# GSP and Statistical Analysis of Single Line Failure Impact and Spread in Power Systems

Naeem Md Sami
*Department of Electrical Engineering*
*University of South Florida*
Tampa, USA
naeemmdsami@usf.edu

Mia Naeini
*Department of Electrical Engineering*
*University of South Florida*
Tampa, USA
mnaeini@usf.edu

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***Abstract*—Power systems are typically designed with resilience against individual failures, yet the preparedness for ensuring reliability in the face of multiple failures and cascading failures necessitates a comprehensive understanding of the consequences of a single line failure on the system and its components. Various power physics-based techniques, including the line outage distribution factor, and simulation-based approaches have been employed to model and characterize the impact and spread of a single line failure on power systems. In the evolving landscape of power system analysis, the integration of Graph Signal Processing (GSP) techniques introduces a novel perspective in such analyses. This study focuses on characterizing the effects of a single-line failure on power systems through multiple statistical and GSP-based measures. By extracting relevant features from the data using these measures, this study aims to quantify the impact and spread of failures on graph signal values. It also examines how the most impactful and spreadable failures contribute to cascading failures. The insights gained from this study offer a deeper understanding of failure impact and spread, which are crucial for analyzing more complex scenarios, such as cascading failures.**

***Index Terms*—Transmission Line Failure, Failure Impact, Failure Spreadability, Graph Signal Processing, Power Flow, Cascading Failures**

## I. INTRODUCTION

Power systems are sophisticated dynamic systems, which are reliable yet sensitive to external and internal stresses. Small perturbations and failures, if not monitored and mitigated in time, can result in disastrous consequences such as large system failures and blackouts. The characterization and quantification of the impact of such perturbations enable the undertaking of appropriate mitigating measures in a timely manner. Although such perturbations may be caused by various events, this work considers single-line failure as a perturbation to the grid and attempts to understand its impact and spread of its effects on the overall system. While power systems are generally designed to withstand a single failure (i.e., N–1 criterion), ensuring reliability amid multiple or cascading failures requires a thorough understanding of the impact that a single line failure can have on the entire system and its components, which defines the goal of this paper.

A power system can be modeled as a graph based on its physical structure. The associated attributes to the vertices over the defined graph form the *graph signal*. Such representation of power system measurements enables the implementation of different graph signal processing (GSP) tools and techniques to support different functions in the power system [1] while capturing the grid topology and the interconnection and interactions among its components. Examples of GSP-based applications in power system problems include stress and cyber attack detection [2]–[5], state estimation and recovery [6], [7], and impact analysis for load changes [8], [9]. This work will focus on characterizing and quantifying the impact and spread of single-line failures on the power systems attributes through a set of statistical analyses and GSP features. Specifically, by investigating various GSP features including graph signal total variation and graph Fourier transform (GFT), this work offers new insights into how failures affect the power components' attributes. Additionally, a novel statistical metric for measuring the spread of failures is proposed to assess the significance and extent of a failure's impact on the system's graph signal.

Here, the power system is specifically modeled as a *line graph* with nodes representing the power lines in the system. The line graph enables defining graph signals based on the attributes of the line and avoids diconnectivities due to line failures. Two main graph signals are considered on the derived line graph; one defined based on the difference between the real power values in the power lines before and after a failure, and one based on the Line Outage Distribution Factor (LODF) [10]. Although these graph signals are closely similar, the measurement signal allows capturing the real nonlinear dynamics of the system using data from sensors and Phasor Measurement Units (PMUs) in the system for GSP applications and the LODF serves as a benchmark based on the DC power flow model. The GSP features including total variation and GFT of these two graph signals are evaluated to propose a metric to quantify the most impactful line failures. A spreadability metric has also been proposed for measuring the highest distance of impact, which has been compared with a network science-based spreadability measure proposed in the literature [11]. The analyses have been performed over the IEEE 118 and the IEEE 300 test cases and the most impactful and spreadable line failures have been identified. Finally, the role of the most impactful and the most spreadable line failures in generating large cascades has been investigated through simulation.

