction compared to classical friction. Conversely, (ii) graphite exhibits

859   unexpectedly high friction compared to graphene caused by the distinct electronic structure due to the coupling

860 between its layers; this coupling leads to the emergence of low-energy plasmon modes in graphite, which are

861 absent in single-layer graphene. These low-energy excitations, particularly the surface plasmon modes, strongly

862 interact with water molecules at the interface. In graphite, the low-energy plasmon mode has a frequency of around

863 50 meV and is polarized perpendicularly to the layers. This mode has a flat dispersion over a range of momenta,

864 meaning it can interact more effectively with the fluctuating electric fields of water molecules, particularly with

865 the Debye mode of water. The strong interaction between the graphite plasmon modes and the water Debye mode

866 leads to enhanced quantum friction at the interface. Thus, the overall friction at the interface of graphite-water is

867 higher than the graphene-water interface. Furthermore, Bui et al.[144] applied a classical model that adjusts the

868 dielectric properties of a solid using a simple model of charge density fluctuations in a carbon substrate. Their

869 findings showed an increase in interfacial friction consistent with recent theories of quantum friction, with friction

870 rising as the solid's dielectric spectrum features overlap with the librational and Debye modes of water.

871 **5.2 Solid-liquid affinity characterized via wettability and liquid structuring effects on slip**

872 In nanoconfined liquids, surface effects are predominant and one of the most significant is the solid-liquid

873 affinity. A macroscopic outcome of such affinity can be characterized using the contact angle (surface wettability).

874 From an experimental point of view, it is very difficult to change the wettability of a surface; however, MD

875 simulations offer several options to do this, e.g., (i) modifying the solid-liquid atomic force field, (ii) manipulating

876 the electrostatic interactions between solid and liquid particles, (iii) varying the surface atomic density (density),

877 (iv) modifying the simulation temperature. Although these simulations are limited to atomically smooth surfaces,

878 important investigations have been conducted in this area.[145–147]

879 Voronov et al.[96,97] used standard EMD simulations to determine the contact angle of simple Lennard-Jones

880 (LJ) fluid on a graphite-like solid (i.e., droplet wettability). A parametric analysis was conducted in which the

881 solid-liquid LJ parameters were independently varied to assess their effects on the calculated contact angle.

882 Increasing the value of the solid-liquid energy parameter (ε_{sl}) generated more hydrophilic surfaces and prompted

883 liquid particles near the wall to mimic the solid structure, as reported by Thompson et al.[98] A linear dependence

884 of the contact angle on ε_{sl} was found. Alternatively, the LJ length parameter (σ_{sl}) produced changes in the surface

885 energy landscape. Larger values of σ_{sl} mimicked smoother and more hydrophilic surfaces than smaller values of

886 this parameter, similar observations are reported by Zhang et al.[124] Thus, two opposite trends were found

887 depending on how the surface wettability is altered, and caution was advised for modeling slip surfaces. L_S

888 increased as the contact angle increased when one modifies ε_{sl}; however, the L_S decreased as the contact angle

889 decreased when σ_{sl} was altered. Two different mechanisms are responsible for such behaviors, one is pertinent to

890 a binding energy effect (ε_{sl}), and the other is relevant to the surface energy landscape granularity (σ_{sl}). Hydrophilic

891 surfaces generated by smooth energy landscapes cause large slip; alternatively, hydrophilic surfaces generated by

892 strong solid-liquid affinity led to small slip. Thus, liquids can slip over hydrophilic surfaces and hydrophobic