The paper is organized as follows. Section II discusses the literature review on the impact and spread analysis on graphs. Section III elaborates on the power systems graph signals used in this paper. Section IV defines the proposed impact and spread measures alongside other available metrics for comparison. Section V performs the comparative analysis of different impact and spread metrics discussed in the previous section, and provides insights into the impactful and spreadable failures on cascade sizes. Finally, Section VI summarizes the research with the notions of future works.

## II. Literature Review

The study of how perturbations impact and spread across various graphs or networks has long been an important area of research in different applications including social network information diffusion, epidemics, and system failures [1]. The analysis of the impact and spread subjected to single perturbation in power systems has also been explored from different avenues. For example, LODF has been introduced to quantify the changes in the power flow in the transmission lines given the failure of a single line [12]–[14].

As power grids can be interpreted as graphs, there are several GSP-based works currently being investigated to obtain new insights into the intensity of a change in node and its spread across the power network. For example, the authors in [8] performed the analysis of single-bus perturbation characterized by a change in real power load or demand in the bus. The work performed a GSP-based analysis of the perturbation due to such changes with respect to the global and local smoothness of the graph signals and parameterized the spreadability of the load or demand change in a single bus. Although the analysis of single-line failure with respect to GSP can help select important lines for vulnerability and reliability analysis, limited works are available on such impact and spread analysis.

Moreover, some works are focused on investigating the failures to identify vulnerable nodes/edges based on network-based centrality measures [15]–[17]. For example, [15] has compared notable centrality-based metrics with a proposed heuristic metric for determining the most critical node in the power system graph. The work in [16] performed a structural analysis of the power grid and communication system graphs using centrality-based vertex and edge importance metrics to leverage their connectivity, spectral, and bottleneckness properties. The authors in [17] have performed the spread analysis of single line failure using the Moore-Penrose pseudoinverse of power grid admittance matrix, and resistance distance, and compare the performance with epidemic and percolation-based models. However, these purely network-based centrality metrics are highly dependent on the structure of the topology and fail to take the intensity of the signal values into consideration. In this paper, the proposed impact and spread metrics will provide insights into the significance of changes in power values and the largest distance that a single-line failure may result in notable changes in power values. This analysis can further aid in the understanding of multiple failures, such as cascading failure scenarios. Specifically, the relation between the cascade size and the impact and spread of the line failures based on the proposed metrics are verified through simulation.

## III. Power System Graph Signals

As mentioned before, electric power systems can be presented in the form of graphs. In the GSP-based analyses of line failures presented in this work, the power system has been modeled as a *line graph* $\mathcal{G}'$ of the original topology of the system $\mathcal{G}$. The original topology of the system with $N$ buses and $M$ transmission lines can be modeled as graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. In the line graph $\mathcal{G}'$, the nodes represent the branches in the original topology, i.e., $\mathcal{V}' = \{e_1, e_2, ..., e_M\}$, where $e_i \in \mathcal{E}$. The nodes in the line graph are connected if they share a node end-point in the original topology. The values associated with the nodes of the line graph can represent different attributes including the power flow through the power lines in the system. By considering the line graph instead of the topology graph, when a power line fails in the system, it does not cause any disconnectivity in the line graph as it only changes the value associated with a particular node of the line graph.

To define the graph signals over the line graph $\mathcal{G}'$, two node attributes are considered. First, the difference between the real power values in the power lines before and after each failure is simulated and considered as the attribute of the nodes in the line graph. This graph signal is named, *difference line flow measurement (DLFM) graph signal*. Secondly, the Line Outage Distribution Factor (LODF) [10] due to a specific failure is derived theoretically and considered as the attribute of the nodes in the graph signals. This graph signal is named, *LODF graph signal*.

The difference line flow measurement signal due to failure of the line $n$ at time $t$ is denoted by $\mathbf{x}^n(t)$ with value at node $x^n(t, e_m)$, or $x^n(t, m)$ in short, for $m \in \mathcal{V}'$ defined as $x^n(t, m) = \frac{|p^n_m(t+\epsilon) - p_m(t-\epsilon)|}{p_m(t-\epsilon)}$, where $p_m(t)$ is the power flow value at node $m$ at time $t$ and $\epsilon$ is a small real number.

The second graph signal (i.e., LODF graph signal) is derived following the approach presented in [13]. Specifically, the LODF graph signal, $w^n$ for the failure of the line $n$ is the $n$-th column of the matrix $\mathbf{W} \in \mathbb{R}^{M\times M}$, which is expressed with respect to the power outage distribution factor (PTDF), $K \in \mathbb{R}^{M\times N}$ and the incident matrix, $C \in \mathbb{R}^{N\times M}$, of the original graph $\mathcal{G}$. The $k$-th element of the signal $w^n$ is calculated as ${w_k}^n = \frac{[K\times C]_{kn}}{1-[K\times C]_{nn}}$. The PTDF matrix is defined based on the diagonal branch susceptance matrix $B_d \in \mathbb{R}^{M\times M}$, pseudoinverse of the nodal susceptance matrix $B \in \mathbb{R}^{N\times N}$, and the incident matrix $C$, and is expressed as $K = B_d C^T B^{-1}$, where $T$ indicates the transpose matrix.

While these graph signals are similar in capturing the changes due to failures, the signals based on the system measurements capture the nonlinear dynamics of the system more closely as they can be obtained from the AC power flow model and the system sensors and PMU measurements; whereas the LODF graph signal is derived based on the DC power flow model. The impact and spread analyses of failures are performed on these signals to verify that the direct application of such measures including GSP and statistical features on collected measurement data from the system can lead to a similar analysis from the analytical derivation of LODF values, which requires the system model.

## IV. Impact and Spread Measures

In this section, the measures related to the impact and spread of a single failure in the graph values of the power system are presented. In the following sections, these measures are applied to the DLFM graph signal to characterize lines with significant impact and spread and the LODF graph signal is used for the verification of the results.

### A. Impact Measures

We start by reviewing some statistical measures on the signals and later introduce GSP-based measures.

**Mean of the graph signal:** The mean of the graph signals (i.e., the DLFM and the LODF graph signals) is the first statistical measure of the average amount of change in the attributes of the nodes in the line graph after a failure. It is a simple aggregated measure; however, it does not provide information on the spread of the effects of the failures. Nonetheless, a high mean value of these graph signals can indicate a high impact of a failure.

**Number of significant values (NSV):** In addition to the mean of the changes in values due to a failure, the number of significant value changes can indicate the impact of a failure. As such, this statistical measure counts the number of values beyond a threshold value for the graph signals.

**Total variation (TV)**: The TV is a GSP-based feature of the graph signal, which provides a notion of the smoothness of the graph signal. TV of the graph signal vector $\mathbf{x}$ (i.e., the vector representation of the graph signal values indexed by nodes) quantifies the changes in the signal as $TV(x) = x^T \mathbf{L} x$, where $\mathbf{L}$ is the graph Laplacian defined as $\mathbf{L} = \mathbf{D} - \mathbf{A}$ and $\mathbf{D}$ being the degree matrix and $\mathbf{A}$ being the adjacency matrix of the graph [18].