893 surfaces can have minimum slip if the surface landscape allows for liquid particles to be trapped.
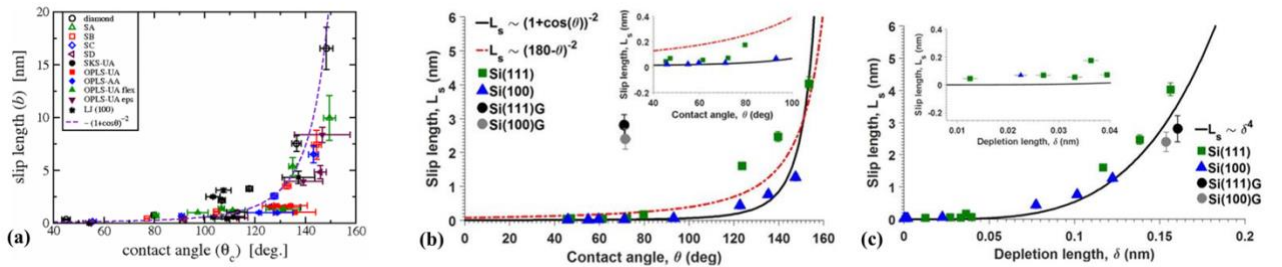
894   L_S is greatly influenced by the magnitude and type of the solid-liquid force field parameters, which define the

895 interface affinity. Previous investigations linked the L_S at various interfaces to surface wettability. In a significant

896 development, Huang et al.[103] made an important contribution by proposing a quasi-universal scaling relation that

897 suggests that L_S is a function of wettability, $L_S \sim (1+cos\theta)^{-2}$, where θ denotes the contact angle, see Figure 12(a).

898 Ho et al.[99] challenged this relation by modifying the wettability of MgO through adjustments to its lattice constant,

899 finding that L_S increased in more hydrophilic surfaces. In a related study, Wang et al.[148] used MD simulations and

900 AFM experiments to determine the friction coefficient at various solid-water interfaces. Their findings revealed a

901 significant limitation of using the contact angle alone to explain variations in friction coefficients at the nanoscale.

902 Despite observing a similar contact angle, the friction coefficient increased 41 times as the surface charge increased

903 from 0e to 0.36e. This rise in friction was attributed to localized potential energy fluctuations due to charge

904 differences, which create additional energy barriers for water molecules, underscoring the limitation of wettability

905 metrics to explain friction coefficient and hydrodynamic slip in nanochannels. Wang et al.[149] further investigated

906 the role of ordered water molecules at the solid-liquid interface of superhydrophilic surfaces using NEMD

907 simulations. They observed that the formation of a hexagonal-like structure in the first water monolayer

908 significantly reduced friction between the monolayer and bulk water above by decreasing the number of hydrogen

909 bonds. The weakened hydrogen bonding led to smoother interlayer movement, thereby considerably reducing

910 overall friction at the interface. Supporting these observations, Xu et al.[150] conducted MD simulations in different

911 polygonal carbon nanotubes (CNTs), demonstrating a similar frictional reduction due to the ordering of water

912 molecules in the first monolayer.

913   These findings indicate that friction and hydrodynamic slip at the solid-liquid interface are governed by liquid

914   structuring at the interface rather than by wettability. To quantify this phenomenon, recent investigations

915   introduced the concept of density depletion length (δ), [93,103,104,151] which quantifies the presence (excess/deficit) of

916   momentum-carrying liquid molecules at the interface (see Figure 3a). The following equation can be used to

917   calculate δ:

918

$$\delta = \int_0^{\infty} \left[1 - \frac{\rho_S(z)}{\rho_S^b} - \frac{\rho_L(z)}{\rho_L^b} \right] dz \tag{32}$$

919

920   where ρ_S and ρ_L represent the solid and liquid density distribution, respectively, with the superscript 'b' denoting

921   a bulk value, which is characteristic of regions far from the interface. A lower δ value indicates a higher

922   concentration and closer proximity of liquid particles to the solid surface, enhancing momentum transfer, while a

923   higher δ value suggests reduced momentum carriers at the interface.

924   In addition to a quasi-universal relationship, Huang et al.[103] and Sendner et al.[104] reported that L_S correlates

925   with the density depletion length as $L_S \sim \delta^4$. This scaling law is based on analyses using a mean-field theory model

926   of wettability and a Green-Kubo-like model of slip, effectively explaining L_S behaviors across different models.

927   However, Ramos-Alvarado et al. [151], in their series of EMD simulations on various Si nanochannels (bare Si (100),

928   Si (111), and graphene-coated Si, noted that the quasi-universal relationship to θ only traced the data trends with

929   limited fidelity and broke down for graphite-coated Si surfaces, as shown in Figure 12(b). However, the scaling

930   law of δ reliably quantified L_S across these diverse nanochannels, see Figure 12(c).
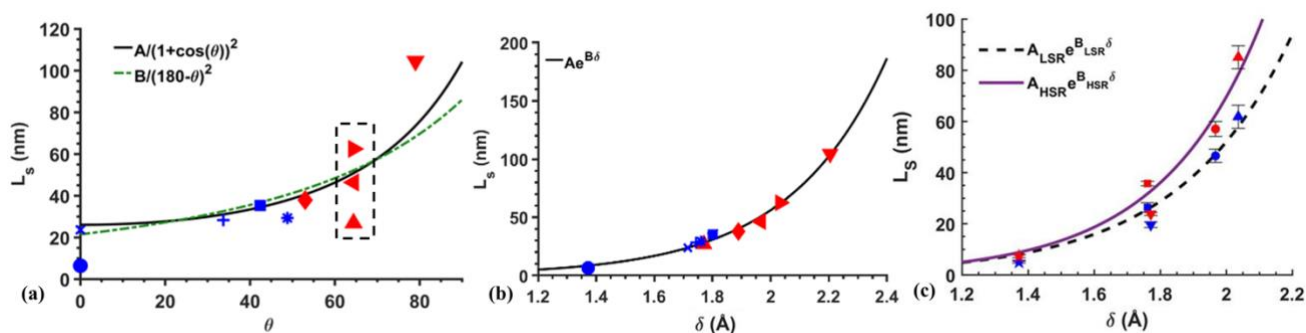

931

932   Figure 12. (a) Quasi-universal relationship, where L_S is a function of contact angle, proposed by Huang et al.

933   Adapted from Huang et al. [103] with permission. L_S of Si and graphene-coated Si channels as a function of (b)

934   contact angle, (c) density depletion length δ. Figure 12(b)-(c) were adapted from Ramos-Alvarado et al. [151] with

935   permission.

936   In more detailed studies, Paniagua et al.[93] utilized a range of Lennard-Jones (LJ) parameters to model

937   graphite-water interactions through EMD simulations. They reported that surface wettability was inadequate in

938   characterizing the L_S. Despite accurately controlling the surface wettability in their MD simulations (symbols

939   inside the rectangular box in Figure 13(a), considerable variations in L_S were observed —26.87 nm, 42.61 nm, and

940   62.48 nm. These slip variations existed even when the contact angle, binding energy, and works of adhesion were

941   similar across the three highlighted interfaces. This variability, driven by different friction coefficients, highlights

942   the inadequacy of using wettability metrics alone to explain hydrodynamic slip. Conversely, as depicted in Figure

943   13(b), δ effectively captured the variations in L_S calculated via EMD across the different interface models, where

944   wettability metrics fell short.

945   Furthermore, Paniagua et al.[93] noted that while the wettability scaling law could generally describe the

946   behavior across most interface conditions, it failed in extreme hydrophobic or hydrophilic scenarios. To address

947   this, they proposed an empirical exponential function ($L_S \sim e^{B\delta}$) that could effectively model L_S across all graphite-

948   water interface conditions. Corroborating these findings, Shuvo et al.[129] conducted NEMD simulations in shear-

949   driven flows within graphite nanochannels, confirming that the exponential function of δ also accurately describes

950   the behavior of L_S under different shear conditions, as illustrated in Figure 13(c).