**High graph frequency content (HGFC) of the graph signal:** GFT is an analogous concept similar to the classical Fourier transform that describes the spectral characteristics of the graph signal. For an undirected graph, the GFT of the signal $x(n)$ is defined as $X(\lambda_k) = \sum_{n=1}^{N} x(n) u_k(n)$, where $u_k(n)$ is the $k$-th eigenvectors of the Laplacian matrix $\mathbf{L}$ forming the basis for the graph frequency domain, $n$ is the index of the $n$-th node in the graph, and $\lambda_k$ is the $k$-th eigenvalue of the Laplacian matrix $\mathbf{L}$. The sorted eigenvalues of the Laplacian matrix $\mathbf{L}$ for a connected graph are $0 = \lambda_1 < \lambda_2 < ... < \lambda_N$ and represent the graph frequencies. In general, larger values corresponding to higher frequencies indicate a signal with larger variations. The high graph frequency content measure denoted by $r$ is defined as $r = \sum_k X(\lambda_k) H(\lambda_k)$, where $H(\lambda)$ is a high-pass graph filter, and its frequency response is $H(\lambda) = \begin{cases} 0 & \text{if } \lambda \leq \lambda_k \\ 1 & \text{if } \lambda > \lambda_k \end{cases}$, where $\lambda_k$ is the cutoff graph frequency. This GSP-based measure is also an aggregated measure and does not directly provide information on the spread of the effects of the failure.

### B. Spread Measures

While the impact measures quantify the amount and significance of changes in the values of the signal due to a failure, spread measures incorporate the distance of the changes with respect to the failure location as well. Here, we present a spread measure to capture the pattern of changes over distance. A network-science-based spread measure is also reviewed at the end of this section, which is used in the presented analysis for comparison.

**Deviation of changes from exponential decay (DCED):** Similar to how waves decay as they move away from where a rock is dropped into water, changes in the graph signal also diminish with distance to the location of the failure [17]. However, due to nonlinear and at-distance interactions among the components in power systems, this decay does not necessarily follow a smooth decreasing pattern, and failures with more spread may result in more significant changes at distance, which is also observed from our simulations. To capture these aspects of spread, this measure focuses on the average changes at each hop distance with respect to the location of the failure and compares its pattern of decay to an exponential decreasing decay to quantify its deviation.

Specifically, for the graph signal $x^n(t)$ due to failure of line $n$ in the system, the average of the signal values at each hop $k$ is calculated as $\bar{v}_k^n = \sum_{i \in \mathcal{N}_k^n} x^n(t,i)$, where $\mathcal{N}_k^n$ is the set of nodes at $k$-hop distance from the failed line $n$ (i.e., node $n$ in the line graph). The deviation of $\bar{v}_k^n$ beyond an exponentially decaying function $y(k) = ae^{-b(k)}$ (with $a$ and $b$ being the parameters of the model after fitting the curve to $\bar{v}_k^n$) are calculated as the measure of spread denoted by $s = \sum_{i=1}^{K} \max((\bar{v}_i^n - y(i)), 0)$, where $K$ is the diameter of the network. Note that this function only adds up the positive deviation from the exponential fitted curve; emphasizing the more significant changes at distance as a measure of spread.

**Closeness centrality-based spread (CCS):** This measure refers to the metric introduced in [11] based on the centrality measure of the nodes and is used in our analysis for comparison purposes. This spread measure denoted by $s'(n)$ is defined as:

$$s'(n) = \varphi'(n) \sum_{i=1}^{M} \frac{x^n(t,i)}{\sum_{j=1}^{M} x^n(t,j)} \mathcal{D}(n,i), \qquad (1)$$

where $\mathcal{D}(n,i)$ is the shortest path length between the failed line $n$ and all other lines $i$ in the line graph. The modified normalized closeness centrality, $\varphi'(n)$ is expressed as $\frac{M}{\sum_{i=1}^{M} \mathcal{D}(n,i)}$.

## V. Results and Discussions

In this section, the measures presented in Section IV are evaluated and compared using the IEEE 118 and the IEEE 300 bus test cases, which have 179 and 409 transmission lines, respectively (excluding duplicate lines). The experiments are performed using MATPOWER [19] for power flow calculations. Due to the similarity of the results for the IEEE 118 and 300 test cases and the space limit, the detailed analyses of these measures are presented only for the IEEE 118 bus system; however, the aggregated analyses and comparisons through the Pearson correlation coefficient (PCC) are presented for both the IEEE 118 and 300 bus systems. The analyses have been performed on both graph signals (i.e., the DLFM and the LODF graph signals) to identify the impactful and spreadable failures. Furthermore, the effects of the identified impactful and spreadable lines on cascading failure size are evaluated using a cascading failure simulation model following [20], [21]. The comparison of the existing metrics with the proposed metrics for both the impact and spread analysis will provide a new perspective on interpreting the power values as graph signals from the lens of GSP.

### *A. Line Failure Impact Analysis*

In this section, the mean, NSV, TV, and HGFC of both the DLFM and the LODF graph signals have been evaluated and compared to determine the most impactful failures in the system. Figure 1 shows the mean, NSV, TV, and HGFC for the DLFM graph signal for the IEEE 118 bus.

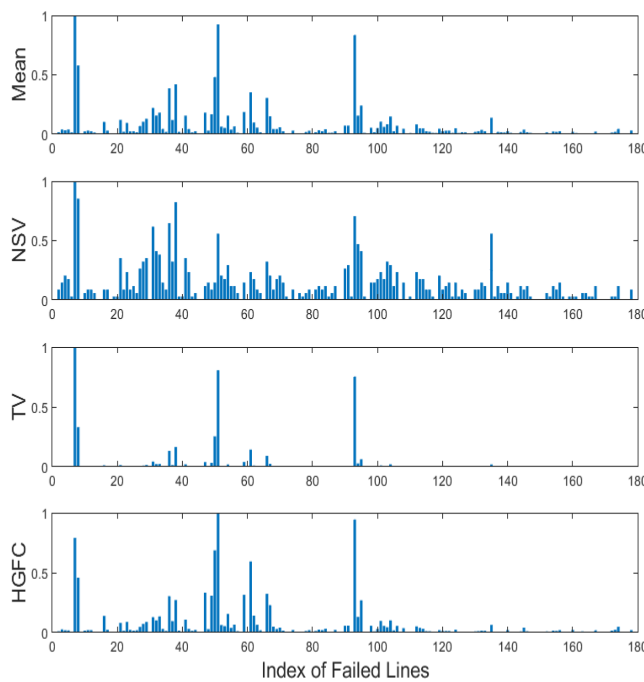



Fig. 1: Measure of the mean, NSV, TV, and HGFC of the DLFS graph signal for the IEEE 118 test case.

Based on these results, it can be observed that these measures (except for the NSV in some cases) agree on the most impactful lines (which are lines 7, 51, and 93 according to the results). These analyses have also been performed on the LODF graph signal and the similarity of the measures for these two signals is summarized using PCC in Table I for both the IEEE 118 and 300 bus systems. Although these measures show notable correlation, the largest correlation is between the HGFC measure on the DLFM signal with all the measures applied to the LODF signal. This suggests that when using measurement signals from the power system, GSP-based measure HGFC can provide more robust analyses similar to the analytically derived LODF signal and can serve as a better metric for measuring the impact of the failures.

### *B. Line Failure Spread Analysis*

The spreadability measures of failures based on the DLFM and LODF graph signals are presented in this subsection. To better clarify the spread measure DCED, the exponential decay function for the value changes at different hops is shown for line failures 50 (in the IEEE 118 test case) as an example in Fig. 2. The deviation of the average values at each hop distance beyond the fitted exponential function, in this case, indicates a more significant impact at a distance and thus more spread.