951

952 Figure 13. L_S as a function of (a) contact angle and (b) density depletion length. Adapted from Paniagua et al.[93]

953 with permission. Copyright 2020, American Chemical Society. (c) Shear-dependent L_S as a function of the

954 depletion length. The solid and dashed lines represent the averaged L_S at high and low shear rates. Adapted from

955 Shuvo et al.[129] with permission. Each symbol represents a different interface model.

956 **5.3 Shear rate effect on hydrodynamic slip**

957 Shear rate is a crucial factor in defining the boundary condition in nanoconfined liquids. The shear force on

958 liquid particles competes with the solid-liquid binding and liquid-liquid cohesive forces. Thompson and Troian[100]

959 noted the absence of a universal boundary condition at the solid-liquid interface. Through NEMD simulations of

960 an LJ liquid, varying parameters like ε_{sl}, σ_{sl}, and solid density across different shear rates in a Couette flow model,

961 they observed that wall corrugation reduced slip as ε_{sl}. L_S remained constant over a certain shear rate range but

962 exhibited rapid growth beyond a critical shear rate γ_c. Despite the variation in shear rates, the shear viscosity

963 showed no significant change, indicating Newtonian behavior. A universal boundary condition was proposed:

964 $L_s = L_s^0 \left(1 - \frac{\gamma}{\gamma_c}\right)^{-\alpha}$, where L_s^0 represents the low shear rate L_S limit and α limit and α is a fitting parameter,

965 suggesting that the Navier slip condition is just a specific case of a broader relationship.

966 The rapid increase in L_S at a critical shear rate was similarly observed in studies by Voronov et al.[96,97] and

967 Chen et al.[33] in shear-driven MD simulations. Kannam et al.[126] investigated hydrodynamic slip for both Poiseuille

968 and Couette flows of graphite-argon, and graphite-methene systems, reporting an exponential (unbounded) growth

969 of the L_S in both flow types. The authors did not address the seemingly infinite growth of the L_S when factors such

970 as the wall friction coefficient and the fluid's viscosity pose a physical limit to solid-liquid friction. Wagemann et

971 al.[152] expanded on this by examining the L_S of water within graphene nanochannels, particularly focusing on the

972 crystallographic directions—zig-zag and arm-chair. Their observations indicated an unbounded growth of L_S in

973 both directions, see Figure 14(a). The authors calculated the wall friction coefficient as a function of shear rates

974 and found that at low shear rates, the friction coefficient remained constant, indicating a stable interaction between

975 the fluid and the solid surface. However, at high shear rates, a rapid reduction in the friction coefficient was

976 observed, suggesting an unbounded growth of L_S. Notably, the authors did not investigate the rheological

977 properties of the liquid, which are crucial because L_S is a function of both fluid viscosity and wall friction

978 coefficient. Recently, Li et. al.[153] investigated the rheology of water in nanoconfined graphite walls and suggested

979 a shear thinning effect at high shear rates.

45

980   Conversely, Martini et al.[139] observed different regimes of slip featuring a bounded growth of L_S after a given

981   critical shear rate, see Figure 14(b), corroborating their molecular mechanisms of slip theory. In the literature

982   concerning the MD modeling of droplet wettability, it was reported that keeping the solid atoms rigid not only

983   allowed to significantly reduce the computing times but also a negligible variation of the contact angle was

984   observed.[33] Thus, several early contributions took a similar approach for their NEMD simulations of slip.[96,97]

985   Martini et al.[106] hypothesized that the unbounded growth of L_S with increasing shear rate, observed in previous

986   simulations, was due to the use of fixed wall atoms, which overlooked momentum transfer between solid and

987   liquid particles. To validate this, they conducted NEMD simulations of Couette flow with both fixed and flexible

988   wall atoms. The results confirmed that rigid walls lead to unbounded L_S growth at high shear rates, whereas flexible

989   walls exhibited a constant L_S beyond a certain shear rate threshold.

990   Pahlavan and Freund[154] suggested reevaluating the high shear rate limit in NEMD simulations by decoupling

991   the effects of the wall and thermostating. Their findings indicated that the solid-liquid vibrational frequency

992   mismatch had a negligible effect on L_S, while a reduction in L_S was attributed to the local temperature rise caused

993   by an increasing number of solid-liquid collisions at high shear rates, see Figure 14(c). Furthermore, the reduction

994   of L_S was also reported by Ramos-Alvarado et al.[155] through NEMD simulation of both Couette and Poiseuille
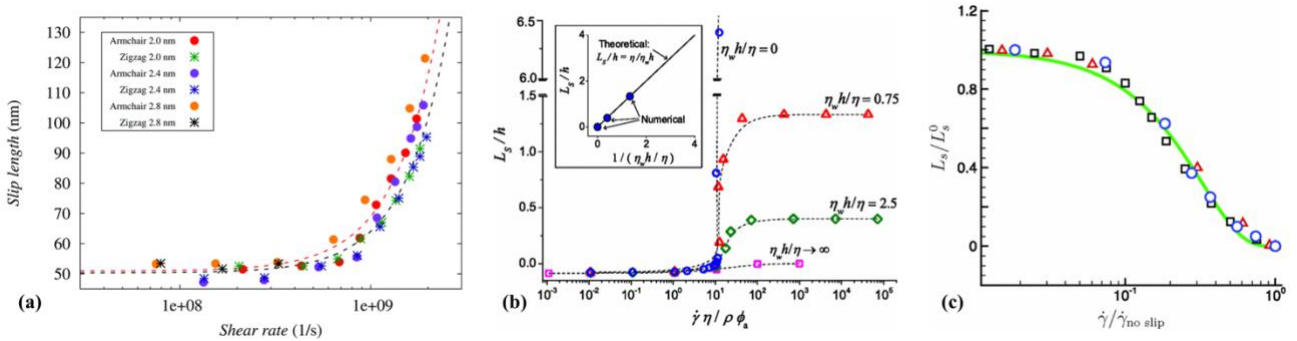
995   flow.

996

997 Figure 14. Shear effect on slip reported through the years: (a) unbounded growth of the L_S, reproduced from Ref.[152]

998 with permission from the Royal Society of Chemistry; (b) bounded growth of the L_S, adapted from Martini et al.[106]

999 with permission; and (c) reduction of the L_S, adapted from Pahlavan et al.[154] with permission.

Further detailed investigations by Shuvo et al.[129] using different interface models of graphite-water interfaces under different shear conditions showed a bounded growth of the L_S at high shear rates, see Figure 15(a), aligning with Martini's findings using the flexible wall model. They explored the rheology of water and the wall friction coefficient to understand the bimodal response of L_S under varying shear conditions. They discovered that both viscosity and friction coefficient decreased at high shear rates but remained constant at lower shear rates, see Figure 15(b). During the transition from low to high shear rates (LSR to HSR), the friction coefficient decreased more rapidly than the shear viscosity, until reaching a new equilibrium. As a result, the L_S was higher and constant at higher shear rates.
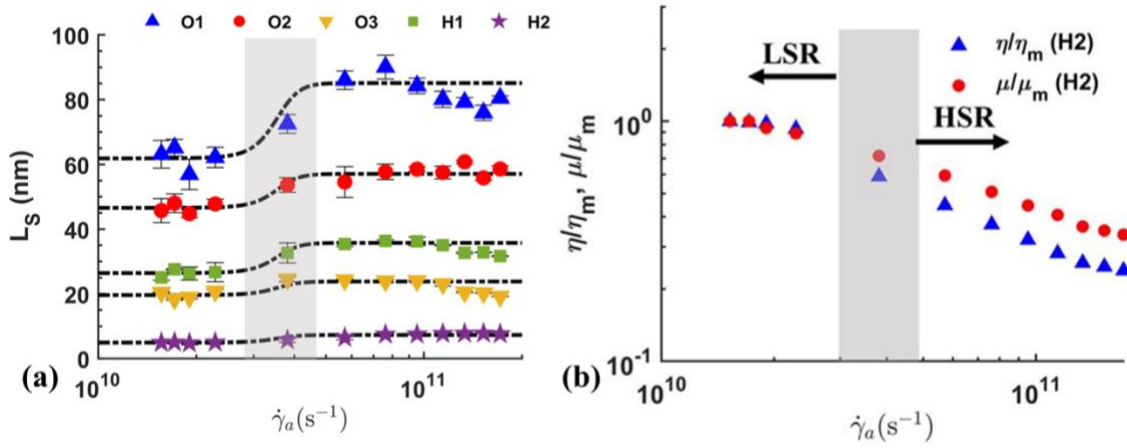