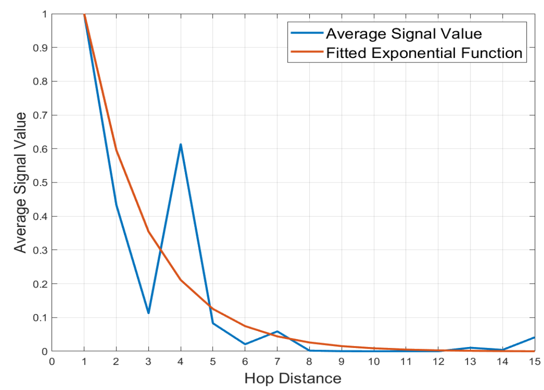



Fig. 2: An exponential function fitted to the average values of the signal at different hop distances for line 50 failure in the IEEE 118 test case.

The DCED value for each of the failures for the DLFM graph signal of the IEEE 118 bus system is shown in Fig. 3(a). To compare the proposed metric with the CCS spread measure from [11], the CCS of the same graph signal is shown in Fig. 3(b). It can be observed that they are similar and share a PCC of 0.8424. The spread measures for both the DLFM and LODF graph signals for both the IEEE 118 and 300 bus systems are enlisted in Table II. Similar to the impact analysis, a notable correlation is observed between both the DCED and CSS metrics on both the DLFM and LODF graph signals; particularly, between the DCED measure on the DLFM graph signal with all the measures applied to the LODF graph signal. Thus the DCED measure can be interpreted as an effective metric for quantifying the spread of the failures.

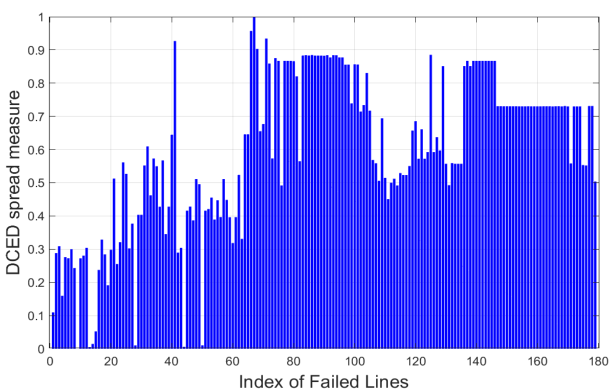
### *C. Effects on Cascading Failures*

The new insights into impactful and spreadable failures in power systems can enhance our understanding of the complex cascading phenomena within these systems. In this section, we test the hypothesis that impactful and spreadable failures play a more significant role in cascading failures and show up in cascade scenarios with a larger size or number of failures.

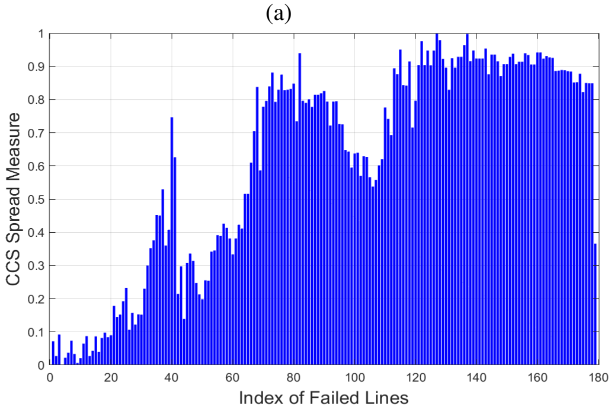
Following [20], cascading failures are simulated based on the line overloading due to failure and redistribution of the power flow in the system calculated using MATPOWER [19].

TABLE I: Pearson Correlation Coefficients of various impact measures for the IEEE 118 and 300 bus systems.

| IEEE 118 Bus System | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **Mean-DLFM** | **NSV-DLFM** | **TV-DLFM** | **HGFC-DLFM** | **Mean-LODF** | **NSV-LODF** | **TV-LODF** | **HGFC-LODF** |
| **Mean-DLFM** | **1.00** | 0.8167 | 0.9314 | 0.9585 | 0.9398 | 0.8361 | 0.8903 | 0.9765 |
| **NSV-DLFM** | 0.8167 | **1.00** | 0.6509 | 0.6774 | 0.7629 | 0.9825 | 0.5289 | 0.7142 |
| **TV-DLFM** | 0.9314 | 0.6509 | **1.00** | 0.8809 | 0.8710 | 0.6757 | 0.9193 | 0.9117 |
| **HGFC-DLFM** | 0.9585 | 0.6774 | 0.8809 | **1.00** | 0.9093 | 0.6986 | 0.9403 | 0.9943 |
| **Mean-LODF** | 0.9398 | 0.7629 | 0.8710 | 0.9093 | **1.00** | 0.7806 | 0.8414 | 0.9235 |
| **NSV-LODF** | 0.8361 | 0.9825 | 0.6757 | 0.6986 | 0.7806 | **1.00** | 0.5561 | 0.7375 |
| **TV-LODF** | 0.8903 | 0.5289 | 0.9193 | 0.9403 | 0.8414 | 0.5561 | **1.00** | 0.9404 |
| **HGFC-LODF** | 0.9765 | 0.7142 | 0.9117 | 0.9943 | 0.9235 | 0.7375 | 0.9404 | **1.00** |
| **IEEE 300 Bus System** | | | | | | | | |
| | **Mean-DLFM** | **NSV-DLFM** | **TV-DLFM** | **HGFC-DLFM** | **Mean-LODF** | **NSV-LODF** | **TV-LODF** | **HGFC-LODF** |
| **Mean-DLFM** | **1.00** | 0.8417 | 0.8940 | 0.9549 | 0.7344 | 0.8509 | 0.7745 | 0.9831 |
| **NSV-DLFM** | 0.8417 | **1.00** | 0.5953 | 0.7197 | 0.7167 | 0.9790 | 0.6085 | 0.7710 |
| **TV-DLFM** | 0.8940 | 0.5953 | **1.00** | 0.8364 | 0.7025 | 0.6139 | 0.9611 | 0.8870 |
| **HGFC-DLFM** | 0.9549 | 0.7197 | 0.8364 | **1.00** | 0.7279 | 0.7392 | 0.7480 | 0.9815 |
| **Mean-LODF** | 0.7344 | 0.7167 | 0.7025 | 0.7279 | **1.00** | 0.7132 | 0.6470 | 0.7349 |
| **NSV-LODF** | 0.8509 | 0.9790 | 0.6139 | 0.7392 | 0.7132 | **1.00** | 0.7349 | 0.7883 |
| **TV-LODF** | 0.7745 | 0.6085 | 0.9611 | 0.7480 | 0.6470 | 0.7349 | **1.00** | 0.7892 |
| **HGFC-LODF** | 0.9831 | 0.7710 | 0.8870 | 0.9815 | 0.7349 | 0.7883 | 0.7892 | **1.00** |




(a)



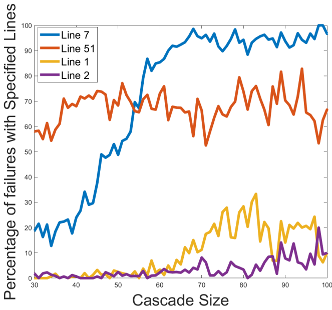

(b)

Fig. 3: Measure of (a) the DCED spread measure and (b) CCS spread measure [11] for the DLFS graph signal for the IEEE 118 test case.

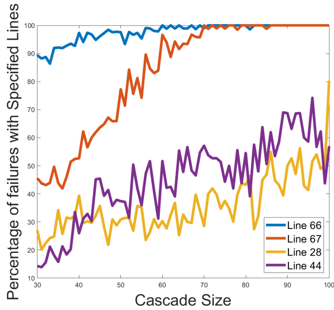
TABLE II: Pearson Correlation Coefficients of various spread measures for the IEEE 118 and 300 bus systems.