Figure 15. (a) The bimodal response of L_S under different shear conditions, (b) normalized viscosity and friction coefficient in the shear-driven flow. Adapted from Shuvo et al. [129] with permission. Figure 15(a) was redrawn with sigmoid fits using the data reported by Shuvo et al. [129]

As discussed in Section 4.1, NEMD simulations are pivotal in calculating the transport properties of molecular systems by emulating experimental setups. However, achieving a good signal-to-noise ratio necessitates applying an external perturbation that is significantly larger than those typically used in experiments. This approach helps overcome the limitations of short simulation timescales and smaller length scales compared to those in experimental settings, but it also presents a challenge for directly validating the simulation results. Addressing this limitation, Maffioli et al.[156] developed the TTCF4LAMMPS technique, which combines direct NEMD simulations with the Transient Time Correlation Function (TTCF). This integration facilitates the exploration of fluid responses at shear rates achievable in experiments. TTCF relies on the correlation between the

47

1020 initial rate of energy dissipation and the response of any phase variable following an external perturbation,

1021 described mathematically as:

$$\langle B(t) \rangle = B(0) + \int_0^t \langle \Omega(0)B(s) \rangle ds \tag{33}$$

1022 where B(t) represents an arbitrary dynamic variable of interest, and Ω denotes the dissipation function related to

1023 the system energy changes due to external perturbations. $\langle \Omega(0)B(s) \rangle$ represents the cross-correlation between the

1024 initial dissipation and the variable at time s. In the TTCF methodology, mother and daughter trajectories are

1025 essential for analyzing system properties. Initially, a mother trajectory is established through EMD simulations,

1026 allowing the system to evolve under equilibrium conditions to provide a statistical baseline. From this, several

1027 daughter trajectories are generated at varied intervals, each inheriting initial states from the mother trajectory but

1028 experiencing specific external perturbations. These daughter trajectories are crucial for examining the system's

1029 response to these perturbations, with their transient responses averaged to determine the desired transport

1030 properties of the system with a good signal-to-noise ratio at low (realistic) shear rates.

1031 Despite the advantages of TTCF over traditional NEMD in terms of accessing experimentally relevant shear

1032 rates, its adoption remains limited, possibly due to the complexity of its implementation and the high

1033 computational demands associated with it.

1034 **6. Summary and Outlook**

1035 Research into fluid dynamics at the nanoscale has revealed notable deviations from continuum fluid behavior,

1036 particularly regarding the phenomenologically reported no-slip condition at the solid-liquid interface. At such

1037 scales, surface effects—such as roughness, wettability, and molecular interactions—become increasingly

1038 significant due to the dimensions being on the order of molecular mean free paths. This leads to unique properties

1039 like altered viscosity and density profiles near the interface, complicating the understanding of flow dynamics.

1040 In Section 3, experimental methodologies including SFA, AFM, and μ-PIV were introduced for the

1041 direct measurement of hydrodynamic slip length. SFA and AFM quantify the viscous drag force, which

1042 is balanced by the restorative force of the AFM cantilever based on the separation between the surface

1043 and the AFM tip. μ-PIV measures the velocity of liquid near a solid wall by tracking particle movement.

1044 Additionally, new approaches using microchannel resonators and hybrid nanochannels with varying flow

1045 resistances were presented. These techniques are highly valuable for studying nanoscale hydrodynamic

1046 phenomena at the solid-liquid interface, but they also have limitations in providing detailed information

1047 on interfacial interactions necessary to fully understand the origins of hydrodynamic slip. In Section 3.5,

1048 SFG and XRR were introduced as typical interfacial analysis techniques to study water molecules at

1049 solid surfaces. However, it remains unclear how the structural and chemical interactions of interfacial

1050 water molecules influence hydrodynamic slip behavior. These challenges have steered the field towards

1051 computational methods, where NEMD and EMD simulations play pivotal roles. NEMD simulations, which

1052 simulate shear rate effects in a manner akin to experimental setups, often require large velocity gradients to

1053 mitigate statistical noise, which can lead to unphysical conditions. On the other hand, EMD simulations focus on

1054 tracking the linear response of systems, providing a more reliable means of computing transport properties, such

1055 as interfacial friction coefficients. However, the application of Green-Kubo relations in these simulations must be

1056 handled with care to avoid introducing non-physical parameters that could skew the interpretation of nanoconfined

1057 flow characteristics.

1058 The effect of shear rate on hydrodynamic slip varies widely depending on simulation conditions, with reports

1059 of unbounded, bounded, and even reduced L_S. The unbounded L_S is often an artifact of simulations that neglect

1060 momentum transfer between solid and liquid, while reduced L_S is linked to local temperature increases due to

1061 frequent solid-liquid collisions at high shear rates. The bounded growth of L_S has been supported by MD

1062 simulations with flexible solid wall models where momentum transfer between solid and liquid is allowed,

1063 experiments, and theoretical models like the vdFK. Furthermore, EMD-computed L_S matches with NEMD results

1064 in low shear rates. A significant limitation of traditional NEMD simulations is their reliance on very large velocity

1065 gradients. The recent development of the TCFF4LAMMPS technique enables the generation of numerical data

1066 with high signal-to-noise ratios at velocity gradients that are accessible in experiments, thereby facilitating the

1067 validation of computational models with experimental findings.

1068   Significant advancements have been made in MD simulations unveiling atomistic details of hydrodynamic

1069   slip behavior at the solid-liquid interface, yet several fundamental questions remain unresolved. The challenge lies

1070   in developing a physics-informed boundary condition that accounts for complex interfacial interactions, including

1071   surface chemistry and interfacial liquid structuring. Current research indicates that a liquid structuring parameter

1072   may play more significant roles in hydrodynamic slip than wettability metrics, a hypothesis requiring verification.

1073   Inter-particle interactions in the liquid near the solid surface cannot be fully modeled with the parameters

1074   determined from the bulk phase properties. The solid-liquid interaction right at the interaction can influence the

1075   intermolecular interaction in the next layer, which will propagate further into the liquid phase. How fast or slow

1076   this interaction decays with the distance from the surface could be another important parameter that governs (at

1077   least affects) how effectively the momentum will be transferred from the bulk liquid to the solid surface. Although

1078   indirect, such interactions could be extracted from advanced characterization methods that are sensitive to

1079   structural order or density change in the liquid phase in proximity to the solid surface. By integrating the solid-

1080   liquid interaction parameters with the boundary conditions, MD simulations will be able to predict and explain

1081   hydrodynamic slip behavior at the liquid/solid interface.

1082

1083   **Author Contributions**

1084   **Abdul Aziz Shuvo:** Data curation (experimental and numerical part, equal), Formal analysis (experimental

1085   and numerical part, equal), Investigation (experimental and numerical part, equal), Writing-original draft

1086   (experimental and numerical part, equal)

1087   **Luis E. Paniagua-Guerra:** Data curation (experimental and numerical part, equal), Formal analysis (equal),

1088   Investigation (experimental and numerical part, equal), Writing-original draft (experimental and numerical part,

1089   equal), Writing -review & editing (equal)

1090   **Juseok Choi:** Data curation (experimental part, equal), Formal analysis (experimental part, equal),

1091   Investigation (experimental part, equal), Writing-original draft (experimental part, equal)

1092 **Seong H. Kim:** Data curation (experimental part, equal), Formal analysis (experimental part, equal),

1093 Investigation (experimental part, equal), Writing -review & editing (equal), Funding acquisition (equal),

1094 Supervision (equal)

1095 **Bladimir Ramos-Alvarado:** Data curation (experimental and numerical part, equal), Formal analysis

1096 (experimental and numerical part, equal), Investigation (experimental and numerical part, equal), Writing-original

1097 draft (experimental and numerical part, equal), Writing -review & editing (equal), Funding acquisition (equal),

1098 Supervision (equal)

1099 **Conflicts of interest**

1100 There are no conflicts to declare.

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