| IEEE 118 Bus System | | | | |
|---|---|---|---|---|
| | **DCED-DLFM** | **CCS-DLFM** | **DCED-LODF** | **CCS-LODF** |
| **DCED-DLFM** | **1.00** | 0.8424 | 0.9334 | 0.7775 |
| **CCS-DLFM** | 0.8424 | **1.00** | 0.7848 | 0.9784 |
| **DCED-LODF** | 0.9334 | 0.7848 | **1.00** | 0.7399 |
| **CCS-LODF** | 0.7775 | 0.9784 | 0.7399 | **1.00** |
| **IEEE 300 Bus System** | | | | |
| | **DCED-DLFM** | **CCS-DLFM** | **DCED-LODF** | **CCS-LODF** |
| **DCED-DLFM** | **1.00** | 0.6135 | 0.7568 | 0.6285 |
| **CCS-DLFM** | 0.6135 | **1.00** | 0.7831 | 0.9950 |
| **DCED-LODF** | 0.7568 | 0.7831 | **1.00** | 0.7675 |
| **CCS-LODF** | 0.6285 | 0.9950 | 0.7675 | **1.00** |

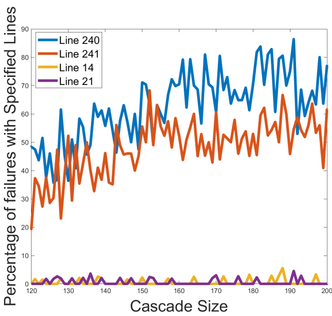
The operating parameters [21] including the stress level on the system (i.e., total load over the generation capacity) have been varied to generate a large number of cascade scenarios (10,000 cascading) initiated with two random initial failures in the IEEE 118 and 300 systems. Next, the frequency of appearance of the identified impactful and spreadable lines in different sizes of generated cascades is evaluated. As can be seen in Fig. 4, the frequency of appearance of impactful lines (lines 7 and 51) based on the HGFC on the DLFM graph signal in the IEEE 118 case increases with the size of the cascade compared to lines with small impact measure such as lines 1 and 2. Similarly, the frequency of appearance of spreadable line failures (lines 66 and 67) based on the DCED on the DLFM graph signal increases with the size of the cascade compared to lines with small spread measures such as lines 28 and 44. A similar behavior has been observed for the IEEE 300 test case with impactful lines being lines 240 and 241 based on the proposed HGFC on the DLFM graph signal and spreadable lines being 169 and 176 based on the proposed DCED measure on the DLFM graph signal respectively.



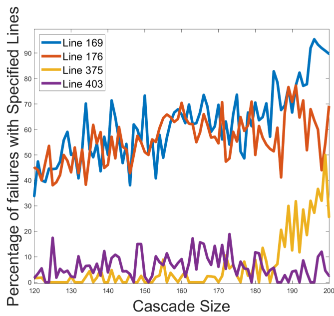

(a) Impactful lines of IEEE 118




(b) Spreadable lines of IEEE 118




(c) Impactful lines of IEEE 300




(d) Spreadable lines of IEEE 300

Fig. 4: Effects of impactful and spreadable lines on cascading failures in IEEE 118 and 300 bus systems.

## VI. Conclusion

The analysis of the effects of a single line failure on the power system graph signals is crucial for understanding the grid's resilience and reliability as well as more complex phenomena such as cascading failures. This article explored the impact and spread of a single-line failure in the power system from the perspective of GSP-based and statistical measures. Novel metrics to measure the impact and spread have been proposed, evaluated, and compared with existing statistical and network-theoretical metrics. The results of this study demonstrate a notable correlation between the existing statistical metrics and the proposed measures. The results also suggested that the identified most impactful and spreadable lines based on the proposed metrics show a higher influence on the size of the cascading failure. Specifically, it is observed that the selected impactful and spreadable lines are present in scenarios that cause large cascading failures, corroborating their contributions to the increasing size of the cascades. The insights gained from this study provide a deeper understanding of failure impact and spread with respect to graph signals, which will be essential for analyzing and quantifying more complex scenarios, such as cascading failures from a GSP perspective, in future research.

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